Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.3.1-a+b-sec-^m-d-sec-^n-A+B-sec-

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [634]. This is test number [123].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (634)	% 0. (0)
Mathematica	% 100. (634)	% 0. (0)
Maple	% 92.43 (586)	% 7.57 (48)
Maxima	% 30.44 (193)	% 69.56 (441)
Fricas	% 47.16 (299)	% 52.84 (335)
Sympy	% 1.1 (7)	% 98.9 (627)
Giac	% 32.18 (204)	% 67.82 (430)

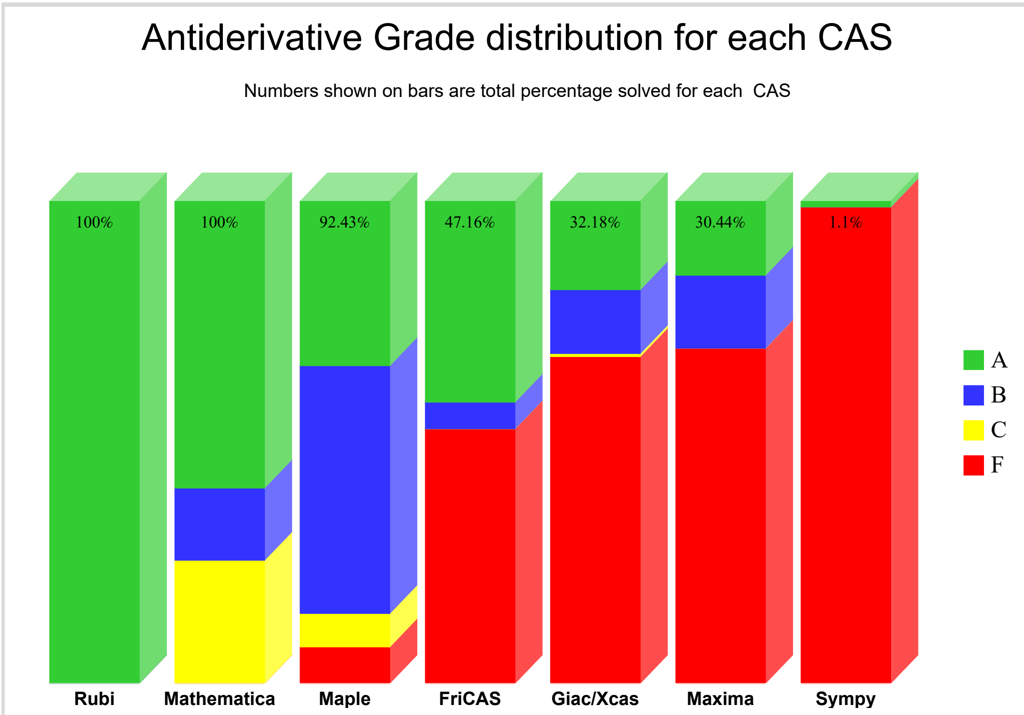
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

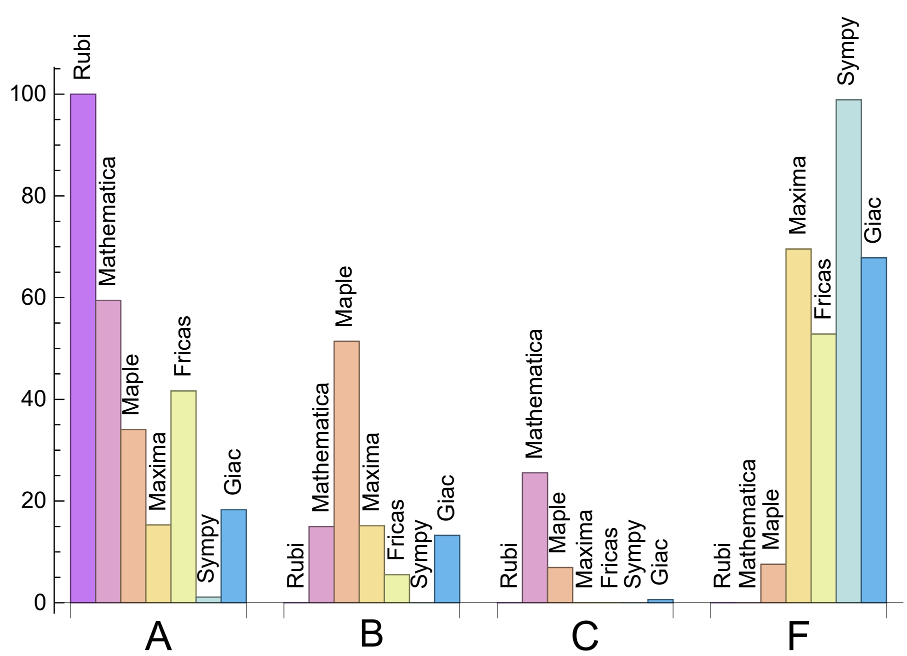
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	59.46	14.98	25.55	0.
Maple	34.07	51.42	6.94	7.57
Maxima	15.3	15.14	0.	69.56
Fricas	41.64	5.52	0.	52.84
Sympy	1.1	0.	0.	98.9
Giac	18.3	13.25	0.63	67.82

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.56	217.71	0.99	187.	1.
Mathematica	5.49	3124.54	9.61	224.	1.16
Maple	1.31	1002.6	3.55	435.	2.62
Maxima	1.87	1476.83	8.37	375.	2.48
Fricas	2.82	951.4	5.43	797.	5.17
Sympy	4.23	20.71	0.68	0.	0.
Giac	2.7	441.69	2.7	307.	2.3

1.4 list of integrals that has no closed form antiderivative

{474, 475, 476, 477, 478}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {10, 11, 15, 16, 37, 38, 82, 99, 149, 156, 157, 164, 165, 193, 220, 223, 234, 235, 242, 243, 244, 251, 252, 259, 260, 266, 267, 269, 270, 271, 272, 273, 274, 275, 320, 329, 348, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 375, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 415, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 441, 443, 444, 448, 449, 450, 451, 452, 456, 468, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 527, 535, 536, 548, 549, 550, 553, 554, 555, 556, 557, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

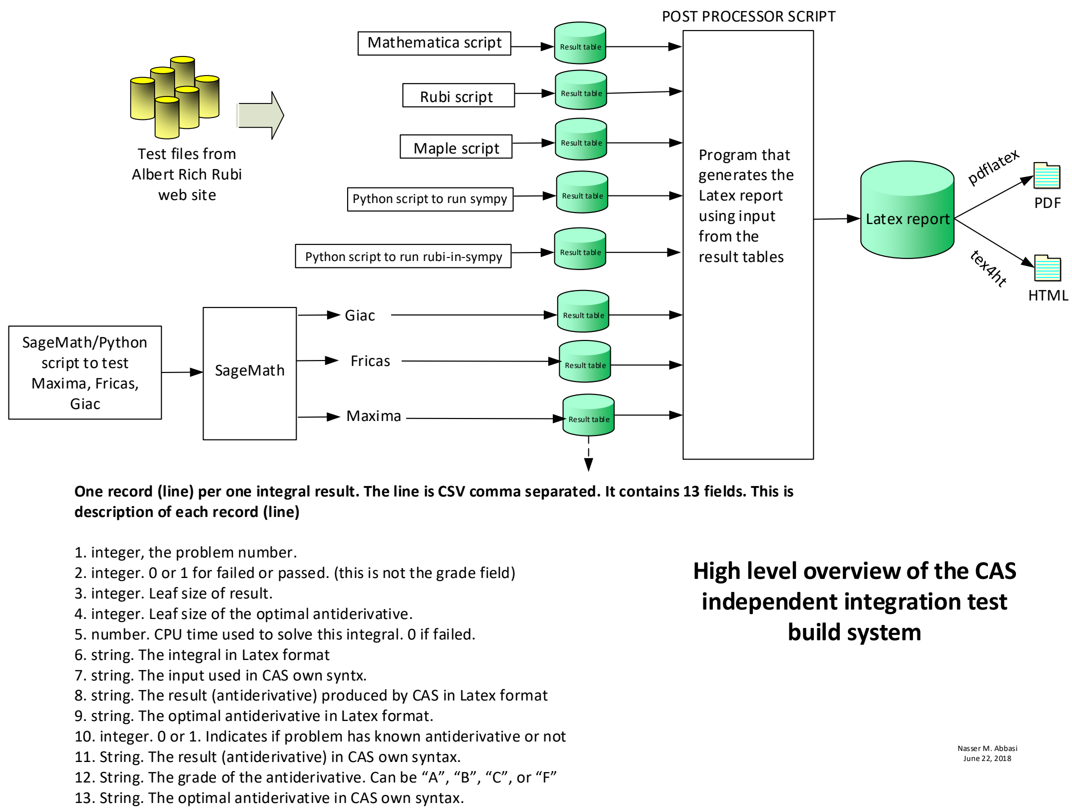
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 87, 94, 101, 102, 103, 110, 111, 112, 113, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 160, 161, 162, 179, 180, 181, 182, 183, 184, 185, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 347, 350, 351, 358, 372, 373, 374, 379, 380, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 420, 422, 427, 428, 429, 430, 433, 434, 437, 438, 439, 440, 445, 446, 447, 453, 454, 455, 458, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593 }

B grade: { 55, 56, 57, 64, 65, 66, 67, 74, 75, 76, 77, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 135, 255, 262, 263, 269, 270, 271, 272, 273, 274, 275, 297, 305, 311, 312, 341, 342, 343, 348, 349, 354, 355, 356, 357, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 377, 378, 382, 384, 385, 386, 387, 388, 390, 392, 415, 421, 423, 424, 425, 426, 431, 432, 560, 561, 578 }

C grade: { 124, 125, 126, 141, 142, 143, 149, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 352, 353, 361, 375, 376, 381, 383, 389, 435, 436, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 462, 463, 468, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

F grade: { }

2.1.3 Maple

A grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 87, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 135, 136, 137, 167, 168, 182, 185, 190, 191, 192, 198, 199, 200, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 238, 244, 245, 246, 247, 251, 252, 253, 254, 260, 261, 268, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 323, 331, 332, 338, 339, 340, 345, 347, 374, 396, 399, 416, 417, 418, 419, 474, 475, 476, 477, 478, 489, 490, 496, 497, 498, 499, 500, 502, 509, 515, 516, 517, 518, 523, 524, 525, 526, 527, 532, 533, 534, 535, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 555, 556, 557, 558, 576, 577 }

B grade: { 82, 83, 84, 88, 89, 90, 91, 98, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 189, 193, 194, 195, 196, 197, 201, 202, 209, 210, 216, 217, 224, 225, 226, 227, 231, 232, 233, 234, 239, 240, 241, 242, 243, 248, 249, 250, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 311, 312, 313, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 341, 342, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 440, 445, 446, 447, 453, 454, 455, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 483, 484, 485, 486, 487, 488, 491, 492, 493, 494, 495, 501, 503, 504, 505, 506, 507, 508, 510, 511, 512, 513, 514, 519, 520, 521, 522, 528, 529, 530, 531, 536, 537, 538, 539, 540, 546, 547, 553, 554, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 601, 602, 607, 608, 609, 615, 616, 617, 621, 622, 623, 624, 628, 629, 630, 631, 632 }

C grade: { 1, 2, 3, 4, 5, 6, 435, 436, 437, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 458, 462, 463, 468, 597, 598, 599, 603, 604, 605, 606, 610, 611, 612, 613, 614, 618, 619, 620, 625, 626, 627, 633, 634 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 269, 270, 271, 272, 273, 274, 275, 276, 479, 480, 481, 482 }

2.1.4 Maxima

A grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 75, 76, 77, 78, 79, 80, 81, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 228, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 347, 474, 475, 476, 477, 478, 518, 544

B grade: { 62, 63, 64, 72, 73, 74, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 107, 122, 123, 124, 125, 126, 130, 131, 138, 139, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 259, 265, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 538, 539, 540, 541, 542, 543, 545, 546, 547, 550, 551, 553, 558, 559 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 118, 119, 120, 121, 127, 128, 129, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 242, 243, 255, 257, 258, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 537, 548, 549, 552, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.5 FriCAS

A grade: { 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 316, 317, 318, 319, 323, 326, 327, 344, 345, 346, 347, 478, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561 }

B grade: { 47, 84, 155, 156, 164, 171, 175, 226, 227, 257, 263, 280, 311, 312, 313, 320, 321, 322, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 269, 270, 271, 272, 273, 274, 275, 276, 336, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.6 Sympy

A grade: { 47, 280, 345, 474, 475, 476, 477 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.7 Giac

A grade: { 43, 44, 45, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 127, 128, 129, 135, 136, 137, 144, 145, 146, 152, 153, 154, 160, 161, 162, 163, 171, 173, 174, 175, 176, 177, 178, 298, 307, 313, 314, 315, 316, 317, 320, 322, 323, 324, 326, 327, 334, 346, 347, 474, 475, 476, 477, 478 }

B grade: { 46, 47, 48, 49, 56, 57, 121, 123, 124, 125, 126, 131, 132, 133, 134, 139, 140, 141, 142, 143, 147, 148, 149, 150, 151, 155, 156, 164, 172, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 318, 319, 321, 325, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345 }

C grade: { 167, 168, 169, 170 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 122, 130, 138, 157, 158, 159, 165, 166, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	99	518	0	0	0	0
normalized size	1	1.	0.58	3.03	0.	0.	0.	0.
time (sec)	N/A	0.119	0.476	0.275	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	87	500	0	0	0	0
normalized size	1	1.	0.64	3.68	0.	0.	0.	0.
time (sec)	N/A	0.101	0.275	0.252	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	73	453	0	0	0	0
normalized size	1	1.	0.7	4.36	0.	0.	0.	0.
time (sec)	N/A	0.082	0.12	0.287	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	445	0	0	0	0
normalized size	1	1.	0.66	5.43	0.	0.	0.	0.
time (sec)	N/A	0.068	0.088	0.238	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	86	470	0	0	0	0
normalized size	1	1.	0.74	4.05	0.	0.	0.	0.
time (sec)	N/A	0.091	0.181	0.216	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	88	482	0	0	0	0
normalized size	1	1.	0.6	3.28	0.	0.	0.	0.
time (sec)	N/A	0.104	0.508	0.198	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	90	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.256	0.117	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	91	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.118	0.105	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.087	0.107	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	88	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.093	0.237	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	88	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.138	0.358	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	90	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.303	0.118	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	91	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.149	0.105	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.111	0.107	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	87	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.112	0.177	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	88	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.096	0.412	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	90	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.253	0.117	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	90	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.114	0.108	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.093	0.15	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	90	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.026	0.007	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	90	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.023	0.006	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.236	0.121	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	91	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.099	0.137	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.145	0.109	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	91	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.076	0.004	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.172	0.008	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.382	0.15	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.215	0.143	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.267	0.142	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.245	0.138	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.233	0.138	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.334	0.131	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	126	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.213	0.995	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	119	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.221	0.924	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	119	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.248	0.842	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.15	0.605	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	107	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.158	0.897	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	114	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.316	0.894	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.27	0.186	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.233	0.179	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	135	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.353	0.188	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.315	0.177	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	87	213	270	381	0	289
normalized size	1	1.	0.65	1.59	2.01	2.84	0.	2.16
time (sec)	N/A	0.141	0.744	0.045	0.985	0.497	0.	1.367

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	171	220	339	0	254
normalized size	1	1.	0.73	1.61	2.08	3.2	0.	2.4
time (sec)	N/A	0.123	0.389	0.041	0.982	0.492	0.	1.285

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	56	128	171	288	0	208
normalized size	1	1.	0.65	1.49	1.99	3.35	0.	2.42
time (sec)	N/A	0.115	0.325	0.039	0.967	0.485	0.	1.261

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	119	239	0	167
normalized size	1	1.	1.34	1.54	2.12	4.27	0.	2.98
time (sec)	N/A	0.067	0.026	0.035	0.965	0.48	0.	1.345

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	65	76	220	71	113
normalized size	1	1.	1.34	2.03	2.38	6.88	2.22	3.53
time (sec)	N/A	0.033	0.016	0.032	1.	0.489	13.457	1.257

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	46	56	78	139	0	107
normalized size	1	1.	1.44	1.75	2.44	4.34	0.	3.34
time (sec)	N/A	0.047	0.027	0.064	1.022	0.487	0.	1.241

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	74	99	0	126
normalized size	1	1.	0.94	1.21	1.57	2.11	0.	2.68
time (sec)	N/A	0.086	0.096	0.065	0.98	0.459	0.	1.27

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	107	146	0	167
normalized size	1	1.	0.84	1.1	1.39	1.9	0.	2.17
time (sec)	N/A	0.108	0.169	0.074	0.982	0.465	0.	1.445

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	136	193	0	211
normalized size	1	1.	0.77	1.1	1.4	1.99	0.	2.18
time (sec)	N/A	0.119	0.237	0.081	0.978	0.474	0.	1.279

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	77	128	167	239	0	248
normalized size	1	1.	0.62	1.02	1.34	1.91	0.	1.98
time (sec)	N/A	0.134	0.243	0.094	0.988	0.479	0.	1.242

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	280	235	375	421	0	332
normalized size	1	1.	1.66	1.39	2.22	2.49	0.	1.96
time (sec)	N/A	0.244	1.333	0.05	1.003	0.496	0.	1.347

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	262	187	311	362	0	286
normalized size	1	1.	1.9	1.36	2.25	2.62	0.	2.07
time (sec)	N/A	0.228	1.172	0.043	1.009	0.494	0.	1.364

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	481	141	225	315	0	240
normalized size	1	1.	4.67	1.37	2.18	3.06	0.	2.33
time (sec)	N/A	0.113	6.109	0.039	0.996	0.494	0.	1.25

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	307	113	173	297	0	208
normalized size	1	1.	3.74	1.38	2.11	3.62	0.	2.54
time (sec)	N/A	0.084	1.255	0.038	0.98	0.496	0.	1.435

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	258	107	142	278	0	212
normalized size	1	1.	3.53	1.47	1.95	3.81	0.	2.9
time (sec)	N/A	0.13	1.605	0.062	1.026	0.495	0.	1.268

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	96	108	136	194	0	196
normalized size	1	1.	1.09	1.23	1.55	2.2	0.	2.23
time (sec)	N/A	0.145	0.159	0.076	0.994	0.496	0.	1.354

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	61	116	149	165	0	192
normalized size	1	1.	0.6	1.14	1.46	1.62	0.	1.88
time (sec)	N/A	0.153	0.172	0.078	1.013	0.463	0.	1.403

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	154	194	213	0	238
normalized size	1	1.	0.64	1.14	1.44	1.58	0.	1.76
time (sec)	N/A	0.231	0.355	0.087	1.01	0.474	0.	1.212

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	240	271	0	284
normalized size	1	1.	0.68	1.16	1.5	1.69	0.	1.78
time (sec)	N/A	0.252	0.476	0.094	1.014	0.48	0.	1.303

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	346	281	547	485	0	378
normalized size	1	1.	1.65	1.34	2.6	2.31	0.	1.8
time (sec)	N/A	0.399	2.041	0.057	1.029	0.507	0.	1.395

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	294	234	455	431	0	332
normalized size	1	1.	1.8	1.44	2.79	2.64	0.	2.04
time (sec)	N/A	0.268	1.474	0.049	1.017	0.501	0.	1.341

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	273	188	354	366	0	286
normalized size	1	1.	2.18	1.5	2.83	2.93	0.	2.29
time (sec)	N/A	0.143	1.293	0.048	1.	0.492	0.	1.355

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	1056	158	267	356	0	255
normalized size	1	1.	9.51	1.42	2.41	3.21	0.	2.3
time (sec)	N/A	0.144	6.387	0.046	0.995	0.502	0.	1.327

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	335	144	223	342	0	259
normalized size	1	1.	3.1	1.33	2.06	3.17	0.	2.4
time (sec)	N/A	0.238	2.526	0.073	0.989	0.504	0.	1.384

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	302	145	189	323	0	259
normalized size	1	1.	2.58	1.24	1.62	2.76	0.	2.21
time (sec)	N/A	0.264	4.602	0.078	1.001	0.503	0.	1.424

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	113	153	200	254	0	243
normalized size	1	1.	0.9	1.22	1.6	2.03	0.	1.94
time (sec)	N/A	0.271	0.239	0.084	1.025	0.501	0.	1.375

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	86	176	225	216	0	238
normalized size	1	1.	0.69	1.42	1.81	1.74	0.	1.92
time (sec)	N/A	0.17	0.27	0.086	1.	0.482	0.	1.346

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	108	223	288	278	0	284
normalized size	1	1.	0.61	1.27	1.64	1.58	0.	1.61
time (sec)	N/A	0.372	0.433	0.096	1.034	0.484	0.	1.296

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	354	332	0	329
normalized size	1	1.	0.67	1.32	1.76	1.65	0.	1.64
time (sec)	N/A	0.41	0.536	0.1	1.003	0.491	0.	1.451

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	358	280	626	481	0	378
normalized size	1	1.	1.85	1.44	3.23	2.48	0.	1.95
time (sec)	N/A	0.318	2.234	0.054	1.005	0.512	0.	1.363

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	306	234	498	431	0	332
normalized size	1	1.	1.92	1.47	3.13	2.71	0.	2.09
time (sec)	N/A	0.179	1.615	0.056	1.032	0.499	0.	1.249

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	326	204	396	408	0	301
normalized size	1	1.	2.16	1.35	2.62	2.7	0.	1.99
time (sec)	N/A	0.214	1.757	0.055	1.042	0.512	0.	1.34

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	1202	189	317	405	0	306
normalized size	1	1.	7.96	1.25	2.1	2.68	0.	2.03
time (sec)	N/A	0.368	6.45	0.084	1.013	0.51	0.	1.342

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	373	182	269	390	0	311
normalized size	1	1.	2.33	1.14	1.68	2.44	0.	1.94
time (sec)	N/A	0.39	4.835	0.086	1.032	0.513	0.	1.4

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	342	190	252	383	0	305
normalized size	1	1.	2.07	1.15	1.53	2.32	0.	1.85
time (sec)	N/A	0.41	1.85	0.081	1.043	0.514	0.	1.304

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	138	199	277	306	0	289
normalized size	1	1.	0.8	1.15	1.6	1.77	0.	1.67
time (sec)	N/A	0.403	0.34	0.091	1.03	0.511	0.	1.318

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	108	248	319	279	0	284
normalized size	1	1.	0.68	1.57	2.02	1.77	0.	1.8
time (sec)	N/A	0.202	0.326	0.095	1.027	0.486	0.	1.331

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	134	306	401	329	0	329
normalized size	1	1.	0.61	1.39	1.82	1.5	0.	1.5
time (sec)	N/A	0.532	0.585	0.107	1.016	0.492	0.	1.401

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	156	358	481	396	0	375
normalized size	1	1.	0.65	1.49	2.	1.64	0.	1.56
time (sec)	N/A	0.567	0.696	0.116	1.032	0.504	0.	1.444

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	489	340	497	417	0	246
normalized size	1	1.	3.73	2.6	3.79	3.18	0.	1.88
time (sec)	N/A	0.171	5.953	0.06	1.051	0.493	0.	1.349

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	311	252	381	386	0	211
normalized size	1	1.	2.88	2.33	3.53	3.57	0.	1.95
time (sec)	N/A	0.163	3.518	0.055	1.009	0.49	0.	1.334

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	224	163	265	319	0	147
normalized size	1	1.	3.61	2.63	4.27	5.15	0.	2.37
time (sec)	N/A	0.117	1.195	0.044	1.01	0.482	0.	1.326

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	109	78	134	197	0	95
normalized size	1	1.	2.53	1.81	3.12	4.58	0.	2.21
time (sec)	N/A	0.082	0.252	0.044	0.98	0.47	0.	1.333

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	72	56	99	105	0	59
normalized size	1	1.	2.06	1.6	2.83	3.	0.	1.69
time (sec)	N/A	0.059	0.138	0.05	1.469	0.451	0.	1.185

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	76	108	193	149	0	107
normalized size	1	1.	1.27	1.8	3.22	2.48	0.	1.78
time (sec)	N/A	0.109	0.37	0.076	1.493	0.46	0.	1.282

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	197	211	304	203	0	166
normalized size	1	1.	2.01	2.15	3.1	2.07	0.	1.69
time (sec)	N/A	0.15	0.444	0.082	1.448	0.466	0.	1.297

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	249	281	419	243	0	204
normalized size	1	1.	2.04	2.3	3.43	1.99	0.	1.67
time (sec)	N/A	0.159	0.668	0.083	1.669	0.472	0.	1.221

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	764	382	574	616	0	305
normalized size	1	1.	4.27	2.13	3.21	3.44	0.	1.7
time (sec)	N/A	0.321	6.352	0.065	1.013	0.506	0.	1.351

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	496	294	454	566	0	267
normalized size	1	1.	3.18	1.88	2.91	3.63	0.	1.71
time (sec)	N/A	0.306	4.057	0.063	1.017	0.496	0.	1.415

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	292	205	329	501	0	204
normalized size	1	1.	2.7	1.9	3.05	4.64	0.	1.89
time (sec)	N/A	0.257	1.684	0.053	1.022	0.491	0.	1.37

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	169	119	196	338	0	151
normalized size	1	1.	2.14	1.51	2.48	4.28	0.	1.91
time (sec)	N/A	0.187	0.532	0.051	0.984	0.478	0.	1.302

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	126	144	0	81
normalized size	1	1.	1.17	0.92	1.94	2.22	0.	1.25
time (sec)	N/A	0.08	0.199	0.049	0.978	0.438	0.	1.239

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	153	97	162	228	0	115
normalized size	1	1.	2.19	1.39	2.31	3.26	0.	1.64
time (sec)	N/A	0.112	0.349	0.057	1.474	0.451	0.	1.242

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	245	149	258	296	0	163
normalized size	1	1.	2.5	1.52	2.63	3.02	0.	1.66
time (sec)	N/A	0.23	0.586	0.082	1.491	0.465	0.	1.208

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	315	252	382	342	0	221
normalized size	1	1.	2.2	1.76	2.67	2.39	0.	1.55
time (sec)	N/A	0.3	0.741	0.091	1.513	0.472	0.	1.266

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	369	322	502	389	0	259
normalized size	1	1.	2.17	1.89	2.95	2.29	0.	1.52
time (sec)	N/A	0.319	0.714	0.11	1.53	0.481	0.	1.299

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	610	334	509	757	0	315
normalized size	1	1.	3.02	1.65	2.52	3.75	0.	1.56
time (sec)	N/A	0.475	6.161	0.075	1.07	0.506	0.	1.345

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	480	245	386	668	0	251
normalized size	1	1.	3.08	1.57	2.47	4.28	0.	1.61
time (sec)	N/A	0.429	3.979	0.057	1.027	0.498	0.	1.363

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	197	159	252	481	0	198
normalized size	1	1.	1.58	1.27	2.02	3.85	0.	1.58
time (sec)	N/A	0.315	0.894	0.054	1.03	0.486	0.	1.386

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	155	227	0	101
normalized size	1	1.	0.94	0.63	1.52	2.23	0.	0.99
time (sec)	N/A	0.203	0.291	0.052	1.062	0.439	0.	1.212

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	155	227	0	101
normalized size	1	1.	1.32	0.63	1.52	2.23	0.	0.99
time (sec)	N/A	0.114	0.324	0.061	1.012	0.445	0.	1.321

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	241	137	216	351	0	163
normalized size	1	1.	2.23	1.27	2.	3.25	0.	1.51
time (sec)	N/A	0.186	0.567	0.065	1.468	0.461	0.	1.299

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	365	189	312	431	0	212
normalized size	1	1.	2.68	1.39	2.29	3.17	0.	1.56
time (sec)	N/A	0.367	1.029	0.093	1.49	0.475	0.	1.423

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	435	292	435	495	0	270
normalized size	1	1.	2.33	1.56	2.33	2.65	0.	1.44
time (sec)	N/A	0.47	0.787	0.101	1.507	0.484	0.	1.275

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	491	362	556	540	0	308
normalized size	1	1.	2.25	1.66	2.55	2.48	0.	1.41
time (sec)	N/A	0.495	1.043	0.099	1.51	0.498	0.	1.342

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	880	374	566	940	0	360
normalized size	1	1.	3.7	1.57	2.38	3.95	0.	1.51
time (sec)	N/A	0.656	6.473	0.069	0.999	0.516	0.	1.391

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	754	285	440	837	0	297
normalized size	1	1.	3.89	1.47	2.27	4.31	0.	1.53
time (sec)	N/A	0.616	6.386	0.062	1.02	0.504	0.	1.331

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	239	199	308	624	0	244
normalized size	1	1.	1.47	1.22	1.89	3.83	0.	1.5
time (sec)	N/A	0.475	1.404	0.06	1.021	0.491	0.	1.241

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	109	88	236	311	0	158
normalized size	1	1.	0.75	0.6	1.62	2.13	0.	1.08
time (sec)	N/A	0.229	0.34	0.058	1.001	0.445	0.	1.382

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	163	88	235	309	0	158
normalized size	1	1.	1.18	0.64	1.7	2.24	0.	1.14
time (sec)	N/A	0.265	0.377	0.057	1.007	0.451	0.	1.364

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	193	90	236	308	0	158
normalized size	1	1.	1.4	0.65	1.71	2.23	0.	1.14
time (sec)	N/A	0.151	0.444	0.059	1.001	0.451	0.	1.276

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	329	177	271	475	0	208
normalized size	1	1.	2.38	1.28	1.96	3.44	0.	1.51
time (sec)	N/A	0.268	0.755	0.068	1.477	0.471	0.	1.206

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	485	229	366	574	0	257
normalized size	1	1.	2.92	1.38	2.2	3.46	0.	1.55
time (sec)	N/A	0.573	1.039	0.1	1.527	0.486	0.	1.322

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	555	332	491	645	0	315
normalized size	1	1.	2.49	1.49	2.2	2.89	0.	1.41
time (sec)	N/A	0.649	1.111	0.1	1.525	0.499	0.	1.48

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	611	402	610	701	0	352
normalized size	1	1.	2.39	1.57	2.38	2.74	0.	1.38
time (sec)	N/A	0.705	1.628	0.111	1.641	0.509	0.	1.251

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	98	138	0	308	0	362
normalized size	1	1.	0.52	0.74	0.	1.65	0.	1.94
time (sec)	N/A	0.338	0.538	0.324	0.	0.474	0.	4.95

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	81	116	0	265	0	300
normalized size	1	1.	0.56	0.81	0.	1.84	0.	2.08
time (sec)	N/A	0.277	0.265	0.28	0.	0.471	0.	4.882

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	80	94	0	217	0	238
normalized size	1	1.	0.79	0.93	0.	2.15	0.	2.36
time (sec)	N/A	0.228	0.278	0.331	0.	0.464	0.	4.817

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	53	70	0	169	0	174
normalized size	1	1.	0.85	1.13	0.	2.73	0.	2.81
time (sec)	N/A	0.094	0.158	0.273	0.	0.462	0.	4.606

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	198	620	0	0
normalized size	1	1.	1.15	1.79	3.	9.39	0.	0.
time (sec)	N/A	0.088	0.301	0.253	1.664	0.512	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	93	198	1268	694	0	450
normalized size	1	1.	1.37	2.91	18.65	10.21	0.	6.62
time (sec)	N/A	0.106	0.235	0.292	2.001	0.607	0.	6.473

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	398	2499	801	0	851
normalized size	1	1.	1.	3.4	21.36	6.85	0.	7.27
time (sec)	N/A	0.177	0.388	0.34	2.339	0.618	0.	6.802

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	70	580	4024	898	0	1156
normalized size	1	1.	0.44	3.62	25.15	5.61	0.	7.22
time (sec)	N/A	0.242	0.173	0.405	2.886	0.633	0.	7.136

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	70	762	11557	995	0	1458
normalized size	1	1.	0.34	3.75	56.93	4.9	0.	7.18
time (sec)	N/A	0.298	0.171	0.343	4.102	0.715	0.	7.293

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	100	139	0	329	0	362
normalized size	1	1.	0.53	0.74	0.	1.74	0.	1.92
time (sec)	N/A	0.461	0.735	0.282	0.	0.486	0.	5.126

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	82	117	0	279	0	300
normalized size	1	1.	0.59	0.85	0.	2.02	0.	2.17
time (sec)	N/A	0.297	0.38	0.241	0.	0.476	0.	5.009

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	95	0	225	0	238
normalized size	1	1.	0.69	0.94	0.	2.23	0.	2.36
time (sec)	N/A	0.14	0.285	0.227	0.	0.468	0.	5.318

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	1347	814	0	0
normalized size	1	1.	0.97	2.26	12.83	7.75	0.	0.
time (sec)	N/A	0.146	0.571	0.25	1.952	0.528	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	97	212	2431	755	0	544
normalized size	1	1.	0.94	2.06	23.6	7.33	0.	5.28
time (sec)	N/A	0.241	0.423	0.278	2.303	0.616	0.	6.778

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	399	0	833	0	863
normalized size	1	1.	0.93	3.35	0.	7.	0.	7.25
time (sec)	N/A	0.273	0.597	0.296	0.	0.617	0.	7.109

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	137	581	0	949	0	1166
normalized size	1	1.	0.84	3.54	0.	5.79	0.	7.11
time (sec)	N/A	0.365	0.998	0.365	0.	0.627	0.	7.227

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	154	763	0	1049	0	1469
normalized size	1	1.	0.74	3.65	0.	5.02	0.	7.03
time (sec)	N/A	0.449	1.301	0.294	0.	0.717	0.	7.8

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	487	163	0	409	0	424
normalized size	1	1.	2.05	0.69	0.	1.73	0.	1.79
time (sec)	N/A	0.657	6.166	0.276	0.	0.493	0.	5.738

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	96	141	0	346	0	362
normalized size	1	1.	0.55	0.81	0.	1.98	0.	2.07
time (sec)	N/A	0.353	0.643	0.254	0.	0.487	0.	5.284

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	89	119	0	292	0	300
normalized size	1	1.	0.64	0.86	0.	2.12	0.	2.17
time (sec)	N/A	0.184	0.476	0.231	0.	0.48	0.	5.074

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1885	954	0	0
normalized size	1	1.	0.9	2.4	13.27	6.72	0.	0.
time (sec)	N/A	0.223	1.043	0.262	2.086	0.54	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	126	256	3753	968	0	644
normalized size	1	1.	0.88	1.79	26.24	6.77	0.	4.5
time (sec)	N/A	0.411	0.837	0.292	2.521	0.628	0.	7.159

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	116	410	0	890	0	957
normalized size	1	1.	0.75	2.66	0.	5.78	0.	6.21
time (sec)	N/A	0.419	0.807	0.325	0.	0.632	0.	7.373

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	312	583	0	976	0	1177
normalized size	1	1.	1.9	3.55	0.	5.95	0.	7.18
time (sec)	N/A	0.456	1.045	0.342	0.	0.636	0.	7.649

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	366	765	0	1103	0	1480
normalized size	1	1.	1.75	3.66	0.	5.28	0.	7.08
time (sec)	N/A	0.58	1.292	0.285	0.	0.72	0.	8.437

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	416	947	0	1227	0	1782
normalized size	1	1.	1.64	3.73	0.	4.83	0.	7.02
time (sec)	N/A	0.654	1.786	0.323	0.	0.737	0.	8.601

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	140	785	0	1116	0	387
normalized size	1	1.	0.69	3.89	0.	5.52	0.	1.92
time (sec)	N/A	0.606	0.526	0.34	0.	0.598	0.	9.475

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	123	595	0	1019	0	366
normalized size	1	1.	0.77	3.74	0.	6.41	0.	2.3
time (sec)	N/A	0.42	0.374	0.306	0.	0.58	0.	9.607

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	106	405	0	917	0	251
normalized size	1	1.	0.9	3.43	0.	7.77	0.	2.13
time (sec)	N/A	0.257	0.293	0.293	0.	0.576	0.	9.417

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	88	200	0	751	0	194
normalized size	1	1.	1.13	2.56	0.	9.63	0.	2.49
time (sec)	N/A	0.107	0.166	0.24	0.	0.566	0.	9.196

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	814	0	302
normalized size	1	1.	1.01	2.13	0.	8.95	0.	3.32
time (sec)	N/A	0.107	0.281	0.236	0.	2.455	0.	11.281

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	10104	353	0	1214	0	531
normalized size	1	1.	84.91	2.97	0.	10.2	0.	4.46
time (sec)	N/A	0.229	26.457	0.299	0.	3.143	0.	11.284

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	135	717	0	1323	0	876
normalized size	1	1.	0.82	4.35	0.	8.02	0.	5.31
time (sec)	N/A	0.369	0.415	0.374	0.	5.703	0.	11.575

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	150	1067	0	1426	0	1142
normalized size	1	1.	0.73	5.18	0.	6.92	0.	5.54
time (sec)	N/A	0.555	0.675	0.312	0.	5.722	0.	11.901

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	160	793	0	1315	0	421
normalized size	1	1.	0.74	3.67	0.	6.09	0.	1.95
time (sec)	N/A	0.633	2.372	0.299	0.	0.61	0.	9.688

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	141	603	0	1189	0	400
normalized size	1	1.	0.82	3.53	0.	6.95	0.	2.34
time (sec)	N/A	0.461	1.366	0.26	0.	0.592	0.	9.465

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	125	405	0	1003	0	257
normalized size	1	1.	1.06	3.43	0.	8.5	0.	2.18
time (sec)	N/A	0.259	0.75	0.241	0.	0.576	0.	9.311

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	127	402	0	957	0	208
normalized size	1	1.	1.46	4.62	0.	11.	0.	2.39
time (sec)	N/A	0.122	0.799	0.19	0.	0.572	0.	9.226

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	10115	554	0	1416	0	417
normalized size	1	1.	79.65	4.36	0.	11.15	0.	3.28
time (sec)	N/A	0.181	26.548	0.201	0.	7.627	0.	11.323

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	10898	713	0	1575	0	0
normalized size	1	1.	64.11	4.19	0.	9.26	0.	0.
time (sec)	N/A	0.405	26.727	0.279	0.	9.971	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	395	1075	0	1673	0	0
normalized size	1	1.	1.79	4.86	0.	7.57	0.	0.
time (sec)	N/A	0.583	2.286	0.333	0.	14.673	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	502	1425	0	1789	0	0
normalized size	1	1.	1.87	5.32	0.	6.68	0.	0.
time (sec)	N/A	0.78	6.143	0.292	0.	14.712	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	161	795	0	1474	0	420
normalized size	1	1.	0.75	3.68	0.	6.82	0.	1.94
time (sec)	N/A	0.655	2.571	0.275	0.	0.614	0.	10.154

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	144	597	0	1273	0	390
normalized size	1	1.	0.85	3.53	0.	7.53	0.	2.31
time (sec)	N/A	0.455	1.503	0.259	0.	0.592	0.	9.935

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	131	602	0	1233	0	258
normalized size	1	1.	1.04	4.78	0.	9.79	0.	2.05
time (sec)	N/A	0.276	1.574	0.244	0.	0.582	0.	9.836

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	206	594	0	1219	0	258
normalized size	1	1.	1.63	4.71	0.	9.67	0.	2.05
time (sec)	N/A	0.165	1.506	0.191	0.	0.589	0.	9.396

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	10177	824	0	1754	0	471
normalized size	1	1.	62.05	5.02	0.	10.7	0.	2.87
time (sec)	N/A	0.254	26.677	0.204	0.	16.161	0.	11.376

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	10956	1065	0	1948	0	0
normalized size	1	1.	52.93	5.14	0.	9.41	0.	0.
time (sec)	N/A	0.558	26.939	0.303	0.	21.255	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	512	1427	0	2059	0	0
normalized size	1	1.	1.94	5.41	0.	7.8	0.	0.
time (sec)	N/A	0.79	6.164	0.388	0.	28.205	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	140	120	0	780	0	225
normalized size	1	1.	1.57	1.35	0.	8.76	0.	2.53
time (sec)	N/A	0.146	0.538	0.243	0.	0.506	0.	1.884

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	269	155	0	1112	0	346
normalized size	1	1.	2.34	1.35	0.	9.67	0.	3.01
time (sec)	N/A	0.221	1.5	0.299	0.	0.528	0.	1.92

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	297	367	0	1188	0	379
normalized size	1	1.	1.92	2.37	0.	7.66	0.	2.45
time (sec)	N/A	0.362	1.629	0.345	0.	0.539	0.	2.019

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	330	625	0	1258	0	412
normalized size	1	1.	1.72	3.26	0.	6.55	0.	2.15
time (sec)	N/A	0.524	1.888	0.404	0.	0.548	0.	2.217

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	133	322	298	0	1285	0	262
normalized size	1	1.15	2.78	2.57	0.	11.08	0.	2.26
time (sec)	N/A	0.2	6.595	0.23	0.	0.531	0.	1.876

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	361	462	0	1345	0	344
normalized size	1	1.	2.47	3.16	0.	9.21	0.	2.36
time (sec)	N/A	0.355	6.617	0.283	0.	0.541	0.	2.069

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	408	883	0	1415	0	393
normalized size	1	1.	2.1	4.55	0.	7.29	0.	2.03
time (sec)	N/A	0.529	6.665	0.351	0.	0.556	0.	2.174

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	452	1104	0	1490	0	425
normalized size	1	1.	1.92	4.68	0.	6.31	0.	1.8
time (sec)	N/A	0.701	6.687	0.326	0.	0.572	0.	2.231

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	185	387	695	0	1544	0	296
normalized size	1	1.22	2.55	4.57	0.	10.16	0.	1.95
time (sec)	N/A	0.206	6.739	0.257	0.	0.543	0.	2.031

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	423	788	0	1609	0	393
normalized size	1	1.	2.3	4.28	0.	8.74	0.	2.14
time (sec)	N/A	0.505	6.791	0.342	0.	0.556	0.	2.26

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	458	1475	0	1671	0	409
normalized size	1	1.	1.94	6.25	0.	7.08	0.	1.73
time (sec)	N/A	0.731	6.815	0.34	0.	0.575	0.	3.159

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	514	1964	0	1744	0	459
normalized size	1	1.	1.84	7.01	0.	6.23	0.	1.64
time (sec)	N/A	0.906	6.84	0.44	0.	0.613	0.	3.049

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	200	691	0	0	0	0
normalized size	1	1.	1.01	3.47	0.	0.	0.	0.
time (sec)	N/A	0.179	0.74	5.554	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	168	662	0	0	0	0
normalized size	1	1.	0.98	3.85	0.	0.	0.	0.
time (sec)	N/A	0.165	0.643	5.197	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	94	427	0	0	0	0
normalized size	1	1.	0.7	3.16	0.	0.	0.	0.
time (sec)	N/A	0.144	0.516	4.205	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	240	0	0	0	0
normalized size	1	1.	0.73	2.26	0.	0.	0.	0.
time (sec)	N/A	0.134	0.299	1.761	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	83	321	0	0	0	0
normalized size	1	1.	0.75	2.92	0.	0.	0.	0.
time (sec)	N/A	0.13	0.307	1.844	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	99	355	0	0	0	0
normalized size	1	1.	0.7	2.52	0.	0.	0.	0.
time (sec)	N/A	0.153	0.563	1.652	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	113	383	0	0	0	0
normalized size	1	1.	0.66	2.23	0.	0.	0.	0.
time (sec)	N/A	0.161	0.969	1.702	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	463	852	0	0	0	0
normalized size	1	1.	1.98	3.64	0.	0.	0.	0.
time (sec)	N/A	0.342	4.459	6.524	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	321	743	0	0	0	0
normalized size	1	1.	1.61	3.73	0.	0.	0.	0.
time (sec)	N/A	0.299	6.285	5.715	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	310	513	0	0	0	0
normalized size	1	1.	1.94	3.21	0.	0.	0.	0.
time (sec)	N/A	0.278	3.08	2.067	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	320	388	0	0	0	0
normalized size	1	1.	2.03	2.46	0.	0.	0.	0.
time (sec)	N/A	0.256	2.406	1.832	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	153	357	0	0	0	0
normalized size	1	1.	0.92	2.15	0.	0.	0.	0.
time (sec)	N/A	0.26	1.786	1.663	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	193	385	0	0	0	0
normalized size	1	1.	0.96	1.92	0.	0.	0.	0.
time (sec)	N/A	0.29	2.351	1.915	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	217	413	0	0	0	0
normalized size	1	1.	0.93	1.76	0.	0.	0.	0.
time (sec)	N/A	0.32	2.944	1.755	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	793	1180	0	0	0	0
normalized size	1	1.	2.86	4.26	0.	0.	0.	0.
time (sec)	N/A	0.541	6.806	8.211	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	465	931	0	0	0	0
normalized size	1	1.	1.91	3.82	0.	0.	0.	0.
time (sec)	N/A	0.439	5.137	6.857	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	244	916	0	0	0	0
normalized size	1	1.	1.16	4.34	0.	0.	0.	0.
time (sec)	N/A	0.416	2.319	5.977	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	202	654	0	0	0	0
normalized size	1	1.	1.02	3.29	0.	0.	0.	0.
time (sec)	N/A	0.409	1.945	2.202	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	207	519	0	0	0	0
normalized size	1	1.	0.98	2.46	0.	0.	0.	0.
time (sec)	N/A	0.413	1.695	1.976	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	194	385	0	0	0	0
normalized size	1	1.	0.92	1.82	0.	0.	0.	0.
time (sec)	N/A	0.437	2.52	1.741	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	196	413	0	0	0	0
normalized size	1	1.	0.8	1.69	0.	0.	0.	0.
time (sec)	N/A	0.477	2.785	1.717	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	239	441	0	0	0	0
normalized size	1	1.	0.86	1.59	0.	0.	0.	0.
time (sec)	N/A	0.51	3.492	1.659	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	814	806	0	0	0	0
normalized size	1	1.	3.55	3.52	0.	0.	0.	0.
time (sec)	N/A	0.248	7.512	6.423	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	372	493	0	0	0	0
normalized size	1	1.	1.94	2.57	0.	0.	0.	0.
time (sec)	N/A	0.227	3.299	5.281	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	420	318	0	0	0	0
normalized size	1	1.	2.75	2.08	0.	0.	0.	0.
time (sec)	N/A	0.186	4.372	3.819	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	200	243	0	0	0	0
normalized size	1	1.	1.63	1.98	0.	0.	0.	0.
time (sec)	N/A	0.171	1.128	1.793	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	445	244	0	0	0	0
normalized size	1	1.	3.48	1.91	0.	0.	0.	0.
time (sec)	N/A	0.178	2.631	1.739	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	232	262	0	0	0	0
normalized size	1	1.	1.41	1.6	0.	0.	0.	0.
time (sec)	N/A	0.194	2.363	1.688	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	540	282	0	0	0	0
normalized size	1	1.	2.74	1.43	0.	0.	0.	0.
time (sec)	N/A	0.213	3.755	1.807	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	568	300	0	0	0	0
normalized size	1	1.	2.47	1.3	0.	0.	0.	0.
time (sec)	N/A	0.23	3.938	1.687	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	865	750	0	0	0	0
normalized size	1	1.	3.65	3.16	0.	0.	0.	0.
time (sec)	N/A	0.371	7.809	6.082	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	455	492	0	0	0	0
normalized size	1	1.	2.23	2.41	0.	0.	0.	0.
time (sec)	N/A	0.345	6.64	2.295	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	256	350	0	0	0	0
normalized size	1	1.	1.59	2.17	0.	0.	0.	0.
time (sec)	N/A	0.303	2.604	1.937	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	256	350	0	0	0	0
normalized size	1	1.	1.52	2.08	0.	0.	0.	0.
time (sec)	N/A	0.31	3.242	1.894	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	854	421	0	0	0	0
normalized size	1	1.	4.82	2.38	0.	0.	0.	0.
time (sec)	N/A	0.326	6.752	2.047	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	899	435	0	0	0	0
normalized size	1	1.	4.26	2.06	0.	0.	0.	0.
time (sec)	N/A	0.357	6.801	1.951	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	946	465	0	0	0	0
normalized size	1	1.	3.88	1.91	0.	0.	0.	0.
time (sec)	N/A	0.382	6.907	2.023	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	953	876	0	0	0	0
normalized size	1	1.	3.26	3.	0.	0.	0.	0.
time (sec)	N/A	0.56	7.963	2.949	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	924	685	0	0	0	0
normalized size	1	1.	3.54	2.62	0.	0.	0.	0.
time (sec)	N/A	0.536	7.248	2.524	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	919	451	0	0	0	0
normalized size	1	1.	4.18	2.05	0.	0.	0.	0.
time (sec)	N/A	0.49	6.885	2.099	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	918	451	0	0	0	0
normalized size	1	1.	4.25	2.09	0.	0.	0.	0.
time (sec)	N/A	0.484	6.868	2.077	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	919	451	0	0	0	0
normalized size	1	1.	4.14	2.03	0.	0.	0.	0.
time (sec)	N/A	0.489	6.96	1.961	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	364	451	0	0	0	0
normalized size	1	1.	1.6	1.98	0.	0.	0.	0.
time (sec)	N/A	0.499	6.518	1.895	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	377	465	0	0	0	0
normalized size	1	1.	1.44	1.78	0.	0.	0.	0.
time (sec)	N/A	0.553	6.899	2.198	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	1032	493	0	0	0	0
normalized size	1	1.	3.51	1.68	0.	0.	0.	0.
time (sec)	N/A	0.57	7.362	2.108	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	131	408	4512	1152	0	0
normalized size	1	1.	0.74	2.32	25.64	6.55	0.	0.
time (sec)	N/A	0.288	1.382	0.355	2.734	0.761	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	106	344	2601	1045	0	0
normalized size	1	1.	0.81	2.63	19.85	7.98	0.	0.
time (sec)	N/A	0.237	0.488	0.363	2.45	0.756	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	89	278	1222	863	0	0
normalized size	1	1.	1.14	3.56	15.67	11.06	0.	0.
time (sec)	N/A	0.16	0.263	0.328	2.359	0.744	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	177	354	819	0	0
normalized size	1	1.	1.09	2.33	4.66	10.78	0.	0.
time (sec)	N/A	0.157	0.378	0.286	2.061	0.57	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	56	75	181	197	0	0
normalized size	1	1.	0.68	0.91	2.21	2.4	0.	0.
time (sec)	N/A	0.158	0.219	0.307	2.01	0.46	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	71	96	428	243	0	0
normalized size	1	1.	0.55	0.74	3.29	1.87	0.	0.
time (sec)	N/A	0.222	0.286	0.314	2.128	0.461	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	91	118	672	290	0	0
normalized size	1	1.	0.52	0.67	3.84	1.66	0.	0.
time (sec)	N/A	0.288	0.311	0.349	2.158	0.467	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	153	479	7937	1297	0	0
normalized size	1	1.	0.67	2.11	34.96	5.71	0.	0.
time (sec)	N/A	0.547	1.37	0.314	3.926	1.004	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	134	415	6218	1197	0	0
normalized size	1	1.	0.74	2.31	34.54	6.65	0.	0.
time (sec)	N/A	0.419	1.187	0.309	2.966	0.765	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	107	355	4575	1072	0	0
normalized size	1	1.	0.8	2.67	34.4	8.06	0.	0.
time (sec)	N/A	0.336	0.67	0.332	2.61	0.758	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	107	346	1913	957	0	0
normalized size	1	1.	0.86	2.79	15.43	7.72	0.	0.
time (sec)	N/A	0.314	1.521	0.34	2.275	0.76	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	109	211	424	971	0	0
normalized size	1	1.	0.87	1.69	3.39	7.77	0.	0.
time (sec)	N/A	0.339	0.586	0.311	2.087	0.576	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	73	97	338	251	0	0
normalized size	1	1.	0.56	0.74	2.58	1.92	0.	0.
time (sec)	N/A	0.256	0.473	0.302	2.012	0.464	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	92	119	694	305	0	0
normalized size	1	1.	0.51	0.66	3.83	1.69	0.	0.
time (sec)	N/A	0.438	0.457	0.304	2.178	0.47	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	110	141	945	355	0	0
normalized size	1	1.	0.48	0.62	4.14	1.56	0.	0.
time (sec)	N/A	0.508	0.659	0.316	2.235	0.477	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	178	543	12477	1476	0	0
normalized size	1	1.	0.65	1.98	45.54	5.39	0.	0.
time (sec)	N/A	0.693	2.098	0.333	6.935	1.023	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	154	479	9897	1351	0	0
normalized size	1	1.	0.68	2.11	43.6	5.95	0.	0.
time (sec)	N/A	0.595	1.409	0.329	4.424	1.014	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	133	419	8501	1224	0	0
normalized size	1	1.	0.74	2.33	47.23	6.8	0.	0.
time (sec)	N/A	0.513	1.248	0.306	22.095	0.772	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	137	386	0	1169	0	0
normalized size	1	1.	0.76	2.14	0.	6.49	0.	0.
time (sec)	N/A	0.504	1.811	0.367	0.	0.769	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	133	376	0	1085	0	0
normalized size	1	1.	0.75	2.12	0.	6.13	0.	0.
time (sec)	N/A	0.505	0.966	0.348	0.	0.773	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	127	235	884	1100	0	0
normalized size	1	1.	0.74	1.37	5.14	6.4	0.	0.
time (sec)	N/A	0.489	1.864	0.333	2.226	0.588	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	91	121	520	317	0	0
normalized size	1	1.	0.51	0.68	2.92	1.78	0.	0.
time (sec)	N/A	0.317	0.551	0.309	2.116	0.468	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	108	143	1007	371	0	0
normalized size	1	1.	0.47	0.63	4.42	1.63	0.	0.
time (sec)	N/A	0.632	0.764	0.306	2.202	0.479	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	127	165	1276	435	0	0
normalized size	1	1.	0.46	0.6	4.64	1.58	0.	0.
time (sec)	N/A	0.699	4.273	0.329	2.265	0.488	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	125	423	3407	1620	0	0
normalized size	1	1.	0.66	2.23	17.93	8.53	0.	0.
time (sec)	N/A	0.573	0.867	0.385	2.529	0.862	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	106	353	1827	1401	0	0
normalized size	1	1.	0.75	2.5	12.96	9.94	0.	0.
time (sec)	N/A	0.387	0.409	0.36	2.424	0.846	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	210	765	965	0	0
normalized size	1	1.	0.95	2.1	7.65	9.65	0.	0.
time (sec)	N/A	0.232	0.191	0.322	2.395	0.616	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	114	150	263	813	0	0
normalized size	1	1.	1.15	1.52	2.66	8.21	0.	0.
time (sec)	N/A	0.185	0.276	0.271	2.027	0.512	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	132	183	522	940	0	0
normalized size	1	1.	0.93	1.29	3.68	6.62	0.	0.
time (sec)	N/A	0.331	0.374	0.33	2.15	0.518	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	133	205	864	1030	0	0
normalized size	1	1.	0.71	1.1	4.62	5.51	0.	0.
time (sec)	N/A	0.507	1.097	0.347	2.26	0.522	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	152	227	1087	1129	0	0
normalized size	1	1.	0.66	0.99	4.73	4.91	0.	0.
time (sec)	N/A	0.689	1.6	0.38	2.3	0.532	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	497	541	0	1993	0	0
normalized size	1	1.	2.01	2.19	0.	8.07	0.	0.
time (sec)	N/A	0.783	4.446	0.308	0.	1.002	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	132	477	9527	1750	0	0
normalized size	1	1.	0.67	2.42	48.36	8.88	0.	0.
time (sec)	N/A	0.601	1.75	0.328	3.607	0.987	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	113	316	0	1597	0	0
normalized size	1	1.	0.78	2.18	0.	11.01	0.	0.
time (sec)	N/A	0.395	0.799	0.302	0.	0.669	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	84	219	0	995	0	0
normalized size	1	1.	0.79	2.05	0.	9.3	0.	0.
time (sec)	N/A	0.195	0.219	0.292	0.	0.513	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	174	287	11081	1114	0	0
normalized size	1	1.	1.12	1.84	71.03	7.14	0.	0.
time (sec)	N/A	0.362	1.429	0.3	2.445	0.527	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	173	317	0	1218	0	0
normalized size	1	1.	0.85	1.56	0.	6.	0.	0.
time (sec)	N/A	0.553	1.725	0.312	0.	0.529	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	171	339	0	1328	0	0
normalized size	1	1.	0.68	1.36	0.	5.31	0.	0.
time (sec)	N/A	0.734	1.406	0.324	0.	0.543	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	941	831	0	2122	0	0
normalized size	1	1.	3.83	3.38	0.	8.63	0.	0.
time (sec)	N/A	0.82	6.165	0.335	0.	1.114	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	570	550	0	1972	0	0
normalized size	1	1.	2.94	2.84	0.	10.16	0.	0.
time (sec)	N/A	0.588	5.668	0.309	0.	0.714	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	106	349	0	1289	0	0
normalized size	1	1.	0.68	2.24	0.	8.26	0.	0.
time (sec)	N/A	0.272	0.678	0.302	0.	0.517	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	203	103	347	7997	1303	0	0
normalized size	1	1.3	0.66	2.22	51.26	8.35	0.	0.
time (sec)	N/A	0.572	1.053	0.309	5.064	0.52	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	206	419	0	1384	0	0
normalized size	1	1.	1.01	2.06	0.	6.82	0.	0.
time (sec)	N/A	0.571	2.302	0.315	0.	0.535	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	193	449	0	1503	0	0
normalized size	1	1.	0.77	1.8	0.	6.01	0.	0.
time (sec)	N/A	0.761	1.735	0.323	0.	0.544	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	196	471	0	1611	0	0
normalized size	1	1.	0.66	1.59	0.	5.42	0.	0.
time (sec)	N/A	0.956	2.13	0.348	0.	0.557	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	4445	0	0	0	0	0
normalized size	1	1.	10.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.632	20.04	0.145	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	354	354	2709	0	0	0	0	0
normalized size	1	1.	7.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.366	19.124	0.176	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	415	415	2901	0	0	0	0	0
normalized size	1	1.	6.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.433	19.222	0.144	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	787	787	4110	0	0	0	0	0
normalized size	1	1.	5.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.839	19.299	0.141	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	739	739	5094	0	0	0	0	0
normalized size	1	1.	6.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.702	21.178	0.144	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	764	764	4066	0	0	0	0	0
normalized size	1	1.	5.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.733	19.17	0.147	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	4897	0	0	0	0	0
normalized size	1	1.	24.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.361	22.405	1.236	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	111	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	1.077	1.14	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	85	171	220	352	0	410
normalized size	1	1.	0.75	1.5	1.93	3.09	0.	3.6
time (sec)	N/A	0.145	0.618	0.033	0.981	0.826	0.	1.269

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	67	128	171	298	0	284
normalized size	1	1.	0.72	1.38	1.84	3.2	0.	3.05
time (sec)	N/A	0.133	0.269	0.027	0.975	0.98	0.	1.249

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	119	247	0	207
normalized size	1	1.	1.23	1.41	1.95	4.05	0.	3.39
time (sec)	N/A	0.078	0.022	0.029	0.963	0.808	0.	1.241

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	65	76	225	71	113
normalized size	1	1.	1.23	1.86	2.17	6.43	2.03	3.23
time (sec)	N/A	0.035	0.01	0.029	0.983	0.503	10.113	1.214

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	56	78	142	0	107
normalized size	1	1.	1.31	1.6	2.23	4.06	0.	3.06
time (sec)	N/A	0.055	0.028	0.047	0.964	0.494	0.	1.22

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	74	104	0	163
normalized size	1	1.	0.98	1.1	1.42	2.	0.	3.13
time (sec)	N/A	0.096	0.082	0.051	0.96	0.468	0.	1.169

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	107	149	0	243
normalized size	1	1.	0.89	1.01	1.27	1.77	0.	2.89
time (sec)	N/A	0.125	0.156	0.059	0.964	0.474	0.	1.161

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	136	205	0	367
normalized size	1	1.	0.87	1.02	1.3	1.95	0.	3.5
time (sec)	N/A	0.139	0.235	0.063	0.961	0.476	0.	1.232

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	150	312	373	521	0	713
normalized size	1	1.	0.76	1.58	1.88	2.63	0.	3.6
time (sec)	N/A	0.291	1.514	0.042	0.979	0.526	0.	1.257

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	120	241	308	443	0	645
normalized size	1	1.	0.67	1.35	1.72	2.47	0.	3.6
time (sec)	N/A	0.322	0.735	0.038	1.031	0.52	0.	1.223

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	92	174	223	371	0	397
normalized size	1	1.	0.79	1.5	1.92	3.2	0.	3.42
time (sec)	N/A	0.18	0.466	0.035	1.003	0.504	0.	1.202

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	133	170	335	0	259
normalized size	1	1.	0.78	1.55	1.98	3.9	0.	3.01
time (sec)	N/A	0.081	0.263	0.032	0.965	0.506	0.	1.202

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	109	104	139	294	0	208
normalized size	1	1.	1.82	1.73	2.32	4.9	0.	3.47
time (sec)	N/A	0.102	0.479	0.045	0.989	0.503	0.	1.24

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	120	120	134	213	0	240
normalized size	1	1.	1.5	1.5	1.68	2.66	0.	3.
time (sec)	N/A	0.174	0.212	0.054	0.96	0.51	0.	1.194

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	146	201	0	343
normalized size	1	1.	0.84	1.07	1.36	1.88	0.	3.21
time (sec)	N/A	0.216	0.224	0.059	0.968	0.48	0.	1.23

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	152	192	274	0	590
normalized size	1	1.	0.87	1.12	1.41	2.01	0.	4.34
time (sec)	N/A	0.26	0.448	0.068	0.992	0.493	0.	1.195

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	146	184	238	350	0	657
normalized size	1	1.	0.81	1.02	1.32	1.94	0.	3.65
time (sec)	N/A	0.268	0.545	0.068	0.963	0.502	0.	1.193

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	181	382	460	612	0	975
normalized size	1	1.	0.72	1.52	1.83	2.43	0.	3.87
time (sec)	N/A	0.479	3.341	0.042	1.004	0.546	0.	1.264

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	140	290	359	510	0	791
normalized size	1	1.	0.78	1.61	1.99	2.83	0.	4.39
time (sec)	N/A	0.333	0.95	0.041	0.99	0.528	0.	1.254

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	108	223	273	458	0	454
normalized size	1	1.	0.79	1.63	1.99	3.34	0.	3.31
time (sec)	N/A	0.19	0.572	0.04	0.975	0.55	0.	1.265

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	131	399	172	228	401	0	325
normalized size	1	1.1	3.35	1.45	1.92	3.37	0.	2.73
time (sec)	N/A	0.223	0.963	0.054	0.983	0.562	0.	1.225

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	217	168	194	369	0	316
normalized size	1	1.	1.75	1.35	1.56	2.98	0.	2.55
time (sec)	N/A	0.333	0.681	0.056	0.98	0.53	0.	1.291

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	159	207	205	317	0	424
normalized size	1	1.	1.1	1.43	1.41	2.19	0.	2.92
time (sec)	N/A	0.347	0.355	0.06	0.963	0.542	0.	1.267

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	140	180	231	321	0	724
normalized size	1	1.	0.78	1.01	1.29	1.79	0.	4.04
time (sec)	N/A	0.423	0.408	0.063	0.967	0.525	0.	1.224

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	176	227	293	423	0	907
normalized size	1	1.	0.8	1.03	1.33	1.91	0.	4.1
time (sec)	N/A	0.494	0.698	0.071	0.972	0.549	0.	1.252

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	244	550	640	797	0	1601
normalized size	1	1.	0.73	1.65	1.92	2.39	0.	4.79
time (sec)	N/A	0.711	2.864	0.047	0.997	0.702	0.	1.284

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	198	431	512	687	0	1148
normalized size	1	1.	0.79	1.72	2.05	2.75	0.	4.59
time (sec)	N/A	0.52	3.914	0.049	0.983	0.594	0.	1.277

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	160	338	409	603	0	857
normalized size	1	1.	0.8	1.69	2.04	3.02	0.	4.28
time (sec)	N/A	0.327	1.027	0.049	0.98	0.605	0.	1.315

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	1051	262	331	524	0	522
normalized size	1	1.	5.39	1.34	1.7	2.69	0.	2.68
time (sec)	N/A	0.368	6.288	0.063	0.99	0.574	0.	1.245

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	310	236	282	479	0	713
normalized size	1	1.	1.48	1.13	1.35	2.29	0.	3.41
time (sec)	N/A	0.463	1.898	0.064	0.981	0.606	0.	1.304

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	257	255	266	471	0	501
normalized size	1	1.	1.3	1.29	1.34	2.38	0.	2.53
time (sec)	N/A	0.591	1.075	0.063	0.989	0.593	0.	1.288

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	210	319	290	447	0	814
normalized size	1	1.	0.97	1.48	1.34	2.07	0.	3.77
time (sec)	N/A	0.609	0.601	0.071	0.983	0.595	0.	1.303

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	263	258	332	478	0	1068
normalized size	1	1.	1.02	1.	1.29	1.85	0.	4.14
time (sec)	N/A	0.691	0.633	0.073	0.985	0.594	0.	1.313

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	333	316	414	587	0	1521
normalized size	1	1.	1.08	1.02	1.34	1.9	0.	4.92
time (sec)	N/A	0.82	1.228	0.079	0.987	0.609	0.	1.284

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	422	688	0	1650	0	556
normalized size	1	1.	2.26	3.68	0.	8.82	0.	2.97
time (sec)	N/A	0.676	2.366	0.083	0.	2.357	0.	1.263

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	300	410	0	1353	0	363
normalized size	1	1.	2.1	2.87	0.	9.46	0.	2.54
time (sec)	N/A	0.395	1.775	0.074	0.	11.326	0.	1.309

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	130	228	0	1065	0	238
normalized size	1	1.	1.33	2.33	0.	10.87	0.	2.43
time (sec)	N/A	0.229	0.573	0.061	0.	0.876	0.	1.227

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	707	0	171
normalized size	1	1.	1.47	1.78	0.	9.3	0.	2.25
time (sec)	N/A	0.126	0.179	0.063	0.	1.919	0.	1.232

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	540	0	136
normalized size	1	1.	1.01	1.69	0.	8.06	0.	2.03
time (sec)	N/A	0.099	0.123	0.072	0.	0.521	0.	1.2

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	172	0	702	0	190
normalized size	1	1.	0.94	1.91	0.	7.8	0.	2.11
time (sec)	N/A	0.149	0.208	0.097	0.	0.543	0.	1.173

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	367	0	934	0	306
normalized size	1	1.	0.9	2.74	0.	6.97	0.	2.28
time (sec)	N/A	0.403	0.329	0.099	0.	0.566	0.	1.214

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	152	641	0	1177	0	486
normalized size	1	1.	0.85	3.6	0.	6.61	0.	2.73
time (sec)	N/A	0.642	0.487	0.102	0.	0.584	0.	1.223

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	202	1212	0	1504	0	867
normalized size	1	1.	0.84	5.05	0.	6.27	0.	3.61
time (sec)	N/A	0.982	0.64	0.108	0.	0.669	0.	1.2

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	438	698	0	2969	0	518
normalized size	1	1.	1.61	2.57	0.	10.92	0.	1.9
time (sec)	N/A	0.866	6.272	0.098	0.	49.393	0.	1.253

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	240	510	0	2433	0	545
normalized size	1	1.	1.46	3.11	0.	14.84	0.	3.32
time (sec)	N/A	0.578	2.084	0.085	0.	31.363	0.	1.269

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	191	350	0	1551	0	312
normalized size	1	1.	1.46	2.67	0.	11.84	0.	2.38
time (sec)	N/A	0.301	0.697	0.079	0.	9.579	0.	1.262

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	861	0	232
normalized size	1	1.	0.97	1.32	0.	8.61	0.	2.32
time (sec)	N/A	0.134	0.346	0.078	0.	0.55	0.	1.22

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	155	328	0	1226	0	271
normalized size	1	1.	1.25	2.65	0.	9.89	0.	2.19
time (sec)	N/A	0.207	0.656	0.091	0.	0.585	0.	1.215

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	221	453	0	1715	0	505
normalized size	1	1.	1.23	2.52	0.	9.53	0.	2.81
time (sec)	N/A	0.569	1.093	0.116	0.	0.66	0.	1.398

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	184	651	0	2136	0	459
normalized size	1	1.	0.7	2.49	0.	8.18	0.	1.76
time (sec)	N/A	0.891	1.074	0.114	0.	0.741	0.	1.506

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	224	926	0	2592	0	639
normalized size	1	1.	0.65	2.68	0.	7.49	0.	1.85
time (sec)	N/A	1.275	1.374	0.12	0.	0.815	0.	1.49

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	507	1599	0	5414	0	1878
normalized size	1	1.	1.25	3.93	0.	13.3	0.	4.61
time (sec)	N/A	1.959	2.947	0.106	0.	171.058	0.	1.709

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	418	1406	0	4591	0	784
normalized size	1	1.	1.45	4.87	0.	15.89	0.	2.71
time (sec)	N/A	1.424	6.47	0.097	0.	114.913	0.	1.584

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	270	1085	0	3051	0	656
normalized size	1	1.	1.23	4.93	0.	13.87	0.	2.98
time (sec)	N/A	0.686	1.827	0.095	0.	43.553	0.	1.519

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	157	238	0	1631	0	540
normalized size	1	1.	0.87	1.32	0.	9.06	0.	3.
time (sec)	N/A	0.336	0.678	0.084	0.	0.656	0.	1.443

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	172	236	0	1631	0	539
normalized size	1	1.	1.05	1.44	0.	9.95	0.	3.29
time (sec)	N/A	0.264	0.856	0.084	0.	0.665	0.	1.48

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	267	1063	0	2479	0	617
normalized size	1	1.	1.3	5.19	0.	12.09	0.	3.01
time (sec)	N/A	0.536	1.417	0.099	0.	0.751	0.	1.44

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	306	1349	0	3394	0	737
normalized size	1	1.	1.06	4.65	0.	11.7	0.	2.54
time (sec)	N/A	1.535	1.969	0.123	0.	0.916	0.	1.469

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	734	1552	0	4018	0	1827
normalized size	1	1.	1.87	3.95	0.	10.22	0.	4.65
time (sec)	N/A	1.999	4.256	0.134	0.	1.111	0.	1.537

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	548	2948	0	0	0	1357
normalized size	1	1.	1.31	7.05	0.	0.	0.	3.25
time (sec)	N/A	5.273	3.	0.108	0.	0.	0.	1.614

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	369	2264	0	5023	0	1139
normalized size	1	1.	1.19	7.3	0.	16.2	0.	3.67
time (sec)	N/A	1.366	1.81	0.122	0.	116.544	0.	1.659

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	226	375	0	2707	0	936
normalized size	1	1.	0.82	1.37	0.	9.88	0.	3.42
time (sec)	N/A	0.7	2.232	0.094	0.	0.799	0.	1.555

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	252	388	0	2715	0	980
normalized size	1	1.	0.96	1.48	0.	10.32	0.	3.73
time (sec)	N/A	0.615	1.157	0.089	0.	0.845	0.	1.589

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	404	376	0	2707	0	936
normalized size	1	1.	1.7	1.59	0.	11.42	0.	3.95
time (sec)	N/A	0.51	1.05	0.096	0.	0.84	0.	1.545

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	769	2242	0	4111	0	1099
normalized size	1	1.	2.63	7.68	0.	14.08	0.	3.76
time (sec)	N/A	1.068	3.38	0.108	0.	1.044	0.	1.471

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	1205	2891	0	5736	0	1304
normalized size	1	1.	2.93	7.03	0.	13.96	0.	3.17
time (sec)	N/A	5.595	6.084	0.144	0.	1.361	0.	1.563

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	538	538	1452	3099	0	6657	0	1420
normalized size	1	1.	2.7	5.76	0.	12.37	0.	2.64
time (sec)	N/A	6.844	5.824	0.148	0.	1.56	0.	1.634

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	116	0	454	0	143
normalized size	1	1.	1.	1.9	0.	7.44	0.	2.34
time (sec)	N/A	0.115	0.144	0.077	0.	0.536	0.	1.313

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	9	3	18
normalized size	1	1.	1.	1.17	0.	1.5	0.5	3.
time (sec)	N/A	0.001	0.001	0.007	0.	0.43	6.069	1.385

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	97	163	0	670	0	188
normalized size	1	1.	1.13	1.9	0.	7.79	0.	2.19
time (sec)	N/A	0.18	0.355	0.092	0.	0.551	0.	1.373

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	39	39	108	225	0	78
normalized size	1	1.	0.45	0.45	1.24	2.59	0.	0.9
time (sec)	N/A	0.074	0.069	0.074	1.429	0.497	0.	1.395

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	3734	4394	0	0	0	0
normalized size	1	1.	7.7	9.06	0.	0.	0.	0.
time (sec)	N/A	1.437	25.544	1.664	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	397	3330	3438	0	0	0	0
normalized size	1	1.	8.39	8.66	0.	0.	0.	0.
time (sec)	N/A	0.933	24.424	1.091	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	434	2498	0	0	0	0
normalized size	1	1.	1.38	7.96	0.	0.	0.	0.
time (sec)	N/A	0.598	18.454	0.718	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	408	1752	0	0	0	0
normalized size	1	1.	1.59	6.84	0.	0.	0.	0.
time (sec)	N/A	0.34	14.937	0.49	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	913	1372	0	0	0	0
normalized size	1	1.	2.85	4.29	0.	0.	0.	0.
time (sec)	N/A	0.29	17.887	0.403	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	1107	1389	0	0	0	0
normalized size	1	1.	3.22	4.04	0.	0.	0.	0.
time (sec)	N/A	0.369	17.92	0.385	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	1161	2065	0	0	0	0
normalized size	1	1.	2.71	4.81	0.	0.	0.	0.
time (sec)	N/A	0.733	18.83	0.379	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	1565	2954	0	0	0	0
normalized size	1	1.	3.07	5.8	0.	0.	0.	0.
time (sec)	N/A	1.129	20.186	0.453	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	475	475	3766	4394	0	0	0	0
normalized size	1	1.	7.93	9.25	0.	0.	0.	0.
time (sec)	N/A	1.225	25.765	1.639	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	388	388	3342	3424	0	0	0	0
normalized size	1	1.	8.61	8.82	0.	0.	0.	0.
time (sec)	N/A	0.829	24.546	1.07	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	502	2683	0	0	0	0
normalized size	1	1.	1.61	8.6	0.	0.	0.	0.
time (sec)	N/A	0.57	18.755	0.71	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	6093	2340	0	0	0	0
normalized size	1	1.	15.99	6.14	0.	0.	0.	0.
time (sec)	N/A	0.465	24.118	0.476	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	979	2199	0	0	0	0
normalized size	1	1.	2.71	6.09	0.	0.	0.	0.
time (sec)	N/A	0.451	18.375	0.464	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	428	428	1598	2439	0	0	0	0
normalized size	1	1.	3.73	5.7	0.	0.	0.	0.
time (sec)	N/A	0.792	19.438	0.379	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	520	520	1551	3142	0	0	0	0
normalized size	1	1.	2.98	6.04	0.	0.	0.	0.
time (sec)	N/A	1.275	19.034	0.448	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	566	566	4227	5368	0	0	0	0
normalized size	1	1.	7.47	9.48	0.	0.	0.	0.
time (sec)	N/A	1.784	26.593	2.49	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	3781	4395	0	0	0	0
normalized size	1	1.	8.06	9.37	0.	0.	0.	0.
time (sec)	N/A	1.183	26.294	1.639	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	2957	3637	0	0	0	0
normalized size	1	1.	7.7	9.47	0.	0.	0.	0.
time (sec)	N/A	0.807	23.279	1.082	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	442	442	7168	3285	0	0	0	0
normalized size	1	1.	16.22	7.43	0.	0.	0.	0.
time (sec)	N/A	0.656	25.207	0.749	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1146	3215	0	0	0	0
normalized size	1	1.	2.65	7.42	0.	0.	0.	0.
time (sec)	N/A	0.703	19.289	0.639	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	1338	3271	0	0	0	0
normalized size	1	1.	2.97	7.27	0.	0.	0.	0.
time (sec)	N/A	0.835	19.344	0.648	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	1567	3511	0	0	0	0
normalized size	1	1.	3.03	6.78	0.	0.	0.	0.
time (sec)	N/A	1.365	19.486	0.489	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	617	617	5186	4231	0	0	0	0
normalized size	1	1.	8.41	6.86	0.	0.	0.	0.
time (sec)	N/A	1.832	24.539	0.612	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	3000	2499	0	0	0	0
normalized size	1	1.	9.12	7.6	0.	0.	0.	0.
time (sec)	N/A	0.618	23.224	0.756	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	372	1567	0	0	0	0
normalized size	1	1.	1.43	6.	0.	0.	0.	0.
time (sec)	N/A	0.397	16.407	0.495	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	356	829	0	0	0	0
normalized size	1	1.	1.7	3.95	0.	0.	0.	0.
time (sec)	N/A	0.206	14.516	0.403	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	147	215	0	0	0	0
normalized size	1	1.	0.71	1.03	0.	0.	0.	0.
time (sec)	N/A	0.124	2.232	0.347	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	348	348	1027	1028	0	0	0	0
normalized size	1	1.	2.95	2.95	0.	0.	0.	0.
time (sec)	N/A	0.405	17.024	0.39	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	1639	1885	0	0	0	0
normalized size	1	1.	3.77	4.33	0.	0.	0.	0.
time (sec)	N/A	0.724	15.906	0.388	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	525	525	1585	2954	0	0	0	0
normalized size	1	1.	3.02	5.63	0.	0.	0.	0.
time (sec)	N/A	1.168	19.87	0.471	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	3460	3333	0	0	0	0
normalized size	1	1.	10.52	10.13	0.	0.	0.	0.
time (sec)	N/A	0.72	24.791	0.743	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	467	2276	0	0	0	0
normalized size	1	1.	1.7	8.28	0.	0.	0.	0.
time (sec)	N/A	0.464	18.717	0.469	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	468	1634	0	0	0	0
normalized size	1	1.	1.84	6.43	0.	0.	0.	0.
time (sec)	N/A	0.348	15.577	0.381	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	1491	2009	0	0	0	0
normalized size	1	1.	3.97	5.34	0.	0.	0.	0.
time (sec)	N/A	0.433	14.712	0.385	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	1613	2871	0	0	0	0
normalized size	1	1.	3.78	6.72	0.	0.	0.	0.
time (sec)	N/A	0.701	19.701	0.372	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	531	531	2667	3980	0	0	0	0
normalized size	1	1.	5.02	7.5	0.	0.	0.	0.
time (sec)	N/A	1.139	18.56	0.525	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	630	630	2343	5086	0	0	0	0
normalized size	1	1.	3.72	8.07	0.	0.	0.	0.
time (sec)	N/A	1.669	22.385	0.717	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	510	4342	8044	0	0	0	0
normalized size	1	1.	8.51	15.77	0.	0.	0.	0.
time (sec)	N/A	1.588	26.921	1.589	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	3920	6455	0	0	0	0
normalized size	1	1.	9.4	15.48	0.	0.	0.	0.
time (sec)	N/A	1.016	26.453	0.794	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	3514	5170	0	0	0	0
normalized size	1	1.	9.08	13.36	0.	0.	0.	0.
time (sec)	N/A	0.694	24.887	0.42	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	3225	4213	0	0	0	0
normalized size	1	1.	9.14	11.93	0.	0.	0.	0.
time (sec)	N/A	0.598	22.685	0.387	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	2083	5710	0	0	0	0
normalized size	1	1.	4.21	11.54	0.	0.	0.	0.
time (sec)	N/A	0.767	17.16	0.414	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	582	582	2390	8545	0	0	0	0
normalized size	1	1.	4.11	14.68	0.	0.	0.	0.
time (sec)	N/A	1.214	22.046	0.677	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	686	686	823	10322	0	0	0	0
normalized size	1	1.	1.2	15.05	0.	0.	0.	0.
time (sec)	N/A	2.05	14.717	0.957	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	248	642	0	0	0	0
normalized size	1	1.	2.36	6.11	0.	0.	0.	0.
time (sec)	N/A	0.081	10.552	0.432	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	211	457	0	0	0	0
normalized size	1	1.	1.97	4.27	0.	0.	0.	0.
time (sec)	N/A	0.085	8.097	0.427	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	132	663	0	0	0	0
normalized size	1	1.	0.73	3.68	0.	0.	0.	0.
time (sec)	N/A	0.183	1.871	5.766	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	104	428	0	0	0	0
normalized size	1	1.	0.73	2.99	0.	0.	0.	0.
time (sec)	N/A	0.147	0.913	4.188	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	84	244	0	0	0	0
normalized size	1	1.	0.76	2.2	0.	0.	0.	0.
time (sec)	N/A	0.139	0.295	1.928	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	90	326	0	0	0	0
normalized size	1	1.	0.78	2.83	0.	0.	0.	0.
time (sec)	N/A	0.147	0.255	1.785	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	108	371	0	0	0	0
normalized size	1	1.	0.73	2.51	0.	0.	0.	0.
time (sec)	N/A	0.163	0.604	1.806	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	125	413	0	0	0	0
normalized size	1	1.	0.69	2.29	0.	0.	0.	0.
time (sec)	N/A	0.181	1.055	1.948	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	221	859	0	0	0	0
normalized size	1	1.	0.84	3.27	0.	0.	0.	0.
time (sec)	N/A	0.373	4.602	7.429	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	171	750	0	0	0	0
normalized size	1	1.	0.77	3.39	0.	0.	0.	0.
time (sec)	N/A	0.315	2.69	6.353	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	125	677	0	0	0	0
normalized size	1	1.	0.71	3.82	0.	0.	0.	0.
time (sec)	N/A	0.27	1.263	4.855	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	404	0	0	0	0
normalized size	1	1.	0.77	2.51	0.	0.	0.	0.
time (sec)	N/A	0.248	0.757	2.187	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	128	487	0	0	0	0
normalized size	1	1.	0.75	2.85	0.	0.	0.	0.
time (sec)	N/A	0.26	0.972	1.87	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	161	548	0	0	0	0
normalized size	1	1.	0.76	2.57	0.	0.	0.	0.
time (sec)	N/A	0.289	1.448	2.148	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	189	610	0	0	0	0
normalized size	1	1.	0.74	2.4	0.	0.	0.	0.
time (sec)	N/A	0.337	1.94	1.985	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	452	1193	0	0	0	0
normalized size	1	1.	1.31	3.46	0.	0.	0.	0.
time (sec)	N/A	0.572	6.553	9.93	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	225	944	0	0	0	0
normalized size	1	1.	0.76	3.2	0.	0.	0.	0.
time (sec)	N/A	0.504	3.88	8.33	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	190	997	0	0	0	0
normalized size	1	1.	0.78	4.09	0.	0.	0.	0.
time (sec)	N/A	0.483	2.456	6.648	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	166	1212	0	0	0	0
normalized size	1	1.	0.69	5.07	0.	0.	0.	0.
time (sec)	N/A	0.51	1.986	5.698	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	172	867	0	0	0	0
normalized size	1	1.	0.73	3.67	0.	0.	0.	0.
time (sec)	N/A	0.461	1.586	2.298	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	180	664	0	0	0	0
normalized size	1	1.	0.73	2.71	0.	0.	0.	0.
time (sec)	N/A	0.467	1.358	2.01	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	219	745	0	0	0	0
normalized size	1	1.	0.74	2.53	0.	0.	0.	0.
time (sec)	N/A	0.538	2.096	2.046	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	256	825	0	0	0	0
normalized size	1	1.	0.74	2.39	0.	0.	0.	0.
time (sec)	N/A	0.574	3.096	1.988	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	669	785	0	0	0	0
normalized size	1	1.	2.42	2.83	0.	0.	0.	0.
time (sec)	N/A	1.014	6.962	6.75	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	229	466	0	0	0	0
normalized size	1	1.	1.09	2.22	0.	0.	0.	0.
time (sec)	N/A	0.713	3.659	5.356	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	125	325	0	0	0	0
normalized size	1	1.	0.99	2.58	0.	0.	0.	0.
time (sec)	N/A	0.401	1.336	3.801	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	217	0	0	0	0
normalized size	1	1.	0.77	2.15	0.	0.	0.	0.
time (sec)	N/A	0.198	0.581	2.02	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	224	295	0	0	0	0
normalized size	1	1.	1.5	1.98	0.	0.	0.	0.
time (sec)	N/A	0.257	6.941	2.069	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	282	786	0	0	0	0
normalized size	1	1.	1.44	4.01	0.	0.	0.	0.
time (sec)	N/A	0.467	6.524	2.248	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	617	1074	0	0	0	0
normalized size	1	1.	2.55	4.44	0.	0.	0.	0.
time (sec)	N/A	0.755	6.939	2.22	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	738	1024	0	0	0	0
normalized size	1	1.	1.82	2.52	0.	0.	0.	0.
time (sec)	N/A	1.162	7.145	9.448	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	685	877	0	0	0	0
normalized size	1	1.	2.17	2.78	0.	0.	0.	0.
time (sec)	N/A	0.837	6.984	6.326	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	643	715	0	0	0	0
normalized size	1	1.	2.5	2.78	0.	0.	0.	0.
time (sec)	N/A	0.527	6.889	5.18	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	727	802	0	0	0	0
normalized size	1	1.	2.76	3.05	0.	0.	0.	0.
time (sec)	N/A	0.506	6.905	5.786	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	657	843	0	0	0	0
normalized size	1	1.	2.32	2.98	0.	0.	0.	0.
time (sec)	N/A	0.569	6.975	6.439	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	704	1059	0	0	0	0
normalized size	1	1.	1.93	2.9	0.	0.	0.	0.
time (sec)	N/A	0.859	7.146	7.161	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	583	583	902	2178	0	0	0	0
normalized size	1	1.	1.55	3.74	0.	0.	0.	0.
time (sec)	N/A	1.779	7.455	15.662	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	480	847	2024	0	0	0	0
normalized size	1	1.	1.76	4.22	0.	0.	0.	0.
time (sec)	N/A	1.378	7.271	10.555	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	800	1768	0	0	0	0
normalized size	1	1.	1.99	4.4	0.	0.	0.	0.
time (sec)	N/A	0.914	7.017	8.45	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	887	1872	0	0	0	0
normalized size	1	1.	2.21	4.66	0.	0.	0.	0.
time (sec)	N/A	0.914	7.015	8.338	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	890	1959	0	0	0	0
normalized size	1	1.	2.21	4.87	0.	0.	0.	0.
time (sec)	N/A	0.865	6.986	8.849	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	823	2000	0	0	0	0
normalized size	1	1.	1.93	4.68	0.	0.	0.	0.
time (sec)	N/A	0.998	7.388	9.803	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	868	2216	0	0	0	0
normalized size	1	1.	1.67	4.25	0.	0.	0.	0.
time (sec)	N/A	1.444	7.353	11.085	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	422	2521	0	0	0	0
normalized size	1	1.	1.26	7.5	0.	0.	0.	0.
time (sec)	N/A	1.108	5.34	0.492	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	377	1431	0	0	0	0
normalized size	1	1.	1.49	5.66	0.	0.	0.	0.
time (sec)	N/A	0.783	6.096	0.503	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	122	1549	0	0	0	0
normalized size	1	1.	0.59	7.45	0.	0.	0.	0.
time (sec)	N/A	0.543	2.549	0.491	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	165	1926	0	0	0	0
normalized size	1	1.	0.82	9.58	0.	0.	0.	0.
time (sec)	N/A	0.479	0.754	0.36	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	200	2739	0	0	0	0
normalized size	1	1.	0.75	10.26	0.	0.	0.	0.
time (sec)	N/A	0.748	1.181	0.494	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	208	3778	0	0	0	0
normalized size	1	1.	0.61	11.01	0.	0.	0.	0.
time (sec)	N/A	1.035	1.311	0.582	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	673	4051	0	0	0	0
normalized size	1	1.	1.6	9.62	0.	0.	0.	0.
time (sec)	N/A	1.597	6.783	0.575	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	595	2947	0	0	0	0
normalized size	1	1.	1.76	8.69	0.	0.	0.	0.
time (sec)	N/A	1.207	6.72	0.402	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	554	2595	0	0	0	0
normalized size	1	1.	2.04	9.54	0.	0.	0.	0.
time (sec)	N/A	0.868	6.685	0.502	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	437	2552	0	0	0	0
normalized size	1	1.	1.58	9.25	0.	0.	0.	0.
time (sec)	N/A	0.918	4.431	0.391	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	201	2915	0	0	0	0
normalized size	1	1.	0.76	10.96	0.	0.	0.	0.
time (sec)	N/A	0.792	1.578	0.441	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	255	3752	0	0	0	0
normalized size	1	1.	0.75	10.97	0.	0.	0.	0.
time (sec)	N/A	1.129	1.935	0.568	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	313	4846	0	0	0	0
normalized size	1	1.	0.73	11.35	0.	0.	0.	0.
time (sec)	N/A	1.495	2.072	0.763	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	513	513	768	5392	0	0	0	0
normalized size	1	1.	1.5	10.51	0.	0.	0.	0.
time (sec)	N/A	1.998	6.91	0.791	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	678	4258	0	0	0	0
normalized size	1	1.	1.61	10.09	0.	0.	0.	0.
time (sec)	N/A	1.594	6.923	0.515	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	628	3939	0	0	0	0
normalized size	1	1.	1.75	10.97	0.	0.	0.	0.
time (sec)	N/A	1.249	7.002	0.523	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	599	3663	0	0	0	0
normalized size	1	1.	1.72	10.5	0.	0.	0.	0.
time (sec)	N/A	1.25	6.95	0.412	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	616	3564	0	0	0	0
normalized size	1	1.	1.8	10.42	0.	0.	0.	0.
time (sec)	N/A	1.223	6.966	0.466	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	257	3980	0	0	0	0
normalized size	1	1.	0.76	11.71	0.	0.	0.	0.
time (sec)	N/A	1.155	1.791	0.557	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	313	4847	0	0	0	0
normalized size	1	1.	0.74	11.4	0.	0.	0.	0.
time (sec)	N/A	1.518	2.497	0.744	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	380	5946	0	0	0	0
normalized size	1	1.	0.73	11.46	0.	0.	0.	0.
time (sec)	N/A	1.96	3.553	1.022	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	344	344	451	2737	0	0	0	0
normalized size	1	1.	1.31	7.96	0.	0.	0.	0.
time (sec)	N/A	1.111	3.992	0.46	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	339	1440	0	0	0	0
normalized size	1	1.	1.32	5.62	0.	0.	0.	0.
time (sec)	N/A	0.731	6.754	0.43	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	91	283	0	0	0	0
normalized size	1	1.	0.66	2.05	0.	0.	0.	0.
time (sec)	N/A	0.392	0.258	0.395	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	103	940	0	0	0	0
normalized size	1	1.	0.69	6.27	0.	0.	0.	0.
time (sec)	N/A	0.31	3.651	0.382	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	161	1731	0	0	0	0
normalized size	1	1.	0.76	8.17	0.	0.	0.	0.
time (sec)	N/A	0.48	0.878	0.401	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	198	2739	0	0	0	0
normalized size	1	1.	0.71	9.78	0.	0.	0.	0.
time (sec)	N/A	0.75	1.259	0.46	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	518	2655	0	0	0	0
normalized size	1	1.	1.4	7.16	0.	0.	0.	0.
time (sec)	N/A	1.269	5.746	0.359	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	464	1585	0	0	0	0
normalized size	1	1.	2.11	7.2	0.	0.	0.	0.
time (sec)	N/A	0.625	4.584	0.405	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	161	941	0	0	0	0
normalized size	1	1.	0.75	4.38	0.	0.	0.	0.
time (sec)	N/A	0.572	0.695	0.392	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	178	1452	0	0	0	0
normalized size	1	1.	0.76	6.18	0.	0.	0.	0.
time (sec)	N/A	0.579	1.001	0.421	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	252	2285	0	0	0	0
normalized size	1	1.	0.77	7.01	0.	0.	0.	0.
time (sec)	N/A	0.836	1.576	0.37	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	316	3156	0	0	0	0
normalized size	1	1.	0.75	7.46	0.	0.	0.	0.
time (sec)	N/A	1.222	2.173	0.458	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	726	5195	0	0	0	0
normalized size	1	1.	1.82	13.02	0.	0.	0.	0.
time (sec)	N/A	1.374	6.827	0.408	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	217	3138	0	0	0	0
normalized size	1	1.	0.66	9.54	0.	0.	0.	0.
time (sec)	N/A	0.842	2.245	0.401	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	245	3865	0	0	0	0
normalized size	1	1.	0.71	11.17	0.	0.	0.	0.
time (sec)	N/A	0.822	2.042	0.411	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	297	5169	0	0	0	0
normalized size	1	1.	0.81	14.05	0.	0.	0.	0.
time (sec)	N/A	0.937	2.352	0.464	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	353	6745	0	0	0	0
normalized size	1	1.	0.75	14.29	0.	0.	0.	0.
time (sec)	N/A	1.409	2.909	0.541	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	392	8251	0	0	0	0
normalized size	1	1.	0.67	14.03	0.	0.	0.	0.
time (sec)	N/A	1.879	3.742	0.81	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	21.182	0.145	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	17.829	0.139	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	3.338	0.165	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	3.33	0.181	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	4.27	1.197	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	544	365	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	1.63	4.423	0.804	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	307	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.786	2.208	0.623	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	239	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.406	0.964	1.058	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	168	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.387	0.559	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	872	383	0	0	0	0
normalized size	1	1.	6.61	2.9	0.	0.	0.	0.
time (sec)	N/A	0.229	6.284	1.923	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	830	355	0	0	0	0
normalized size	1	1.	8.22	3.51	0.	0.	0.	0.
time (sec)	N/A	0.21	6.204	2.14	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	309	321	0	0	0	0
normalized size	1	1.	4.41	4.59	0.	0.	0.	0.
time (sec)	N/A	0.186	5.918	2.037	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	252	240	0	0	0	0
normalized size	1	1.	3.82	3.64	0.	0.	0.	0.
time (sec)	N/A	0.194	6.074	2.013	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	813	426	0	0	0	0
normalized size	1	1.	8.56	4.48	0.	0.	0.	0.
time (sec)	N/A	0.217	6.338	4.806	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	865	661	0	0	0	0
normalized size	1	1.	6.55	5.01	0.	0.	0.	0.
time (sec)	N/A	0.232	6.387	5.993	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	1086	413	0	0	0	0
normalized size	1	1.	5.6	2.13	0.	0.	0.	0.
time (sec)	N/A	0.404	6.341	1.789	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	1040	385	0	0	0	0
normalized size	1	1.	6.46	2.39	0.	0.	0.	0.
time (sec)	N/A	0.362	6.249	1.744	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	994	357	0	0	0	0
normalized size	1	1.	7.89	2.83	0.	0.	0.	0.
time (sec)	N/A	0.347	6.301	1.776	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	735	388	0	0	0	0
normalized size	1	1.	6.34	3.34	0.	0.	0.	0.
time (sec)	N/A	0.337	6.374	1.988	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	736	513	0	0	0	0
normalized size	1	1.	6.13	4.28	0.	0.	0.	0.
time (sec)	N/A	0.35	6.426	2.253	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	1025	741	0	0	0	0
normalized size	1	1.	6.45	4.66	0.	0.	0.	0.
time (sec)	N/A	0.381	6.52	6.256	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	1067	851	0	0	0	0
normalized size	1	1.	5.5	4.39	0.	0.	0.	0.
time (sec)	N/A	0.415	6.555	7.114	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	1292	282	0	0	0	0
normalized size	1	1.	8.23	1.8	0.	0.	0.	0.
time (sec)	N/A	0.265	6.601	1.886	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	1239	262	0	0	0	0
normalized size	1	1.	9.99	2.11	0.	0.	0.	0.
time (sec)	N/A	0.244	6.533	2.127	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	1208	244	0	0	0	0
normalized size	1	1.	13.73	2.77	0.	0.	0.	0.
time (sec)	N/A	0.22	6.476	1.775	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	1204	243	0	0	0	0
normalized size	1	1.	14.51	2.93	0.	0.	0.	0.
time (sec)	N/A	0.222	6.451	1.912	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	1240	318	0	0	0	0
normalized size	1	1.	10.97	2.81	0.	0.	0.	0.
time (sec)	N/A	0.24	6.663	4.232	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	1277	493	0	0	0	0
normalized size	1	1.	8.4	3.24	0.	0.	0.	0.
time (sec)	N/A	0.26	6.987	5.406	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	1396	465	0	0	0	0
normalized size	1	1.	6.84	2.28	0.	0.	0.	0.
time (sec)	N/A	0.419	6.833	2.219	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	1352	435	0	0	0	0
normalized size	1	1.	7.91	2.54	0.	0.	0.	0.
time (sec)	N/A	0.401	6.718	2.084	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	1318	421	0	0	0	0
normalized size	1	1.	9.62	3.07	0.	0.	0.	0.
time (sec)	N/A	0.375	6.632	1.839	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	921	350	0	0	0	0
normalized size	1	1.	7.61	2.89	0.	0.	0.	0.
time (sec)	N/A	0.346	6.518	2.116	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	921	350	0	0	0	0
normalized size	1	1.	7.61	2.89	0.	0.	0.	0.
time (sec)	N/A	0.355	6.513	1.891	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	1351	492	0	0	0	0
normalized size	1	1.	8.24	3.	0.	0.	0.	0.
time (sec)	N/A	0.397	6.727	2.314	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	1392	750	0	0	0	0
normalized size	1	1.	7.07	3.81	0.	0.	0.	0.
time (sec)	N/A	0.428	7.232	6.874	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	1448	465	0	0	0	0
normalized size	1	1.	6.55	2.1	0.	0.	0.	0.
time (sec)	N/A	0.582	6.906	2.1	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	1415	451	0	0	0	0
normalized size	1	1.	7.53	2.4	0.	0.	0.	0.
time (sec)	N/A	0.548	6.824	2.265	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	1407	451	0	0	0	0
normalized size	1	1.	7.73	2.48	0.	0.	0.	0.
time (sec)	N/A	0.539	6.746	2.089	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	1406	451	0	0	0	0
normalized size	1	1.	7.9	2.53	0.	0.	0.	0.
time (sec)	N/A	0.522	6.655	2.285	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	1407	451	0	0	0	0
normalized size	1	1.	7.82	2.51	0.	0.	0.	0.
time (sec)	N/A	0.533	6.705	2.002	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	1447	685	0	0	0	0
normalized size	1	1.	6.55	3.1	0.	0.	0.	0.
time (sec)	N/A	0.577	6.973	2.533	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	119	130	738	309	0	0
normalized size	1	1.	0.54	0.59	3.35	1.4	0.	0.
time (sec)	N/A	0.476	0.485	0.34	2.067	0.486	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	96	108	564	265	0	0
normalized size	1	1.	0.55	0.62	3.22	1.51	0.	0.
time (sec)	N/A	0.403	0.339	0.276	2.026	0.48	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	79	86	400	216	0	0
normalized size	1	1.	0.61	0.66	3.08	1.66	0.	0.
time (sec)	N/A	0.332	0.165	0.3	1.988	0.473	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	56	65	190	171	0	0
normalized size	1	1.	0.68	0.79	2.32	2.09	0.	0.
time (sec)	N/A	0.262	0.175	0.265	1.942	0.471	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	94	169	354	799	0	0
normalized size	1	1.	0.98	1.76	3.69	8.32	0.	0.
time (sec)	N/A	0.259	0.312	0.301	1.913	0.552	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	89	275	1222	929	0	0
normalized size	1	1.	0.91	2.81	12.47	9.48	0.	0.
time (sec)	N/A	0.257	0.377	0.297	2.124	0.675	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	106	342	2601	1035	0	0
normalized size	1	1.	0.7	2.26	17.23	6.85	0.	0.
time (sec)	N/A	0.331	0.623	0.344	2.272	0.681	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	131	404	4512	1131	0	0
normalized size	1	1.	0.67	2.06	23.02	5.77	0.	0.
time (sec)	N/A	0.396	1.047	0.353	2.606	0.688	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	131	153	949	397	0	0
normalized size	1	1.	0.48	0.56	3.45	1.44	0.	0.
time (sec)	N/A	0.714	0.548	0.303	2.132	0.494	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	118	131	753	332	0	0
normalized size	1	1.	0.52	0.57	3.3	1.46	0.	0.
time (sec)	N/A	0.686	0.422	0.315	2.094	0.485	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	100	109	609	282	0	0
normalized size	1	1.	0.55	0.6	3.36	1.56	0.	0.
time (sec)	N/A	0.617	0.316	0.273	2.063	0.482	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	87	373	228	0	0
normalized size	1	1.	0.61	0.66	2.85	1.74	0.	0.
time (sec)	N/A	0.383	0.267	0.272	2.003	0.475	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	101	201	787	910	0	0
normalized size	1	1.	0.7	1.39	5.43	6.28	0.	0.
time (sec)	N/A	0.444	0.43	0.296	2.056	0.561	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	133	306	1913	1018	0	0
normalized size	1	1.	0.92	2.12	13.28	7.07	0.	0.
time (sec)	N/A	0.433	0.766	0.309	2.16	0.682	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	107	343	4575	1062	0	0
normalized size	1	1.	0.7	2.24	29.9	6.94	0.	0.
time (sec)	N/A	0.454	0.837	0.312	2.457	0.681	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	134	405	6218	1177	0	0
normalized size	1	1.	0.67	2.02	31.09	5.88	0.	0.
time (sec)	N/A	0.54	1.282	0.289	2.832	0.69	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	153	467	7937	1277	0	0
normalized size	1	1.	0.62	1.89	32.13	5.17	0.	0.
time (sec)	N/A	0.634	1.849	0.33	3.813	0.83	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	137	155	1018	412	0	0
normalized size	1	1.	0.5	0.56	3.7	1.5	0.	0.
time (sec)	N/A	0.831	0.584	0.34	2.163	0.495	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	116	133	805	348	0	0
normalized size	1	1.	0.51	0.58	3.53	1.53	0.	0.
time (sec)	N/A	0.758	0.457	0.293	2.101	0.487	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	99	111	651	294	0	0
normalized size	1	1.	0.56	0.62	3.66	1.65	0.	0.
time (sec)	N/A	0.458	0.351	0.274	2.068	0.482	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	118	225	475	1040	0	0
normalized size	1	1.	0.61	1.17	2.47	5.42	0.	0.
time (sec)	N/A	0.619	0.635	0.311	2.002	0.572	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	117	368	3495	1146	0	0
normalized size	1	1.	0.59	1.87	17.74	5.82	0.	0.
time (sec)	N/A	0.626	0.662	0.332	2.387	0.695	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	173	376	0	1160	0	0
normalized size	1	1.	0.86	1.88	0.	5.8	0.	0.
time (sec)	N/A	0.627	0.923	0.338	0.	0.693	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	133	407	8501	1204	0	0
normalized size	1	1.	0.66	2.04	42.5	6.02	0.	0.
time (sec)	N/A	0.651	1.3	0.289	21.4	0.695	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	154	469	9897	1331	0	0
normalized size	1	1.	0.62	1.9	40.07	5.39	0.	0.
time (sec)	N/A	0.766	1.902	0.302	4.215	0.833	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	178	531	12477	1455	0	0
normalized size	1	1.	0.61	1.81	42.44	4.95	0.	0.
time (sec)	N/A	0.857	2.821	0.323	6.729	0.848	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	170	217	1011	1081	0	0
normalized size	1	1.	0.68	0.87	4.04	4.32	0.	0.
time (sec)	N/A	0.843	1.138	0.289	2.201	0.539	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	154	195	784	983	0	0
normalized size	1	1.	0.74	0.94	3.79	4.75	0.	0.
time (sec)	N/A	0.632	0.751	0.415	2.147	0.534	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	124	173	645	892	0	0
normalized size	1	1.	0.77	1.07	3.98	5.51	0.	0.
time (sec)	N/A	0.447	0.326	0.335	2.09	0.524	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	140	142	263	813	0	0
normalized size	1	1.	1.18	1.19	2.21	6.83	0.	0.
time (sec)	N/A	0.286	0.298	0.323	1.967	0.516	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	201	944	945	0	0
normalized size	1	1.	0.82	1.44	6.74	6.75	0.	0.
time (sec)	N/A	0.343	0.214	0.311	2.08	0.59	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	114	342	2037	1507	0	0
normalized size	1	1.	0.63	1.89	11.25	8.33	0.	0.
time (sec)	N/A	0.503	0.529	0.319	2.285	0.753	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	137	413	3650	1615	0	0
normalized size	1	1.	0.6	1.8	15.87	7.02	0.	0.
time (sec)	N/A	0.7	1.023	0.354	2.448	0.766	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	178	329	0	1283	0	0
normalized size	1	1.	0.66	1.22	0.	4.75	0.	0.
time (sec)	N/A	0.866	1.292	0.403	0.	0.547	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	155	307	0	1172	0	0
normalized size	1	1.	0.7	1.38	0.	5.26	0.	0.
time (sec)	N/A	0.687	1.232	0.315	0.	0.542	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	198	235	11081	1068	0	0
normalized size	1	1.	1.12	1.34	62.96	6.07	0.	0.
time (sec)	N/A	0.49	1.876	0.385	2.425	0.53	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	86	209	2924	995	0	0
normalized size	1	1.	0.68	1.65	23.02	7.83	0.	0.
time (sec)	N/A	0.317	0.505	0.287	2.182	0.521	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	113	303	0	1577	0	0
normalized size	1	1.	0.61	1.64	0.	8.52	0.	0.
time (sec)	N/A	0.531	0.924	0.298	0.	0.655	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	288	467	9527	1862	0	0
normalized size	1	1.	1.22	1.97	40.2	7.86	0.	0.
time (sec)	N/A	0.744	2.17	0.321	3.479	0.841	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	328	531	0	1989	0	0
normalized size	1	1.	1.14	1.85	0.	6.93	0.	0.
time (sec)	N/A	0.947	3.301	0.291	0.	0.862	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	207	461	0	1565	0	0
normalized size	1	1.	0.65	1.45	0.	4.94	0.	0.
time (sec)	N/A	1.123	2.095	0.325	0.	0.567	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	183	439	0	1457	0	0
normalized size	1	1.	0.68	1.63	0.	5.4	0.	0.
time (sec)	N/A	0.91	1.665	0.325	0.	0.558	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	228	365	0	1338	0	0
normalized size	1	1.	1.02	1.64	0.	6.	0.	0.
time (sec)	N/A	0.7	2.461	0.444	0.	0.546	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	108	339	7997	1257	0	0
normalized size	1	1.	0.48	1.52	35.86	5.64	0.	0.
time (sec)	N/A	0.712	0.988	0.324	4.878	0.53	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	108	340	7231	1243	0	0
normalized size	1	1.	0.61	1.93	41.09	7.06	0.	0.
time (sec)	N/A	0.403	0.808	0.338	4.138	0.528	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	965	540	0	1906	0	0
normalized size	1	1.	4.12	2.31	0.	8.15	0.	0.
time (sec)	N/A	0.714	6.156	0.321	0.	0.688	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	1061	821	0	2234	0	0
normalized size	1	1.	3.71	2.87	0.	7.81	0.	0.
time (sec)	N/A	0.958	6.172	0.333	0.	0.925	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	0	0	0
normalized size	1	1.	0.74	2.95	0.	0.	0.	0.
time (sec)	N/A	0.231	0.893	1.888	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	0	0	0
normalized size	1	1.	0.8	3.44	0.	0.	0.	0.
time (sec)	N/A	0.214	0.441	1.894	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	0	0	0
normalized size	1	1.	0.89	4.35	0.	0.	0.	0.
time (sec)	N/A	0.193	0.259	1.946	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	244	0	0	0	0
normalized size	1	1.	0.9	3.44	0.	0.	0.	0.
time (sec)	N/A	0.193	0.369	2.018	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	107	428	0	0	0	0
normalized size	1	1.	1.04	4.16	0.	0.	0.	0.
time (sec)	N/A	0.212	0.49	4.615	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	134	663	0	0	0	0
normalized size	1	1.	0.96	4.74	0.	0.	0.	0.
time (sec)	N/A	0.231	0.842	5.971	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.368	1.216	1.966	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	0	0	0
normalized size	1	1.	0.76	3.48	0.	0.	0.	0.
time (sec)	N/A	0.332	0.622	2.096	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	404	0	0	0	0
normalized size	1	1.	0.84	3.34	0.	0.	0.	0.
time (sec)	N/A	0.318	0.663	2.04	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	105	677	0	0	0	0
normalized size	1	1.	0.83	5.37	0.	0.	0.	0.
time (sec)	N/A	0.335	1.197	4.912	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	175	750	0	0	0	0
normalized size	1	1.	1.02	4.36	0.	0.	0.	0.
time (sec)	N/A	0.375	1.128	6.644	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	191	859	0	0	0	0
normalized size	1	1.	0.89	4.01	0.	0.	0.	0.
time (sec)	N/A	0.396	4.61	8.288	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	263	1074	0	0	0	0
normalized size	1	1.	1.45	5.9	0.	0.	0.	0.
time (sec)	N/A	0.855	2.543	2.38	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	210	786	0	0	0	0
normalized size	1	1.	1.54	5.78	0.	0.	0.	0.
time (sec)	N/A	0.586	1.208	2.597	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	131	295	0	0	0	0
normalized size	1	1.	1.47	3.31	0.	0.	0.	0.
time (sec)	N/A	0.276	0.964	2.124	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0
normalized size	1	1.	0.95	3.56	0.	0.	0.	0.
time (sec)	N/A	0.219	0.222	1.973	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	208	325	0	0	0	0
normalized size	1	1.	2.42	3.78	0.	0.	0.	0.
time (sec)	N/A	0.393	2.5	4.391	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	263	466	0	0	0	0
normalized size	1	1.	1.75	3.11	0.	0.	0.	0.
time (sec)	N/A	0.836	2.319	5.868	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	328	785	0	0	0	0
normalized size	1	1.	1.51	3.62	0.	0.	0.	0.
time (sec)	N/A	1.188	4.583	7.738	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	320	1059	0	0	0	0
normalized size	1	1.	1.05	3.47	0.	0.	0.	0.
time (sec)	N/A	1.018	3.4	7.568	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	283	843	0	0	0	0
normalized size	1	1.	1.27	3.78	0.	0.	0.	0.
time (sec)	N/A	0.702	2.813	6.337	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	263	802	0	0	0	0
normalized size	1	1.	1.3	3.95	0.	0.	0.	0.
time (sec)	N/A	0.614	2.506	5.03	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	276	715	0	0	0	0
normalized size	1	1.	1.4	3.63	0.	0.	0.	0.
time (sec)	N/A	0.667	2.721	4.72	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	319	877	0	0	0	0
normalized size	1	1.	1.25	3.44	0.	0.	0.	0.
time (sec)	N/A	0.944	4.374	6.725	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	429	1024	0	0	0	0
normalized size	1	1.	1.24	2.96	0.	0.	0.	0.
time (sec)	N/A	1.311	6.907	9.885	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	463	2216	0	0	0	0
normalized size	1	1.	1.	4.81	0.	0.	0.	0.
time (sec)	N/A	1.585	6.011	12.205	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	394	2000	0	0	0	0
normalized size	1	1.	1.07	5.45	0.	0.	0.	0.
time (sec)	N/A	1.108	4.701	10.316	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	361	1959	0	0	0	0
normalized size	1	1.	1.04	5.66	0.	0.	0.	0.
time (sec)	N/A	1.102	4.496	8.999	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	368	1872	0	0	0	0
normalized size	1	1.	1.09	5.54	0.	0.	0.	0.
time (sec)	N/A	1.006	4.719	8.612	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	387	1768	0	0	0	0
normalized size	1	1.	1.13	5.17	0.	0.	0.	0.
time (sec)	N/A	1.096	4.893	8.816	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	463	2024	0	0	0	0
normalized size	1	1.	1.1	4.82	0.	0.	0.	0.
time (sec)	N/A	1.478	5.698	10.895	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	572	2178	0	0	0	0
normalized size	1	1.	1.09	4.16	0.	0.	0.	0.
time (sec)	N/A	1.98	7.301	16.947	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	455	2364	0	0	0	0
normalized size	1	1.	1.33	6.89	0.	0.	0.	0.
time (sec)	N/A	1.219	17.282	0.603	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	353	1701	0	0	0	0
normalized size	1	1.	1.32	6.37	0.	0.	0.	0.
time (sec)	N/A	0.914	14.752	0.487	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	305	1162	0	0	0	0
normalized size	1	1.	1.52	5.78	0.	0.	0.	0.
time (sec)	N/A	0.62	9.013	0.38	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	25347	822	0	0	0	0
normalized size	1	1.	121.86	3.95	0.	0.	0.	0.
time (sec)	N/A	0.683	29.407	0.389	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	52603	789	0	0	0	0
normalized size	1	1.	207.92	3.12	0.	0.	0.	0.
time (sec)	N/A	0.935	32.289	0.421	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	77879	1475	0	0	0	0
normalized size	1	1.	231.78	4.39	0.	0.	0.	0.
time (sec)	N/A	1.262	32.981	0.417	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	540	3068	0	0	0	0
normalized size	1	1.	1.26	7.19	0.	0.	0.	0.
time (sec)	N/A	1.708	18.385	0.702	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	466	2326	0	0	0	0
normalized size	1	1.	1.36	6.8	0.	0.	0.	0.
time (sec)	N/A	1.304	16.942	0.493	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	369	1749	0	0	0	0
normalized size	1	1.	1.39	6.58	0.	0.	0.	0.
time (sec)	N/A	0.97	14.26	0.481	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	45958	1429	0	0	0	0
normalized size	1	1.	166.51	5.18	0.	0.	0.	0.
time (sec)	N/A	1.093	33.895	0.364	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	66581	1410	0	0	0	0
normalized size	1	1.	244.78	5.18	0.	0.	0.	0.
time (sec)	N/A	1.023	32.949	0.439	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	79375	1659	0	0	0	0
normalized size	1	1.	234.14	4.89	0.	0.	0.	0.
time (sec)	N/A	1.418	32.955	0.414	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	104716	2351	0	0	0	0
normalized size	1	1.	248.73	5.58	0.	0.	0.	0.
time (sec)	N/A	1.802	33.385	0.508	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	519	519	626	3816	0	0	0	0
normalized size	1	1.	1.21	7.35	0.	0.	0.	0.
time (sec)	N/A	2.174	19.966	1.01	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	425	425	542	3069	0	0	0	0
normalized size	1	1.	1.28	7.22	0.	0.	0.	0.
time (sec)	N/A	1.716	18.308	0.689	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	340	470	2450	0	0	0	0
normalized size	1	1.	1.38	7.21	0.	0.	0.	0.
time (sec)	N/A	1.322	17.262	0.597	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	49609	2052	0	0	0	0
normalized size	1	1.	145.06	6.	0.	0.	0.	0.
time (sec)	N/A	1.393	34.447	0.43	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	73332	2073	0	0	0	0
normalized size	1	1.	210.12	5.94	0.	0.	0.	0.
time (sec)	N/A	1.426	33.574	0.444	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	97208	2216	0	0	0	0
normalized size	1	1.	270.77	6.17	0.	0.	0.	0.
time (sec)	N/A	1.429	33.906	0.605	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	106199	2441	0	0	0	0
normalized size	1	1.	251.66	5.78	0.	0.	0.	0.
time (sec)	N/A	1.795	33.62	0.528	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	513	513	131553	3175	0	0	0	0
normalized size	1	1.	256.44	6.19	0.	0.	0.	0.
time (sec)	N/A	2.248	34.253	0.724	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	363	1701	0	0	0	0
normalized size	1	1.	1.3	6.08	0.	0.	0.	0.
time (sec)	N/A	0.916	14.789	0.445	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	311	1080	0	0	0	0
normalized size	1	1.	1.47	5.09	0.	0.	0.	0.
time (sec)	N/A	0.632	8.965	0.384	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	260	564	0	0	0	0
normalized size	1	1.	1.73	3.76	0.	0.	0.	0.
time (sec)	N/A	0.435	6.739	0.382	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	16611	257	0	0	0	0
normalized size	1	1.	120.37	1.86	0.	0.	0.	0.
time (sec)	N/A	0.526	28.334	0.361	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	51168	776	0	0	0	0
normalized size	1	1.	199.88	3.03	0.	0.	0.	0.
time (sec)	N/A	0.889	32.261	0.376	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	344	344	77909	1568	0	0	0	0
normalized size	1	1.	226.48	4.56	0.	0.	0.	0.
time (sec)	N/A	1.281	32.654	0.409	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	423	423	533	2084	0	0	0	0
normalized size	1	1.	1.26	4.93	0.	0.	0.	0.
time (sec)	N/A	1.404	18.958	0.435	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	417	1460	0	0	0	0
normalized size	1	1.	1.28	4.48	0.	0.	0.	0.
time (sec)	N/A	1.032	17.014	0.342	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	365	889	0	0	0	0
normalized size	1	1.	1.55	3.78	0.	0.	0.	0.
time (sec)	N/A	0.72	15.146	0.407	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	328	564	0	0	0	0
normalized size	1	1.	1.53	2.62	0.	0.	0.	0.
time (sec)	N/A	0.665	10.363	0.386	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	50122	840	0	0	0	0
normalized size	1	1.	227.83	3.82	0.	0.	0.	0.
time (sec)	N/A	0.788	32.167	0.336	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	95694	1441	0	0	0	0
normalized size	1	1.	257.94	3.88	0.	0.	0.	0.
time (sec)	N/A	1.446	33.481	0.367	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	487	487	140027	2295	0	0	0	0
normalized size	1	1.	287.53	4.71	0.	0.	0.	0.
time (sec)	N/A	1.859	34.871	0.449	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	588	588	4179	5675	0	0	0	0
normalized size	1	1.	7.11	9.65	0.	0.	0.	0.
time (sec)	N/A	2.067	24.638	0.734	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	472	472	626	4480	0	0	0	0
normalized size	1	1.	1.33	9.49	0.	0.	0.	0.
time (sec)	N/A	1.54	20.078	0.548	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	368	368	621	3337	0	0	0	0
normalized size	1	1.	1.69	9.07	0.	0.	0.	0.
time (sec)	N/A	1.105	18.54	0.677	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	346	346	463	2416	0	0	0	0
normalized size	1	1.	1.34	6.98	0.	0.	0.	0.
time (sec)	N/A	1.008	16.369	0.495	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	487	1921	0	0	0	0
normalized size	1	1.	1.48	5.84	0.	0.	0.	0.
time (sec)	N/A	1.043	16.032	0.454	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	97528	3159	0	0	0	0
normalized size	1	1.	244.43	7.92	0.	0.	0.	0.
time (sec)	N/A	1.5	33.595	0.535	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	526	526	184379	5358	0	0	0	0
normalized size	1	1.	350.53	10.19	0.	0.	0.	0.
time (sec)	N/A	1.99	35.973	0.714	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [272] had the largest ratio of [0.44]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	5	1.	23	0.217
2	A	7	5	1.	23	0.217
3	A	6	5	1.	23	0.217
4	A	5	4	1.	23	0.174
5	A	6	5	1.	23	0.217
6	A	7	5	1.	23	0.217
7	A	6	4	1.	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	6	4	1.	29	0.138
9	A	5	3	1.	23	0.13
10	A	6	4	1.	29	0.138
11	A	6	4	1.	31	0.129
12	A	6	4	1.	31	0.129
13	A	6	4	1.	29	0.138
14	A	5	3	1.	23	0.13
15	A	6	4	1.	29	0.138
16	A	6	4	1.	31	0.129
17	A	6	4	1.	31	0.129
18	A	6	4	1.	29	0.138
19	A	5	3	1.	23	0.13
20	A	6	4	1.	29	0.138
21	A	6	4	1.	31	0.129
22	A	6	4	1.	31	0.129
23	A	6	4	1.	29	0.138
24	A	5	3	1.	23	0.13
25	A	6	4	1.	29	0.138
26	A	6	4	1.	31	0.129
27	A	6	4	1.	31	0.129
28	A	6	4	1.	31	0.129
29	A	6	4	1.	31	0.129
30	A	6	4	1.	31	0.129
31	A	6	4	1.	31	0.129
32	A	6	4	1.	31	0.129
33	A	6	4	1.	29	0.138
34	A	6	4	1.	29	0.138
35	A	6	4	1.	27	0.148
36	A	5	3	1.	21	0.143
37	A	6	4	1.	27	0.148
38	A	6	4	1.	29	0.138
39	A	6	4	1.	31	0.129
40	A	6	4	1.	31	0.129
41	A	6	4	1.	31	0.129
42	A	6	4	1.	31	0.129
43	A	7	5	1.	29	0.172
44	A	6	5	1.	29	0.172
45	A	6	6	1.	29	0.207
46	A	5	5	1.	27	0.185
47	A	4	4	1.	21	0.19
48	A	3	2	1.	27	0.074
49	A	4	4	1.	29	0.138
50	A	5	5	1.	29	0.172
51	A	6	5	1.	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
52	A	7	5	1.	29	0.172
53	A	7	6	1.	31	0.194
54	A	7	7	1.	31	0.226
55	A	6	6	1.	29	0.207
56	A	5	5	1.	23	0.217
57	A	4	3	1.	29	0.103
58	A	4	3	1.	31	0.097
59	A	5	5	1.	31	0.161
60	A	6	6	1.	31	0.194
61	A	7	6	1.	31	0.194
62	A	8	6	1.	31	0.194
63	A	11	7	1.	31	0.226
64	A	10	6	1.	29	0.207
65	A	6	5	1.	23	0.217
66	A	5	3	1.	29	0.103
67	A	5	4	1.	31	0.129
68	A	5	3	1.	31	0.097
69	A	8	6	1.	31	0.194
70	A	7	6	1.	31	0.194
71	A	8	6	1.	31	0.194
72	A	14	7	1.	31	0.226
73	A	13	6	1.	29	0.207
74	A	7	5	1.	23	0.217
75	A	6	3	1.	29	0.103
76	A	6	4	1.	31	0.129
77	A	6	4	1.	31	0.129
78	A	6	3	1.	31	0.097
79	A	11	6	1.	31	0.194
80	A	8	6	1.	31	0.194
81	A	9	6	1.	31	0.194
82	A	6	5	1.	31	0.161
83	A	6	6	1.	31	0.194
84	A	5	5	1.	31	0.161
85	A	3	3	1.	29	0.103
86	A	2	2	1.	23	0.087
87	A	4	4	1.	29	0.138
88	A	5	5	1.	31	0.161
89	A	6	5	1.	31	0.161
90	A	7	5	1.	31	0.161
91	A	7	6	1.	31	0.194
92	A	6	6	1.	31	0.194
93	A	4	4	1.	31	0.129
94	A	2	2	1.	29	0.069
95	A	3	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	5	4	1.	29	0.138
97	A	6	5	1.	31	0.161
98	A	7	5	1.	31	0.161
99	A	8	6	1.	31	0.194
100	A	7	6	1.	31	0.194
101	A	5	5	1.	31	0.161
102	A	3	3	1.	31	0.097
103	A	3	3	1.	29	0.103
104	A	4	3	1.	23	0.13
105	A	6	4	1.	29	0.138
106	A	7	5	1.	31	0.161
107	A	8	5	1.	31	0.161
108	A	9	6	1.	31	0.194
109	A	8	6	1.	31	0.194
110	A	6	5	1.	31	0.161
111	A	4	4	1.	31	0.129
112	A	4	4	1.	31	0.129
113	A	4	3	1.	29	0.103
114	A	5	3	1.	23	0.13
115	A	7	4	1.	29	0.138
116	A	8	5	1.	31	0.161
117	A	9	5	1.	31	0.161
118	A	5	5	1.	33	0.152
119	A	4	4	1.	33	0.121
120	A	3	3	1.	33	0.091
121	A	2	2	1.	31	0.065
122	A	4	4	1.	25	0.16
123	A	3	3	1.	31	0.097
124	A	4	4	1.	33	0.121
125	A	5	4	1.	33	0.121
126	A	6	4	1.	33	0.121
127	A	5	5	1.	33	0.152
128	A	4	4	1.	33	0.121
129	A	3	3	1.	31	0.097
130	A	5	5	1.	25	0.2
131	A	4	4	1.	31	0.129
132	A	4	4	1.	33	0.121
133	A	5	5	1.	33	0.152
134	A	6	5	1.	33	0.152
135	A	6	5	1.	33	0.152
136	A	5	4	1.	33	0.121
137	A	4	3	1.	31	0.097
138	A	6	5	1.	25	0.2
139	A	5	4	1.	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
140	A	5	5	1.	33	0.152
141	A	5	4	1.	33	0.121
142	A	6	5	1.	33	0.152
143	A	7	5	1.	33	0.152
144	A	6	5	1.	33	0.152
145	A	5	5	1.	33	0.152
146	A	4	4	1.	33	0.121
147	A	3	3	1.	31	0.097
148	A	5	4	1.	25	0.16
149	A	6	5	1.	31	0.161
150	A	7	5	1.	33	0.152
151	A	8	5	1.	33	0.152
152	A	6	6	1.	33	0.182
153	A	5	5	1.	33	0.152
154	A	4	4	1.	33	0.121
155	A	3	3	1.	31	0.097
156	A	6	5	1.	25	0.2
157	A	7	6	1.	31	0.194
158	A	8	6	1.	33	0.182
159	A	9	6	1.	33	0.182
160	A	6	5	1.	33	0.152
161	A	5	5	1.	33	0.152
162	A	4	4	1.	33	0.121
163	A	4	4	1.	31	0.129
164	A	7	5	1.	25	0.2
165	A	8	6	1.	31	0.194
166	A	9	6	1.	33	0.182
167	A	5	4	1.	26	0.154
168	A	6	5	1.	32	0.156
169	A	7	5	1.	34	0.147
170	A	8	5	1.	34	0.147
171	A	6	5	1.15	26	0.192
172	A	7	6	1.	32	0.188
173	A	8	6	1.	34	0.176
174	A	9	6	1.	34	0.176
175	A	7	6	1.22	26	0.231
176	A	8	6	1.	32	0.188
177	A	9	6	1.	34	0.176
178	A	10	6	1.	34	0.176
179	A	9	6	1.	31	0.194
180	A	8	6	1.	31	0.194
181	A	7	6	1.	31	0.194
182	A	6	5	1.	31	0.161
183	A	6	5	1.	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	7	6	1.	31	0.194
185	A	8	6	1.	31	0.194
186	A	9	7	1.	33	0.212
187	A	8	7	1.	33	0.212
188	A	7	6	1.	33	0.182
189	A	7	6	1.	33	0.182
190	A	7	6	1.	33	0.182
191	A	8	7	1.	33	0.212
192	A	9	7	1.	33	0.212
193	A	10	7	1.	33	0.212
194	A	9	7	1.	33	0.212
195	A	8	6	1.	33	0.182
196	A	8	7	1.	33	0.212
197	A	8	6	1.	33	0.182
198	A	8	6	1.	33	0.182
199	A	9	7	1.	33	0.212
200	A	10	7	1.	33	0.212
201	A	9	6	1.	33	0.182
202	A	8	6	1.	33	0.182
203	A	7	6	1.	33	0.182
204	A	6	5	1.	33	0.152
205	A	6	5	1.	33	0.152
206	A	7	6	1.	33	0.182
207	A	8	6	1.	33	0.182
208	A	9	6	1.	33	0.182
209	A	9	6	1.	33	0.182
210	A	8	6	1.	33	0.182
211	A	7	5	1.	33	0.152
212	A	7	6	1.	33	0.182
213	A	7	5	1.	33	0.152
214	A	8	6	1.	33	0.182
215	A	9	6	1.	33	0.182
216	A	10	6	1.	33	0.182
217	A	9	6	1.	33	0.182
218	A	8	5	1.	33	0.152
219	A	8	6	1.	33	0.182
220	A	8	6	1.	33	0.182
221	A	8	5	1.	33	0.152
222	A	9	6	1.	33	0.182
223	A	10	6	1.	33	0.182
224	A	5	4	1.	35	0.114
225	A	4	4	1.	35	0.114
226	A	3	3	1.	35	0.086
227	A	3	3	1.	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
228	A	2	2	1.	35	0.057
229	A	3	3	1.	35	0.086
230	A	4	3	1.	35	0.086
231	A	6	5	1.	35	0.143
232	A	5	5	1.	35	0.143
233	A	4	4	1.	35	0.114
234	A	4	4	1.	35	0.114
235	A	4	4	1.	35	0.114
236	A	3	3	1.	35	0.086
237	A	4	4	1.	35	0.114
238	A	5	4	1.	35	0.114
239	A	7	5	1.	35	0.143
240	A	6	5	1.	35	0.143
241	A	5	4	1.	35	0.114
242	A	5	4	1.	35	0.114
243	A	5	5	1.	35	0.143
244	A	5	4	1.	35	0.114
245	A	4	3	1.	35	0.086
246	A	5	4	1.	35	0.114
247	A	6	4	1.	35	0.114
248	A	7	6	1.	35	0.171
249	A	6	6	1.	35	0.171
250	A	5	5	1.	35	0.143
251	A	3	3	1.	35	0.086
252	A	4	4	1.	35	0.114
253	A	5	4	1.	35	0.114
254	A	6	4	1.	35	0.114
255	A	8	7	1.	35	0.2
256	A	7	7	1.	35	0.2
257	A	6	6	1.	35	0.171
258	A	3	3	1.	35	0.086
259	A	4	4	1.	35	0.114
260	A	5	5	1.	35	0.143
261	A	6	5	1.	35	0.143
262	A	8	7	1.	35	0.2
263	A	7	6	1.	35	0.171
264	A	4	4	1.	35	0.114
265	A	5	5	1.3	35	0.143
266	A	5	4	1.	35	0.114
267	A	6	5	1.	35	0.143
268	A	7	5	1.	35	0.143
269	A	9	9	1.	25	0.36
270	A	8	8	1.	25	0.32
271	A	9	9	1.	25	0.36

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	11	11	1.	25	0.44
273	A	10	10	1.	25	0.4
274	A	11	11	1.	25	0.44
275	A	7	4	1.	33	0.121
276	A	4	4	1.	35	0.114
277	A	6	5	1.	29	0.172
278	A	6	6	1.	29	0.207
279	A	5	5	1.	27	0.185
280	A	4	4	1.	21	0.19
281	A	3	2	1.	27	0.074
282	A	4	4	1.	29	0.138
283	A	5	5	1.	29	0.172
284	A	6	5	1.	29	0.172
285	A	7	6	1.	31	0.194
286	A	7	7	1.	31	0.226
287	A	6	6	1.	29	0.207
288	A	5	4	1.	23	0.174
289	A	5	4	1.	29	0.138
290	A	5	5	1.	31	0.161
291	A	5	5	1.	31	0.161
292	A	7	6	1.	31	0.194
293	A	7	6	1.	31	0.194
294	A	8	7	1.	31	0.226
295	A	7	6	1.	29	0.207
296	A	6	5	1.	23	0.217
297	A	6	5	1.1	29	0.172
298	A	6	6	1.	31	0.194
299	A	6	6	1.	31	0.194
300	A	6	6	1.	31	0.194
301	A	8	7	1.	31	0.226
302	A	9	7	1.	31	0.226
303	A	8	6	1.	29	0.207
304	A	7	6	1.	23	0.261
305	A	7	6	1.	29	0.207
306	A	7	6	1.	31	0.194
307	A	7	7	1.	31	0.226
308	A	7	7	1.	31	0.226
309	A	7	7	1.	31	0.226
310	A	9	8	1.	31	0.258
311	A	8	8	1.	31	0.258
312	A	7	7	1.	31	0.226
313	A	7	7	1.	31	0.226
314	A	5	5	1.	29	0.172
315	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	5	5	1.	29	0.172
317	A	6	6	1.	31	0.194
318	A	7	6	1.	31	0.194
319	A	8	6	1.	31	0.194
320	A	8	8	1.	31	0.258
321	A	7	7	1.	31	0.226
322	A	6	6	1.	31	0.194
323	A	5	5	1.	29	0.172
324	A	5	5	1.	23	0.217
325	A	6	6	1.	29	0.207
326	A	7	6	1.	31	0.194
327	A	8	6	1.	31	0.194
328	A	9	9	1.	31	0.29
329	A	8	8	1.	31	0.258
330	A	7	7	1.	31	0.226
331	A	6	6	1.	31	0.194
332	A	6	5	1.	29	0.172
333	A	6	6	1.	23	0.261
334	A	7	7	1.	29	0.241
335	A	8	7	1.	31	0.226
336	A	9	9	1.	31	0.29
337	A	8	8	1.	31	0.258
338	A	7	7	1.	31	0.226
339	A	7	6	1.	31	0.194
340	A	7	5	1.	29	0.172
341	A	7	6	1.	23	0.261
342	A	8	7	1.	29	0.241
343	A	9	7	1.	31	0.226
344	A	4	4	1.	28	0.143
345	A	2	2	1.	28	0.071
346	A	5	5	1.	23	0.217
347	A	4	4	1.	21	0.19
348	A	7	7	1.	33	0.212
349	A	6	6	1.	33	0.182
350	A	5	5	1.	33	0.152
351	A	4	4	1.	31	0.129
352	A	5	5	1.	25	0.2
353	A	6	6	1.	31	0.194
354	A	7	7	1.	33	0.212
355	A	8	7	1.	33	0.212
356	A	7	6	1.	33	0.182
357	A	6	5	1.	33	0.152
358	A	5	4	1.	31	0.129
359	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	6	6	1.	31	0.194
361	A	7	7	1.	33	0.212
362	A	8	7	1.	33	0.212
363	A	8	6	1.	33	0.182
364	A	7	5	1.	33	0.152
365	A	6	4	1.	31	0.129
366	A	7	7	1.	25	0.28
367	A	7	7	1.	31	0.226
368	A	7	7	1.	33	0.212
369	A	8	8	1.	33	0.242
370	A	9	8	1.	33	0.242
371	A	5	5	1.	33	0.152
372	A	4	4	1.	33	0.121
373	A	3	3	1.	31	0.097
374	A	3	3	1.	25	0.12
375	A	6	6	1.	31	0.194
376	A	7	7	1.	33	0.212
377	A	8	7	1.	33	0.212
378	A	5	5	1.	33	0.152
379	A	4	4	1.	33	0.121
380	A	4	4	1.	31	0.129
381	A	6	6	1.	25	0.24
382	A	7	7	1.	31	0.226
383	A	8	8	1.	33	0.242
384	A	9	8	1.	33	0.242
385	A	6	6	1.	33	0.182
386	A	5	5	1.	33	0.152
387	A	5	5	1.	33	0.152
388	A	5	4	1.	31	0.129
389	A	7	7	1.	25	0.28
390	A	8	8	1.	31	0.258
391	A	9	8	1.	33	0.242
392	A	1	1	1.	31	0.032
393	A	1	1	1.	32	0.031
394	A	8	6	1.	31	0.194
395	A	7	6	1.	31	0.194
396	A	6	5	1.	31	0.161
397	A	6	5	1.	31	0.161
398	A	7	6	1.	31	0.194
399	A	8	6	1.	31	0.194
400	A	9	7	1.	33	0.212
401	A	8	7	1.	33	0.212
402	A	7	6	1.	33	0.182
403	A	7	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	A	7	6	1.	33	0.182
405	A	8	7	1.	33	0.212
406	A	9	7	1.	33	0.212
407	A	10	8	1.	33	0.242
408	A	9	8	1.	33	0.242
409	A	8	7	1.	33	0.212
410	A	8	7	1.	33	0.212
411	A	8	7	1.	33	0.212
412	A	8	7	1.	33	0.212
413	A	9	8	1.	33	0.242
414	A	10	8	1.	33	0.242
415	A	11	9	1.	33	0.273
416	A	10	9	1.	33	0.273
417	A	7	7	1.	33	0.212
418	A	5	5	1.	33	0.152
419	A	7	7	1.	33	0.212
420	A	9	8	1.	33	0.242
421	A	10	9	1.	33	0.273
422	A	11	9	1.	33	0.273
423	A	10	9	1.	33	0.273
424	A	9	8	1.	33	0.242
425	A	9	8	1.	33	0.242
426	A	9	8	1.	33	0.242
427	A	10	9	1.	33	0.273
428	A	12	10	1.	33	0.303
429	A	11	10	1.	33	0.303
430	A	10	9	1.	33	0.273
431	A	10	9	1.	33	0.273
432	A	10	9	1.	33	0.273
433	A	10	9	1.	33	0.273
434	A	11	10	1.	33	0.303
435	A	13	13	1.	35	0.371
436	A	12	12	1.	35	0.343
437	A	11	11	1.	35	0.314
438	A	8	8	1.	35	0.229
439	A	9	9	1.	35	0.257
440	A	10	9	1.	35	0.257
441	A	14	13	1.	35	0.371
442	A	13	13	1.	35	0.371
443	A	12	12	1.	35	0.343
444	A	12	12	1.	35	0.343
445	A	9	9	1.	35	0.257
446	A	10	9	1.	35	0.257
447	A	11	9	1.	35	0.257

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	15	14	1.	35	0.4
449	A	14	14	1.	35	0.4
450	A	13	13	1.	35	0.371
451	A	13	13	1.	35	0.371
452	A	13	13	1.	35	0.371
453	A	10	10	1.	35	0.286
454	A	11	10	1.	35	0.286
455	A	12	10	1.	35	0.286
456	A	13	13	1.	35	0.371
457	A	12	12	1.	35	0.343
458	A	7	7	1.	35	0.2
459	A	7	7	1.	35	0.2
460	A	8	8	1.	35	0.229
461	A	9	9	1.	35	0.257
462	A	13	13	1.	35	0.371
463	A	9	9	1.	35	0.257
464	A	8	8	1.	35	0.229
465	A	8	8	1.	35	0.229
466	A	9	9	1.	35	0.257
467	A	10	9	1.	35	0.257
468	A	13	13	1.	35	0.371
469	A	9	9	1.	35	0.257
470	A	9	9	1.	35	0.257
471	A	9	9	1.	35	0.257
472	A	10	10	1.	35	0.286
473	A	11	10	1.	35	0.286
474	A	0	0	0.	0	0.
475	A	0	0	0.	0	0.
476	A	0	0	0.	0	0.
477	A	0	0	0.	0	0.
478	A	0	0	0.	0	0.
479	A	9	7	1.	31	0.226
480	A	8	6	1.	31	0.194
481	A	7	5	1.	31	0.161
482	A	6	4	1.	29	0.138
483	A	8	7	1.	31	0.226
484	A	7	7	1.	31	0.226
485	A	6	6	1.	31	0.194
486	A	6	6	1.	31	0.194
487	A	7	7	1.	31	0.226
488	A	8	7	1.	31	0.226
489	A	9	8	1.	33	0.242
490	A	8	8	1.	33	0.242
491	A	7	7	1.	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
492	A	7	7	1.	33	0.212
493	A	7	7	1.	33	0.212
494	A	8	8	1.	33	0.242
495	A	9	8	1.	33	0.242
496	A	7	6	1.	33	0.182
497	A	6	6	1.	33	0.182
498	A	5	5	1.	33	0.152
499	A	5	5	1.	33	0.152
500	A	6	6	1.	33	0.182
501	A	7	6	1.	33	0.182
502	A	8	6	1.	33	0.182
503	A	7	6	1.	33	0.182
504	A	6	5	1.	33	0.152
505	A	6	6	1.	33	0.182
506	A	6	5	1.	33	0.152
507	A	7	6	1.	33	0.182
508	A	8	6	1.	33	0.182
509	A	8	6	1.	33	0.182
510	A	7	5	1.	33	0.152
511	A	7	6	1.	33	0.182
512	A	7	6	1.	33	0.182
513	A	7	5	1.	33	0.152
514	A	8	6	1.	33	0.182
515	A	6	4	1.	35	0.114
516	A	5	4	1.	35	0.114
517	A	4	4	1.	35	0.114
518	A	3	3	1.	35	0.086
519	A	4	4	1.	35	0.114
520	A	4	4	1.	35	0.114
521	A	5	5	1.	35	0.143
522	A	6	5	1.	35	0.143
523	A	7	5	1.	35	0.143
524	A	6	5	1.	35	0.143
525	A	5	5	1.	35	0.143
526	A	4	4	1.	35	0.114
527	A	5	5	1.	35	0.143
528	A	5	5	1.	35	0.143
529	A	5	5	1.	35	0.143
530	A	6	6	1.	35	0.171
531	A	7	6	1.	35	0.171
532	A	7	5	1.	35	0.143
533	A	6	5	1.	35	0.143
534	A	5	4	1.	35	0.114
535	A	6	5	1.	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
536	A	6	6	1.	35	0.171
537	A	6	5	1.	35	0.143
538	A	6	5	1.	35	0.143
539	A	7	6	1.	35	0.171
540	A	8	6	1.	35	0.171
541	A	7	5	1.	35	0.143
542	A	6	5	1.	35	0.143
543	A	5	5	1.	35	0.143
544	A	4	4	1.	35	0.114
545	A	6	6	1.	35	0.171
546	A	7	7	1.	35	0.2
547	A	8	7	1.	35	0.2
548	A	7	6	1.	35	0.171
549	A	6	6	1.	35	0.171
550	A	5	5	1.	35	0.143
551	A	4	4	1.	35	0.114
552	A	7	7	1.	35	0.2
553	A	8	8	1.	35	0.229
554	A	9	8	1.	35	0.229
555	A	8	6	1.	35	0.171
556	A	7	6	1.	35	0.171
557	A	6	5	1.	35	0.143
558	A	6	6	1.	35	0.171
559	A	5	5	1.	35	0.143
560	A	8	7	1.	35	0.2
561	A	9	8	1.	35	0.229
562	A	8	7	1.	31	0.226
563	A	7	7	1.	31	0.226
564	A	6	6	1.	31	0.194
565	A	6	6	1.	31	0.194
566	A	7	7	1.	31	0.226
567	A	8	7	1.	31	0.226
568	A	7	7	1.	33	0.212
569	A	6	6	1.	33	0.182
570	A	6	6	1.	33	0.182
571	A	6	6	1.	33	0.182
572	A	7	7	1.	33	0.212
573	A	8	7	1.	33	0.212
574	A	8	8	1.	33	0.242
575	A	7	7	1.	33	0.212
576	A	6	6	1.	33	0.182
577	A	4	4	1.	33	0.121
578	A	6	6	1.	33	0.182
579	A	8	8	1.	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
580	A	9	8	1.	33	0.242
581	A	8	8	1.	33	0.242
582	A	7	7	1.	33	0.212
583	A	7	7	1.	33	0.212
584	A	7	7	1.	33	0.212
585	A	8	8	1.	33	0.242
586	A	9	8	1.	33	0.242
587	A	9	9	1.	33	0.273
588	A	8	8	1.	33	0.242
589	A	8	8	1.	33	0.242
590	A	8	8	1.	33	0.242
591	A	8	8	1.	33	0.242
592	A	9	8	1.	33	0.242
593	A	10	8	1.	33	0.242
594	A	11	10	1.	35	0.286
595	A	10	10	1.	35	0.286
596	A	9	9	1.	35	0.257
597	A	12	12	1.	35	0.343
598	A	13	13	1.	35	0.371
599	A	14	14	1.	35	0.4
600	A	12	10	1.	35	0.286
601	A	11	10	1.	35	0.286
602	A	10	10	1.	35	0.286
603	A	13	13	1.	35	0.371
604	A	13	13	1.	35	0.371
605	A	14	14	1.	35	0.4
606	A	15	14	1.	35	0.4
607	A	13	11	1.	35	0.314
608	A	12	11	1.	35	0.314
609	A	11	11	1.	35	0.314
610	A	14	14	1.	35	0.4
611	A	14	14	1.	35	0.4
612	A	14	14	1.	35	0.4
613	A	15	15	1.	35	0.429
614	A	16	15	1.	35	0.429
615	A	10	10	1.	35	0.286
616	A	9	9	1.	35	0.257
617	A	8	8	1.	35	0.229
618	A	8	8	1.	35	0.229
619	A	13	13	1.	35	0.371
620	A	14	14	1.	35	0.4
621	A	11	10	1.	35	0.286
622	A	10	10	1.	35	0.286
623	A	9	9	1.	35	0.257

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	9	9	1.	35	0.257
625	A	10	10	1.	35	0.286
626	A	14	14	1.	35	0.4
627	A	15	14	1.	35	0.4
628	A	12	11	1.	35	0.314
629	A	11	11	1.	35	0.314
630	A	10	10	1.	35	0.286
631	A	10	10	1.	35	0.286
632	A	10	10	1.	35	0.286
633	A	14	14	1.	35	0.4
634	A	15	15	1.	35	0.429

Chapter 3

Listing of integrals

3.1 $\int (b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{2Ab^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2Ab\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{6b^2B\sin(c+dx)\sqrt{b\,\text{s}}}{5d}$$

```
[Out] (-6*b^3*B*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
 d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(3*d) + (6*b^2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A
*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*B*(b*Sec[c + d*x])^(5/2)
*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.118789, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3787, 3768, 3771, 2641, 2639}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2Ab\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{6b^2B\sin(c+dx)\sqrt{b\sec(c+}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-6*b^3*B*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
 d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(3*d) + (6*b^2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A
*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*B*(b*Sec[c + d*x])^(5/2)
*Sin[c + d*x])/(5*d)
```


Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{5/2}\,dx + \frac{B\int (b\sec(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2Ab(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2B(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{3}\Big(A$$
$$= \frac{6b^2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2Ab(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2B(b$$
$$= \frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{6b^2B\sqrt{b\sec(c+dx)}\sin}{5d}$$
$$= -\frac{6b^3BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec}}{3d}$$

Mathematica [A] time = 0.475985, size = 99, normalized size = 0.58

$$\frac{(b\sec(c+dx))^{5/2}\left(20A\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10A\sin(2(c+dx))+21B\sin(c+dx)+9B\sin(3(c+dx))\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(-36*B*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
 + 20*A*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*B*Sin[c + d*x] +
10*A*Sin[2*(c + d*x)] + 9*B*Sin[3*(c + d*x)]))/(30*d)
```

Maple [C] time = 0.275, size = 518, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/15/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(5*I*A*sin(d*x+c)*cos(d*x+c)^3*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+co
s(d*x+c))/sin(d*x+c),I)+9*I*B*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),I)-9*I*B*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*A*sin(d*x+
c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*I*B*sin(d*x+c)*cos(d*x+c)^2*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos
(d*x+c))/sin(d*x+c),I)-9*I*B*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)-5*A*cos(d*x+c)^3-9*B*cos(d*x+c)^3+6*B*cos(d*x+c)^2+5*A*cos(d*x+c)+3*B)*
(b/cos(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2\sec(dx+c)^3+Ab^2\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*b^2*sec(d*x + c)^2)*sqrt(b*sec(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(5/2), x)
```

3.2 $\int (b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{2bB\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\sec(c}}{d}$$

```
[Out] (-2*A*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(3*d) + (2*A*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*B*(b*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.10093, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3787, 3768, 3771, 2639, 2641}

$$-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{2bB\sqrt{\cos(c+c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*A*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(3*d) + (2*A*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*B*(b*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{3/2}\,dx + \frac{B\int (b\sec(c+dx))^{5/2}\,dx}{b}$$
$$= \frac{2Ab\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} - \left(Ab^2\right)$$
$$= \frac{2Ab\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} - \frac{\left(Ab^2\right)}{\sqrt{\cos(c}}$$
$$= -\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(}}{3d}$$

Mathematica [A] time = 0.274528, size = 87, normalized size = 0.64

$$\frac{(b\sec(c+dx))^{3/2}\left(2B\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)(3A\cos(c+dx)+B)-6A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(-6*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ 2*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*A*Cos[c + d*x
])*Sin[c + d*x]))/(3*d)
```

Maple [C] time = 0.252, size = 500, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*A*sin(d*x+c)*cos(d*x+c)^2*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+co
s(d*x+c))/sin(d*x+c),I)-3*I*A*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),I)-I*B*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*A*cos(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1
+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*A*cos(d*x+c)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+
c),I)*sin(d*x+c)-I*B*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*A*co
s(d*x+c)^2+B*cos(d*x+c)^2-3*A*cos(d*x+c)-B)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)
^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^2+Ab\sec(dx+c)\right)\sqrt{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*sqrt(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^{\frac{3}{2}}\,(A+B\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*(A + B*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x)
```

3.3 $\int \sqrt{b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=104

$$\frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*b*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]
])/d + (2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0822444, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3787, 3771, 2641, 3768, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*b*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]
])/d + (2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\sec(c+dx)}(A+B\sec(c+dx))\,dx = A\int \sqrt{b\sec(c+dx)}\,dx + \frac{B\int (b\sec(c+dx))^{3/2}\,dx}{b}$$
$$= \frac{2B\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} - (bB)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx + \left(A\sqrt{\cos(c+d\right.$$
$$= \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d} + \frac{2B\sqrt{b\sec(c+dx)}\sin(c}{d}$$
$$= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(}{d}$$

Mathematica [A] time = 0.119837, size = 73, normalized size = 0.7

$$\frac{2\sqrt{b\sec(c+dx)}\left(A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+B\sin(c+dx)-B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(-(B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
+ A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x]))/d
```

Maple [C] time = 0.287, size = 453, normalized size = 4.4

$$2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{d(\sin(dx+c))^5}\sqrt{\frac{b}{\cos(dx+c)}}\left(iA\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sqrt{(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/d*(b/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*A*EllipticF(
I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*B*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)*sin(d*x+c)+I*B*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+I*A*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(
d*x+c))/sin(d*x+c),I)*sin(d*x+c)-I*B*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I
*B*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-B*cos(d*x+c)+B)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)\sqrt{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(c+dx)}\,(A+B\sec(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*(A + B*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

3.4 $\int \frac{A+B\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
 + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(b*d)
```

Rubi [A] time = 0.0678043, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3787, 3771, 2639, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
 + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(b*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = A\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx + \frac{B\int \sqrt{b\sec(c+dx)}\,dx}{b}$$
$$= \frac{A\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Mathematica [A] time = 0.088494, size = 54, normalized size = 0.66

$$\frac{2\left(B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+AE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(A*EllipticE[(c + d*x)/2, 2] + B*EllipticF[(c + d*x)/2, 2]))/(d*Sqrt[Cos
[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.238, size = 445, normalized size = 5.4

$$2\frac{1}{d\sin(dx+c)\,b}\left(iA\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*A*EllipticE(I*(-
1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*B*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c
),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-I*A*EllipticE(I*(-1+cos
(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+I*B*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-A*cos(d*x+c)^2+A
*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)}}{b\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c))/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)
```

3.5 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2A\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]
)/(3*b^2*d) + (2*A*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0913051, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]
)/(3*b^2*d) + (2*A*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= A\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx + \frac{B\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b} \\
&= \frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} + \frac{A\int \sqrt{b\sec(c+dx)}\,dx}{3b^2} + \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int}{3b^2} \\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2A}{3bd\sqrt{}}
\end{aligned}$$

Mathematica [A] time = 0.181428, size = 86, normalized size = 0.74

$$\frac{\sec^2(c+dx)\left(A\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)+6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*(2*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d*(b*Se
c[c + d*x])^(3/2))
```

Maple [C] time = 0.216, size = 470, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/3/d*(I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*I*B*EllipticF(
I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*B*EllipticE(I*(-1+cos(d*x+c))/sin
(d*x+c),I)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*I*B*EllipticF(I
*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-3*I*B*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*cos(
d*x+c)^3-3*B*cos(d*x+c)^2+A*cos(d*x+c)+3*B*cos(d*x+c))/sin(d*x+c)/cos(d*x+c
)^2/(b/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)}}{b^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c))/(b^2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x)
```

3.6 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2B\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+$$

```
[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d
*x]])/(3*b^3*d) + (2*A*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)) + (2*B*
Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.10415, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3787, 3769, 3771, 2639, 2641}

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d
*x]])/(3*b^3*d) + (2*A*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)) + (2*B*
Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= A\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx+\frac{B\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b}\\ &= \frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{(3A)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2}+\frac{B\int \sqrt{b\sec(c+dx)}\,dx}{3b^3}\\ &= \frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{(3A)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{\left(B\sqrt{\cos(c+\right.}{}\\ &= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2A}{5bd(b}\end{aligned}$$

Mathematica [A] time = 0.508454, size = 88, normalized size = 0.6

$$\frac{2\left(5B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3A\cos(c+dx)+5B)+9AE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*A*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*B + 3*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d*Cos[c + d*x]^(5/2
)*(b*Sec[c + d*x])^(5/2))
```

Maple [C] time = 0.198, size = 482, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/15/d*(9*I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*I*A*sin(d*x
+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*B*EllipticF(I*(-1+cos(d*x+c))/
sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+9*I*A*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-9*I*A*sin(d*
x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I
*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*B*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3
*A*cos(d*x+c)^4-5*B*cos(d*x+c)^3-6*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*
x+c))/sin(d*x+c)/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)}}{b^3\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x)
```

3.7 $\int \sec^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{8/3}\text{Hyperg}}{8b^2d\sqrt{\text{si}}}$$

```
[Out] (3*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/
3)*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-4/3
, 1/2, -1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(8/3)*Sin[c + d*x])/(8*b^2*d*
Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0982565, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{8/3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+d\right.}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/
3)*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-4/3
, 1/2, -1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(8/3)*Sin[c + d*x])/(8*b^2*d*
Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{8/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{8/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{11/3}\,dx}{b^3}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{8/3}}\,dx}{b^2} + \frac{\left(B\left(\right.\right.}{}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{5/3}\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.256236, size = 90, normalized size = 0.76

$$-\frac{3\left(-\tan^2(c+dx)\right)^{3/2}\csc^3(c+dx)(b\sec(c+dx))^{2/3}\left(11A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sec^2(c+dx)\right)+\right.}{88d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(11*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sec[
c + d*x]^2] + 8*B*Hypergeometric2F1[1/2, 11/6, 17/6, Sec[c + d*x]^2])*(b*Se
c[c + d*x])^(2/3)*(-Tan[c + d*x]^2)^(3/2))/(88*d)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\sec(dx+c))^{\frac{2}{3}} (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

3.8 $\int \sec(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{5/3}\text{Hyp}}{5bd\sqrt{}}$$

```
[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/
3)*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-5/6,
1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x])/(5*b*d*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0984483, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+\right.}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/
3)*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-5/6,
1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x])/(5*b*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{5/3}(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\sec(c+dx))^{5/3}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{8/3}\,dx}{b^2}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{5/3}}\,dx}{b} + \frac{\left(B\left(\frac{\cos(c+}{b}\right.\right.}{}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}} + \frac{3}{}$$

Mathematica [A] time = 0.117802, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{5/3}\left(8A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(c+dx)\right)+5B\text{Hype}\right.}{40bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(8*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c +
 d*x]^2] + 5*B*Hypergeometric2F1[1/2, 4/3, 7/3, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(5/3)*Sqrt[-Tan[c + d*x]^2])/(40*b*d)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \sec(dx+c)(b\sec(dx+c))^{\frac{2}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^2+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c), x)
```

3.9 $\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=112

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}-\frac{3Ab\sin(c+dx)\text{Hypergeometric2F1}\left(\right.}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c}$$

```
[Out] (-3*A*b*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*d*Sqrt[Si
n[c + d*x]^2])
```

Rubi [A] time = 0.0862797, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3787, 3772, 2643}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}-\frac{3Ab\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{2/3}\,dx + \frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b}$$

$$= \left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right.\right.}{}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}} - \frac{3A\cos(c}{}$$

Mathematica [A] time = 0.0870232, size = 88, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{2/3}\left(5A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)+2B\text{Hy}\right.}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c +
d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*d)
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{2}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (c+dx))^{\frac{2}{3}}\,(A+B\sec (c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)\,(b\sec (dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

3.10 $\int \cos(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{3Ab^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3bB\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos\right.}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*A*b^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4
*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Hypergeometric2F1[
1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.104279, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3bB\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4
*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Hypergeometric2F1[
1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = b\int \frac{A+B\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$
$$= (Ab)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx + B\int (b\sec(c+dx))^{2/3}\,dx$$
$$= \left(Ab\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx + ($$
$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin}{d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.092931, size = 88, normalized size = 0.77

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{2/3}\left(2A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)-B\text{Hyp}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*Cot[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c
+ d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*(b*Sec[c +
d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*d)
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b\sec(dx+c))^{\frac{2}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\,(b\sec(dx+c))^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*(b*sec(d*x + c))^(2
/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c), x)
```

3.11 $\int \cos^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3Ab^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}-\frac{3b^2B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(\right.}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*A*b^3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
7*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3)
*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.121113, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3Ab^3\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}-\frac{3b^2B\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```



```
[Out] (-3*A*b^3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
7*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3)
*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = b^2\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx + (bB)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx$$
$$= \left(Ab^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int\left(\frac{\cos(c+dx)}{b}\right)^{4/}$$
$$= -\frac{3B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/}}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.137663, size = 88, normalized size = 0.74

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)+4B\text{Hypergeometri}\right.}{4d\sqrt[3]{b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*b*Cot[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c
+ d*x]^2] + 4*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Ta
n[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^{\frac{2}{3}} (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*(b*sec(d*x + c)
)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

3.12 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{10/3}\text{H}}{10b^2}$$

```
[Out] (3*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7
/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-5/
3, 1/2, -2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(10/3)*Sin[c + d*x])/(10*b^2
*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.101489, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{10/3}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(\right.}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7
/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-5/
3, 1/2, -2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(10/3)*Sin[c + d*x])/(10*b^2
*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{10/3}(A+B\sec(c+dx))\,dx}{b^2}$$

$$= \frac{A\int (b\sec(c+dx))^{10/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{13/3}\,dx}{b^3}$$

$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10/3}}dx}{b^2} + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\right.}{}$$

$$= \frac{3Ab\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,\text{ta}}{7d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.302891, size = 90, normalized size = 0.76

$$-\frac{3\left(-\tan^2(c+dx)\right)^{3/2}\csc^3(c+dx)(b\sec(c+dx))^{4/3}\left(13A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\sec^2(c+dx)\right)+10B\right.}{130d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(13*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[
c + d*x]^2] + 10*B*Hypergeometric2F1[1/2, 13/6, 19/6, Sec[c + d*x]^2])*(b*S
ec[c + d*x])^(4/3)*(-Tan[c + d*x]^2)^(3/2))/(130*d)
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\sec(dx+c))^{\frac{4}{3}} (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^4+Ab\sec(dx+c)^3\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^4 + A*b*sec(d*x + c)^3)*(b*sec(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.13 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{7/3}\text{Hyperg}}{7bd\sqrt{\sin}}$$

```
[Out] (3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/
3)*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-7/6,
1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d*x])/(7*b*d*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0985424, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+d\right.}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/
3)*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-7/6,
1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d*x])/(7*b*d*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{7/3}(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\sec(c+dx))^{7/3}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{10/3}\,dx}{b^2}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}\,dx}{b} + \frac{\left(B\sqrt[3]{\frac{\cos(c+d}{b}}\right.}{}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.148966, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{7/3}\left(10A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sec^2(c+dx)\right)+7BH\right.}{70bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(10*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c
+ d*x]^2] + 7*B*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(70*b*d)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \sec(dx+c)(b\sec(dx+c))^{\frac{4}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec\left(dx+c\right)^{3}+Ab\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*sec(d*x + c)^3 + A*b*sec(d*x + c)^2)*(b*sec(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.14 $\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=112

$$\frac{3Ab\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hype}}{4d\sqrt{\text{si}}}$$

```
[Out] (3*A*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(
1/3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*d*Sqrt[Si
n[c + d*x]^2])
```

Rubi [A] time = 0.0880377, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3787, 3772, 2643}

$$\frac{3Ab\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+d\right.}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(
1/3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{4/3}\,dx + \frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b}$$

$$= \left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec\right.}{}$$

$$= \frac{3Ab\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2}\right.}{}$$

Mathematica [A] time = 0.111403, size = 88, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{4/3}\left(7A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)+4B\text{Hyper}\right.}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c +
d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(28*d)
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^2+Ab\sec(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x)
```

3.15 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=115

$$\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3Ab^2\sin(c+dx)\text{Hypergeometric2F1}\left(\right.}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+}$$

```
[Out] (-3*A*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[
-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.104678, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[
-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = b\int \sqrt[3]{b\sec(c+dx)}(A+B\sec(c+dx))\,dx$$
$$= (Ab)\int \sqrt[3]{b\sec(c+dx)}\,dx + B\int (b\sec(c+dx))^{4/3}\,dx$$
$$= \left(Ab\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx + \left(B\sqrt[3]{\frac{c}{}}\right.$$
$$= \frac{3bB\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} -$$

Mathematica [A] time = 0.111987, size = 87, normalized size = 0.76

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{4/3}\left(4A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)+B\text{Hyp}\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Cot[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c +
d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*
x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,(A+B\sec(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)\sec(dx+c)^2+Ab\cos(dx+c)\sec(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)*sec(d*x + c)^2 + A*b*cos(d*x + c)*sec(d*x + c))*
(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.16 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3Ab^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3b^2B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\text{cc}\right.}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*A*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hypergeometric2
F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/3)
*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.123223, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3Ab^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hypergeometric2
F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/3)
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = b^2\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx + (bB)\int \sqrt[3]{b\sec(c+dx)}\,dx$$
$$= \left(Ab^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}\,dx + \Big(b$$
$$= -\frac{3bB\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin}{2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0955387, size = 88, normalized size = 0.74

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)-2B\text{Hype}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*b*Cot[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c
+ d*x]^2] - 2*B*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(2*d)
```

Maple [F] time = 0.412, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2\,(b\sec(dx+c))^{\frac{4}{3}}\,(A+B\sec(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*b*cos(d*x + c)^2*sec(d*x +
c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.17 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hyperge}}{4b^2d\sqrt{\sin^2(}}$$

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0969972, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```


```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqr
t[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{4/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b^3}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx}{b^2} + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx}\right.}{b^3}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\right.}{}$$

Mathematica [A] time = 0.25324, size = 90, normalized size = 0.77

$$-\frac{3\left(-\tan^2(c+dx)\right)^{3/2}\csc^3(c+dx)\left(7A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)+4B\text{Hypergeomet}\right.}{28d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c
 + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(-Tan[c
 + d*x]^2)^(3/2))/(28*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (A+B\sec(dx+c)) (b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

3.18 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\text{Hypergeometric2F1}\left(\right.}{2d\sqrt{\sin^2(c+dx)}(b\sec(c}$$

```
[Out] (-3*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Si
n[c + d*x]^2])
```

Rubi [A] time = 0.0931026, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\sec(c+dx)}(A+B\sec(c+dx))\,dx}{b}$$

$$= \frac{A\int \sqrt[3]{b\sec(c+dx)}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{4/3}\,dx}{b^2}$$

$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx}{b} + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int -}{b^2}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - \frac{3A\cos(c+dx)\,{}_2F_1\left(\right.}{}$$

Mathematica [A] time = 0.114028, size = 90, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sqrt[3]{b\sec(c+dx)}\left(4A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)+B\text{Hypergeo}\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c +
d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*
x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \sec(dx+c)(A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec(c+dx)}{(b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

3.19 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3Ab\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*A*b*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/3
, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0865656, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3787, 3772, 2643}

$$-\frac{3Ab\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*b*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/3
, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = A\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx + \frac{B\int \sqrt[3]{b\sec(c+dx)}\,dx}{b}$$

$$= \left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}dx + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int}{b}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}} - \frac{3A\cos^2(c+dx)\,{}_2}{}$$

Mathematica [A] time = 0.092692, size = 87, normalized size = 0.76

$$-\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\left(A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)-2B\text{Hypergeometric2}\right.}{2d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c +
d*x]^2] - 2*B*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*Sqrt[-Tan[c
 + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int (A+B\sec(dx+c))\,(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}}{b\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x)
```

3.20 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\text{Hypergeometric2F1}\left(\right.}{2d\sqrt{\sin^2(c+dx)}(b\sec(c\right.}$$

```
[Out] (-3*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Si
n[c + d*x]^2])
```

Rubi [A] time = 0.0933538, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\sec(c+dx)}(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int \sqrt[3]{b\sec(c+dx)}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{4/3}\,dx}{b^2}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx}{b} + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int -}{b^2}$$
$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - \frac{3A\cos(c+dx)\,{}_2F_1\Big(}{}$$

Mathematica [A] time = 0.0255202, size = 90, normalized size = 0.79

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sqrt[3]{b\sec(c+dx)}\left(4A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)+B\text{Hyperge}\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c +
d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*
x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \sec(dx+c)(A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec(c+dx)}{(b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

3.21 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hyperge}}{4b^2d\sqrt{\sin^2(}}$$

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0977646, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```


```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqr
t[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{4/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b^3}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx}{b^2} + \frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right.}{b^3}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\right.}{}$$

Mathematica [A] time = 0.0229351, size = 90, normalized size = 0.77

$$-\frac{3\left(-\tan^2(c+dx)\right)^{3/2}\csc^3(c+dx)\left(7A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)+4B\text{Hypergeomet}\right.}{28d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c
 + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(-Tan[c
 + d*x]^2)^(3/2))/(28*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (A+B\sec(dx+c)) (b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

3.22 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\text{Hypergeometric2F}}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\,\text{se}}}$$

```
[Out] (-3*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.096767, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```


```
[Out] (-3*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*b^2*d*Sqr
t[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{2/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b^3}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b^2} + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c\right.}{}$$
$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A\cos(c+dx)\,{}_2}{}$$

Mathematica [A] time = 0.23562, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{2/3}\left(5A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)+2B\text{Hyper}\right.}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c +
d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*b^2*d)
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{2}{3}}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

3.23 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3A\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+d\right.}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/6, 1
/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0931761, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/6, 1
/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{A\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx}{b} + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c\right.}{}$$
$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A}{}$$

Mathematica [A] time = 0.098687, size = 91, normalized size = 0.8

$$-\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\left(2A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)-B\text{Hypergeometric2}\right.}{2bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c
+ d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*Sqrt[-Tan[c
 + d*x]^2])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \sec(dx+c)(A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}}{b^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

3.24 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3Ab\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*A*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2
, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0890296, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3787, 3772, 2643}

$$-\frac{3Ab\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2
, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3)*Sqrt[
Sin[c + d*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = A\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx + \frac{B\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$= \left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int\left(\frac{\cos(c+dx)}{b}\right)^{4/3}dx + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx\right.}{b}$$

$$= -\frac{3B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A\cos^3(c+dx)\,{}_2}{}$$

Mathematica [A] time = 0.144719, size = 87, normalized size = 0.76

$$-\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\left(A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)+4B\text{Hypergeometric2F1}\right.}{4d(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c +
d*x]^2] + 4*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Tan[
c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int (A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}}{b^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec{(c + dx)}}{(b\sec{(c + dx)})^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec{(dx + c)} + A}{(b\sec{(dx + c)})^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x)
```

3.25 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3A\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+d\right.}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/6, 1
/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0931583, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/6, 1
/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$= \frac{A\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$

$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx}{b} + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c\right.}{}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A}{}$$

Mathematica [A] time = 0.076464, size = 91, normalized size = 0.8

$$-\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\left(2A\cos(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right) - B\text{Hypergeometric2}\right.}{2bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c
+ d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*Sqrt[-Tan[c
 + d*x]^2])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int \sec(dx+c)(A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}}{b^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

3.26 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\text{Hypergeometric2F}}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\,\text{se}}}$$

```
[Out] (-3*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0971938, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3787, 3772, 2643}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{2/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b^3}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b^2} + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c\right.}{}$$
$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A\cos(c+dx)\,{}_2}{}$$

Mathematica [A] time = 0.172233, size = 91, normalized size = 0.78

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{2/3}\left(5A\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)+2B\text{Hyper}\right.}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c +
d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*b^2*d)
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (A+B\sec(dx+c)) (b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{2}{3}}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

3.27 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=167

$$\frac{3Ab\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-1),\frac{1}{6}(5-3m),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}+\frac{3bB\sin(c\ldots}{}$$

```
[Out] (3*A*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Se
c[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c +
 d*x]^2]) + (3*b*B*Hypergeometric2F1[1/2, (-4 - 3*m)/6, (2 - 3*m)/6, Cos[c
+ d*x]^2]*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 +
 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.120175, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$\frac{3Ab\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}+\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+d\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Se
c[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c +
 d*x]^2]) + (3*b*B*Hypergeometric2F1[1/2, (-4 - 3*m)/6, (2 - 3*m)/6, Cos[c
+ d*x]^2]*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 +
 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{\left(Ab\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}} + \frac{\left(bB\sqrt[3]{b\sec(c+\right.}{\sqrt[3]{}}$$

$$= \left(Ab\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\right)\int \cos^{-\frac{4}{3}-}$$

$$= \frac{3Ab\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1-3m);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)\sec^m(c+}{d(1+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.381677, size = 140, normalized size = 0.84

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{4/3}\sec^m(c+dx)\left(A(3m+7)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-\right.\right.}{d(3m+4)(3m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + 3*m)/
6, 5/3 + m/2, Sec[c + d*x]^2] + B*(4 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m
)/6, (13 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*S
qrt[-Tan[c + d*x]^2])/(d*(4 + 3*m)*(7 + 3*m))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{\frac{4}{3}} (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Bb\sec\left(dx+c\right)^{2}+Ab\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3)*sec
(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

3.28 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-2),\frac{1}{6}(4-3m),\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}}-\frac{3A\sin\cdots}{\cdots}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Si
n[c + d*x]^2]) + (3*B*Hypergeometric2F1[1/2, (-2 - 3*m)/6, (4 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(2 + 3*
m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.117112, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-2);\frac{1}{6}(4-3m);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)(b\sec(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Si
n[c + d*x]^2]) + (3*B*Hypergeometric2F1[1/2, (-2 - 3*m)/6, (4 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(2 + 3*
m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{\sec^{\frac{2}{3}}(c+dx)}$$

$$= \frac{\left(A(b\sec(c+dx))^{2/3}\right)\int \sec^{\frac{2}{3}+m}(c+dx)\,dx}{\sec^{\frac{2}{3}}(c+dx)} + \frac{\left(B(b\sec(c+dx))^{2}\right.}{\sec}$$

$$= \left(A\cos^{\frac{2}{3}+m}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^{2/3}\right)\int \cos^{-\frac{2}{3}-m}($$

$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+d}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.215456, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{2/3}\sec^m(c+dx)\left(A(3m+5)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2)\right.\right.}{d(3m+2)(3m+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(5 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + 3*m)/
6, (8 + 3*m)/6, Sec[c + d*x]^2] + B*(2 + 3*m)*Hypergeometric2F1[1/2, (5 + 3
*m)/6, (11 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)
*Sqrt[-Tan[c + d*x]^2])/(d*(2 + 3*m)*(5 + 3*m))
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{\frac{2}{3}} (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

3.29 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-1),\frac{1}{6}(5-3m),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+}{}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Si
n[c + d*x]^2]) + (3*B*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*
m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.111724, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Si
n[c + d*x]^2]) + (3*B*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*
m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{\sqrt[3]{\sec(c+dx)}}$$
$$= \frac{\left(A\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}} + \frac{\left(B\sqrt[3]{b\sec(c+dx)}\right)}{\sqrt[3]{\sec}}$$
$$= \left(A\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\right)\int \cos^{-\frac{1}{3}-m}(c$$
$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.2672, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\left(A(3m+4)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1\right.\right.}{d(3m+1)(3m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(4 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + 3*m)/
6, (7 + 3*m)/6, Sec[c + d*x]^2] + B*(1 + 3*m)*Hypergeometric2F1[1/2, (4 + 3
*m)/6, 5/3 + m/2, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sq
rt[-Tan[c + d*x]^2])/(d*(1 + 3*m)*(4 + 3*m))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m\sqrt[3]{b\sec(dx+c)}\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^{m}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec(c+dx)}\,(A+B\sec(c+dx))\sec^{m}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

3.30 $\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{3A\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4-3m),\frac{1}{6}(10-3m),\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^{m}}{}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(1 - 3*m)*(b*Sec[c + d*x])^(1/
3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.109431, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$-\frac{3A\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^{m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(\right.}{d(1-3m)\sqrt{\sin^2(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(1 - 3*m)*(b*Sec[c + d*x])^(1/
3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$= \frac{\left(A\sqrt[3]{\sec(c+dx)}\right)\int \sec^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}} + \frac{\left(B\sqrt[3]{\sec(c+dx)}\right)\int \sec^{\frac{2}{3}+m}(c+dx)\,d$$

$$= \frac{\left(A\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{1}{3}-m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}} + \frac{\left(B\cos^{\frac{2}{3}+m}(c+dx)\,s\right.}{}$$

$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}} -$$

Mathematica [A] time = 0.245183, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^m(c+dx)\left(A(3m+2)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-1),\frac{1}{6}(3m+5),\sec^2\right.\right.}{d(3m-1)(3m+2)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(2 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + 3*m)
/6, (5 + 3*m)/6, Sec[c + d*x]^2] + B*(-1 + 3*m)*Hypergeometric2F1[1/2, (2 +
 3*m)/6, (8 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^2]
)/(d*(-1 + 3*m)*(2 + 3*m)*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (A+B\sec(dx+c))\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m}{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(
d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

3.31 $\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=165

$$-\frac{3A\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5-3m),\frac{1}{6}(11-3m),\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sec^m(c\cdots}{}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[S
in[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(2 - 3*m)*(b*Sec[c + d*x])^(2/
3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.115001, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$-\frac{3A\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-\cdots\right.}{d(2-3m)\sqrt{\sin^2(c+dx)}(b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[S
in[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(2 - 3*m)*(b*Sec[c + d*x])^(2/
3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{(b\sec(c+dx))^{2/3}}$$

$$= \frac{\left(A\sec^{\frac{2}{3}}(c+dx)\right)\int \sec^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}} + \frac{\left(B\sec^{\frac{2}{3}}(c+dx)\right)\int \sec^{\frac{1}{3}+m}(c+dx}{(b\sec(c+dx))^{2/3}}$$

$$= \frac{\left(A\cos^{\frac{1}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{2}{3}-m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}} + \frac{\left(B\cos^{\frac{1}{3}+m}(c+d\right.}{}$$

$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+d}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.23309, size = 140, normalized size = 0.85

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^m(c+dx)\left(A(3m+1)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-2),\frac{1}{6}(3m+4),\text{s}\right.\right.}{d(3m-2)(3m+1)(b\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + 3*m)
/6, (4 + 3*m)/6, Sec[c + d*x]^2] + B*(-2 + 3*m)*Hypergeometric2F1[1/2, (1 +
 3*m)/6, (7 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^2]
)/(d*(-2 + 3*m)*(1 + 3*m)*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (A+B\sec(dx+c))(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^{m}}{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(
d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec^{m}(c+dx)}{(b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sec(dx+c)^{m}}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

3.32 $\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=173

$$-\frac{3A\sin(c+dx)\sec^{m-2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7-3m),\frac{1}{6}(13-3m),\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^{m}}{}$$

```
[Out] (-3*A*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6,
Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(b*d*(4 - 3*m)*(b*Sec[c
 + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.11961, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$-\frac{3A\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\right.}{bd(4-3m)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6,
Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(b*d*(4 - 3*m)*(b*Sec[c
 + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(A\sqrt[3]{\sec(c+dx)}\right)\int \sec^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}} + \frac{\left(B\sqrt[3]{\sec(c+dx)}\right)\int \sec^{-\frac{1}{3}+m}(c+dx)\,d}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(A\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{4}{3}-m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}} + \frac{\left(B\cos^{\frac{2}{3}+m}(c+dx)\,s\right.}{}$$
$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}} -$$

Mathematica [A] time = 0.33367, size = 140, normalized size = 0.81

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^m(c+dx)\left(A(3m-1)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-4),\frac{1}{6}(3m+2),\sec^2(\right.\right.}{d(3m-4)(3m-1)(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(-1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-4 + 3*m
)/6, (2 + 3*m)/6, Sec[c + d*x]^2] + B*(-4 + 3*m)*Hypergeometric2F1[1/2, (-1
 + 3*m)/6, (5 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^
2])/(d*(-4 + 3*m)*(-1 + 3*m)*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (A+B\sec(dx+c)) (b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\,(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m}{b^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*se
c(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

3.33 $\int \sec^m(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=172

$$\frac{B\sin(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-n),\frac{1}{2}(-m-n+2),\cos^2(c+dx)\right)}{d(m+n)\sqrt{\sin^2(c+dx)}}-\frac{A\sin(c$$

```
[Out] -((A*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*S
ec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[S
in[c + d*x]^2])) + (B*Hypergeometric2F1[1/2, (-m - n)/2, (2 - m - n)/2, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(m + n)*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.110761, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {20, 3787, 3772, 2643}

$$\frac{B\sin(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n);\frac{1}{2}(-m-n+2);\cos^2(c+dx)\right)}{d(m+n)\sqrt{\sin^2(c+dx)}}-\frac{A\sin(c+dx)\sec^{m-1}(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*S
ec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[S
in[c + d*x]^2])) + (B*Hypergeometric2F1[1/2, (-m - n)/2, (2 - m - n)/2, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(m + n)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{m+n}(c+dx)(A+B\sec($$
$$= (A\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{m+n}(c+dx)\,dx + (B\,\mathrm{s}$$
$$= (A\cos^{m+n}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n)\int \cos^{-m-n}($$
$$= -\frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-m-n);\frac{1}{2}(3-m-n);\cos^2(c+dx)\right)\sec^{-1+m}}{d(1-m-n)\sqrt{\sin^2(c+d}}$$

Mathematica [A] time = 0.213094, size = 126, normalized size = 0.73

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\left(A(m+n+1)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n}{2},\frac{1}{2}\right.\right.}{d(m+n)(m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(A*(1 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2,
 (2 + m + n)/2, Sec[c + d*x]^2] + B*(m + n)*Hypergeometric2F1[1/2, (1 + m +
 n)/2, (3 + m + n)/2, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*Sq
rt[-Tan[c + d*x]^2])/(d*(m + n)*(1 + m + n))
```

Maple [F] time = 0.995, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^n (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)(b\sec(dx+c))^n\sec(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n (A+B\sec(c+dx))\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

3.34 $\int \sec^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{A\sin(c+dx)(b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-n-1),\frac{1-n}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))}{}$$

```
[Out] (A*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]) + (B*Hyperg
eometric2F1[1/2, (-2 - n)/2, -n/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2 + n)
*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.126553, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$\frac{A\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-\right.}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]) + (B*Hyperg
eometric2F1[1/2, (-2 - n)/2, -n/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2 + n)
*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{2+n}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{2+n}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{3+n}\,dx}{b^3}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-2-n}dx}{b^2} + \frac{\left(B\left(\frac{\cos(c}{b}\right.\right.}{}$$
$$= \frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{1+n}\sin(c+}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.220685, size = 119, normalized size = 0.83

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)(b\sec(c+dx))^n\left(A(n+3)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sec^2(c+dx)\right)+B(\right.}{d(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]*(b*Sec[c + d*x])^n*(A*(3 + n)*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2] + B*(2 + n)*Hypergeometric2F1[1/
2, (3 + n)/2, (5 + n)/2, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2
])/(d*(2 + n)*(3 + n))
```

Maple [F] time = 0.924, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\sec(dx+c))^n (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n \sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(c+dx\right)\right)^{n}\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

3.35 $\int \sec(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{A\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n+1}\text{Hyperg}}{bd(n+1}$$

```
[Out] (A*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])
^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (-1
 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/
(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.115049, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {16, 3787, 3772, 2643}

$$\frac{A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2\right.}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])
^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (-1
 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/
(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{1+n}(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\sec(c+dx))^{1+n}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2+n}\,dx}{b^2}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-1-n}dx}{b} + \frac{\left(B\left(\frac{\mathrm{co}}{}\right.\right.}{}$$
$$= \frac{A\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}} +$$

Mathematica [A] time = 0.248129, size = 119, normalized size = 0.88

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)(b\sec(c+dx))^n\left(A(n+2)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\mathrm{sec}\right.\right.}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(A*(2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3
+ n)/2, Sec[c + d*x]^2] + B*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sec[c + d*x]^2])*Sec[c + d*x]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2
])/(d*(1 + n)*(2 + n))
```

Maple [F] time = 0.842, size = 0, normalized size = 0.

$$\int \sec(dx+c)(b\sec(dx+c))^n(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(c+dx\right)\right)^{n}\left(A+B\sec\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)
```

3.36 $\int (b\sec(c+dx))^n (A+B\sec(c+dx))\,dx$

Optimal. Leaf size=137

$$\frac{B\sin(c+dx)(b\sec(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{Ab\sin(c+dx)(b\sec(c+dx))^{n-1}\text{Hyp}}{d(1-n}$$

```
[Out] -((A*b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])) + (B*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin
[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0962599, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3787, 3772, 2643}

$$\frac{B\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{Ab\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])) + (B*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin
[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^n\,dx + \frac{B\int (b\sec(c+dx))^{1+n}\,dx}{b}$$

$$= \left(A\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-n} dx + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^n \right.}{}$$

$$= -\frac{A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}} + $$

Mathematica [A] time = 0.14961, size = 107, normalized size = 0.78

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\left(A(n+1)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\sec^2(c+dx)\right)+Bn\text{H}\right.}{dn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(A*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2
, Sec[c + d*x]^2] + B*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c
+ d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*n*(1 + n))
```

Maple [F] time = 0.605, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n (A+B\sec(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)(b\sec(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (c + dx))^n (A + B \sec (c + dx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B \sec (dx + c) + A)(b \sec (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

3.37 $\int \cos(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$-\frac{Ab^2\sin(c+dx)(b\sec(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}-\frac{bB\sin(c+dx)(b\sec(c+dx))^{n-1}\text{H}}{d(1}$$

```
[Out] -((A*b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])) - (b*B
*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*
x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.126733, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {16, 3787, 3772, 2643}

$$-\frac{Ab^2\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}-\frac{bB\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])) - (b*B
*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*
x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = b\int (b\sec(c+dx))^{-1+n}(A+B\sec(c+dx))\,dx$$
$$= (Ab)\int (b\sec(c+dx))^{-1+n}\,dx + B\int (b\sec(c+dx))^n\,dx$$
$$= \left(Ab\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{1-n} dx -$$
$$= -\frac{B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.157987, size = 107, normalized size = 0.71

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^n\left(An\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\sec^2(c+dx)\right)+B(n\right.}{d(n-1)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cot[c + d*x]*(A*n*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/
2, Sec[c + d*x]^2] + B*(-1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[
c + d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + n)*n)
```

Maple [F] time = 0.897, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b\sec(dx+c))^n(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))(b\sec(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*(b*sec(d*x + c))^n,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (c+dx))^n (A+B\sec (c+dx))\cos (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)(b\sec (dx+c))^n\cos (dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)
```

3.38 $\int \cos^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=153

$$-\frac{Ab^3\sin(c+dx)(b\sec(c+dx))^{n-3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-n}{2},\frac{5-n}{2},\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2B\sin(c+dx)(b\sec(c+dx))^{\prime}}{}$$

```
[Out] -((A*b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])) - (b^2
*B*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.142149, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 3787, 3772, 2643}

$$-\frac{Ab^3\sin(c+dx)(b\sec(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2B\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2}\right.}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])) - (b^2
*B*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = b^2\int (b\sec(c+dx))^{-2+n}(A+B\sec(c+dx))\,dx$$
$$= \left(Ab^2\right)\int (b\sec(c+dx))^{-2+n}\,dx + (bB)\int (b\sec(c+dx))^{-1+n}\,dx$$
$$= \left(Ab^2\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2-n} dx +$$
$$= -\frac{B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n \text{s}}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.316128, size = 114, normalized size = 0.75

$$\frac{b\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{n-1}\left(A(n-1)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\sec^2(c+dx)\right)+B\right.}{d(n-2)(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*Cot[c + d*x]*(A*(-1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2,
 n/2, Sec[c + d*x]^2] + B*(-2 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
n)/2, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*
(-2 + n)*(-1 + n))
```

Maple [F] time = 0.894, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^n (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^2\sec(dx+c)+A\cos(dx+c)^2\right)(b\sec(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*(b*sec(d*x + c)
)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

3.39 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-1),\frac{1}{4}(3-2n),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(c+$$

```
[Out] (2*A*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt
[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*
x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.116446, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*A*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt
[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*
x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{\frac{3}{2}+n}(c+dx)(A+B\sec($$
$$= (A\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{\frac{3}{2}+n}(c+dx)\,dx + (B\,\mathrm{s}$$
$$= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{-\frac{3}{2}-n}($$
$$= \frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.270016, size = 140, normalized size = 0.86

$$\frac{2\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\left(A(2n+5)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3\right.\right.}{d(2n+3)(2n+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x]*(A*(5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/
4, (7 + 2*n)/4, Sec[c + d*x]^2] + B*(3 + 2*n)*Hypergeometric2F1[1/2, (5 + 2
*n)/4, (9 + 2*n)/4, Sec[c + d*x]^2])*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*
Sqrt[-Tan[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{3}{2}}(b\sec(dx+c))^n(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^2 + A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^n\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^n*sqrt(sec(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right)\right)^n \sec\left(dx+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

3.40 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-1),\frac{1}{4}(3-2n),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2A\sin(c$$

```
[Out] (-2*A*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*
x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.111319, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2A\sin(c+dx)(b\sec(c+dx}{d(1-2n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*A*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*
x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx &= \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(c+dx)(A+B\sec(c+\\
&= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(c+dx)\,dx+(B\sec\\
&= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{-\frac{1}{2}-n}(c+\\
&= -\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n}{d(1-2n)\sqrt{\sec(c+dx)}\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.232687, size = 140, normalized size = 0.86

$$
\frac{2\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\left(A(2n+3)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\sec^2(c+dx)\right)+\right.}{d(2n+1)(2n+3)\sqrt{\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (1 +
 2*n)/4, (5 + 2*n)/4, Sec[c + d*x]^2] + B*(1 + 2*n)*Hypergeometric2F1[1/2,
(3 + 2*n)/4, (7 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^
2])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sec[c + d*x]])
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$
\int (b\sec(dx+c))^n (A+B\sec(dx+c))\sqrt{\sec(dx+c)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (B\sec(dx+c)+A)(b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)\,(b\sec(dx+c))^n\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\,(b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

3.41 $\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3-2n),\frac{1}{4}(7-2n),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(c+$$

```
[Out] (-2*A*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.112593, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-\right.}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-2*A*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{-\frac{1}{2}+n}(c+dx)(A+B\sec(c+dx))\,dx$$

$$= (A\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{-\frac{1}{2}+n}(c+dx)\,dx + (B\sec^{-n}(c+dx$$

$$= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{\frac{1}{2}-n}(c+dx)\,dx + ($$

$$= -\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+d}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.352953, size = 135, normalized size = 0.83

$$\frac{2\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\left(A(2n+1)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\sec^2(c+dx)\right)\right.}{d\left(4n^2-1\right)\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-1
+ 2*n)/4, (3 + 2*n)/4, Sec[c + d*x]^2] + B*(-1 + 2*n)*Hypergeometric2F1[1/2
, (1 + 2*n)/4, (5 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x
]^2])/(d*(-1 + 4*n^2)*Sec[c + d*x]^(3/2))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n (A+B\sec(dx+c))\frac{1}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

3.42 $\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5-2n),\frac{1}{4}(9-2n),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(\ldots}{\ldots}$$

```
[Out] (-2*A*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.112862, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 3787, 3772, 2643}

$$-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(\ldots\right)}{d(3-2n)\sqrt{\sin^2(c+d\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*A*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[
Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)(A+B\sec(c+dx))\,dx$$
$$= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)\,dx + \left(B\sec^{-n}(c+dx)(b\right.$$
$$= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{\frac{3}{2}-n}(c+dx)\,dx + \left(B\,c\right.$$
$$= -\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.315445, size = 140, normalized size = 0.86

$$\frac{2\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\left(A(2n-1)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\sec^2(c+dx)\right)+\right.}{d(2n-3)(2n-1)\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-3
 + 2*n)/4, (1 + 2*n)/4, Sec[c + d*x]^2] + B*(-3 + 2*n)*Hypergeometric2F1[1/
2, (-1 + 2*n)/4, (3 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d
*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Sec[c + d*x]^(5/2))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n (A+B\sec(dx+c))(\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

3.43 $\int \sec^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{a(5A+4B)\tan^3(c+dx)}{15d}+\frac{a(5A+4B)\tan(c+dx)}{5d}+\frac{3a(A+B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+dx)\sec^3(c+d}{4d}$$

```
[Out] (3*a*(A + B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 4*B)*Tan[c + d*x])/(5
*d) + (3*a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A + B)*Sec[c + d*
x]^3*Tan[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(5*
A + 4*B)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.141087, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3997, 3787, 3767, 3768, 3770}

$$\frac{a(5A+4B)\tan^3(c+dx)}{15d}+\frac{a(5A+4B)\tan(c+dx)}{5d}+\frac{3a(A+B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+dx)\sec^3(c+d}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*a*(A + B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 4*B)*Tan[c + d*x])/(5
*d) + (3*a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A + B)*Sec[c + d*
x]^3*Tan[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(5*
A + 4*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aB\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^4(c+dx)(a(5A+4$$
$$= \frac{aB\sec^4(c+dx)\tan(c+dx)}{5d}+(a(A+B))\int \sec^5(c+dx)\,a$$
$$= \frac{a(A+B)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{aB\sec^4(c+dx)\tan(c$$
$$= \frac{a(5A+4B)\tan(c+dx)}{5d}+\frac{3a(A+B)\sec(c+dx)\tan(c+$$
$$= \frac{3a(A+B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(5A+4B)\tan(c+dx)}{5d}$$

Mathematica [A] time = 0.744289, size = 87, normalized size = 0.65

$$\frac{a\left(45(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5(A+2B)\tan^2(c+dx)+15(A+B)+3B\tan^4(c+dx)\right)+30(A\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(45*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(45*(A + B)*Sec[c + d*x
] + 30*(A + B)*Sec[c + d*x]^3 + 8*(15*(A + B) + 5*(A + 2*B)*Tan[c + d*x]^2
+ 3*B*Tan[c + d*x]^4))))/(120*d)
```

Maple [A] time = 0.045, size = 213, normalized size = 1.6

$$\frac{2\,Aa\tan(dx+c)}{3\,d}+\frac{Aa\tan(dx+c)(\sec(dx+c))^2}{3\,d}+\frac{Ba(\sec(dx+c))^3\tan(dx+c)}{4\,d}+\frac{3\,Ba\sec(dx+c)\tan(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/3/d*A*a*tan(d*x+c)+1/3/d*A*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*B*sec(d*x+c)^3
*tan(d*x+c)/d+3/8*a*B*sec(d*x+c)*tan(d*x+c)/d+3/8/d*B*a*ln(sec(d*x+c)+tan(d
*x+c))+1/4/d*A*a*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a*tan(d*x+c)*sec(d*x+c)+3/
8/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+8/15*a*B*tan(d*x+c)/d+1/5*a*B*sec(d*x+c)^
4*tan(d*x+c)/d+4/15*a*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.984584, size = 270, normalized size = 2.01

$$80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba-15\,Aa\left(\frac{2\left(3\sin(\right.}{\sin(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a - 15*A*a*(2*(3*sin(d*x + c)^3 - 5*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) - 15*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*lo
g(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.497098, size = 381, normalized size = 2.84

$$\frac{45\,(A+B)a\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-45\,(A+B)a\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(5\,A+4\,B)a\,\text{co}\right.}{240\,d\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(45*(A + B)*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*(A + B)*a*cos
(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*A + 4*B)*a*cos(d*x + c)^4 + 4
5*(A + B)*a*cos(d*x + c)^3 + 8*(5*A + 4*B)*a*cos(d*x + c)^2 + 30*(A + B)*a*
cos(d*x + c) + 24*B*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^{4}\left(c+dx\right)dx+\int A\sec^{5}\left(c+dx\right)dx+\int B\sec^{5}\left(c+dx\right)dx+\int B\sec^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integr
al(B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A] time = 1.36729, size = 289, normalized size = 2.16

$$45\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+45\,Ba\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(45*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*(A*a + B*a)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 45*B
*a*tan(1/2*d*x + 1/2*c)^9 - 290*A*a*tan(1/2*d*x + 1/2*c)^7 - 130*B*a*tan(1/
2*d*x + 1/2*c)^7 + 400*A*a*tan(1/2*d*x + 1/2*c)^5 + 464*B*a*tan(1/2*d*x + 1
/2*c)^5 - 350*A*a*tan(1/2*d*x + 1/2*c)^3 - 190*B*a*tan(1/2*d*x + 1/2*c)^3 +
 195*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^5)/d
```

3.44 $\int \sec^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=106

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3B)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (a*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.123469, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3997, 3787, 3768, 3770, 3767}

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3B)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int \sec^3(c+dx)(a(4A+3$$
$$= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d} + (a(A+B))\int \sec^4(c+dx)\,a$$
$$= \frac{a(4A+3B)\sec(c+dx)\tan(c+dx)}{8d} + \frac{aB\sec^3(c+dx)\tan}{4d}$$
$$= \frac{a(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a(A+B)\tan(c+dx)}{d}$$

Mathematica [A] time = 0.38896, size = 77, normalized size = 0.73

$$\frac{a\left(3(4A+3B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(A+B)(\cos(2(c+dx))+2)\sec(c+dx)+12A+6B\,\mathrm{s}\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(3*(4*A + 3*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*A + 9*B + 8*(A +
 B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*B*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

Maple [A] time = 0.041, size = 171, normalized size = 1.6

$$\frac{Aa\tan(dx+c)\sec(dx+c)}{2d} + \frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{2Ba\tan(dx+c)}{3d} + \frac{Ba(\sec(dx+c))^2\tan(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*A*a*tan(d*x+c)*sec(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*B
*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d+2/3/d*A*a*tan(d*x+c)+1/3/d*
A*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*B*sec(d
*x+c)*tan(d*x+c)/d+3/8/d*B*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.982083, size = 220, normalized size = 2.08

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba-3Ba\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a - 3*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.491569, size = 339, normalized size = 3.2

$$\frac{3\,(4\,A+3\,B)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,A+3\,B)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(A+B)a\,\mathrm{c}\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(4*A + 3*B)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A + 3*B)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(A + B)*a*cos(d*x + c)^3 +
3*(4*A + 3*B)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos(d*x + c) + 6*B*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^{3}\left(c+dx\right)dx+\int A\sec^{4}\left(c+dx\right)dx+\int B\sec^{4}\left(c+dx\right)dx+\int B\sec^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integr
al(B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))
```

Giac [A] time = 1.28472, size = 254, normalized size = 2.4

$$\frac{3\,(4\,Aa+3\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,Aa+3\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9\,Ba\tan\left(\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(4*A*a + 3*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*B
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*A*a*tan(1/2*d*x + 1/2*c)^7 +
 9*B*a*tan(1/2*d*x + 1/2*c)^7 - 28*A*a*tan(1/2*d*x + 1/2*c)^5 - 49*B*a*tan(
1/2*d*x + 1/2*c)^5 + 52*A*a*tan(1/2*d*x + 1/2*c)^3 + 31*B*a*tan(1/2*d*x + 1
/2*c)^3 - 36*A*a*tan(1/2*d*x + 1/2*c) - 39*B*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.45 $\int \sec^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a(3A+2B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)\sec^2(c$$

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 2*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.115444, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3997, 3787, 3767, 8, 3768, 3770}

$$\frac{a(3A+2B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)\sec^2(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 2*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec^2(c+dx)(a(3A+2B)$$
$$= \frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}+(a(A+B))\int \sec^3(c+dx)\,dx+$$
$$= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}$$
$$= \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(3A+2B)\tan(c+dx)}{3d}+$$

Mathematica [A] time = 0.324826, size = 56, normalized size = 0.65

$$\frac{a\left(3(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(A+B)\sec(c+dx)+6(A+B)+2B\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*S
ec[c + d*x] + 2*B*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.039, size = 128, normalized size = 1.5

$$\frac{Aa\tan(dx+c)}{d}+\frac{Ba\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Aa\tan(dx+c)\sec(dx+c)}{2d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a*tan(d*x+c)+1/2*a*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*A*a*tan(d*x+c)*sec(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x
+c))+2/3*a*B*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.967454, size = 171, normalized size = 1.99

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba-3\,Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*A*a*tan(d*x + c))/d
```

Fricas [A] time = 0.484705, size = 288, normalized size = 3.35

$$\frac{3\,(A+B)a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(A+B)a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(3\,A+2\,B)a\,\mathrm{co}\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(A + B)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A + 2*B)*a*cos(d*x + c)^2 + 3*(A
+ B)*a*cos(d*x + c) + 2*B*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^{2}\left(c+dx\right)dx+\int A\sec^{3}\left(c+dx\right)dx+\int B\sec^{3}\left(c+dx\right)dx+\int B\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integr
al(B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))
```

Giac [A] time = 1.26102, size = 208, normalized size = 2.42

$$\frac{3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*tan(1/2*d*x + 1
/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

3.46 $\int \sec(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=56

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*(2*A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0673097, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(2*A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aB\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \sec(c+dx)(a(2A+B)+$$
$$= \frac{aB\sec(c+dx)\tan(c+dx)}{2d} + (a(A+B))\int \sec^2(c+dx)\,dx$$
$$= \frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aB\sec(c+dx)\tan(c+d}{2d}$$
$$= \frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(A+B)\tan(c+dx)}{d} + $$

Mathematica [A] time = 0.0258427, size = 75, normalized size = 1.34

$$\frac{aA\tan(c+dx)}{d} + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\tan(c+dx)}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/(2*d) + (a*A*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.035, size = 86, normalized size = 1.5

$$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Ba\tan(dx+c)}{d} + \frac{Aa\tan(dx+c)}{d} + \frac{Ba\sec(dx+c)\tan(dx+c)}{2d} + \frac{Ba\ln(\sec(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/d+1/d*A*a*tan(d*x+c)+1/2*a
*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.964841, size = 119, normalized size = 2.12

$$-\frac{Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4\,Aa\log(\sec(dx+c)+\tan(dx+c)) - 4\,Aa\tan(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/4*(B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*A*a*log(sec(d*x + c) + tan(d*x + c)) - 4*A*a*tan(d
*x + c) - 4*B*a*tan(d*x + c))/d
```

Fricas [A] time = 0.47976, size = 239, normalized size = 4.27

$$\frac{(2A+B)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(2A+B)a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2(A+B)a\cos(dx+}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*A + B)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A + B)*a*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(A + B)*a*cos(d*x + c) + B*a)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec(c+dx)\,dx+\int A\sec^2(c+dx)\,dx+\int B\sec^2(c+dx)\,dx+\int B\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sec(c + d*x)**2, x) + Integral(
B*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**3, x))
```

Giac [B] time = 1.34501, size = 167, normalized size = 2.98

$$\frac{(2Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(2Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((2*A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1
/2*d*x + 1/2*c)^3 - 2*A*a*tan(1/2*d*x + 1/2*c) - 3*B*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.47 $\int (a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=32

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{aB\tan(c+dx)}{d}$$

```
[Out] a*A*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rubi [A] time = 0.0331723, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3914, 3767, 8, 3770}

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{aB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= aAx+(aB)\int \sec^2(c+dx)\,dx+(a(A+B))\int \sec(c+dx)\,dx\\ &= aAx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(aB)\operatorname{Subst}(\int 1\,dx,x,-\tan(c+\\ &= aAx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0162481, size = 43, normalized size = 1.34

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{aB\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
B*Tan[c + d*x])/d
```

Maple [A] time = 0.032, size = 65, normalized size = 2.

$$aAx+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Aac}{d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*A*x+1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*c+1/d*B*a*ln(sec(d*x+c)+tan
(d*x+c))+a*B*tan(d*x+c)/d
```

Maxima [A] time = 1.00014, size = 76, normalized size = 2.38

$$\frac{(dx+c)Aa+Aa\log(\sec(dx+c)+\tan(dx+c))+Ba\log(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*A*a + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*a*log(sec(d*x + c
) + tan(d*x + c)) + B*a*tan(d*x + c))/d
```

Fricas [B] time = 0.489232, size = 220, normalized size = 6.88

$$\frac{2\,Aadx\cos(dx+c)+(A+B)a\cos(dx+c)\log(\sin(dx+c)+1)-(A+B)a\cos(dx+c)\log(-\sin(dx+c)+1)+2\,Ba\,\mathrm{s}}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*a*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [A] time = 13.4574, size = 71, normalized size = 2.22

$$\begin{cases} \frac{Aa(c+dx)+Aa\log(\tan(c+dx)+\sec(c+dx))+Ba\log(\tan(c+dx)+\sec(c+dx))+Ba\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(A+B\sec(c))(a\sec(c)+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*(c + d*x) + A*a*log(tan(c + d*x) + sec(c + d*x)) + B*a*log(
tan(c + d*x) + sec(c + d*x)) + B*a*tan(c + d*x))/d, Ne(d, 0)), (x*(A + B*se
c(c))*(a*sec(c) + a), True))
```

Giac [B] time = 1.25726, size = 113, normalized size = 3.53

$$\frac{(dx+c)Aa+(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A*a + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.48 $\int \cos(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=32

$$ax(A+B)+\frac{aA\sin(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*(A + B)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d
```

Rubi [A] time = 0.0473172, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {3996, 3770}

$$ax(A+B)+\frac{aA\sin(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(A + B)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{aA\sin(c+dx)}{d}-\int(-a(A+B)-aB\sec(c+dx))\,dx\\ &= a(A+B)x+\frac{aA\sin(c+dx)}{d}+(aB)\int\sec(c+dx)\,dx\\ &= a(A+B)x+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0266833, size = 46, normalized size = 1.44

$$\frac{aA\sin(c)\cos(dx)}{d}+\frac{aA\cos(c)\sin(dx)}{d}+aAx+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+aBx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + a*B*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (
a*A*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.064, size = 56, normalized size = 1.8

$$aAx + Bax + \frac{Aa\sin(dx+c)}{d} + \frac{Aac}{d} + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*A*x+B*a*x+a*A*sin(d*x+c)/d+1/d*A*a*c+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/
d*B*a*c
```

Maxima [A] time = 1.02222, size = 78, normalized size = 2.44

$$\frac{2(dx+c)Aa + 2(dx+c)Ba + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + 2*(d*x + c)*B*a + B*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*A*a*sin(d*x + c))/d
```

Fricas [A] time = 0.48656, size = 139, normalized size = 4.34

$$\frac{2(A+B)adx + Ba\log(\sin(dx+c)+1) - Ba\log(-\sin(dx+c)+1) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(A + B)*a*d*x + B*a*log(sin(d*x + c) + 1) - B*a*log(-sin(d*x + c) +
1) + 2*A*a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\cos(c+dx)\,dx + \int A\cos(c+dx)\sec(c+dx)\,dx + \int B\cos(c+dx)\sec(c+dx)\,dx + \int B\cos(c+dx)\sec^2($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c +
 d*x)**2, x))
```

Giac [B] time = 1.2413, size = 107, normalized size = 3.34

$$\frac{Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Aa+Ba)(dx+c)+\frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (A*a + B*a)*(d*x + c) + 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.49 $\int \cos^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(A+2B)+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (a*(A + 2*B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0864057, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3996, 3787, 2637, 8}

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(A+2B)+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(A + 2*B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{aA\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}\int \cos(c+dx)(-2a(A+B \\ &= \frac{aA\cos(c+dx)\sin(c+dx)}{2d}+(a(A+B))\int \cos(c+dx)\,dx \\ &= \frac{1}{2}a(A+2B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{aA\cos(c+dx)\sin\end{aligned}$$

Mathematica [A] time = 0.0962903, size = 44, normalized size = 0.94

$$\frac{a(4(A+B)\sin(c+dx)+A\sin(2(c+dx))+2Ac+2Adx+4Bdx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(2*A*c + 2*A*d*x + 4*B*d*x + 4*(A + B)*Sin[c + d*x] + A*Sin[2*(c + d*x)]
))/(4*d)
```

Maple [A] time = 0.065, size = 57, normalized size = 1.2

$$\frac{1}{d}\left(Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)+Ba\sin(dx+c)+Ba(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*a*sin(d*x+c)+B*a*sin(d
*x+c)+B*a*(d*x+c))
```

Maxima [A] time = 0.979916, size = 74, normalized size = 1.57

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa+4\,(dx+c)Ba+4\,Aa\sin(dx+c)+4\,Ba\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*B*a + 4*A*a*sin(d*x
 + c) + 4*B*a*sin(d*x + c))/d
```

Fricas [A] time = 0.458909, size = 99, normalized size = 2.11

$$\frac{(A+2\,B)adx+(Aa\cos(dx+c)+2\,(A+B)a)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((A + 2*B)*a*d*x + (A*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\cos^2(c+dx)\,dx+\int A\cos^2(c+dx)\sec(c+dx)\,dx+\int B\cos^2(c+dx)\sec(c+dx)\,dx+\int B\cos^2(c+dx)\,\mathrm{se}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x)**2, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)
, x) + Integral(B*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**2, x))
```

Giac [B] time = 1.26993, size = 126, normalized size = 2.68

$$\frac{(Aa+2\,Ba)(dx+c)+\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*a)*(d*x + c) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/
2*d*x + 1/2*c)^3 + 3*A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.50 $\int \cos^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{a(2A+3B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] (a*(A + B)*x)/2 + (a*(2*A + 3*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.10831, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3996, 3787, 2635, 8, 2637}

$$\frac{a(2A+3B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(A + B)*x)/2 + (a*(2*A + 3*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)(-3a(A+$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} + (a(A+B))\int \cos^2(c+dx)$$
$$= \frac{a(2A+3B)\sin(c+dx)}{3d} + \frac{a(A+B)\cos(c+dx)\sin(c+dx}{2d}$$
$$= \frac{1}{2}a(A+B)x + \frac{a(2A+3B)\sin(c+dx)}{3d} + \frac{a(A+B)\cos(c+}{2}$$

Mathematica [A] time = 0.169215, size = 65, normalized size = 0.84

$$\frac{a(3(3A+4B)\sin(c+dx)+3(A+B)\sin(2(c+dx))+A\sin(3(c+dx))+6Ac+6Adx+6Bc+6Bdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(3*A + 4*B)*Sin[c + d*x] + 3*(A +
 B)*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.074, size = 85, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c))
```

Maxima [A] time = 0.982049, size = 107, normalized size = 1.39

$$-\frac{4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Aa - 3(2dx+2c+\sin(2dx+2c))Aa - 3(2dx+2c+\sin(2dx+2c))Ba - 12Ba}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 12*B*a*sin(d*x + c)
)/d
```

Fricas [A] time = 0.464526, size = 146, normalized size = 1.9

$$\frac{3\,(A+B)adx+\left(2\,Aa\cos\left(dx+c\right)^{2}+3\,(A+B)a\cos\left(dx+c\right)+2\,(2\,A+3\,B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A + B)*a*d*x + (2*A*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2
*(2*A + 3*B)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44524, size = 167, normalized size = 2.17

$$\frac{3\,(Aa+Ba)(dx+c)+\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*(d*x + c) + 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan(
1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*tan(1/2*d*x + 1/
2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3)/d
```

3.51 $\int \cos^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(3A+4B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(3A+4B)+\frac{aA\sin(c+$$

```
[Out] (a*(3*A + 4*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(3*A + 4*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.118785, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3996, 3787, 2633, 2635, 8}

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(3A+4B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(3A+4B)+\frac{aA\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(3*A + 4*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(3*A + 4*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)(-4a(A+B)$$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} + (a(A+B))\int \cos^3(c+dx)\,dx$$
$$= \frac{a(3A+4B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA\cos^3(c+dx)\sin(c+}{4d}$$
$$= \frac{1}{8}a(3A+4B)x + \frac{a(A+B)\sin(c+dx)}{d} + \frac{a(3A+4B)\cos(c+}{8d}$$

Mathematica [A] time = 0.236748, size = 75, normalized size = 0.77

$$\frac{a\left(-32(A+B)\sin^3(c+dx)+96(A+B)\sin(c+dx)+24(A+B)\sin(2(c+dx))+3A\sin(4(c+dx))+36Ac+36Adx+4\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(36*A*c + 48*B*c + 36*A*d*x + 48*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(A
 + B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))
/(96*d)
```

Maple [A] time = 0.081, size = 107, normalized size = 1.1

$$\frac{1}{d}\left(Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{Ba\left(2+(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.97799, size = 136, normalized size = 1.4

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa+32\left(\sin(dx+c)^3-3\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))
```

```
*B*a - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a)/d
```

Fricas [A] time = 0.474213, size = 193, normalized size = 1.99

$$\frac{3\,(3\,A+4\,B)adx+\left(6\,Aa\cos\left(dx+c\right)^{3}+8\,(A+B)a\cos\left(dx+c\right)^{2}+3\,(3\,A+4\,B)a\cos\left(dx+c\right)+16\,(A+B)a\right)\sin\left(dx\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(3*A + 4*B)*a*d*x + (6*A*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d*x + c
)^2 + 3*(3*A + 4*B)*a*cos(d*x + c) + 16*(A + B)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27944, size = 211, normalized size = 2.18

$$\frac{3\,(3\,Aa+4\,Ba)(dx+c)+\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+49\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+28\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a)*(d*x + c) + 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 + 12*B*
a*tan(1/2*d*x + 1/2*c)^7 + 49*A*a*tan(1/2*d*x + 1/2*c)^5 + 28*B*a*tan(1/2*d
*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x + 1/2*c)
^3 + 39*A*a*tan(1/2*d*x + 1/2*c) + 36*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

3.52 $\int \cos^5(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$-\frac{a(4A+5B)\sin^3(c+dx)}{15d}+\frac{a(4A+5B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)\cos(c$$

```
[Out] (3*a*(A + B)*x)/8 + (a*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*B)*Sin[c + d*x]^3)
/(15*d)
```

Rubi [A] time = 0.133528, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3996, 3787, 2635, 8, 2633}

$$-\frac{a(4A+5B)\sin^3(c+dx)}{15d}+\frac{a(4A+5B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*a*(A + B)*x)/8 + (a*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*B)*Sin[c + d*x]^3)
/(15*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)(-5a(A+$$
$$= \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \cos^4(c+dx)\,a$$
$$= \frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aA\cos^4(c+dx)\sin(c$$
$$= \frac{a(4A+5B)\sin(c+dx)}{5d} + \frac{3a(A+B)\cos(c+dx)\sin(c+d}{8d}$$
$$= \frac{3}{8}a(A+B)x + \frac{a(4A+5B)\sin(c+dx)}{5d} + \frac{3a(A+B)\cos(c}{8}$$

Mathematica [A] time = 0.243168, size = 77, normalized size = 0.62

$$\frac{a\left(-160(2A+B)\sin^3(c+dx)+480(A+B)\sin(c+dx)+15(A+B)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))+\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(480*(A + B)*Sin[c + d*x] - 160*(2*A + B)*Sin[c + d*x]^3 + 96*A*Sin[c +
d*x]^5 + 15*(A + B)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
)/(480*d)
```

Maple [A] time = 0.094, size = 128, normalized size = 1.

$$\frac{1}{d}\left(\frac{Aa\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A] time = 0.988108, size = 167, normalized size = 1.34

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 160*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*B*a + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*si
n(2*d*x + 2*c))*B*a)/d
```

Fricas [A] time = 0.479236, size = 239, normalized size = 1.91

$$\frac{45\,(A+B)adx + \left(24\,Aa\cos{(dx+c)}^4 + 30\,(A+B)a\cos{(dx+c)}^3 + 8\,(4\,A+5\,B)a\cos{(dx+c)}^2 + 45\,(A+B)a\cos{(dx+}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(45*(A + B)*a*d*x + (24*A*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x + c
)^3 + 8*(4*A + 5*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(4*A
+ 5*B)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24219, size = 248, normalized size = 1.98

$$\frac{45\,(Aa+Ba)(dx+c) + \frac{2\left(45\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 45\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 130\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 290\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 464\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 40\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(45*(A*a + B*a)*(d*x + c) + 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 45*B*a
*tan(1/2*d*x + 1/2*c)^9 + 130*A*a*tan(1/2*d*x + 1/2*c)^7 + 290*B*a*tan(1/2*
d*x + 1/2*c)^7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*B*a*tan(1/2*d*x + 1/2
*c)^5 + 190*A*a*tan(1/2*d*x + 1/2*c)^3 + 350*B*a*tan(1/2*d*x + 1/2*c)^3 + 1
95*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 + 1)^5)/d
```

3.53 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=169

$$\frac{a^2(10A+9B)\tan^3(c+dx)}{15d}+\frac{a^2(10A+9B)\tan(c+dx)}{5d}+\frac{a^2(7A+6B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+6B)\tan(c+\cdots}{20d}$$

```
[Out] (a^2*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(10*A + 9*B)*Tan[c + d
*x])/(5*d) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A
+ 6*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*Sec[c + d*x]^3*(a^2 + a^2*S
ec[c + d*x])*Tan[c + d*x])/(5*d) + (a^2*(10*A + 9*B)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.244465, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^2(10A+9B)\tan^3(c+dx)}{15d}+\frac{a^2(10A+9B)\tan(c+dx)}{5d}+\frac{a^2(7A+6B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+6B)\tan(c+\cdots}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(10*A + 9*B)*Tan[c + d
*x])/(5*d) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A
+ 6*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*Sec[c + d*x]^3*(a^2 + a^2*S
ec[c + d*x])*Tan[c + d*x])/(5*d) + (a^2*(10*A + 9*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B\sec^3(c+dx)\left(a^2+a^2\sec(c+dx)\right)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3$$

$$= \frac{a^2(5A+6B)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{B\sec^3(c+dx)\left(a^2+\right.}{}$$

$$= \frac{a^2(5A+6B)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{B\sec^3(c+dx)\left(a^2+\right.}{}$$

$$= \frac{a^2(7A+6B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(5A+6B)\sec^3(c+}{20d}$$

$$= \frac{a^2(7A+6B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(10A+9B)\tan(c+d}{5d}$$

Mathematica [A] time = 1.33341, size = 280, normalized size = 1.66

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(7A+6B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^5*(240*(7*A + 6*
B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(14*A + 15*B)*Sin[d*x] - 240*(2*A
 + B)*Sin[2*c + d*x] + 330*A*Sin[c + 2*d*x] + 420*B*Sin[c + 2*d*x] + 330*A*
Sin[3*c + 2*d*x] + 420*B*Sin[3*c + 2*d*x] + 800*A*Sin[2*c + 3*d*x] + 720*B*
Sin[2*c + 3*d*x] + 105*A*Sin[3*c + 4*d*x] + 90*B*Sin[3*c + 4*d*x] + 105*A*S
in[5*c + 4*d*x] + 90*B*Sin[5*c + 4*d*x] + 160*A*Sin[4*c + 5*d*x] + 144*B*Si
n[4*c + 5*d*x])))/(7680*d)
```

Maple [A] time = 0.05, size = 235, normalized size = 1.4

$$\frac{7a^2A\sec(dx+c)\tan(dx+c)}{8d}+\frac{7a^2A\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{6Ba^2\tan(dx+c)}{5d}+\frac{3Ba^2\tan(dx+c)(\sec(d}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 7/8/d*a^2*A*sec(d*x+c)*tan(d*x+c)+7/8/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+6/5
/d*B*a^2*tan(d*x+c)+3/5/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+4/3/d*a^2*A*tan(d*x
+c)+2/3/d*a^2*A*tan(d*x+c)*sec(d*x+c)^2+1/2/d*B*a^2*tan(d*x+c)*sec(d*x+c)^3
+3/4/d*B*a^2*sec(d*x+c)*tan(d*x+c)+3/4/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/
4/d*a^2*A*tan(d*x+c)*sec(d*x+c)^3+1/5/d*B*a^2*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 1.00337, size = 375, normalized size = 2.22

$$\frac{160\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa^{2}+16\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)Ba^{2}+80\left(\tan\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(160*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^2 + 80*(tan(d*x + c)^3 + 3*tan(d*
x + c))*B*a^2 - 15*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 30*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.496249, size = 421, normalized size = 2.49

$$\frac{15\,(7\,A+6\,B)a^{2}\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,(7\,A+6\,B)a^{2}\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(7*A + 6*B)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(7*A +
6*B)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(10*A + 9*B)*a^2*cos
(d*x + c)^4 + 15*(7*A + 6*B)*a^2*cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*cos(d*
x + c)^2 + 30*(A + 2*B)*a^2*cos(d*x + c) + 24*B*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int A\sec^{3}\left(c+dx\right)dx+\int 2A\sec^{4}\left(c+dx\right)dx+\int A\sec^{5}\left(c+dx\right)dx+\int B\sec^{4}\left(c+dx\right)dx+\int 2B\sec^{5}\left(c+dx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*sec(c + d*x)**4, x) + I
ntegral(A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**4, x) + Integral(2
*B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A] time = 1.34674, size = 332, normalized size = 1.96

$$\frac{15\left(7\,Aa^2+6\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(7\,Aa^2+6\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(7*A*a^2 + 6*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(7*A*
a^2 + 6*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(105*A*a^2*tan(1/2*d*
x + 1/2*c)^9 + 90*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 490*A*a^2*tan(1/2*d*x + 1/
2*c)^7 - 420*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 800*A*a^2*tan(1/2*d*x + 1/2*c)^
5 + 864*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 790*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 5
40*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 375*A*a^2*tan(1/2*d*x + 1/2*c) + 390*B*a^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.54 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{a^2(8A+7B)\tan(c+dx)}{6d}+\frac{a^2(8A+7B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8A+7B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(4A-B)\tan}{}$$

```
[Out] (a^2*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*A + 7*B)*Tan[c + d*
x])/(6*d) + (a^2*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A - B)
*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + a*Sec[c + d*x])^3*Ta
n[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.228472, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(8A+7B)\tan(c+dx)}{6d}+\frac{a^2(8A+7B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8A+7B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(4A-B)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*A + 7*B)*Tan[c + d*
x])/(6*d) + (a^2*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A - B)
*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + a*Sec[c + d*x])^3*Ta
n[c + d*x])/(4*a*d)
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4ad} + \frac{\int \sec(c+dx)(a+a\sec(}{}$$
$$= \frac{(4A-B)(a+a\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{B(a+a\sec(c+}{4}$$
$$= \frac{(4A-B)(a+a\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{B(a+a\sec(c+}{4}$$
$$= \frac{a^2(8A+7B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(4A-B)(a+a\sec(c}{12}$$
$$= \frac{a^2(8A+7B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(8A+7B)\tan(c+dx}{6d}$$

Mathematica [A] time = 1.17248, size = 262, normalized size = 1.9

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(24(8A+7B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(8*A + 7*B
)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(5*A + 4*B)*Sin[c] + 3*(8*A + 15*
B)*Sin[d*x] + 24*A*Sin[2*c + d*x] + 45*B*Sin[2*c + d*x] + 136*A*Sin[c + 2*d
*x] + 128*B*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] + 24*A*Sin[2*c + 3*d*x]
+ 21*B*Sin[2*c + 3*d*x] + 24*A*Sin[4*c + 3*d*x] + 21*B*Sin[4*c + 3*d*x] + 4
0*A*Sin[3*c + 4*d*x] + 32*B*Sin[3*c + 4*d*x])))/(768*d)
```

Maple [A] time = 0.043, size = 187, normalized size = 1.4

$$\frac{5a^2A\tan(dx+c)}{3d} + \frac{7Ba^2\sec(dx+c)\tan(dx+c)}{8d} + \frac{7Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{a^2A\sec(dx+c)\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 5/3/d*a^2*A*tan(d*x+c)+7/8/d*B*a^2*sec(d*x+c)*tan(d*x+c)+7/8/d*B*a^2*ln(sec
(d*x+c)+tan(d*x+c))+1/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)
+tan(d*x+c))+4/3/d*B*a^2*tan(d*x+c)+2/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+1/3
/d*a^2*A*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*a^2*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 1.00926, size = 311, normalized size = 2.25

$$16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2-3\,Ba^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2 - 3*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^2*tan(d*x +
 c))/d
```

Fricas [A] time = 0.494235, size = 362, normalized size = 2.62

$$\frac{3(8A+7B)a^2\cos(dx+c)^4\log(\sin(dx+c)+1)-3(8A+7B)a^2\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8(5A+\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*A + 7*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*A + 7*B
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(5*A + 4*B)*a^2*cos(d*x
+ c)^3 + 3*(8*A + 7*B)*a^2*cos(d*x + c)^2 + 8*(A + 2*B)*a^2*cos(d*x + c) +
6*B*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\sec^2(c+dx)\,dx+\int 2A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int B\sec^3(c+dx)\,dx+\int 2B\sec^4(c+dx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*sec(c + d*x)**3, x) + I
ntegral(A*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**3, x) + Integral(2
*B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))
```

Giac [A] time = 1.36399, size = 286, normalized size = 2.07

$$\frac{3\left(8\,Aa^2+7\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(8\,Aa^2+7\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,B\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(8*A*a^2 + 7*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*A*a^2
 + 7*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*A*a^2*tan(1/2*d*x +
1/2*c)^7 + 21*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 88*A*a^2*tan(1/2*d*x + 1/2*c)^
5 - 77*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 136*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 83
*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*tan(1/2*d*x + 1/2*c) - 75*B*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.55 $\int \sec(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{2a^2(3A+2B)\tan(c+dx)}{3d}+\frac{a^2(3A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3A+2B)\tan(c+dx)\sec(c+dx)}{6d}+\frac{B\tan(c+dx}{}$$

```
[Out] (a^2*(3*A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*(3*A + 2*B)*Tan[c +
d*x])/(3*d) + (a^2*(3*A + 2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (B*(a + a
*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.113348, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4001, 3788, 3767, 8, 4046, 3770}

$$\frac{2a^2(3A+2B)\tan(c+dx)}{3d}+\frac{a^2(3A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3A+2B)\tan(c+dx)\sec(c+dx)}{6d}+\frac{B\tan(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*(3*A + 2*B)*Tan[c +
d*x])/(3*d) + (a^2*(3*A + 2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (B*(a + a
*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}(3A+2B)\int \sec(c+d$$
$$= \frac{B(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}(3A+2B)\int \sec(c+d$$
$$= \frac{a^2(3A+2B)\sec(c+dx)\tan(c+dx)}{6d} + \frac{B(a+a\sec(c+dx))^2}{3d}$$
$$= \frac{a^2(3A+2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2(3A+2B)\tan(c+dx}{3d}$$

Mathematica [B] time = 6.1085, size = 481, normalized size = 4.67

$$a^2\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(\frac{4(6A+5B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{\left(\sin\left(\frac{c}{2}\right)+\cos\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*(-6*(3*A + 2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(3*A + 2
*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*B*Sin[(d*x)/2])/((Cos[c/2
] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((3*A + 7*B)*Cos[c
/2] - (3*A + 5*B)*Sin[c/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2) + (4*(6*A + 5*B)*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*B*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/
2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - ((3*A + 7*B)*Cos[c/2] + (3*A
 + 5*B)*Sin[c/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) + (4*(6*A + 5*B)*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]))))/(48*d*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.039, size = 141, normalized size = 1.4

$$\frac{3a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{5Ba^2\tan(dx+c)}{3d} + 2\frac{a^2A\tan(dx+c)}{d} + \frac{Ba^2\sec(dx+c)\tan(dx+c)}{d} + \frac{Ba^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 3/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+5/3/d*B*a^2*tan(d*x+c)+2/d*a^2*A*tan(
d*x+c)+1/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+
```

```
1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.995663, size = 225, normalized size = 2.18

$$4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2-3Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-6Ba^2\left(\frac{2}{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 - 3*A*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 24*A*a^2*tan(d
*x + c) + 12*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.493973, size = 315, normalized size = 3.06

$$\frac{3(3A+2B)a^2\cos(dx+c)^3\log(\sin(dx+c)+1)-3(3A+2B)a^2\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2(6A+\right.}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(3*A + 2*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*A + 2*B
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(6*A + 5*B)*a^2*cos(d*x
+ c)^2 + 3*(A + 2*B)*a^2*cos(d*x + c) + 2*B*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\sec(c+dx)\,dx+\int 2A\sec^2(c+dx)\,dx+\int A\sec^3(c+dx)\,dx+\int B\sec^2(c+dx)\,dx+\int 2B\sec^3(c+dx)\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sec(c + d*x)**2, x) + Inte
gral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**2, x) + Integral(2*B*
sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))
```

Giac [A] time = 1.24975, size = 240, normalized size = 2.33

$$\frac{3\left(3Aa^2+2Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(3Aa^2+2Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(9Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(3*A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^2
+ 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*tan(1/2*d*x + 1/2*c)^3 -
 16*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 18*B*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.56 $\int (a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=82

$$\frac{a^2(2A+3B)\tan(c+dx)}{2d}+\frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{B\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}$$

```
[Out] a^2*A*x + (a^2*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A + 3*B)*
Tan[c + d*x])/(2*d) + (B*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0836368, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3917, 3914, 3767, 8, 3770}

$$\frac{a^2(2A+3B)\tan(c+dx)}{2d}+\frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{B\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*A*x + (a^2*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A + 3*B)*
Tan[c + d*x])/(2*d) + (B*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B\left(a^2+a^2\sec(c+dx)\right)\tan(c+dx)}{2d} + \frac{1}{2}\int (a+a\sec(c+dx))(2aA+a(2$$
$$= a^2Ax + \frac{B\left(a^2+a^2\sec(c+dx)\right)\tan(c+dx)}{2d} + \frac{1}{2}\left(a^2(2A+3B)\right)\int \sec^2(c$$
$$= a^2Ax + \frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{B\left(a^2+a^2\sec(c+dx)\right)\tan(c}{2d}$$
$$= a^2Ax + \frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(2A+3B)\tan(c+dx)}{2d} + $$

Mathematica [B] time = 1.25495, size = 307, normalized size = 3.74

$$a^2\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(\frac{4(A+2B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{d\left(\sin\left(\frac{c}{2}\right)+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*(4*A*x - (2*(4*A + 3*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d +
 (2*(4*A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + B/(d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(A + 2*B)*Sin[(d*x)/2])/(d*(Cos[c/2]
- Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2) + (4*(A + 2*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(16*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.038, size = 113, normalized size = 1.4

$$a^2Ax + \frac{Aa^2c}{d} + \frac{3\,Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + 2\,\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\,\frac{Ba^2\tan(dx+c)}{d} + \frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] a^2*A*x+1/d*A*a^2*c+3/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*A*ln(sec(
d*x+c)+tan(d*x+c))+2/d*B*a^2*tan(d*x+c)+1/d*a^2*A*tan(d*x+c)+1/2/d*B*a^2*se
c(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.979652, size = 173, normalized size = 2.11

$$\frac{4\,(dx+c)Aa^2 - Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 8\,Aa^2\log(\sec(dx+c)+\tan(dx+c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^2 - B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 8*A*a^2*log(sec(d*x + c) + tan(
d*x + c)) + 4*B*a^2*log(sec(d*x + c) + tan(d*x + c)) + 4*A*a^2*tan(d*x + c)
 + 8*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.496223, size = 297, normalized size = 3.62

$$\frac{4\,Aa^2dx\cos\left(dx+c\right)^2+\left(4\,A+3\,B\right)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,A+3\,B\right)a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*A*a^2*d*x*cos(d*x + c)^2 + (4*A + 3*B)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (4*A + 3*B)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(A + 2*B)*a^2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\,dx+\int 2A\sec\left(c+dx\right)dx+\int A\sec^2\left(c+dx\right)dx+\int B\sec\left(c+dx\right)dx+\int 2B\sec^2\left(c+dx\right)dx+\int B\sec^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A, x) + Integral(2*A*sec(c + d*x), x) + Integral(A*sec(c + d
*x)**2, x) + Integral(B*sec(c + d*x), x) + Integral(2*B*sec(c + d*x)**2, x)
 + Integral(B*sec(c + d*x)**3, x))
```

Giac [B] time = 1.4353, size = 208, normalized size = 2.54

$$\frac{2\left(dx+c\right)Aa^2+\left(4\,Aa^2+3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(4\,Aa^2+3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Aa^2\,\mathrm{ta}\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + (4*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (4*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^2*
tan(1/2*d*x + 1/2*c)^3 + 3*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^2*tan(1/2*d
*x + 1/2*c) - 5*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

3.57 $\int \cos(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(A+2B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2A+B)+\frac{B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{d}$$

```
[Out] a^2*(2*A + B)*x + (a^2*(A + 2*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A - B)*Si
n[c + d*x])/d + (B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/d
```

Rubi [A] time = 0.129521, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4018, 3996, 3770}

$$\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(A+2B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2A+B)+\frac{B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*(2*A + B)*x + (a^2*(A + 2*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A - B)*Si
n[c + d*x])/d + (B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{d} + \int \cos(c+dx)(a+a$$

$$= \frac{a^2(A-B)\sin(c+dx)}{d} + \frac{B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx}{d}$$

$$= a^2(2A+B)x + \frac{a^2(A-B)\sin(c+dx)}{d} + \frac{B\left(a^2+a^2\sec(c+\right.}{d}$$

$$= a^2(2A+B)x + \frac{a^2(A+2B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2(A-}{}$$

Mathematica [B] time = 1.60464, size = 258, normalized size = 3.53

$$a^2\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(-\frac{(A+2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{(A+2B)\log(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*((2*A + B)*x - ((A + 2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d
 + ((A + 2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (A*Cos[d*x]*Sin
[c])/d + (A*Cos[c]*Sin[d*x])/d + (B*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (B*Sin[(d*x)/2])/(d*(Cos[c/2] + Si
n[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(4*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.062, size = 107, normalized size = 1.5

$$2a^2Ax + Ba^2x + \frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2A\sin(dx+c)}{d} + 2\frac{Aa^2c}{d} + 2\frac{Ba^2\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 2*a^2*A*x+B*a^2*x+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*A*sin(d*x+c)/d+2/
d*A*a^2*c+2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^2*tan(d*x+c)+1/d*B*a^
2*c
```

Maxima [A] time = 1.02609, size = 142, normalized size = 1.95

$$\frac{4(dx+c)Aa^2 + 2(dx+c)Ba^2 + Aa^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Ba^2(\log(\sin(dx+c)+1) - \log}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(4*(d*x + c)*A*a^2 + 2*(d*x + c)*B*a^2 + A*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
```

```
- 1)) + 2*A*a^2*sin(d*x + c) + 2*B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.495015, size = 278, normalized size = 3.81

$$\frac{2\,(2\,A+B)a^{2}dx\cos\left(dx+c\right)+(A+2\,B)a^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(A+2\,B)a^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right.\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*(2*A + B)*a^2*d*x*cos(d*x + c) + (A + 2*B)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (A + 2*B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a^
2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int A\cos\left(c+dx\right)dx+\int 2A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*cos(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(A*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)
*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)*sec(c + d*x)**2, x) + Integra
l(B*cos(c + d*x)*sec(c + d*x)**3, x))
```

Giac [B] time = 1.26769, size = 212, normalized size = 2.9

$$\frac{\left(2\,Aa^{2}+Ba^{2}\right)(dx+c)+\left(Aa^{2}+2\,Ba^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(Aa^{2}+2\,Ba^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^{2}\,t\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((2*A*a^2 + B*a^2)*(d*x + c) + (A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x +
 1/2*c) - B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.58 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=88

$$\frac{a^2(3A+2B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(3A+4B)+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{a^2B\tanh^{-1}(\sin(c+d$$

```
[Out] (a^2*(3*A + 4*B)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*A + 2*B)*
Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(2*d)
```

Rubi [A] time = 0.144874, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4017, 3996, 3770}

$$\frac{a^2(3A+2B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(3A+4B)+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{a^2B\tanh^{-1}(\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 4*B)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*A + 2*B)*
Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(2*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{A\cos(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c$$

$$= \frac{a^2(3A+2B)\sin(c+dx)}{2d} + \frac{A\cos(c+dx)\left(a^2+a^2\sec(c+dx\right.}{2d}$$

$$= \frac{1}{2}a^2(3A+4B)x + \frac{a^2(3A+2B)\sin(c+dx)}{2d} + \frac{A\cos(c+dx)}{}$$

$$= \frac{1}{2}a^2(3A+4B)x + \frac{a^2B\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2(3A+2B)\,\mathrm{s}}{2d}$$

Mathematica [A] time = 0.159389, size = 96, normalized size = 1.09

$$\frac{a^2\left(4(2A+B)\sin(c+dx)+A\sin(2(c+dx))+6Adx-4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4B\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(6*A*d*x + 8*B*d*x - 4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(2*A + B)*Sin[c + d*x] + A*S
in[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.076, size = 108, normalized size = 1.2

$$\frac{a^2A\cos(dx+c)\sin(dx+c)}{2d} + \frac{3a^2Ax}{2} + \frac{3a^2Ac}{2d} + \frac{Ba^2\sin(dx+c)}{d} + 2\frac{a^2A\sin(dx+c)}{d} + 2Ba^2x + 2\frac{Ba^2c}{d} + \frac{Ba^2\ln(\mathrm{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2*a^2*A*cos(d*x+c)*sin(d*x+c)/d+3/2*a^2*A*x+3/2/d*A*a^2*c+1/d*B*a^2*sin(d
*x+c)+2*a^2*A*sin(d*x+c)/d+2*B*a^2*x+2/d*B*a^2*c+1/d*B*a^2*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 0.994061, size = 136, normalized size = 1.55

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+4\,(dx+c)Aa^2+8\,(dx+c)Ba^2+2\,Ba^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*A*a^2 + 8*(d*x +
c)*B*a^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*A*a^
2*sin(d*x + c) + 4*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.496096, size = 194, normalized size = 2.2

$$\frac{(3\,A+4\,B)a^2dx+Ba^2\log\left(\sin\left(dx+c\right)+1\right)-Ba^2\log\left(-\sin\left(dx+c\right)+1\right)+\left(Aa^2\cos\left(dx+c\right)+2\,(2\,A+B)a^2\right)\sin\left(dx\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((3*A + 4*B)*a^2*d*x + B*a^2*log(sin(d*x + c) + 1) - B*a^2*log(-sin(d*x
 + c) + 1) + (A*a^2*cos(d*x + c) + 2*(2*A + B)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3544, size = 196, normalized size = 2.23

$$\frac{2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(3\,Aa^2+4\,Ba^2\right)(dx+c)+\frac{2\left(3\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*B*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (3*A*a^2 + 4*B*a^2)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*tan(1/2*d*x + 1/2*c)
+ 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.59 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{2a^2(2A+3B)\sin(c+dx)}{3d}+\frac{a^2(2A+3B)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(2A+3B)+\frac{A\sin(c+dx)\cos^2(c+dx)(a\sec(c$$

```
[Out] (a^2*(2*A + 3*B)*x)/2 + (2*a^2*(2*A + 3*B)*Sin[c + d*x])/(3*d) + (a^2*(2*A
+ 3*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.153287, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4013, 3788, 2637, 4045, 8}

$$\frac{2a^2(2A+3B)\sin(c+dx)}{3d}+\frac{a^2(2A+3B)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(2A+3B)+\frac{A\sin(c+dx)\cos^2(c+dx)(a\sec(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(2*A + 3*B)*x)/2 + (2*a^2*(2*A + 3*B)*Sin[c + d*x])/(3*d) + (a^2*(2*A
+ 3*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}(2A+$$
$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}(2A+$$
$$= \frac{2a^2(2A+3B)\sin(c+dx)}{3d} + \frac{a^2(2A+3B)\cos(c+dx)\sin}{6d}$$
$$= \frac{1}{2}a^2(2A+3B)x + \frac{2a^2(2A+3B)\sin(c+dx)}{3d} + \frac{a^2(2A+3}{}$$

Mathematica [A] time = 0.171502, size = 61, normalized size = 0.6

$$\frac{a^2(3(7A+8B)\sin(c+dx)+3(2A+B)\sin(2(c+dx))+A\sin(3(c+dx))+12Adx+18Bdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(12*A*d*x + 18*B*d*x + 3*(7*A + 8*B)*Sin[c + d*x] + 3*(2*A + B)*Sin[2*
(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.078, size = 116, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2A\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + 2a^2A\,(1/2\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2) + Ba^2\left(\frac{\cos(dx+c)\sin}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*A*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*A*sin
(d*x+c)+2*B*a^2*sin(d*x+c)+B*a^2*(d*x+c))
```

Maxima [A] time = 1.01302, size = 149, normalized size = 1.46

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2-12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 6*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 12*(d*x + c)*
```

```
B*a^2 - 12*A*a^2*sin(d*x + c) - 24*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.462819, size = 165, normalized size = 1.62

$$\frac{3\,(2\,A+3\,B)a^2dx+\left(2\,Aa^2\cos\left(dx+c\right)^2+3\,(2\,A+B)a^2\cos\left(dx+c\right)+2\,(5\,A+6\,B)a^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(2*A + 3*B)*a^2*d*x + (2*A*a^2*cos(d*x + c)^2 + 3*(2*A + B)*a^2*cos(
d*x + c) + 2*(5*A + 6*B)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40303, size = 192, normalized size = 1.88

$$\frac{3\left(2\,Aa^2+3\,Ba^2\right)(dx+c)+\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(2*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 +
9*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a^2
*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.60 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(5A+4B)\sin(c+dx)\cos^2(c+dx)}{12d}+\frac{a^2(7A+8B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^2x(7A+$$

```
[Out] (a^2*(7*A + 8*B)*x)/8 + (a^2*(4*A + 5*B)*Sin[c + d*x])/(3*d) + (a^2*(7*A +
8*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 4*B)*Cos[c + d*x]^2*Sin
[c + d*x])/(12*d) + (A*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(4*d)
```

Rubi [A] time = 0.230517, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(5A+4B)\sin(c+dx)\cos^2(c+dx)}{12d}+\frac{a^2(7A+8B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^2x(7A+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^2*(7*A + 8*B)*x)/8 + (a^2*(4*A + 5*B)*Sin[c + d*x])/(3*d) + (a^2*(7*A +
8*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 4*B)*Cos[c + d*x]^2*Sin
[c + d*x])/(12*d) + (A*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(4*d)
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{A\cos^3(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3\\ &= \frac{a^2(5A+4B)\cos^2(c+dx)\sin(c+dx)}{12d}+\frac{A\cos^3(c+dx)\left(a^2+\right.}{}\\ &= \frac{a^2(5A+4B)\cos^2(c+dx)\sin(c+dx)}{12d}+\frac{A\cos^3(c+dx)\left(a^2+\right.}{}\\ &= \frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(7A+8B)\cos(c+dx)\sin(c+}{8d}\\ &= \frac{1}{8}a^2(7A+8B)x+\frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(7A+8B)\cos}{}\end{aligned}$$

Mathematica [A] time = 0.354537, size = 86, normalized size = 0.64

$$\frac{a^2(24(6A+7B)\sin(c+dx)+48(A+B)\sin(2(c+dx))+16A\sin(3(c+dx))+3A\sin(4(c+dx))+84Ac+84Adx+8B}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(84*A*c + 84*A*d*x + 96*B*d*x + 24*(6*A + 7*B)*Sin[c + d*x] + 48*(A +
B)*Sin[2*(c + d*x)] + 16*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Si
n[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.087, size = 154, normalized size = 1.1

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ba^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2a^2A\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*
B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*A*(1/2*cos(d*x+c)*sin(d
```

```
*x+c)+1/2*d*x+1/2*c)+B*a^2*sin(d*x+c))
```

Maxima [A] time = 1.00995, size = 194, normalized size = 1.44

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{2}-24\left(2\,dx+2\,c+\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*
c))*A*a^2 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 48*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^2 - 96*B*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.474397, size = 213, normalized size = 1.58

$$\frac{3\left(7\,A+8\,B\right)a^{2}dx+\left(6\,Aa^{2}\cos\left(dx+c\right)^{3}+8\left(2\,A+B\right)a^{2}\cos\left(dx+c\right)^{2}+3\left(7\,A+8\,B\right)a^{2}\cos\left(dx+c\right)+8\left(4\,A+5\,B\right)a^{2}\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(7*A + 8*B)*a^2*d*x + (6*A*a^2*cos(d*x + c)^3 + 8*(2*A + B)*a^2*cos
(d*x + c)^2 + 3*(7*A + 8*B)*a^2*cos(d*x + c) + 8*(4*A + 5*B)*a^2)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21159, size = 238, normalized size = 1.76

$$\frac{3\left(7\,Aa^{2}+8\,Ba^{2}\right)\left(dx+c\right)+\frac{2\left(21\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+24\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+77\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+88\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+83\,Aa^{2}\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2)*(d*x + c) + 2*(21*A*a^2*tan(1/2*d*x + 1/2*c)^7
+ 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 88*B*
a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136*B*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 75*A*a^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.61 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=160

$$-\frac{a^2(9A+10B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+10B)\sin(c+dx)}{5d}+\frac{a^2(6A+5B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(6A+7B)\sin(c}{8}$$

```
[Out] (a^2*(6*A + 7*B)*x)/8 + (a^2*(9*A + 10*B)*Sin[c + d*x])/(5*d) + (a^2*(6*A +
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(6*A + 5*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(9*A + 10*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.251574, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^2(9A+10B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+10B)\sin(c+dx)}{5d}+\frac{a^2(6A+5B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(6A+7B)\sin(c}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^2*(6*A + 7*B)*x)/8 + (a^2*(9*A + 10*B)*Sin[c + d*x])/(5*d) + (a^2*(6*A +
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(6*A + 5*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(9*A + 10*B)*Sin[c + d*x]^3)/(15*d)
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{A\cos^4(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4$$
$$= \frac{a^2(6A+5B)\cos^3(c+dx)\sin(c+dx)}{20d}+\frac{A\cos^4(c+dx)\left(a^2+\right.}{}$$
$$= \frac{a^2(6A+5B)\cos^3(c+dx)\sin(c+dx)}{20d}+\frac{A\cos^4(c+dx)\left(a^2+\right.}{}$$
$$= \frac{a^2(6A+7B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2(6A+5B)\cos^3(c+}{20d}$$
$$= \frac{1}{8}a^2(6A+7B)x+\frac{a^2(9A+10B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)}{}$$

Mathematica [A] time = 0.475953, size = 108, normalized size = 0.68

$$\frac{a^2(60(11A+12B)\sin(c+dx)+240(A+B)\sin(2(c+dx))+90A\sin(3(c+dx))+30A\sin(4(c+dx))+6A\sin(5(c+dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(360*A*c + 360*A*d*x + 420*B*d*x + 60*(11*A + 12*B)*Sin[c + d*x] + 240
*(A + B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 80*B*Sin[3*(c + d*x)] +
 30*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(48
0*d)
```

Maple [A] time = 0.094, size = 186, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*A*(1/4*(cos(d*x
```

```
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*B*a^2*(2+cos(d*x+c)^2)*
sin(d*x+c)+1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.01353, size = 240, normalized size = 1.5

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{2}-160\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{2}+30\,(12\,dx+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 30*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))
*B*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2)/d
```

Fricas [A] time = 0.480349, size = 271, normalized size = 1.69

$$\frac{15\,(6\,A+7\,B)a^{2}dx+\left(24\,Aa^{2}\cos\left(dx+c\right)^{4}+30\,(2\,A+B)a^{2}\cos\left(dx+c\right)^{3}+8\,(9\,A+10\,B)a^{2}\cos\left(dx+c\right)^{2}+15\,(6\,A+}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(6*A + 7*B)*a^2*d*x + (24*A*a^2*cos(d*x + c)^4 + 30*(2*A + B)*a^2
*cos(d*x + c)^3 + 8*(9*A + 10*B)*a^2*cos(d*x + c)^2 + 15*(6*A + 7*B)*a^2*co
s(d*x + c) + 16*(9*A + 10*B)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30281, size = 284, normalized size = 1.78

$$\frac{15\left(6\,Aa^{2}+7\,Ba^{2}\right)(dx+c)+\frac{2\left(90\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+105\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+420\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+490\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+864\,Aa^{2}\,\mathrm{ta}}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(6*A*a^2 + 7*B*a^2)*(d*x + c) + 2*(90*A*a^2*tan(1/2*d*x + 1/2*c)^
9 + 105*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 420*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 4
90*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*
a^2*tan(1/2*d*x + 1/2*c)^5 + 540*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 790*B*a^2*t
an(1/2*d*x + 1/2*c)^3 + 390*A*a^2*tan(1/2*d*x + 1/2*c) + 375*B*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.62 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=210

$$\frac{a^3(19A+17B)\tan^3(c+dx)}{15d}+\frac{a^3(19A+17B)\tan(c+dx)}{5d}+\frac{a^3(26A+23B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(22A+21B)\text{ta}}{}$$

```
[Out] (a^3*(26*A + 23*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(19*A + 17*B)*Tan[c
 + d*x])/(5*d) + (a^3*(26*A + 23*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^
3*(22*A + 21*B)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*B*Sec[c + d*x]^3*(
a + a*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((3*A + 4*B)*Sec[c + d*x]^3*(a^
3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(15*d) + (a^3*(19*A + 17*B)*Tan[c + d*x
]^3)/(15*d)
```

Rubi [A] time = 0.399342, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^3(19A+17B)\tan^3(c+dx)}{15d}+\frac{a^3(19A+17B)\tan(c+dx)}{5d}+\frac{a^3(26A+23B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(22A+21B)\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^3*(26*A + 23*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(19*A + 17*B)*Tan[c
 + d*x])/(5*d) + (a^3*(26*A + 23*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^
3*(22*A + 21*B)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*B*Sec[c + d*x]^3*(
a + a*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((3*A + 4*B)*Sec[c + d*x]^3*(a^
3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(15*d) + (a^3*(19*A + 17*B)*Tan[c + d*x
]^3)/(15*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aB\sec^3(c+dx)(a+a\sec(c+dx))^2\tan(c+dx)}{6d}+\frac{1}{6}\int \sec^3($$

$$= \frac{aB\sec^3(c+dx)(a+a\sec(c+dx))^2\tan(c+dx)}{6d}+\frac{(3A+4B)}{}$$

$$= \frac{a^3(22A+21B)\sec^3(c+dx)\tan(c+dx)}{40d}+\frac{aB\sec^3(c+dx)(a}{}$$

$$= \frac{a^3(22A+21B)\sec^3(c+dx)\tan(c+dx)}{40d}+\frac{aB\sec^3(c+dx)(a}{}$$

$$= \frac{a^3(26A+23B)\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^3(22A+21B)\sec}{}$$

$$= \frac{a^3(26A+23B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(19A+17B)\tan(c}{5d}$$

Mathematica [A] time = 2.041, size = 346, normalized size = 1.65

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(480(26A+23B)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(480*(26*A + 2
3*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-320*(19*A + 17*B)*Sin[c] + 750*(2
*A + 3*B)*Sin[d*x] + 1500*A*Sin[2*c + d*x] + 2250*B*Sin[2*c + d*x] + 7680*A
*Sin[c + 2*d*x] + 7680*B*Sin[c + 2*d*x] - 1440*A*Sin[3*c + 2*d*x] - 480*B*S
in[3*c + 2*d*x] + 1890*A*Sin[2*c + 3*d*x] + 1955*B*Sin[2*c + 3*d*x] + 1890*
A*Sin[4*c + 3*d*x] + 1955*B*Sin[4*c + 3*d*x] + 3648*A*Sin[3*c + 4*d*x] + 32
64*B*Sin[3*c + 4*d*x] + 390*A*Sin[4*c + 5*d*x] + 345*B*Sin[4*c + 5*d*x] + 3
90*A*Sin[6*c + 5*d*x] + 345*B*Sin[6*c + 5*d*x] + 608*A*Sin[5*c + 6*d*x] + 5
44*B*Sin[5*c + 6*d*x])))/(61440*d)
```

Maple [A] time = 0.057, size = 281, normalized size = 1.3

$$\frac{13\,Aa^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{13\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{34\,Ba^3\tan(dx+c)}{15d}+\frac{17\,Ba^3\tan(dx+c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 13/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3
4/15/d*B*a^3*tan(d*x+c)+17/15/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+38/15/d*A*a^3
*tan(d*x+c)+19/15/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+23/24/d*B*a^3*tan(d*x+c)*
sec(d*x+c)^3+23/16/d*B*a^3*sec(d*x+c)*tan(d*x+c)+23/16/d*B*a^3*ln(sec(d*x+c
)+tan(d*x+c))+3/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+3/5/d*B*a^3*tan(d*x+c)*se
c(d*x+c)^4+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+1/6/d*B*a^3*tan(d*x+c)*sec(d
*x+c)^5
```

Maxima [B] time = 1.02866, size = 547, normalized size = 2.6

$$32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+96\left(3\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 96*(3*tan(d*x + c)^5 + 10*tan
(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 160*(tan(d*x + c)^3 + 3*tan(d*x + c)
)*B*a^3 - 5*B*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 90*A*a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) - 90*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 120*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 -
1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.50713, size = 485, normalized size = 2.31

$$15\,(26\,A+23\,B)a^3\cos(dx+c)^6\log(\sin(dx+c)+1)-15\,(26\,A+23\,B)a^3\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(26*A + 23*B)*a^3*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(26*A
 + 23*B)*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(32*(19*A + 17*B)*a^
```

```
3*cos(d*x + c)^5 + 15*(26*A + 23*B)*a^3*cos(d*x + c)^4 + 16*(19*A + 17*B)*a
^3*cos(d*x + c)^3 + 10*(18*A + 23*B)*a^3*cos(d*x + c)^2 + 48*(A + 3*B)*a^3*
cos(d*x + c) + 40*B*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\sec^3(c+dx)\,dx + \int 3A\sec^4(c+dx)\,dx + \int 3A\sec^5(c+dx)\,dx + \int A\sec^6(c+dx)\,dx + \int B\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**3, x) + Integral(3*A*sec(c + d*x)**4, x) + I
ntegral(3*A*sec(c + d*x)**5, x) + Integral(A*sec(c + d*x)**6, x) + Integral
(B*sec(c + d*x)**4, x) + Integral(3*B*sec(c + d*x)**5, x) + Integral(3*B*se
c(c + d*x)**6, x) + Integral(B*sec(c + d*x)**7, x))
```

Giac [A] time = 1.39501, size = 378, normalized size = 1.8

$$15\left(26\,Aa^3 + 23\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(26\,Aa^3 + 23\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(390\,Aa^3\tan\left(\frac{1}{2}\,dx + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(26*A*a^3 + 23*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(26
*A*a^3 + 23*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(390*A*a^3*tan(1/
2*d*x + 1/2*c)^11 + 345*B*a^3*tan(1/2*d*x + 1/2*c)^11 - 2210*A*a^3*tan(1/2*
d*x + 1/2*c)^9 - 1955*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 5148*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 + 4554*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 5988*A*a^3*tan(1/2*d*x +
1/2*c)^5 - 5814*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 4190*A*a^3*tan(1/2*d*x + 1/2
*c)^3 + 3165*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 1530*A*a^3*tan(1/2*d*x + 1/2*c)
 - 1575*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.63 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$\frac{a^3(15A+13B)\tan^3(c+dx)}{60d}+\frac{a^3(15A+13B)\tan(c+dx)}{5d}+\frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(15A+13B)\text{t}}{}$$

```
[Out] (a^3*(15*A + 13*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(15*A + 13*B)*Tan[c
+ d*x])/(5*d) + (3*a^3*(15*A + 13*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + ((
5*A - B)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (B*(a + a*Sec[c + d*
x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(15*A + 13*B)*Tan[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.268072, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(15A+13B)\tan^3(c+dx)}{60d}+\frac{a^3(15A+13B)\tan(c+dx)}{5d}+\frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(15A+13B)\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^3*(15*A + 13*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(15*A + 13*B)*Tan[c
+ d*x])/(5*d) + (3*a^3*(15*A + 13*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + ((
5*A - B)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (B*(a + a*Sec[c + d*
x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(15*A + 13*B)*Tan[c + d*x]^3)/(60*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5ad} + \frac{\int \sec(c+dx)(a+a\sec(\ldots}{\ldots}$$

$$= \frac{(5A-B)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{B(a+a\sec(c+\ldots}{5\ldots}$$

$$= \frac{(5A-B)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{B(a+a\sec(c+\ldots}{5\ldots}$$

$$= \frac{(5A-B)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{B(a+a\sec(c+\ldots}{5\ldots}$$

$$= \frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{20d} + \frac{3a^3(15A+13B)\sec(c\ldots}{40d}$$

$$= \frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(15A+13B)\tan(c\ldots}{5d}$$

Mathematica [A] time = 1.47429, size = 294, normalized size = 1.8

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(15A+13B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(15*A + 1
3*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(30*A + 29*B)*Sin[d*x] - 240*(5
*A + 3*B)*Sin[2*c + d*x] + 570*A*Sin[c + 2*d*x] + 750*B*Sin[c + 2*d*x] + 57
0*A*Sin[3*c + 2*d*x] + 750*B*Sin[3*c + 2*d*x] + 1680*A*Sin[2*c + 3*d*x] + 1
520*B*Sin[2*c + 3*d*x] - 120*A*Sin[4*c + 3*d*x] + 225*A*Sin[3*c + 4*d*x] +
195*B*Sin[3*c + 4*d*x] + 225*A*Sin[5*c + 4*d*x] + 195*B*Sin[5*c + 4*d*x] +
360*A*Sin[4*c + 5*d*x] + 304*B*Sin[4*c + 5*d*x])))/(15360*d)
```

Maple [A] time = 0.049, size = 234, normalized size = 1.4

$$3\,{\frac{A{a}^{3}\tan \left(dx+c \right) }{d}}+{\frac{13\,B{a}^{3}\sec \left(dx+c \right) \tan \left(dx+c \right) }{8\,d}}+{\frac{13\,B{a}^{3}\ln \left(\sec \left(dx+c \right) +\tan \left(dx+c \right) \right) }{8\,d}}+{\frac{15\,A{a}^{3}\sec \left(dx+c \right) \mathrm{t}}{8\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 3/d*A*a^3*tan(d*x+c)+13/8/d*B*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*B*a^3*ln(sec
(d*x+c)+tan(d*x+c))+15/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*A*a^3*ln(sec(
d*x+c)+tan(d*x+c))+38/15/d*B*a^3*tan(d*x+c)+19/15/d*B*a^3*tan(d*x+c)*sec(d*
x+c)^2+1/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4/d*B*a^3*tan(d*x+c)*sec(d*x+c)^
3+1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+1/5/d*B*a^3*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [B] time = 1.01712, size = 455, normalized size = 2.79

$$240\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}+16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Ba^{3}+240\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 - 15*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 45*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 180*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.501048, size = 431, normalized size = 2.64

$$15\,(15\,A+13\,B)a^{3}\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,(15\,A+13\,B)a^{3}\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(15*A + 13*B)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(15*A
 + 13*B)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(45*A + 38*B)*a^3
*cos(d*x + c)^4 + 15*(15*A + 13*B)*a^3*cos(d*x + c)^3 + 8*(15*A + 19*B)*a^3
*cos(d*x + c)^2 + 30*(A + 3*B)*a^3*cos(d*x + c) + 24*B*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int A \sec^2(c+dx)\,dx + \int 3A \sec^3(c+dx)\,dx + \int 3A \sec^4(c+dx)\,dx + \int A \sec^5(c+dx)\,dx + \int B \sec^3(c+dx)\,dx \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sec(c + d*x)**3, x) + I
ntegral(3*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral
(B*sec(c + d*x)**3, x) + Integral(3*B*sec(c + d*x)**4, x) + Integral(3*B*se
c(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A] time = 1.34149, size = 332, normalized size = 2.04

$$\frac{15\left(15\,Aa^3 + 13\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(15\,Aa^3 + 13\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(225\,Aa^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(15*A*a^3 + 13*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(15
*A*a^3 + 13*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(225*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 195*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 1050*A*a^3*tan(1/2*d*
x + 1/2*c)^7 - 910*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1920*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 1664*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1830*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1330*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 735*A*a^3*tan(1/2*d*x + 1/2*c)
+ 765*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.64 $\int \sec(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{a^3(4A+3B)\tan^3(c+dx)}{12d}+\frac{a^3(4A+3B)\tan(c+dx)}{d}+\frac{5a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(4A+3B)\tan(c+}{8d}$$

```
[Out] (5*a^3*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(4*A + 3*B)*Tan[c +
d*x])/d + (3*a^3*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (B*(a + a*S
ec[c + d*x])^3*Tan[c + d*x])/(4*d) + (a^3*(4*A + 3*B)*Tan[c + d*x]^3)/(12*d
)
```

Rubi [A] time = 0.142558, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(4A+3B)\tan^3(c+dx)}{12d}+\frac{a^3(4A+3B)\tan(c+dx)}{d}+\frac{5a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(4A+3B)\tan(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(4*A + 3*B)*Tan[c +
d*x])/d + (3*a^3*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (B*(a + a*S
ec[c + d*x])^3*Tan[c + d*x])/(4*d) + (a^3*(4*A + 3*B)*Tan[c + d*x]^3)/(12*d
)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}(4A+3B)\int \sec(c+d$$

$$= \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}(4A+3B)\int \left(a^3\sec(c\right.$$

$$= \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}\left(a^3(4A+3B)\right)\int \sec(c$$

$$= \frac{a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{4d} + \frac{3a^3(4A+3B)\sec(c+dx}{8d}$$

$$= \frac{5a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(4A+3B)\tan(c+dx}{d}$$

Mathematica [B] time = 1.29325, size = 273, normalized size = 2.18

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(120(4A+3B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(120*(4*A + 3*
B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(11*A + 9*B)*Sin[c] + (36*A + 69
*B)*Sin[d*x] + 36*A*Sin[2*c + d*x] + 69*B*Sin[2*c + d*x] + 280*A*Sin[c + 2*
d*x] + 264*B*Sin[c + 2*d*x] - 72*A*Sin[3*c + 2*d*x] - 24*B*Sin[3*c + 2*d*x]
 + 36*A*Sin[2*c + 3*d*x] + 45*B*Sin[2*c + 3*d*x] + 36*A*Sin[4*c + 3*d*x] +
45*B*Sin[4*c + 3*d*x] + 88*A*Sin[3*c + 4*d*x] + 72*B*Sin[3*c + 4*d*x])))/(1
536*d)
```

Maple [A] time = 0.048, size = 188, normalized size = 1.5

$$\frac{5\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + 3\,\frac{Ba^3\tan(dx+c)}{d} + \frac{11\,Aa^3\tan(dx+c)}{3\,d} + \frac{15\,Ba^3\sec(dx+c)\tan(dx+c)}{8\,d} + \frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^3*tan(d*x+c)+11/3/d*A*a^3*tan
(d*x+c)+15/8/d*B*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*B*a^3*ln(sec(d*x+c)+tan(d
```

```
*x+c))+3/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+
1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*a^3*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [B] time = 1.00015, size = 354, normalized size = 2.83

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+48\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3-3\,Ba^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^3 - 3*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^3*log(sec(d
*x + c) + tan(d*x + c)) + 144*A*a^3*tan(d*x + c) + 48*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.491931, size = 366, normalized size = 2.93

$$\frac{15\,(4\,A+3\,B)a^3\cos(dx+c)^4\log(\sin(dx+c)+1)-15\,(4\,A+3\,B)a^3\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8\,(11\right.}{48\,d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*(4*A + 3*B)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(4*A + 3
*B)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(11*A + 9*B)*a^3*cos(d
*x + c)^3 + 9*(4*A + 5*B)*a^3*cos(d*x + c)^2 + 8*(A + 3*B)*a^3*cos(d*x + c)
 + 6*B*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\sec(c+dx)\,dx+\int 3A\sec^2(c+dx)\,dx+\int 3A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int B\sec^2(c+dx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sec(c + d*x)**2, x) + Inte
gral(3*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integral(B*
sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + Integral(3*B*sec(c
 + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))
```

Giac [A] time = 1.35507, size = 286, normalized size = 2.29

$$15\left(4\,Aa^3+3\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(4\,Aa^3+3\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*(4*A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a
^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 45*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 220*A*a^3*tan(1/2*d*x + 1/2*
c)^5 - 165*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 292*A*a^3*tan(1/2*d*x + 1/2*c)^3
+ 219*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 132*A*a^3*tan(1/2*d*x + 1/2*c) - 147*B
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.65 $\int (a + a\sec(c + dx))^3 (A + B\sec(c + dx))\, dx$

Optimal. Leaf size=111

$$\frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(3A+5B)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d} + a^3Ax +$$

```
[Out] a^3*A*x + (a^3*(7*A + 5*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.144216, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3917, 3914, 3767, 8, 3770}

$$\frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(3A+5B)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d} + a^3Ax +$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^3*A*x + (a^3*(7*A + 5*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}\int (a+a\sec(c+dx))^2(3aA+a(3$$
$$= \frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{(3A+5B)\left(a^3+a^3\sec(c+dx)\right)\tan}{6d}$$
$$= a^3Ax + \frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{(3A+5B)\left(a^3+a^3\sec(c+\right.}{6d}$$
$$= a^3Ax + \frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aB(a+a\sec(c+dx))^2\tan(c}{3d}$$
$$= a^3Ax + \frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{al}{}$$

Mathematica [B] time = 6.38664, size = 1056, normalized size = 9.51

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[
c + d*x]))/(8*(B + A*Cos[c + d*x])) + ((-7*A - 5*B)*Cos[c + d*x]^4*Log[Cos[
c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*
x])^3*(A + B*Sec[c + d*x]))/(16*d*(B + A*Cos[c + d*x])) + ((7*A + 5*B)*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(16*d*(B + A*Cos[c + d*x]))
+ (B*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[
c + d*x])*Sin[(d*x)/2])/(48*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x
)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(3*A*Cos[c/2] + 10*B*Cos
[c/2] - 3*A*Sin[c/2] - 8*B*Sin[c/2]))/(96*d*(B + A*Cos[c + d*x])*(Cos[c/2]
- Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^4*
Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(9*A*Sin[(
d*x)/2] + 11*B*Sin[(d*x)/2]))/(24*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (B*Cos[c + d*x]^4*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(48*
d*(B + A*Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^
3*(A + B*Sec[c + d*x])*(-3*A*Cos[c/2] - 10*B*Cos[c/2] - 3*A*Sin[c/2] - 8*B*
Sin[c/2]))/(96*d*(B + A*Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a
*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(9*A*Sin[(d*x)/2] + 11*B*Sin[(d*x)/2]
))/(24*d*(B + A*Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2]))
```

Maple [A] time = 0.046, size = 158, normalized size = 1.4

$$a^3Ax + \frac{Aa^3c}{d} + \frac{5\,Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + \frac{7\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + \frac{11\,Ba^3\tan(dx+c)}{3\,d} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] a^3*A*x+1/d*A*a^3*c+5/2/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*A*a^3*ln(se
c(d*x+c)+tan(d*x+c))+11/3/d*B*a^3*tan(d*x+c)+3/d*A*a^3*tan(d*x+c)+3/2/d*B*a
^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/3/d*B*a^3*tan(
d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.995285, size = 267, normalized size = 2.41

$$\frac{12\,(dx+c)Aa^3+4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^3-3\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 - 3*A*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 36*A*a^3*log(sec(d*x + c) + tan(d*x +
 c)) + 12*B*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*A*a^3*tan(d*x + c) +
36*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.501615, size = 356, normalized size = 3.21

$$\frac{12\,Aa^3dx\cos(dx+c)^3+3\,(7\,A+5\,B)a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(7\,A+5\,B)a^3\cos(dx+c)^3\log(-\sin(\ldots}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*A*a^3*d*x*cos(d*x + c)^3 + 3*(7*A + 5*B)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(7*A + 5*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(9*A + 11*B)*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) + 2*B*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\,dx+\int 3A\sec(c+dx)\,dx+\int 3A\sec^2(c+dx)\,dx+\int A\sec^3(c+dx)\,dx+\int B\sec(c+dx)\,dx+\int 3B\sec\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A, x) + Integral(3*A*sec(c + d*x), x) + Integral(3*A*sec(c +
 d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x), x)
 + Integral(3*B*sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + In
tegral(B*sec(c + d*x)**4, x))
```

Giac [A] time = 1.3272, size = 255, normalized size = 2.3

$$\frac{6\,(dx+c)Aa^3+3\left(7\,Aa^3+5\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Aa^3+5\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,Aa^3\,\mathrm{t}\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^3 + 3*(7*A*a^3 + 5*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(7*A*a^3 + 5*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d
*x + 1/2*c) + 33*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

3.66 $\int \cos(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{a^3(6A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+2B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}+a^3x(3A+B)-\frac{5a^3B\sin(c+dx)}{2d}+$$

```
[Out] a^3*(3*A + B)*x + (a^3*(6*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*B*
Sin[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((A
 + 2*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/d
```

Rubi [A] time = 0.238215, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4018, 3996, 3770}

$$\frac{a^3(6A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+2B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}+a^3x(3A+B)-\frac{5a^3B\sin(c+dx)}{2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] a^3*(3*A + B)*x + (a^3*(6*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*B*
Sin[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((A
 + 2*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/d
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aB(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)(a+a\,\mathrm{se}$$
$$= \frac{aB(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{(A+2B)\left(a^3+a^3\sec(c\right.}{d}$$
$$= -\frac{5a^3B\sin(c+dx)}{2d} + \frac{aB(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{(A}{}$$
$$= a^3(3A+B)x - \frac{5a^3B\sin(c+dx)}{2d} + \frac{aB(a+a\sec(c+dx))^2\sin}{2d}$$
$$= a^3(3A+B)x + \frac{a^3(6A+7B)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^3B\sin(}{2d}$$

Mathematica [B] time = 2.52613, size = 335, normalized size = 3.1

$$a^3\cos^4(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3(A+B\sec(c+dx))\left(\frac{4(A+3B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{d\left(\sin\left(\frac{c}{2}\right)+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c +
d*x])*(4*(3*A + B)*x - (2*(6*A + 7*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]])/d + (2*(6*A + 7*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*A*
Cos[d*x]*Sin[c])/d + (4*A*Cos[c]*Sin[d*x])/d + B/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2) + (4*(A + 3*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2) + (4*(A + 3*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]))))/(32*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.073, size = 144, normalized size = 1.3

$$\frac{Aa^3\sin(dx+c)}{d} + Ba^3x + \frac{Ba^3c}{d} + 3a^3Ax + 3\frac{Aa^3c}{d} + \frac{7Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 3\frac{Aa^3\ln(\sec(dx+c)+\mathrm{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] a^3*A*sin(d*x+c)/d+B*a^3*x+1/d*B*a^3*c+3*a^3*A*x+3/d*A*a^3*c+7/2/d*B*a^3*ln
(sec(d*x+c)+tan(d*x+c))+3/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^3*tan(d
*x+c)+1/d*A*a^3*tan(d*x+c)+1/2/d*B*a^3*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.988914, size = 223, normalized size = 2.06

$$12(dx+c)Aa^3 + 4(dx+c)Ba^3 - Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 6Aa^3(\log(\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(12*(d*x + c)*A*a^3 + 4*(d*x + c)*B*a^3 - B*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c)
+ 12*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.504092, size = 342, normalized size = 3.17

$$\frac{4\,(3\,A+B)a^3dx\cos\left(dx+c\right)^2+(6\,A+7\,B)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(6\,A+7\,B)a^3\cos\left(dx+c\right)^2\log\left(-\text{si}\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(4*(3*A + B)*a^3*d*x*cos(d*x + c)^2 + (6*A + 7*B)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (6*A + 7*B)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*A*a^3*cos(d*x + c)^2 + 2*(A + 3*B)*a^3*cos(d*x + c) + B*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38422, size = 259, normalized size = 2.4

$$\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(3\,Aa^3+Ba^3\right)(dx+c)+\left(6\,Aa^3+7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,Aa^3+7\,Ba^3\right)\log\left(\left|\text{tan}\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*A*a^3
 + B*a^3)*(d*x + c) + (6*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - (6*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^3*tan
(1/2*d*x + 1/2*c)^3 + 5*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1/2*d*x
+ 1/2*c) - 7*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.67 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=117

$$\frac{a^3(A+3B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(A-2B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(7A+6B)+\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA}{\ }$$

```
[Out] (a^3*(7*A + 6*B)*x)/2 + (a^3*(A + 3*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*A*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((A - 2*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.263939, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4017, 4018, 3996, 3770}

$$\frac{a^3(A+3B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(A-2B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(7A+6B)+\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA}{\ }$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(7*A + 6*B)*x)/2 + (a^3*(A + 3*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*A*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((A - 2*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{1}{2}\int \cos$$
$$= \frac{aA\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{(A-2B}{}$$
$$= \frac{5a^3A\sin(c+dx)}{2d} + \frac{aA\cos(c+dx)(a+a\sec(c+dx))^2\sin}{2d}$$
$$= \frac{1}{2}a^3(7A+6B)x + \frac{5a^3A\sin(c+dx)}{2d} + \frac{aA\cos(c+dx)(a+}{}$$
$$= \frac{1}{2}a^3(7A+6B)x + \frac{a^3(A+3B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3}{}$$

Mathematica [B] time = 4.60165, size = 302, normalized size = 2.58

$$\frac{a^3\cos^4(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3(A+B\sec(c+dx))\left(\frac{4(3A+B)\sin(c)\cos(dx)}{d}+\frac{4(3A+B)\cos(c)\sin(dx)}{d}-\frac{4(A+3}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c +
d*x])*(2*(7*A + 6*B)*x - (4*(A + 3*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]])/d + (4*(A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(3*A
 + B)*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])/d + (4*(3*A + B)*Cos[c]*
Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/d + (4*B*Sin[(d*x)/2])/(d*(Cos[c/2] -
 Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*B*Sin[(d*x)/2])/(d*(
Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(32*(B + A*Co
s[c + d*x]))
```

Maple [A] time = 0.078, size = 145, normalized size = 1.2

$$\frac{Aa^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{7a^3Ax}{2} + \frac{7Aa^3c}{2d} + \frac{Ba^3\sin(dx+c)}{d} + 3\frac{Aa^3\sin(dx+c)}{d} + 3Ba^3x + 3\frac{Ba^3c}{d} + 3\frac{Ba^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*A*a^3*cos(d*x+c)*sin(d*x+c)+7/2*a^3*A*x+7/2/d*A*a^3*c+a^3*B*sin(d*x+c
)/d+3*a^3*A*sin(d*x+c)/d+3*B*a^3*x+3/d*B*a^3*c+3/d*B*a^3*ln(sec(d*x+c)+tan(
d*x+c))+1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^3*tan(d*x+c)
```

Maxima [A] time = 1.00086, size = 189, normalized size = 1.62

$$\frac{(2dx+2c+\sin(2dx+2c))Aa^3 + 12(dx+c)Aa^3 + 12(dx+c)Ba^3 + 2Aa^3(\log(\sin(dx+c)+1) - \log(\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*A*a^3 + 12*(d*x
+ c)*B*a^3 + 2*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^3*sin(d*x + c)
 + 4*B*a^3*sin(d*x + c) + 4*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.503484, size = 323, normalized size = 2.76

$$\frac{(7A+6B)a^3dx\cos(dx+c)+(A+3B)a^3\cos(dx+c)\log(\sin(dx+c)+1)-(A+3B)a^3\cos(dx+c)\log(-\sin(dx+c}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((7*A + 6*B)*a^3*d*x*cos(d*x + c) + (A + 3*B)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (A + 3*B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (A*a^3*
cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + 2*B*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42373, size = 259, normalized size = 2.21

$$-\frac{\frac{4Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}-\left(7Aa^3+6Ba^3\right)(dx+c)-2\left(Aa^3+3Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)+2\left(Aa^3+3Ba^3\right)\log\left(\left|\tan\left(\right.\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (7*A*a^3
+ 6*B*a^3)*(d*x + c) - 2*(A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^3*tan(1/2*d*x
 + 1/2*c) + 2*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.68 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(5A+3B)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(5A+7B)+\frac{a^3B\tanh^{-1}(\sin$$

```
[Out] (a^3*(5*A + 7*B)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(3*d) + ((5*A + 3*B)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rubi [A] time = 0.270957, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4017, 3996, 3770}

$$\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(5A+3B)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(5A+7B)+\frac{a^3B\tanh^{-1}(\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(5*A + 7*B)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(3*d) + ((5*A + 3*B)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int \cos^2($$
$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d} + \frac{(5A+3B)}{}$$
$$= \frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))}{3d}$$
$$= \frac{1}{2}a^3(5A+7B)x + \frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{aA\cos^2(c+dx)}{}$$
$$= \frac{1}{2}a^3(5A+7B)x + \frac{a^3B\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3(A+B)\sin}{2d}$$

Mathematica [A] time = 0.238765, size = 113, normalized size = 0.9

$$\frac{a^3\left(9(5A+4B)\sin(c+dx)+3(3A+B)\sin(2(c+dx))+A\sin(3(c+dx))+30Adx-12B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(30*A*d*x + 42*B*d*x - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(5*A + 4*B)*Sin[c + d*x]
 + 3*(3*A + B)*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.084, size = 153, normalized size = 1.2

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^3}{3d} + \frac{11\,Aa^3\sin(dx+c)}{3d} + \frac{Ba^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{7\,Ba^3x}{2} + \frac{7\,Ba^3c}{2d} + \frac{3\,Aa^3\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^3+11/3*a^3*A*sin(d*x+c)/d+1/2/d*B*a^3*cos
(d*x+c)*sin(d*x+c)+7/2*B*a^3*x+7/2/d*B*a^3*c+3/2/d*A*a^3*cos(d*x+c)*sin(d*x
+c)+5/2*a^3*A*x+5/2/d*A*a^3*c+3*a^3*B*sin(d*x+c)/d+1/d*B*a^3*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 1.02456, size = 200, normalized size = 1.6

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-9(2dx+2c+\sin(2dx+2c))Aa^3-12(dx+c)Aa^3-3(2dx+2c+\sin(2dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*
```

```
B*a^3 - 36*(d*x + c)*B*a^3 - 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) - 36*A*a^3*sin(d*x + c) - 36*B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.501243, size = 254, normalized size = 2.03

$$\frac{3\,(5\,A+7\,B)a^3dx+3\,Ba^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Ba^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Aa^3\cos\left(dx+c\right)^2+3\,(3\,A+B)a^3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(5*A + 7*B)*a^3*d*x + 3*B*a^3*log(sin(d*x + c) + 1) - 3*B*a^3*log(-s
in(d*x + c) + 1) + (2*A*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) +
 2*(11*A + 9*B)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37528, size = 243, normalized size = 1.94

$$\frac{6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(5\,Aa^3+7\,Ba^3\right)(dx+c)+\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*B*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(5*A*a^3 + 7*B*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^3*tan(1/2*d*x + 1/
2*c)^3 + 36*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d*x + 1/2*c) +
21*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.69 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=124

$$-\frac{a^3(3A+4B)\sin^3(c+dx)}{12d}+\frac{a^3(3A+4B)\sin(c+dx)}{d}+\frac{3a^3(3A+4B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(3A+4B)+\frac{A\sin}{}$$

```
[Out] (5*a^3*(3*A + 4*B)*x)/8 + (a^3*(3*A + 4*B)*Sin[c + d*x])/d + (3*a^3*(3*A +
4*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d) - (a^3*(3*A + 4*B)*Sin[c + d*x]^3)/(12*d)
```

Rubi [A] time = 0.169572, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{a^3(3A+4B)\sin^3(c+dx)}{12d}+\frac{a^3(3A+4B)\sin(c+dx)}{d}+\frac{3a^3(3A+4B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(3A+4B)+\frac{A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (5*a^3*(3*A + 4*B)*x)/8 + (a^3*(3*A + 4*B)*Sin[c + d*x])/d + (3*a^3*(3*A +
4*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d) - (a^3*(3*A + 4*B)*Sin[c + d*x]^3)/(12*d)
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}(3A+$$
$$= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}(3A+$$
$$= \frac{1}{4}a^3(3A+4B)x + \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin($$
$$= \frac{1}{4}a^3(3A+4B)x + \frac{3a^3(3A+4B)\sin(c+dx)}{4d} + \frac{3a^3(3A+}{}$$
$$= \frac{5}{8}a^3(3A+4B)x + \frac{a^3(3A+4B)\sin(c+dx)}{d} + \frac{3a^3(3A+4}{}$$

Mathematica [A] time = 0.270153, size = 86, normalized size = 0.69

$$\frac{a^3(24(13A+15B)\sin(c+dx)+24(4A+3B)\sin(2(c+dx))+24A\sin(3(c+dx))+3A\sin(4(c+dx))+180Adx+8B}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(180*A*d*x + 240*B*d*x + 24*(13*A + 15*B)*Sin[c + d*x] + 24*(4*A + 3*B
)*Sin[2*(c + d*x)] + 24*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Sin
[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.086, size = 176, normalized size = 1.4

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+Aa^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)+\frac{Ba^3(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a
^3*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^
3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*B*a^3*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+A*a^3*sin(d*x+c)+3*B*a^3*sin(d*x+c)+B*a^3*(d*x+c))
```

Maxima [A] time = 1.00001, size = 225, normalized size = 1.81

$$-\frac{96\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa^3-72(2dx+2c+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/96*(96*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*
c))*A*a^3 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 72*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^3 - 96*(d*x + c)*B*a^3 - 96*A*a^3*sin(d*x + c) - 288*
B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.481609, size = 216, normalized size = 1.74

$$\frac{15\,(3\,A+4\,B)a^3dx+\left(6\,Aa^3\cos\left(dx+c\right)^3+8\,(3\,A+B)a^3\cos\left(dx+c\right)^2+9\,(5\,A+4\,B)a^3\cos\left(dx+c\right)+8\,(9\,A+11\,B)a^3\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(15*(3*A + 4*B)*a^3*d*x + (6*A*a^3*cos(d*x + c)^3 + 8*(3*A + B)*a^3*co
s(d*x + c)^2 + 9*(5*A + 4*B)*a^3*cos(d*x + c) + 8*(9*A + 11*B)*a^3)*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34596, size = 238, normalized size = 1.92

$$\frac{15\left(3\,Aa^3+4\,Ba^3\right)(dx+c)+\frac{2\left(45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+60\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+165\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+220\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+219\,Aa^3\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(15*(3*A*a^3 + 4*B*a^3)*(d*x + c) + 2*(45*A*a^3*tan(1/2*d*x + 1/2*c)^7
 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 165*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 220
*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 292*B*a^
3*tan(1/2*d*x + 1/2*c)^3 + 147*A*a^3*tan(1/2*d*x + 1/2*c) + 132*B*a^3*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.70 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{a^3(38A+45B)\sin(c+dx)}{15d}+\frac{a^3(43A+45B)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^3(13A+15B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{(7A$$

```
[Out] (a^3*(13*A + 15*B)*x)/8 + (a^3*(38*A + 45*B)*Sin[c + d*x])/(15*d) + (a^3*(1
3*A + 15*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*(43*A + 45*B)*Cos[c + d
*x]^2*Sin[c + d*x])/(60*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + ((7*A + 5*B)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin
[c + d*x])/(20*d)
```

Rubi [A] time = 0.372417, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^3(38A+45B)\sin(c+dx)}{15d}+\frac{a^3(43A+45B)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^3(13A+15B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{(7A$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(13*A + 15*B)*x)/8 + (a^3*(38*A + 45*B)*Sin[c + d*x])/(15*d) + (a^3*(1
3*A + 15*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*(43*A + 45*B)*Cos[c + d
*x]^2*Sin[c + d*x])/(60*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + ((7*A + 5*B)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin
[c + d*x])/(20*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4($$
$$= \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{(7A+5B)}{}$$
$$= \frac{a^3(43A+45B)\cos^2(c+dx)\sin(c+dx)}{60d}+\frac{aA\cos^4(c+dx)(}{}$$
$$= \frac{a^3(43A+45B)\cos^2(c+dx)\sin(c+dx)}{60d}+\frac{aA\cos^4(c+dx)(}{}$$
$$= \frac{a^3(38A+45B)\sin(c+dx)}{15d}+\frac{a^3(13A+15B)\cos(c+dx)\sin}{8d}$$
$$= \frac{1}{8}a^3(13A+15B)x+\frac{a^3(38A+45B)\sin(c+dx)}{15d}+\frac{a^3(13A+}{}$$

Mathematica [A] time = 0.433254, size = 108, normalized size = 0.61

$$\frac{a^3(60(23A+26B)\sin(c+dx)+480(A+B)\sin(2(c+dx))+170A\sin(3(c+dx))+45A\sin(4(c+dx))+6A\sin(5(c+d}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(780*A*c + 780*A*d*x + 900*B*d*x + 60*(23*A + 26*B)*Sin[c + d*x] + 480
*(A + B)*Sin[2*(c + d*x)] + 170*A*Sin[3*(c + d*x)] + 120*B*Sin[3*(c + d*x)]
 + 45*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(
480*d)
```

Maple [A] time = 0.096, size = 223, normalized size = 1.3

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^3*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^3*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^3*(2+cos(d*x+c)^2)*sin(
d*x+c)+A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a^3*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^3*sin(d
*x+c))
```

Maxima [A] time = 1.03417, size = 288, normalized size = 1.64

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3+45\,(12\,dx+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 45*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^3 + 480*B*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.483892, size = 278, normalized size = 1.58

$$\frac{15\,(13\,A+15\,B)a^3dx+\left(24\,Aa^3\cos(dx+c)^4+30\,(3\,A+B)a^3\cos(dx+c)^3+8\,(19\,A+15\,B)a^3\cos(dx+c)^2+15\,(13}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(13*A + 15*B)*a^3*d*x + (24*A*a^3*cos(d*x + c)^4 + 30*(3*A + B)*a
^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*a^3*cos(d*x + c)^2 + 15*(13*A + 15*B)*a
^3*cos(d*x + c) + 8*(38*A + 45*B)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29552, size = 284, normalized size = 1.61

$$\frac{15\left(13\,Aa^3+15\,Ba^3\right)(dx+c)+\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+225\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,Aa^3}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3)*(d*x + c) + 2*(195*A*a^3*tan(1/2*d*x + 1/2*
c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1
920*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 1830
*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c) + 735*B*a^3*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.71 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=201

$$-\frac{a^3(17A+19B)\sin^3(c+dx)}{15d}+\frac{a^3(17A+19B)\sin(c+dx)}{5d}+\frac{a^3(21A+22B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^3(23A+26\ldots}{}$$

```
[Out] (a^3*(23*A + 26*B)*x)/16 + (a^3*(17*A + 19*B)*Sin[c + d*x])/(5*d) + (a^3*(2
3*A + 26*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(21*A + 22*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((4*A + 3*B)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(17*A + 19*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.409748, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^3(17A+19B)\sin^3(c+dx)}{15d}+\frac{a^3(17A+19B)\sin(c+dx)}{5d}+\frac{a^3(21A+22B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^3(23A+26\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^3*(23*A + 26*B)*x)/16 + (a^3*(17*A + 19*B)*Sin[c + d*x])/(5*d) + (a^3*(2
3*A + 26*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(21*A + 22*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((4*A + 3*B)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(17*A + 19*B)*Sin[c + d*x]^3)/(15*d)
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^5($$

$$= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d}+\frac{(4A+3B)}{}$$

$$= \frac{a^3(21A+22B)\cos^3(c+dx)\sin(c+dx)}{40d}+\frac{aA\cos^5(c+dx)(a}{}$$

$$= \frac{a^3(21A+22B)\cos^3(c+dx)\sin(c+dx)}{40d}+\frac{aA\cos^5(c+dx)(a}{}$$

$$= \frac{a^3(23A+26B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^3(21A+22B)\cos}{4}$$

$$= \frac{1}{16}a^3(23A+26B)x+\frac{a^3(17A+19B)\sin(c+dx)}{5d}+\frac{a^3(23A}{}$$

Mathematica [A] time = 0.536478, size = 134, normalized size = 0.67

$$\frac{a^3(120(21A+23B)\sin(c+dx)+15(63A+64B)\sin(2(c+dx))+380A\sin(3(c+dx))+135A\sin(4(c+dx))+36A\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(1380*A*c + 1380*A*d*x + 1560*B*d*x + 120*(21*A + 23*B)*Sin[c + d*x] +
 15*(63*A + 64*B)*Sin[2*(c + d*x)] + 380*A*Sin[3*(c + d*x)] + 340*B*Sin[3*(
c + d*x)] + 135*A*Sin[4*(c + d*x)] + 90*B*Sin[4*(c + d*x)] + 36*A*Sin[5*(c
+ d*x)] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.1, size = 266, normalized size = 1.3

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{Ba^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3
/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*B*a^3*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^3*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^3*(2+cos(d*x+c)^2)*sin(d*x+
c)+1/3*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.0032, size = 354, normalized size = 1.76

$$192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 64*(3*sin(d*x + c)^
5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*si
n(d*x + c))*B*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*B*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3)/d
```

Fricas [A] time = 0.490744, size = 332, normalized size = 1.65

$$\frac{15(23A+26B)a^3dx+\left(40Aa^3\cos(dx+c)^5+48(3A+B)a^3\cos(dx+c)^4+10(23A+18B)a^3\cos(dx+c)^3+16(1\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(23*A + 26*B)*a^3*d*x + (40*A*a^3*cos(d*x + c)^5 + 48*(3*A + B)*a
^3*cos(d*x + c)^4 + 10*(23*A + 18*B)*a^3*cos(d*x + c)^3 + 16*(17*A + 19*B)*
a^3*cos(d*x + c)^2 + 15*(23*A + 26*B)*a^3*cos(d*x + c) + 32*(17*A + 19*B)*a
^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45062, size = 329, normalized size = 1.64

$$15\left(23\,Aa^3+26\,Ba^3\right)(dx+c)+\frac{2\left(345\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+390\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2210\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4554\,A\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(23*A*a^3 + 26*B*a^3)*(d*x + c) + 2*(345*A*a^3*tan(1/2*d*x + 1/2*
c)^11 + 390*B*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*A*a^3*tan(1/2*d*x + 1/2*c)
^9 + 2210*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 5148*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 5
988*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 4190
*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^3*tan(1/2*d*x + 1/2*c) + 1530*B*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.72 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{2a^4(8A+7B)\tan^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\tan(c+dx)}{5d}+\frac{7a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(8A+7B)\tan(c+\ldots}{40d}$$

```
[Out] (7*a^4*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(8*A + 7*B)*Tan[c
 + d*x])/(5*d) + (27*a^4*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(80*d) + (a
^4*(8*A + 7*B)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + ((6*A - B)*(a + a*Sec[
c + d*x])^4*Tan[c + d*x])/(30*d) + (B*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/
(6*a*d) + (2*a^4*(8*A + 7*B)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.318361, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{2a^4(8A+7B)\tan^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\tan(c+dx)}{5d}+\frac{7a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(8A+7B)\tan(c+\ldots}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] (7*a^4*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(8*A + 7*B)*Tan[c
 + d*x])/(5*d) + (27*a^4*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(80*d) + (a
^4*(8*A + 7*B)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + ((6*A - B)*(a + a*Sec[
c + d*x])^4*Tan[c + d*x])/(30*d) + (B*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/
(6*a*d) + (2*a^4*(8*A + 7*B)*Tan[c + d*x]^3)/(15*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^5\tan(c+dx)}{6ad} + \frac{\int \sec(c+dx)(a+a\sec(}{}$$

$$= \frac{(6A-B)(a+a\sec(c+dx))^4\tan(c+dx)}{30d} + \frac{B(a+a\sec(c+}{6}$$

$$= \frac{(6A-B)(a+a\sec(c+dx))^4\tan(c+dx)}{30d} + \frac{B(a+a\sec(c+}{6}$$

$$= \frac{(6A-B)(a+a\sec(c+dx))^4\tan(c+dx)}{30d} + \frac{B(a+a\sec(c+}{6}$$

$$= \frac{a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{10d} + \frac{3a^4(8A+7B)\sec(c+d}{10d}$$

$$= \frac{2a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{5d} + \frac{4a^4(8A+7B)\tan(c+}{5d}$$

$$= \frac{7a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{4a^4(8A+7B)\tan(c+}{5d}$$

Mathematica [A] time = 2.23417, size = 358, normalized size = 1.85

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(3360(8A+7B)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360*(8*A + 7
*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(83*A + 72*B)*Sin[c] + 30*(88*
A + 125*B)*Sin[d*x] + 2640*A*Sin[2*c + d*x] + 3750*B*Sin[2*c + d*x] + 15840
*A*Sin[c + 2*d*x] + 15360*B*Sin[c + 2*d*x] - 4080*A*Sin[3*c + 2*d*x] - 1920
*B*Sin[3*c + 2*d*x] + 3480*A*Sin[2*c + 3*d*x] + 3845*B*Sin[2*c + 3*d*x] + 3
480*A*Sin[4*c + 3*d*x] + 3845*B*Sin[4*c + 3*d*x] + 7728*A*Sin[3*c + 4*d*x]
```

```
+ 6912*B*Sin[3*c + 4*d*x] - 240*A*Sin[5*c + 4*d*x] + 840*A*Sin[4*c + 5*d*x]
 + 735*B*Sin[4*c + 5*d*x] + 840*A*Sin[6*c + 5*d*x] + 735*B*Sin[6*c + 5*d*x]
 + 1328*A*Sin[5*c + 6*d*x] + 1152*B*Sin[5*c + 6*d*x])))/(122880*d)
```

Maple [A] time = 0.054, size = 280, normalized size = 1.4

$$\frac{83\,Aa^4\tan(dx+c)}{15\,d}+\frac{49\,Ba^4\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{49\,Ba^4\ln(\sec(dx+c)+\tan(dx+c))}{16\,d}+\frac{7\,Aa^4\sec(dx+c)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 83/15/d*A*a^4*tan(d*x+c)+49/16/d*B*a^4*sec(d*x+c)*tan(d*x+c)+49/16/d*B*a^4*
ln(sec(d*x+c)+tan(d*x+c))+7/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+7/2/d*A*a^4*ln(
sec(d*x+c)+tan(d*x+c))+24/5/d*B*a^4*tan(d*x+c)+12/5/d*B*a^4*tan(d*x+c)*sec(
d*x+c)^2+34/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+41/24/d*B*a^4*tan(d*x+c)*sec
(d*x+c)^3+1/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+4/5/d*B*a^4*tan(d*x+c)*sec(d*x+
c)^4+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/6/d*B*a^4*tan(d*x+c)*sec(d*x+c)^
5
```

Maxima [B] time = 1.00517, size = 626, normalized size = 3.23

$$\frac{32\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Aa^4+960\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+128\left(3\,\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
960*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 128*(3*tan(d*x + c)^5 + 10*ta
n(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c
))*B*a^4 - 5*B*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
 c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(si
n(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*B*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 480*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*B*a^4*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 480*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.511613, size = 481, normalized size = 2.48

$$\frac{105\,(8\,A+7\,B)a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-105\,(8\,A+7\,B)a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\left(16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*(8*A + 7*B)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(8*A
+ 7*B)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(83*A + 72*B)*a^4*
cos(d*x + c)^5 + 105*(8*A + 7*B)*a^4*cos(d*x + c)^4 + 32*(17*A + 18*B)*a^4*
cos(d*x + c)^3 + 10*(24*A + 41*B)*a^4*cos(d*x + c)^2 + 48*(A + 4*B)*a^4*cos
(d*x + c) + 40*B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int A\sec^2(c+dx)\,dx+\int 4A\sec^3(c+dx)\,dx+\int 6A\sec^4(c+dx)\,dx+\int 4A\sec^5(c+dx)\,dx+\int A\sec^6(c+dx)\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + I
ntegral(6*A*sec(c + d*x)**4, x) + Integral(4*A*sec(c + d*x)**5, x) + Integr
al(A*sec(c + d*x)**6, x) + Integral(B*sec(c + d*x)**3, x) + Integral(4*B*se
c(c + d*x)**4, x) + Integral(6*B*sec(c + d*x)**5, x) + Integral(4*B*sec(c +
 d*x)**6, x) + Integral(B*sec(c + d*x)**7, x))
```

Giac [A] time = 1.3626, size = 378, normalized size = 1.95

$$\frac{105\left(8\,Aa^4+7\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(8\,Aa^4+7\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(840\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(8*A*a^4 + 7*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*
A*a^4 + 7*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(840*A*a^4*tan(1/2*
d*x + 1/2*c)^11 + 735*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 4760*A*a^4*tan(1/2*d*
x + 1/2*c)^9 - 4165*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 11088*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 + 9702*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 13488*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 11802*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 9320*A*a^4*tan(1/2*d*x + 1/
2*c)^3 + 7355*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3000*A*a^4*tan(1/2*d*x + 1/2*c
) - 3105*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.73 $\int \sec(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=159

$$\frac{4a^4(5A+4B)\tan^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\tan(c+dx)}{5d}+\frac{7a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(5A+4B)\tan(c+}{20d}$$

```
[Out] (7*a^4*(5*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*(5*A + 4*B)*Tan[c
+ d*x])/(5*d) + (27*a^4*(5*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^
4*(5*A + 4*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*(a + a*Sec[c + d*x])
^4*Tan[c + d*x])/(5*d) + (4*a^4*(5*A + 4*B)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.17923, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4001, 3791, 3770, 3767, 8, 3768}

$$\frac{4a^4(5A+4B)\tan^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\tan(c+dx)}{5d}+\frac{7a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(5A+4B)\tan(c+}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] (7*a^4*(5*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*(5*A + 4*B)*Tan[c
+ d*x])/(5*d) + (27*a^4*(5*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^
4*(5*A + 4*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*(a + a*Sec[c + d*x])
^4*Tan[c + d*x])/(5*d) + (4*a^4*(5*A + 4*B)*Tan[c + d*x]^3)/(15*d)
```


Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{1}{5}(5A+4B)\int \sec(c+d$$
$$= \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{1}{5}(5A+4B)\int \left(a^4\sec(c\right.$$
$$= \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{1}{5}\left(a^4(5A+4B)\right)\int \sec(c$$
$$= \frac{a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{5d} + \frac{3a^4(5A+4B)\sec(c+dx)}{5d}$$
$$= \frac{4a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{5d} + \frac{8a^4(5A+4B)\tan(c+d}{5d}$$
$$= \frac{7a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{8a^4(5A+4B)\tan(c+d}{5d}$$

Mathematica [A] time = 1.61498, size = 306, normalized size = 1.92

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(1680(5A+4B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^5*(1680*(5*A + 4
*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(64*A + 59*B)*Sin[d*x] - 960*(3*
A + 2*B)*Sin[2*c + d*x] + 930*A*Sin[c + 2*d*x] + 1320*B*Sin[c + 2*d*x] + 93
0*A*Sin[3*c + 2*d*x] + 1320*B*Sin[3*c + 2*d*x] + 3520*A*Sin[2*c + 3*d*x] +
3200*B*Sin[2*c + 3*d*x] - 480*A*Sin[4*c + 3*d*x] - 120*B*Sin[4*c + 3*d*x] +
 405*A*Sin[3*c + 4*d*x] + 420*B*Sin[3*c + 4*d*x] + 405*A*Sin[5*c + 4*d*x] +
 420*B*Sin[5*c + 4*d*x] + 800*A*Sin[4*c + 5*d*x] + 664*B*Sin[4*c + 5*d*x]))
)/(30720*d)
```

Maple [A] time = 0.056, size = 234, normalized size = 1.5

$$\frac{35\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{8\,d} + \frac{83\,Ba^4\tan(dx+c)}{15\,d} + \frac{20\,Aa^4\tan(dx+c)}{3\,d} + \frac{7\,Ba^4\sec(dx+c)\tan(dx+c)}{2\,d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 35/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+83/15/d*B*a^4*tan(d*x+c)+20/3/d*A*a^
4*tan(d*x+c)+7/2/d*B*a^4*sec(d*x+c)*tan(d*x+c)+7/2/d*B*a^4*ln(sec(d*x+c)+ta
n(d*x+c))+27/8/d*A*a^4*sec(d*x+c)*tan(d*x+c)+34/15/d*B*a^4*tan(d*x+c)*sec(d
*x+c)^2+4/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/d*B*a^4*tan(d*x+c)*sec(d*x+c)
^3+1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+1/5/d*B*a^4*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [B] time = 1.03204, size = 498, normalized size = 3.13

$$\frac{320\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{4}+16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Ba^{4}+480\left(\tan\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(320*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 480*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 - 15*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 60*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 360*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 240*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^4*log(sec(d*x + c)
 + tan(d*x + c)) + 960*A*a^4*tan(d*x + c) + 240*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.499388, size = 431, normalized size = 2.71

$$\frac{105\,(5\,A+4\,B)a^{4}\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-105\,(5\,A+4\,B)a^{4}\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(1\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(105*(5*A + 4*B)*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*(5*A
+ 4*B)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(100*A + 83*B)*a^4*
cos(d*x + c)^4 + 15*(27*A + 28*B)*a^4*cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4
*cos(d*x + c)^2 + 30*(A + 4*B)*a^4*cos(d*x + c) + 24*B*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{4}\left(\int A\sec\left(c+dx\right)dx+\int 4A\sec^{2}\left(c+dx\right)dx+\int 6A\sec^{3}\left(c+dx\right)dx+\int 4A\sec^{4}\left(c+dx\right)dx+\int A\sec^{5}\left(c+dx\right.\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*sec(c + d*x)**2, x) + Inte
gral(6*A*sec(c + d*x)**3, x) + Integral(4*A*sec(c + d*x)**4, x) + Integral(
A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**2, x) + Integral(4*B*sec(c
 + d*x)**3, x) + Integral(6*B*sec(c + d*x)**4, x) + Integral(4*B*sec(c + d*
x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A] time = 1.24877, size = 332, normalized size = 2.09

$$105\left(5\,Aa^4+4\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(5\,Aa^4+4\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(525\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(105*(5*A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(5*
A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(525*A*a^4*tan(1/2*
d*x + 1/2*c)^9 + 420*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 2450*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 - 1960*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 3584*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 3950*A*a^4*tan(1/2*d*x + 1/2*
c)^3 - 3160*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1395*A*a^4*tan(1/2*d*x + 1/2*c)
+ 1500*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.74 $\int (a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$\frac{5a^4(8A+7B)\tan(c+dx)}{8d}+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4A+7B)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d}+\cdots$$

```
[Out] a^4*A*x + (a^4*(48*A + 35*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(8*A + 7
*B)*Tan[c + d*x])/(8*d) + (a*B*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.214004, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3917, 3914, 3767, 8, 3770}

$$\frac{5a^4(8A+7B)\tan(c+dx)}{8d}+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4A+7B)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^4*A*x + (a^4*(48*A + 35*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(8*A + 7
*B)*Tan[c + d*x])/(8*d) + (a*B*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+a\sec(c+dx))^3(4aA+a(4$$

$$=\frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+a^2\sec(c+dx)\right)^2\text{ta}}{12d}$$

$$=\frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+a^2\sec(c+dx)\right)^2\text{ta}}{12d}$$

$$=a^4Ax+\frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+a^2\sec(c+\right.}{12d}$$

$$=a^4Ax+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{aB(a+a\sec(c+dx))^3\tan}{4d}$$

$$=a^4Ax+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^4(8A+7B)\tan(c+dx)}{8d}$$

Mathematica [B] time = 1.75681, size = 326, normalized size = 2.16

$$a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\sec(c)(48A\sin(2c+dx)+496A\sin(c+2dx)-144A\sin(3c+2dx)+48A\sin(2c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(48*A + 35*B)*Cos[c + d*x
]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + Sec[c]*(72*A*d*x*Cos[c] + 48*A*d*x*Cos[c + 2*d*x] + 48*A*d
*x*Cos[3*c + 2*d*x] + 12*A*d*x*Cos[3*c + 4*d*x] + 12*A*d*x*Cos[5*c + 4*d*x]
 - 480*A*Sin[c] - 480*B*Sin[c] + 48*A*Sin[d*x] + 105*B*Sin[d*x] + 48*A*Sin[
2*c + d*x] + 105*B*Sin[2*c + d*x] + 496*A*Sin[c + 2*d*x] + 544*B*Sin[c + 2*
d*x] - 144*A*Sin[3*c + 2*d*x] - 96*B*Sin[3*c + 2*d*x] + 48*A*Sin[2*c + 3*d*
x] + 81*B*Sin[2*c + 3*d*x] + 48*A*Sin[4*c + 3*d*x] + 81*B*Sin[4*c + 3*d*x]
+ 160*A*Sin[3*c + 4*d*x] + 160*B*Sin[3*c + 4*d*x])))/(3072*d)
```

Maple [A] time = 0.055, size = 204, normalized size = 1.4

$$a^4Ax+\frac{Aa^4c}{d}+\frac{35\,Ba^4\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+6\,\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{20\,Ba^4\tan(dx+c)}{3\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] a^4*A*x+1/d*A*a^4*c+35/8/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+6/d*A*a^4*ln(sec
(d*x+c)+tan(d*x+c))+20/3/d*B*a^4*tan(d*x+c)+20/3/d*A*a^4*tan(d*x+c)+27/8/d*
B*a^4*sec(d*x+c)*tan(d*x+c)+2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+4/3/d*B*a^4*tan
(d*x+c)*sec(d*x+c)^2+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*a^4*tan(d*
```

```
x+c)*sec(d*x+c)^3
```

Maxima [B] time = 1.04173, size = 396, normalized size = 2.62

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{4}+48\left(dx+c\right)Aa^{4}+64\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{4}-3\,Ba^{4}\left(\frac{2\left(3\,\sin(dx+\cdots}{\sin(dx+c)^{4}-\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 48*(d*x + c)*A*a^4 + 64*
(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 - 3*B*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 48*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72*B*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 + 192*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 48*B*a^4*log(sec(d*x + c) +
 tan(d*x + c)) + 288*A*a^4*tan(d*x + c) + 192*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.511573, size = 408, normalized size = 2.7

$$\frac{48\,Aa^{4}dx\cos\left(dx+c\right)^{4}+3\left(48\,A+35\,B\right)a^{4}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(48\,A+35\,B\right)a^{4}\cos\left(dx+c\right)^{4}\log\left(-\cdots\right.}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*A*a^4*d*x*cos(d*x + c)^4 + 3*(48*A + 35*B)*a^4*cos(d*x + c)^4*log(
sin(d*x + c) + 1) - 3*(48*A + 35*B)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 2*(160*(A + B)*a^4*cos(d*x + c)^3 + 3*(16*A + 27*B)*a^4*cos(d*x + c)^2
 + 8*(A + 4*B)*a^4*cos(d*x + c) + 6*B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{4}\left(\int A\,dx+\int 4A\sec\left(c+dx\right)dx+\int 6A\sec^{2}\left(c+dx\right)dx+\int 4A\sec^{3}\left(c+dx\right)dx+\int A\sec^{4}\left(c+dx\right)dx+\int B\,\mathrm{se}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A, x) + Integral(4*A*sec(c + d*x), x) + Integral(6*A*sec(c +
 d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**
4, x) + Integral(B*sec(c + d*x), x) + Integral(4*B*sec(c + d*x)**2, x) + In
tegral(6*B*sec(c + d*x)**3, x) + Integral(4*B*sec(c + d*x)**4, x) + Integra
l(B*sec(c + d*x)**5, x))
```

Giac [A] time = 1.33957, size = 301, normalized size = 1.99

$$24\,(dx+c)Aa^4+3\left(48\,Aa^4+35\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(48\,Aa^4+35\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*A*a^4 + 3*(48*A*a^4 + 35*B*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 3*(48*A*a^4 + 35*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(120*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 105*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 424
*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 385*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 520*A*a^
4*tan(1/2*d*x + 1/2*c)^3 + 511*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 216*A*a^4*tan
(1/2*d*x + 1/2*c) - 279*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^4)/d
```

3.75 $\int \cos(c + dx)(a + a\sec(c + dx))^4(A + B\sec(c + dx))\,dx$

Optimal. Leaf size=151

$$-\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(13A+12B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+2B)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{(9A}{}$$

```
[Out] a^4*(4*A + B)*x + (a^4*(13*A + 12*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(
3*d) + ((A + 2*B)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((9*A +
11*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.367918, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4018, 3996, 3770}

$$-\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(13A+12B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+2B)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{(9A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^4*(4*A + B)*x + (a^4*(13*A + 12*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(
3*d) + ((A + 2*B)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((9*A +
11*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{aB(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{1}{3}\int \cos(c+dx)(a+a\sec
\\
&= \frac{aB(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{(A+2B)\left(a^2+a^2\sec(c\right.}{2d}
\\
&= \frac{aB(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{(A+2B)\left(a^2+a^2\sec(c\right.}{2d}
\\
&= -\frac{5a^4(A+2B)\sin(c+dx)}{2d} + \frac{aB(a+a\sec(c+dx))^3\sin(c+d}{3d}
\\
&= a^4(4A+B)x - \frac{5a^4(A+2B)\sin(c+dx)}{2d} + \frac{aB(a+a\sec(c+d}{3d}
\\
&= a^4(4A+B)x + \frac{a^4(13A+12B)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^4(A}{}
\end{aligned}
$$

Mathematica [B] time = 6.44965, size = 1202, normalized size = 7.96

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*A + B)*x*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x]))/(16*(B + A*Cos[c + d*x])) + ((-13*A - 12*B)*Cos[c + d*x]
^5*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a
*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(32*d*(B + A*Cos[c + d*x])) + ((13*A
 + 12*B)*Cos[c + d*x]^5*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(32*d*(B + A*Co
s[c + d*x])) + (A*Cos[d*x]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
 + d*x])^4*(A + B*Sec[c + d*x])*Sin[c])/(16*d*(B + A*Cos[c + d*x])) + (A*Co
s[c]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[
c + d*x])*Sin[d*x])/(16*d*(B + A*Cos[c + d*x])) + (B*Cos[c + d*x]^5*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(96
*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2
 + (d*x)/2])^3) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])
^4*(A + B*Sec[c + d*x])*(3*A*Cos[c/2] + 13*B*Cos[c/2] - 3*A*Sin[c/2] - 11*B
*Sin[c/2]))/(192*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d
*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a +
 a*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(3*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2
]))/(12*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] -
Sin[c/2 + (d*x)/2])) + (B*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(96*d*(B + A*Cos[c + d*x])*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c +
d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(-3
*A*Cos[c/2] - 13*B*Cos[c/2] - 3*A*Sin[c/2] - 11*B*Sin[c/2]))/(192*d*(B + A*
Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2
])^2) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*
Sec[c + d*x])*(3*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2]))/(12*d*(B + A*Cos[c + d
*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.084, size = 189, normalized size = 1.3

$$
\frac{Aa^4\sin(dx+c)}{d} + Ba^4x + \frac{Ba^4c}{d} + 4\,a^4Ax + 4\,\frac{Aa^4c}{d} + 6\,\frac{Ba^4\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{13\,Aa^4\ln(\sec(dx+c)+t}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+B*a^4*x+1/d*B*a^4*c+4*a^4*A*x+4/d*A*a^4*c+6/d*B*a^4*ln
(sec(d*x+c)+tan(d*x+c))+13/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*B*a^4
*tan(d*x+c)+4/d*A*a^4*tan(d*x+c)+2/d*B*a^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^
4*sec(d*x+c)*tan(d*x+c)+1/3/d*B*a^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.01326, size = 317, normalized size = 2.1

$$\frac{48\,(dx+c)Aa^4 + 4\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Ba^4 + 12\,(dx+c)Ba^4 - 3\,Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(48*(d*x + c)*A*a^4 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 12*(
d*x + c)*B*a^4 - 3*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 12*B*a^4*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*A*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*B*a^4*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 12*A*a^4*sin(d*x + c) + 48*A*a^4*tan(d*x + c) +
 72*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.509542, size = 405, normalized size = 2.68

$$\frac{12\,(4\,A+B)a^4dx\cos(dx+c)^3 + 3\,(13\,A+12\,B)a^4\cos(dx+c)^3\log(\sin(dx+c)+1) - 3\,(13\,A+12\,B)a^4\cos(dx+c)^3 \cdots}{12 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(12*(4*A + B)*a^4*d*x*cos(d*x + c)^3 + 3*(13*A + 12*B)*a^4*cos(d*x + c
)^3*log(sin(d*x + c) + 1) - 3*(13*A + 12*B)*a^4*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(6*A*a^4*cos(d*x + c)^3 + 8*(3*A + 5*B)*a^4*cos(d*x + c)^2 +
 3*(A + 4*B)*a^4*cos(d*x + c) + 2*B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

[Out] Timed out

Giac [A] time = 1.34172, size = 306, normalized size = 2.03

$$\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(4\,Aa^4+Ba^4\right)(dx+c)+3\left(13\,Aa^4+12\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(13\,Aa^4+12\,Ba^4\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(4*A*a^
4 + B*a^4)*(d*x + c) + 3*(13*A*a^4 + 12*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(13*A*a^4 + 12*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21
*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 48*A*a^4*
tan(1/2*d*x + 1/2*c)^3 - 76*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2
*d*x + 1/2*c) + 54*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3)/d
```

3.76 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(8A+13B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(A-B)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{(A+6B)}{}$$

```
[Out] (a^4*(13*A + 8*B)*x)/2 + (a^4*(8*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^
3*Sin[c + d*x])/(2*d) - ((A - B)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(
2*d) + ((A + 6*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.389536, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4017, 4018, 3996, 3770}

$$\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(8A+13B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(A-B)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{(A+6B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(13*A + 8*B)*x)/2 + (a^4*(8*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^
3*Sin[c + d*x])/(2*d) - ((A - B)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(
2*d) + ((A + 6*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c$$

$$=\frac{aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d}-\frac{(A-B)\left(a^2\right.}{}$$

$$=\frac{aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d}-\frac{(A-B)\left(a^2\right.}{}$$

$$=\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{aA\cos(c+dx)(a+a\sec(c+dx))^3}{2d}$$

$$=\frac{1}{2}a^4(13A+8B)x+\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{aA\cos(c+dx)}{}$$

$$=\frac{1}{2}a^4(13A+8B)x+\frac{a^4(8A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^4}{}$$

Mathematica [B] time = 4.83511, size = 373, normalized size = 2.33

$$\frac{a^4\cos^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4(A+B\sec(c+dx))\left(\frac{4(4A+B)\sin(c)\cos(dx)}{d}+\frac{4(4A+B)\cos(c)\sin(dx)}{d}+\frac{}{d\left(\cos\left(\frac{c}{2}\right)-\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*Cos[c + d*x]^5*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(A + B*Sec[c +
d*x])*(2*(13*A + 8*B)*x - (2*(8*A + 13*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]])/d + (2*(8*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(4*(4*A + B)*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])/d + (4*(4*A + B)*
Cos[c]*Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/d + B/(d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^2) + (4*(A + 4*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2) + (4*(A + 4*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]))))/(64*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.086, size = 182, normalized size = 1.1

$$\frac{Aa^4\sin(dx+c)\cos(dx+c)}{2d}+\frac{13a^4Ax}{2}+\frac{13Aa^4c}{2d}+\frac{Ba^4\sin(dx+c)}{d}+4\frac{Aa^4\sin(dx+c)}{d}+4Ba^4x+4\frac{Ba^4c}{d}+\frac{13Ba^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*A*a^4*sin(d*x+c)*cos(d*x+c)+13/2*a^4*A*x+13/2/d*A*a^4*c+1/d*B*a^4*sin
(d*x+c)+4/d*A*a^4*sin(d*x+c)+4*B*a^4*x+4/d*B*a^4*c+13/2/d*B*a^4*ln(sec(d*x+
c)+tan(d*x+c))+4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a^4*tan(d*x+c)+1/d
*A*a^4*tan(d*x+c)+1/2/d*B*a^4*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.03166, size = 269, normalized size = 1.68

$$(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right))Aa^4 + 24\,(dx + c)Aa^4 + 16\,(dx + c)Ba^4 - Ba^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx + c\right) + 1\right) + \log(s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 24*(d*x + c)*A*a^4 + 16*(d*x
+ c)*B*a^4 - B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) + 8*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 12*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 1
6*A*a^4*sin(d*x + c) + 4*B*a^4*sin(d*x + c) + 4*A*a^4*tan(d*x + c) + 16*B*a
^4*tan(d*x + c))/d
```

Fricas [A] time = 0.512709, size = 390, normalized size = 2.44

$$\frac{2\,(13\,A + 8\,B)a^4dx\cos\left(dx + c\right)^2 + (8\,A + 13\,B)a^4\cos\left(dx + c\right)^2\log\left(\sin\left(dx + c\right) + 1\right) - (8\,A + 13\,B)a^4\cos\left(dx + c\right)^2\log}{4\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(13*A + 8*B)*a^4*d*x*cos(d*x + c)^2 + (8*A + 13*B)*a^4*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (8*A + 13*B)*a^4*cos(d*x + c)^2*log(-sin(d*x + c)
 + 1) + 2*(A*a^4*cos(d*x + c)^3 + 2*(4*A + B)*a^4*cos(d*x + c)^2 + 2*(A + 4
*B)*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39989, size = 311, normalized size = 1.94

$$\left(13\,Aa^4+8\,Ba^4\right)(dx+c)+\left(8\,Aa^4+13\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(8\,Aa^4+13\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((13*A*a^4 + 8*B*a^4)*(d*x + c) + (8*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - (8*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 7*
A*a^4*tan(1/2*d*x + 1/2*c)^5 - 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 11*A*a^4*tan(1/2*d*x
 + 1/2*c) + 11*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/
d
```

3.77 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(A+4B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(8A-3B)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{6d}+\frac{(2A+B)}{}$$

```
[Out] (a^4*(12*A + 13*B)*x)/2 + (a^4*(A + 4*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*
Sin[c + d*x])/(3*d) + ((2*A + B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Si
n[c + d*x])/(2*d) - ((8*A - 3*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*
d)
```

Rubi [A] time = 0.409852, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4017, 4018, 3996, 3770}

$$\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(A+4B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(8A-3B)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{6d}+\frac{(2A+B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^4*(12*A + 13*B)*x)/2 + (a^4*(A + 4*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*
Sin[c + d*x])/(3*d) + ((2*A + B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Si
n[c + d*x])/(2*d) - ((8*A - 3*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*
d)
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{1}{3}\int \cos^2($$
$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{(2A+B)}{}$$
$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{(2A+B)}{}$$
$$= \frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{aA\cos^2(c+dx)(a+a\sec(c+dx}{3d}$$
$$= \frac{1}{2}a^4(12A+13B)x + \frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{aA\cos^2(c+}{}$$
$$= \frac{1}{2}a^4(12A+13B)x + \frac{a^4(A+4B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^4(}{}$$

Mathematica [B] time = 1.85038, size = 342, normalized size = 2.07

$$\frac{a^4\cos^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4(A+B\sec(c+dx))\left(\frac{3(27A+16B)\sin(c)\cos(dx)}{d} + \frac{3(4A+B)\sin(2c)\cos(2dx)}{d} + \frac{3(27}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*Cos[c + d*x]^5*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(A + B*Sec[c +
d*x])*(72*A*x + 78*B*x - (12*(A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]])/d + (12*(A + 4*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(2
7*A + 16*B)*Cos[d*x]*Sin[c])/d + (3*(4*A + B)*Cos[2*d*x]*Sin[2*c])/d + (A*C
os[3*d*x]*Sin[3*c])/d + (3*(27*A + 16*B)*Cos[c]*Sin[d*x])/d + (3*(4*A + B)*
Cos[2*c]*Sin[2*d*x])/d + (A*Cos[3*c]*Sin[3*d*x])/d + (12*B*Sin[(d*x)/2])/(d
*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (12*B*Sin[(
d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/
(192*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.081, size = 190, normalized size = 1.2

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^4}{3d} + \frac{20\,Aa^4\sin(dx+c)}{3d} + \frac{Ba^4\sin(dx+c)\cos(dx+c)}{2d} + \frac{13\,Ba^4x}{2} + \frac{13\,Ba^4c}{2d} + 2\,\frac{Aa^4\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+1/2/d*B*a^4*sin
(d*x+c)*cos(d*x+c)+13/2*B*a^4*x+13/2/d*B*a^4*c+2/d*A*a^4*sin(d*x+c)*cos(d*x
+c)+6*a^4*A*x+6/d*A*a^4*c+4/d*B*a^4*sin(d*x+c)+4/d*B*a^4*ln(sec(d*x+c)+tan(
d*x+c))+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^4*tan(d*x+c)
```

Maxima [A] time = 1.04287, size = 252, normalized size = 1.53

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}-12\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{4}-48\left(dx+c\right)Aa^{4}-3\left(2\,dx+2\,c+\sin\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 12*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a^4 - 48*(d*x + c)*A*a^4 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))
*B*a^4 - 72*(d*x + c)*B*a^4 - 6*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) - 24*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 72*
A*a^4*sin(d*x + c) - 48*B*a^4*sin(d*x + c) - 12*B*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.514288, size = 383, normalized size = 2.32

$$\frac{3\left(12\,A+13\,B\right)a^{4}dx\cos\left(dx+c\right)+3\left(A+4\,B\right)a^{4}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(A+4\,B\right)a^{4}\cos\left(dx+c\right)\log\left(-\mathrm{s}\right.}{6\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(12*A + 13*B)*a^4*d*x*cos(d*x + c) + 3*(A + 4*B)*a^4*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - 3*(A + 4*B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 (2*A*a^4*cos(d*x + c)^3 + 3*(4*A + B)*a^4*cos(d*x + c)^2 + 8*(5*A + 3*B)*a
^4*cos(d*x + c) + 6*B*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30382, size = 305, normalized size = 1.85

$$-\frac{\frac{12\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\left(12\,Aa^4+13\,Ba^4\right)(dx+c)-6\left(Aa^4+4\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6\left(Aa^4+4\,Ba^4\right)\log\left(\left|\mathrm{t}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(12*A*
a^4 + 13*B*a^4)*(d*x + c) - 6*(A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) + 6*(A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*A*
a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 76*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2*d*
x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

3.78 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=173

$$\frac{5a^4(7A+8B)\sin(c+dx)}{8d}+\frac{(7A+4B)\sin(c+dx)\cos^2(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d}+\frac{(35A+32B)\sin(c+dx)\cos}{2}$$

```
[Out] (a^4*(35*A + 48*B)*x)/8 + (a^4*B*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(7*A + 8
*B)*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c
+ d*x])/(4*d) + ((7*A + 4*B)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(12*d) + ((35*A + 32*B)*Cos[c + d*x]*(a^4 + a^4*Sec[c + d*x])*Sin
[c + d*x])/(24*d)
```

Rubi [A] time = 0.402583, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4017, 3996, 3770}

$$\frac{5a^4(7A+8B)\sin(c+dx)}{8d}+\frac{(7A+4B)\sin(c+dx)\cos^2(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d}+\frac{(35A+32B)\sin(c+dx)\cos}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(35*A + 48*B)*x)/8 + (a^4*B*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(7*A + 8
*B)*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c
+ d*x])/(4*d) + ((7*A + 4*B)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(12*d) + ((35*A + 32*B)*Cos[c + d*x]*(a^4 + a^4*Sec[c + d*x])*Sin
[c + d*x])/(24*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int \cos^3($$
$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{(7A+4B)}{}$$
$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{(7A+4B)}{}$$
$$= \frac{5a^4(7A+8B)\sin(c+dx)}{8d} + \frac{aA\cos^3(c+dx)(a+a\sec(c+d}{4d}$$
$$= \frac{1}{8}a^4(35A+48B)x + \frac{5a^4(7A+8B)\sin(c+dx)}{8d} + \frac{aA\cos^3(c}{}$$
$$= \frac{1}{8}a^4(35A+48B)x + \frac{a^4B\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^4(7A+8}{8}$$

Mathematica [A] time = 0.339811, size = 138, normalized size = 0.8

$$\frac{a^4\left(24(28A+27B)\sin(c+dx)+24(7A+4B)\sin(2(c+dx))+32A\sin(3(c+dx))+3A\sin(4(c+dx))+420Adx+8B\sin\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(420*A*d*x + 576*B*d*x - 96*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 96*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(28*A + 27*B)*Sin[c +
 d*x] + 24*(7*A + 4*B)*Sin[2*(c + d*x)] + 32*A*Sin[3*(c + d*x)] + 8*B*Sin[3
*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.091, size = 199, normalized size = 1.2

$$\frac{Aa^4\sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{27\,Aa^4\sin(dx+c)\cos(dx+c)}{8d} + \frac{35\,a^4Ax}{8} + \frac{35\,Aa^4c}{8d} + \frac{B\sin(dx+c)(\cos(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+27/8/d*A*a^4*sin(d*x+c)*cos(d*x+c)+35/8
*a^4*A*x+35/8/d*A*a^4*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*B*a^4*si
n(d*x+c)+4/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+2/d*B*
a^4*sin(d*x+c)*cos(d*x+c)+6*B*a^4*x+6/d*B*a^4*c+1/d*B*a^4*ln(sec(d*x+c)+tan
(d*x+c))
```

Maxima [A] time = 1.02961, size = 277, normalized size = 1.6

$$-\frac{128\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Aa^4-144\left(2\,dx+2\,c+\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 144*(2*d*x + 2*c + sin(2*d*x +
2*c))*A*a^4 - 96*(d*x + c)*A*a^4 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a
^4 - 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 384*(d*x + c)*B*a^4 - 48*B
*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 384*A*a^4*sin(d*x +
c) - 576*B*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.510658, size = 306, normalized size = 1.77

$$\frac{3\,(35\,A+48\,B)a^4dx+12\,Ba^4\log\left(\sin\left(dx+c\right)+1\right)-12\,Ba^4\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Aa^4\cos\left(dx+c\right)^3+8\,(4\,A+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(35*A + 48*B)*a^4*d*x + 12*B*a^4*log(sin(d*x + c) + 1) - 12*B*a^4*l
og(-sin(d*x + c) + 1) + (6*A*a^4*cos(d*x + c)^3 + 8*(4*A + B)*a^4*cos(d*x +
 c)^2 + 3*(27*A + 16*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31806, size = 289, normalized size = 1.67

$$24\,Ba^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Ba^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(35\,Aa^4+48\,Ba^4\right)(dx+c)+\frac{2\left(105\,Aa^4\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*B*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*B*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(35*A*a^4 + 48*B*a^4)*(d*x + c) + 2*(105*A*a^4*tan
(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*A*a^4*tan(1/2*
d*x + 1/2*c)^5 + 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*A*a^4*tan(1/2*d*x + 1/2*
c) + 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.79 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=158

$$-\frac{4a^4(4A+5B)\sin^3(c+dx)}{15d}+\frac{8a^4(4A+5B)\sin(c+dx)}{5d}+\frac{a^4(4A+5B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(4A+5B)\sin(c}{40c}$$

```
[Out] (7*a^4*(4*A + 5*B)*x)/8 + (8*a^4*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(4*A + 5*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(4*A + 5*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c
+ d*x])/(5*d) - (4*a^4*(4*A + 5*B)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.201723, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{4a^4(4A+5B)\sin^3(c+dx)}{15d}+\frac{8a^4(4A+5B)\sin(c+dx)}{5d}+\frac{a^4(4A+5B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(4A+5B)\sin(c}{40c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*a^4*(4*A + 5*B)*x)/8 + (8*a^4*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(4*A + 5*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(4*A + 5*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c
+ d*x])/(5*d) - (4*a^4*(4*A + 5*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}(4A+$$
$$= \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}(4A+$$
$$= \frac{1}{5}a^4(4A+5B)x + \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin($$
$$= \frac{1}{5}a^4(4A+5B)x + \frac{4a^4(4A+5B)\sin(c+dx)}{5d} + \frac{3a^4(4A+}{}$$
$$= \frac{4}{5}a^4(4A+5B)x + \frac{8a^4(4A+5B)\sin(c+dx)}{5d} + \frac{27a^4(4A-}{}$$
$$= \frac{7}{8}a^4(4A+5B)x + \frac{8a^4(4A+5B)\sin(c+dx)}{5d} + \frac{27a^4(4A-}{}$$

Mathematica [A] time = 0.325799, size = 108, normalized size = 0.68

$$\frac{a^4(420(7A+8B)\sin(c+dx)+120(8A+7B)\sin(2(c+dx))+290A\sin(3(c+dx))+60A\sin(4(c+dx))+6A\sin(5(c}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(1680*A*d*x + 2100*B*d*x + 420*(7*A + 8*B)*Sin[c + d*x] + 120*(8*A + 7
*B)*Sin[2*(c + d*x)] + 290*A*Sin[3*(c + d*x)] + 160*B*Sin[3*(c + d*x)] + 60
*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(480*d
)
```

Maple [A] time = 0.095, size = 248, normalized size = 1.6

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+4Aa^4\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^4*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*a^4*(2+cos(d*x+c)^2)*si
n(d*x+c)+4/3*B*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c)+6*B*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*a
```

^4*sin(d*x+c)+4*B*a^4*sin(d*x+c)+B*a^4*(d*x+c))

Maxima [A] time = 1.02748, size = 319, normalized size = 2.02

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-960\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4+60\,(12\,dx+12\,c\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 60*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^4 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^4 + 480*(d*x + c)*B*a^4 + 480*A*a^4*sin(d*x + c) + 1920*B*a^4*s
in(d*x + c))/d
```

Fricas [A] time = 0.485643, size = 279, normalized size = 1.77

$$\frac{105\,(4\,A+5\,B)a^4dx+\left(24\,Aa^4\cos(dx+c)^4+30\,(4\,A+B)a^4\cos(dx+c)^3+16\,(17\,A+10\,B)a^4\cos(dx+c)^2+15\,(28\,A\ldots}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(105*(4*A + 5*B)*a^4*d*x + (24*A*a^4*cos(d*x + c)^4 + 30*(4*A + B)*a^
4*cos(d*x + c)^3 + 16*(17*A + 10*B)*a^4*cos(d*x + c)^2 + 15*(28*A + 27*B)*a
^4*cos(d*x + c) + 8*(83*A + 100*B)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33091, size = 284, normalized size = 1.8

$$\frac{105\left(4\,Aa^4+5\,Ba^4\right)(dx+c)+\frac{2\left(420\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+525\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+1960\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2450\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3584\,Aa^4\ldots}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(105*(4*A*a^4 + 5*B*a^4)*(d*x + c) + 2*(420*A*a^4*tan(1/2*d*x + 1/2*c
)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 1960*A*a^4*tan(1/2*d*x + 1/2*c)^7
+ 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 4
480*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 3160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 3950
*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c) + 1395*B*a^
4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.80 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=220

$$\frac{a^4(72A+83B)\sin(c+dx)}{15d}+\frac{a^4(159A+176B)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{7a^4(7A+8B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{(3A\ldots}{}$$

```
[Out] (7*a^4*(7*A + 8*B)*x)/16 + (a^4*(72*A + 83*B)*Sin[c + d*x])/(15*d) + (7*a^4
*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(159*A + 176*B)*Cos[c
 + d*x]^2*Sin[c + d*x])/(120*d) + (a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^
3*Sin[c + d*x])/(6*d) + ((3*A + 2*B)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x]
)^2*Sin[c + d*x])/(10*d) + ((73*A + 72*B)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c +
 d*x])*Sin[c + d*x])/(120*d)
```

Rubi [A] time = 0.532254, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^4(72A+83B)\sin(c+dx)}{15d}+\frac{a^4(159A+176B)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{7a^4(7A+8B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{(3A\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*a^4*(7*A + 8*B)*x)/16 + (a^4*(72*A + 83*B)*Sin[c + d*x])/(15*d) + (7*a^4
*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(159*A + 176*B)*Cos[c
 + d*x]^2*Sin[c + d*x])/(120*d) + (a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^
3*Sin[c + d*x])/(6*d) + ((3*A + 2*B)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x]
)^2*Sin[c + d*x])/(10*d) + ((73*A + 72*B)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c +
 d*x])*Sin[c + d*x])/(120*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{6d}+\frac{1}{6}\int \mathrm{co}$$

$$= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{6d}+\frac{(3A+}{}$$

$$= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{6d}+\frac{(3A+}{}$$

$$= \frac{a^4(159A+176B)\cos^2(c+dx)\sin(c+dx)}{120d}+\frac{aA\cos^5(c+}{}$$

$$= \frac{a^4(159A+176B)\cos^2(c+dx)\sin(c+dx)}{120d}+\frac{aA\cos^5(c+}{}$$

$$= \frac{a^4(72A+83B)\sin(c+dx)}{15d}+\frac{7a^4(7A+8B)\cos(c+dx)\,\mathrm{s}}{16d}$$

$$= \frac{7}{16}a^4(7A+8B)x+\frac{a^4(72A+83B)\sin(c+dx)}{15d}+\frac{7a^4(7A}{}$$

Mathematica [A] time = 0.585357, size = 134, normalized size = 0.61

$$\frac{a^4(120(44A+49B)\sin(c+dx)+15(127A+128B)\sin(2(c+dx))+720A\sin(3(c+dx))+225A\sin(4(c+dx))+48A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(2940*A*c + 2940*A*d*x + 3360*B*d*x + 120*(44*A + 49*B)*Sin[c + d*x] +
 15*(127*A + 128*B)*Sin[2*(c + d*x)] + 720*A*Sin[3*(c + d*x)] + 580*B*Sin[3
*(c + d*x)] + 225*A*Sin[4*(c + d*x)] + 120*B*Sin[4*(c + d*x)] + 48*A*Sin[5*
(c + d*x)] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.107, size = 306, normalized size = 1.4

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{Ba^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*B*a^4*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^4*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*B*a^4*(2+cos(d*x+c)^2)*sin(d*
x+c)+4/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*B*a^4*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^4*sin
(d*x+c))
```

Maxima [A] time = 1.01621, size = 401, normalized size = 1.82

$$\frac{256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 180*(12*d*
x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 240*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*A*a^4 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*B*a^4 - 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 120*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2
*c + sin(2*d*x + 2*c))*B*a^4 + 960*B*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.492452, size = 329, normalized size = 1.5

$$\frac{105(7A+8B)a^4dx+\left(40Aa^4\cos(dx+c)^5+48(4A+B)a^4\cos(dx+c)^4+10(41A+24B)a^4\cos(dx+c)^3+32(18A\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(105*(7*A + 8*B)*a^4*d*x + (40*A*a^4*cos(d*x + c)^5 + 48*(4*A + B)*a^
4*cos(d*x + c)^4 + 10*(41*A + 24*B)*a^4*cos(d*x + c)^3 + 32*(18*A + 17*B)*a
^4*cos(d*x + c)^2 + 105*(7*A + 8*B)*a^4*cos(d*x + c) + 16*(72*A + 83*B)*a^4
)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.40133, size = 329, normalized size = 1.5

$$105\left(7\,Aa^{4}+8\,Ba^{4}\right)(dx+c)+\frac{2\left(735\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+840\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4760\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+970\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4)*(d*x + c) + 2*(735*A*a^4*tan(1/2*d*x + 1/2*c
)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*A*a^4*tan(1/2*d*x + 1/2*c)^
9 + 4760*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x + 1/2*c)^7 +
 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*A*a^4*tan(1/2*d*x + 1/2*c)^5 +
13488*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 93
20*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*A*a^4*tan(1/2*d*x + 1/2*c) + 3000*B*
a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.81 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=241

$$-\frac{a^4(227A+252B)\sin^3(c+dx)}{105d}+\frac{a^4(227A+252B)\sin(c+dx)}{35d}+\frac{a^4(276A+301B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{a^4(44A\cdots}{}$$

```
[Out] (a^4*(44*A + 49*B)*x)/16 + (a^4*(227*A + 252*B)*Sin[c + d*x])/(35*d) + (a^4
*(44*A + 49*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(276*A + 301*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*A*Cos[c + d*x]^6*(a + a*Sec[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((10*A + 7*B)*Cos[c + d*x]^5*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^4*(a^4 + a^4*Sec[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(227*A + 252*B)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.566977, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^4(227A+252B)\sin^3(c+dx)}{105d}+\frac{a^4(227A+252B)\sin(c+dx)}{35d}+\frac{a^4(276A+301B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{a^4(44A\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(44*A + 49*B)*x)/16 + (a^4*(227*A + 252*B)*Sin[c + d*x])/(35*d) + (a^4
*(44*A + 49*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(276*A + 301*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*A*Cos[c + d*x]^6*(a + a*Sec[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((10*A + 7*B)*Cos[c + d*x]^5*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^4*(a^4 + a^4*Sec[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(227*A + 252*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \frac{1}{7}\int \mathrm{c}$$

$$= \frac{aA\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \frac{(10A+}{}$$

$$= \frac{aA\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \frac{(10A+}{}$$

$$= \frac{a^4(276A+301B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{aA\cos^6(c+}{}$$

$$= \frac{a^4(276A+301B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{aA\cos^6(c+}{}$$

$$= \frac{a^4(44A+49B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^4(276A+301}{}$$

$$= \frac{1}{16}a^4(44A+49B)x + \frac{a^4(227A+252B)\sin(c+dx)}{35d} + \frac{a^4(}{}$$

Mathematica [A] time = 0.696459, size = 156, normalized size = 0.65

$$\frac{a^4(105(323A+352B)\sin(c+dx)+105(124A+127B)\sin(2(c+dx))+5495A\sin(3(c+dx))+2100A\sin(4(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(18480*A*c + 18480*A*d*x + 20580*B*d*x + 105*(323*A + 352*B)*Sin[c + d
*x] + 105*(124*A + 127*B)*Sin[2*(c + d*x)] + 5495*A*Sin[3*(c + d*x)] + 5040
*B*Sin[3*(c + d*x)] + 2100*A*Sin[4*(c + d*x)] + 1575*B*Sin[4*(c + d*x)] + 6
51*A*Sin[5*(c + d*x)] + 336*B*Sin[5*(c + d*x)] + 140*A*Sin[6*(c + d*x)] + 3
5*B*Sin[6*(c + d*x)] + 15*A*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.116, size = 358, normalized size = 1.5

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)+Ba^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/7*A*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)+B*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4*A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c
))*sin(d*x+c)+5/16*d*x+5/16*c)+4/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2
)*sin(d*x+c)+6/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*B*a
^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*A*a^4*(1/
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*B*a^4*(2+cos(
d*x+c)^2)*sin(d*x+c)+1/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^4*(1/2*cos(d
*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.03232, size = 481, normalized size = 2.

$$-\frac{192\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)Aa^4-2688\left(3\sin(dx+c)^5-10\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6720*(192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*A*a^4 - 2688*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*A*a^4 + 140*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x +
 4*c) - 48*sin(2*d*x + 2*c))*A*a^4 + 2240*(sin(d*x + c)^3 - 3*sin(d*x + c))
*A*a^4 - 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4
- 1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 + 35*
(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*B*a^4 + 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 - 1260*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 1680*(2*d*x + 2*c +
 sin(2*d*x + 2*c))*B*a^4)/d
```

Fricas [A] time = 0.504256, size = 396, normalized size = 1.64

$$\frac{105(44A+49B)a^4dx+\left(240Aa^4\cos(dx+c)^6+280(4A+B)a^4\cos(dx+c)^5+192(12A+7B)a^4\cos(dx+c)^4+70(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(44*A + 49*B)*a^4*d*x + (240*A*a^4*cos(d*x + c)^6 + 280*(4*A +
B)*a^4*cos(d*x + c)^5 + 192*(12*A + 7*B)*a^4*cos(d*x + c)^4 + 70*(44*A + 41
*B)*a^4*cos(d*x + c)^3 + 16*(227*A + 252*B)*a^4*cos(d*x + c)^2 + 105*(44*A
```

```
+ 49*B)*a^4*cos(d*x + c) + 32*(227*A + 252*B)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out
```

Giac [A] time = 1.44387, size = 375, normalized size = 1.56

$$105\left(44\,Aa^4+49\,Ba^4\right)(dx+c)+\frac{2\left(4620\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5145\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+30800\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+34300\,Ba^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")

[Out] 1/1680*(105*(44*A*a^4 + 49*B*a^4)*(d*x + c) + 2*(4620*A*a^4*tan(1/2*d*x + 1
/2*c)^13 + 5145*B*a^4*tan(1/2*d*x + 1/2*c)^13 + 30800*A*a^4*tan(1/2*d*x + 1
/2*c)^11 + 34300*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 87164*A*a^4*tan(1/2*d*x +
1/2*c)^9 + 97069*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 135168*A*a^4*tan(1/2*d*x +
1/2*c)^7 + 150528*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 126084*A*a^4*tan(1/2*d*x +
 1/2*c)^5 + 134099*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 58800*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 73220*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 22260*A*a^4*tan(1/2*d*x +
1/2*c) + 21735*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/
d
```

3.82 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$-\frac{(3A-4B)\tan^3(c+dx)}{3ad}-\frac{(3A-4B)\tan(c+dx)}{ad}+\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

```
[Out] (3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((3*A - 4*B)*Tan[c + d*x])/(a*d
) + (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d*x]^3
*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*A - 4*B)*Tan[c + d*x]^3)/(3*a
*d)
```

Rubi [A] time = 0.171113, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4019, 3787, 3768, 3770, 3767}

$$-\frac{(3A-4B)\tan^3(c+dx)}{3ad}-\frac{(3A-4B)\tan(c+dx)}{ad}+\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((3*A - 4*B)*Tan[c + d*x])/(a*d
) + (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d*x]^3
*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*A - 4*B)*Tan[c + d*x]^3)/(3*a
*d)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^3(c+dx)(3a(A-B)-a(3A-4B)\,\text{s}}{a^2}$$
$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-4B)\int \sec^4(c+dx)\,dx}{a} + \frac{(3(A-B}{}$$
$$= \frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{(3(A}{}$$
$$= \frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(3A-4B)\tan(c+dx)}{ad} + \frac{3(A-B)\sec(c+}{2ad}$$

Mathematica [B] time = 5.95256, size = 489, normalized size = 3.73

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left(6(A+B)\sin\left(\frac{dx}{2}\right)+(39B-27A)\sin\left(\frac{3dx}{2}\right)+12A\sin\left(c-\frac{dx}{2}\right)+6A\sin\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-144*(A - B)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[
c]*Sec[c + d*x]^3*(6*(A + B)*Sin[(d*x)/2] + (-27*A + 39*B)*Sin[(3*d*x)/2] +
 12*A*Sin[c - (d*x)/2] - 24*B*Sin[c - (d*x)/2] + 6*A*Sin[c + (d*x)/2] - 6*B
*Sin[c + (d*x)/2] + 24*A*Sin[2*c + (d*x)/2] - 24*B*Sin[2*c + (d*x)/2] - 9*A
*Sin[c + (3*d*x)/2] + 21*B*Sin[c + (3*d*x)/2] - 9*A*Sin[2*c + (3*d*x)/2] +
9*B*Sin[2*c + (3*d*x)/2] + 9*A*Sin[3*c + (3*d*x)/2] - 9*B*Sin[3*c + (3*d*x)
/2] - 3*A*Sin[c + (5*d*x)/2] + 7*B*Sin[c + (5*d*x)/2] + 3*A*Sin[2*c + (5*d*
x)/2] + B*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2] - 3*B*Sin[3*c + (
5*d*x)/2] + 9*A*Sin[4*c + (5*d*x)/2] - 9*B*Sin[4*c + (5*d*x)/2] - 12*A*Sin[
2*c + (7*d*x)/2] + 16*B*Sin[2*c + (7*d*x)/2] - 6*A*Sin[3*c + (7*d*x)/2] + 1
0*B*Sin[3*c + (7*d*x)/2] - 6*A*Sin[4*c + (7*d*x)/2] + 6*B*Sin[4*c + (7*d*x)
/2])))/(48*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.06, size = 340, normalized size = 2.6

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{3\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{A}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{B}{ad}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/3/a/d*B/(tan(1/2*d
*x+1/2*c)+1)^3-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*A+1/a/d/(tan(1/2*d*x+1/2*c)
+1)^2*B-5/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*A-3
```

```
/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A-1/3/a/
d*B/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d/(tan(1/2*d*x+1/2*c)-1)^2*B+1/2/a/d/(tan(
1/2*d*x+1/2*c)-1)^2*A+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B-3/2/a/d*ln(tan(1/2
*d*x+1/2*c)-1)*A-5/2/a/d/(tan(1/2*d*x+1/2*c)-1)*B+3/2/a/d/(tan(1/2*d*x+1/2*
c)-1)*A
```

Maxima [B] time = 1.05089, size = 497, normalized size = 3.79

$$\frac{B\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(\cdots}{(\cos(dx\cdots}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin\cdots}{(\cos(\cdots}}\cdots\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(B*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.493217, size = 417, normalized size = 3.18

$$\frac{9\left((A-B)\cos(dx+c)^4+(A-B)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left((A-B)\cos(dx+c)^4+(A-B)\cos(dx+c)^3\right)\cdots}{12\left(ad\cos(dx+c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(sin(d*x + c)
+ 1) - 9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(-sin(d*x + c
) + 1) - 2*(4*(3*A - 4*B)*cos(d*x + c)^3 + (3*A - 7*B)*cos(d*x + c)^2 - (3*
A - B)*cos(d*x + c) - 2*B)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
+ c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**5/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.34909, size = 246, normalized size = 1.88

$$\frac{\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(9A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-15B\tan\left(\frac{1}{2}d\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(tan
(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2
*c))/a + 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 - 12*A
*tan(1/2*d*x + 1/2*c)^3 + 16*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1
/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.83 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{2(A-B)\tan(c+dx)}{ad}-\frac{(2A-3B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(2A-3B)\tan(c+dx)\sec(}{2ad}$$

```
[Out] -((2*A - 3*B)*ArcTanh[Sin[c + d*x]])/(2*a*d) + (2*(A - B)*Tan[c + d*x])/(a*
d) - ((2*A - 3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d*x
]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.162791, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{2(A-B)\tan(c+dx)}{ad}-\frac{(2A-3B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(2A-3B)\tan(c+dx)\sec(}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((2*A - 3*B)*ArcTanh[Sin[c + d*x]])/(2*a*d) + (2*(A - B)*Tan[c + d*x])/(a*
d) - ((2*A - 3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d*x
]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^2(c+dx)(2a(A-B)-a(2A-3B)\ldots}{a^2}$$
$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(2A-3B)\int \sec^3(c+dx)\,dx}{a} + \frac{(2(A-B\ldots}{}$$
$$= -\frac{(2A-3B)\sec(c+dx)\tan(c+dx)}{2ad} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \ldots$$
$$= -\frac{(2A-3B)\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{2(A-B)\tan(c+dx)}{ad} - \frac{(2A-3B)\sec(c\ldots}{2\ldots}$$

Mathematica [B] time = 3.51831, size = 311, normalized size = 2.88

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(4(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(A-\ldots}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+d\ldots\right.\right.}\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(A + B*Sec[c + d*x])*(4*(A - B)*Sec[c/2]*Sin[(d*x)/2] + C
os[(c + d*x)/2]*((4*A - 6*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*A
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 6*B*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + B/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - B/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^2 + (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(C
os[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])))))/(2*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.055, size = 252, normalized size = 2.3

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3\,B}{2\,ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{A}{ad}\ln\left(\tan\left(\frac{dx}{2}+\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)-1/2/a/d/(tan(1/2*d*x+
1/2*c)+1)^2*B+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B-1/a/d*ln(tan(1/2*d*x+1/2*c
)+1)*A+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B-1/a/d/(tan(1/2*d*x+1/2*c)+1)*A+1/2/
```

```
a/d/(tan(1/2*d*x+1/2*c)-1)^2*B+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*B-1/a/d/(tan(
1/2*d*x+1/2*c)-1)*A-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B+1/a/d*ln(tan(1/2*d*x
+1/2*c)-1)*A
```

Maxima [B] time = 1.00888, size = 381, normalized size = 3.53

$$-\frac{B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.489632, size = 386, normalized size = 3.57

$$-\frac{\left((2A-3B)\cos(dx+c)^3+(2A-3B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((2A-3B)\cos(dx+c)^3+(2A-3B)\,\mathrm{co}\right.}{4\left(ad\cos(dx+c)^3+ad\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/4*(((2*A - 3*B)*cos(d*x + c)^3 + (2*A - 3*B)*cos(d*x + c)^2)*log(sin(d*x
 + c) + 1) - ((2*A - 3*B)*cos(d*x + c)^3 + (2*A - 3*B)*cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c) +
 B)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^3(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**4/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.33356, size = 211, normalized size = 1.95

$$-\frac{\frac{(2A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(2A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3B\tan\ldots\right.}{(\ldots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*((2*A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (2*A - 3*B)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x
+ 1/2*c))/a + 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 -
2*A*tan(1/2*d*x + 1/2*c) + B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 - 1)^2*a))/d
```

3.84 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{B\tan(c+dx)}{ad}$$

```
[Out] ((A - B)*ArcTanh[Sin[c + d*x]])/(a*d) + (B*Tan[c + d*x])/(a*d) - ((A - B)*T
an[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.116752, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4008, 3787, 3770, 3767, 8}

$$\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{B\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((A - B)*ArcTanh[Sin[c + d*x]])/(a*d) + (B*Tan[c + d*x])/(a*d) - ((A - B)*T
an[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = -\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec(c+dx)(-a(A-B)-aB\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A-B)\int \sec(c+dx)\,dx}{a} + \frac{B\int \sec^2(c+dx)\,dx}{a}$$
$$= \frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{B\,\text{Subst}(\int 1\,dx, x, -\text{t}}{ad}$$
$$= \frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad} + \frac{B\tan(c+dx)}{ad} - \frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 1.1954, size = 224, normalized size = 3.61

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left((B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{B\text{s}}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+d\right.\right.}\right.\right.}{ad(\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x])*((-A + B)*Sec[c/2]*Sin[(d*x)/2] +
Cos[(c + d*x)/2]*(-((A - B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) + (B*Sin[d*x])/((Cos[c/2] - Sin[c/2
])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])))))/(a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.044, size = 163, normalized size = 2.6

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{A}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d/(tan(1/2*d*x+1
/2*c)+1)*B+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*
B-1/a/d/(tan(1/2*d*x+1/2*c)-1)*B-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*A+1/a/d*ln(
tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [B] time = 1.00997, size = 265, normalized size = 4.27

$$-\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+\right.}{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - A*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [B] time = 0.482214, size = 319, normalized size = 5.15

$$\frac{\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\cdots}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1)
- ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
2*((A - 2*B)*cos(d*x + c) - B)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(
d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^2(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**3/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.32645, size = 147, normalized size = 2.37

$$\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a - 2
*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.85 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=43

$$\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a*d) + ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rubi [A] time = 0.0818521, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3998, 3770, 3794}

$$\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a*d) + ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= (A-B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx+\frac{B\int \sec(c+dx)\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.251622, size = 109, normalized size = 2.53

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+B\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(B*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A - B)*Sec[c/2]*Sin[
(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.044, size = 78, normalized size = 1.8

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B+1/a/d*ln(tan(1/
2*d*x+1/2*c)+1)*B-1/a/d*B*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 0.980288, size = 134, normalized size = 3.12

$$\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + A*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.47021, size = 197, normalized size = 4.58

$$\frac{(B\cos(dx+c)+B)\log(\sin(dx+c)+1)-(B\cos(dx+c)+B)\log(-\sin(dx+c)+1)+2(A-B)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*((B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - (B*cos(d*x + c) + B)*log(
-sin(d*x + c) + 1) + 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{B\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**
2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.33317, size = 95, normalized size = 2.21

$$\frac{\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} + \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - B*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

3.86 $\int \frac{A+B\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{Ax}{a}-\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] (A*x)/a - ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.0591123, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3919, 3794}

$$\frac{Ax}{a}-\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a - ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{Ax}{a}-(A-B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\ &= \frac{Ax}{a}-\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.13754, size = 72, normalized size = 2.06

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2(B-A)\sin\left(\frac{dx}{2}\right)+Adx\cos\left(c+\frac{dx}{2}\right)+Adx\cos\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(A*d*x*Cos[(d*x)/2] + A*d*x*Cos[c + (d*x)/2] + 2
*(-A + B)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.05, size = 56, normalized size = 1.6

$$2\,{\frac {A\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{ad}}-{\frac {A}{ad}\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) }+{\frac {B}{ad}\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/a/d*A*arctan(tan(1/2*d*x+1/2*c))-1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1
/2*d*x+1/2*c)
```

Maxima [B] time = 1.46919, size = 99, normalized size = 2.83

$$\frac{A {\left(\frac{2 \, \arctan\left(\frac{\sin\left(d x + c\right)}{\cos\left(d x + c\right) + 1}\right)}{a} - \frac{\sin\left(d x + c\right)}{a {\left(\cos\left(d x + c\right) + 1\right)}}\right)} + \frac{B \sin\left(d x + c\right)}{a {\left(\cos\left(d x + c\right) + 1\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + B*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.450558, size = 105, normalized size = 3.

$$\frac{A d x \cos\left(d x + c\right) + A d x - {\left(A - B\right)} \sin\left(d x + c\right)}{a d \cos\left(d x + c\right) + a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (A*d*x*cos(d*x + c) + A*d*x - (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec{\left (c + d x \right)} + 1}\, dx + \int \frac{B \sec{\left (c + d x \right)}}{\sec{\left (c + d x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)
+ 1), x))/a
```

Giac [A] time = 1.18523, size = 59, normalized size = 1.69

$$\frac{\frac{(dx+c)A}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

3.87 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{(2A-B)\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(A-B)}{a}$$

```
[Out] -(((A - B)*x)/a) + ((2*A - B)*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/
(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.109468, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4020, 3787, 2637, 8}

$$\frac{(2A-B)\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(A-B)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(((A - B)*x)/a) + ((2*A - B)*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/
(d*(a + a*Sec[c + d*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = -\frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \cos(c+dx)(a(2A-B)-a(A-B)\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int 1\,dx}{a} + \frac{(2A-B)\int \cos(c+dx)\,dx}{a}$$
$$= -\frac{(A-B)x}{a} + \frac{(2A-B)\sin(c+dx)}{ad} - \frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.369742, size = 76, normalized size = 1.27

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)(dx(B-A)+A\sin(c+dx))+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-A
+ B)*d*x + A*Sin[c + d*x])))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.076, size = 108, normalized size = 1.8

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{A\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{A\arctan(\tan(1/2\,dx+c/2))}{ad}+2\,\frac{\arctan(\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+2/a/d*A*tan(1/2*d*x+1
/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*A*arctan(tan(1/2*d*x+1/2*c))+2/a/d*arc
tan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.49272, size = 193, normalized size = 3.22

$$-\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -(A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.459761, size = 149, normalized size = 2.48

$$-\frac{(A-B)dx\cos(dx+c)+(A-B)dx-(A\cos(dx+c)+2A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -((A - B)*d*x*cos(d*x + c) + (A - B)*d*x - (A*cos(d*x + c) + 2*A - B)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.28206, size = 107, normalized size = 1.78

$$-\frac{\frac{(dx+c)(A-B)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.88 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$-\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A-2B)}{2a}$$

```
[Out] ((3*A - 2*B)*x)/(2*a) - (2*(A - B)*Sin[c + d*x])/(a*d) + ((3*A - 2*B)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rubi [A] time = 0.149809, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A-2B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((3*A - 2*B)*x)/(2*a) - (2*(A - B)*Sin[c + d*x])/(a*d) + ((3*A - 2*B)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \cos^2(c+dx)(a(3A-2B)-2a(A-B)\\
&= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{(3A-2B)\int \cos^2(c+dx)\,dx}{a}-\frac{(2(A-B\\
&= -\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A-B)\cos(c\\
&= \frac{(3A-2B)x}{2a}-\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\cos(c+dx)\sin(c+dx)}{2ad}-
\end{aligned}$$

Mathematica [B] time = 0.443721, size = 197, normalized size = 2.01

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4dx(3A-2B)\cos\left(c+\frac{dx}{2}\right)+4dx(3A-2B)\cos\left(\frac{dx}{2}\right)-4A\sin\left(c+\frac{dx}{2}\right)-3A\sin\left(c+\frac{3dx}{2}\right)-3$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(3*A - 2*B)*d*x*Cos[(d*x)/2] + 4*(3*A - 2*B)*
d*x*Cos[c + (d*x)/2] - 20*A*Sin[(d*x)/2] + 20*B*Sin[(d*x)/2] - 4*A*Sin[c +
(d*x)/2] + 4*B*Sin[c + (d*x)/2] - 3*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d
*x)/2] - 3*A*Sin[2*c + (3*d*x)/2] + 4*B*Sin[2*c + (3*d*x)/2] + A*Sin[2*c +
(5*d*x)/2] + A*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.082, size = 211, normalized size = 2.2

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{(\tan(1/2\,dx+c/2))^3A}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+2\,\frac{(\tan(1/2\,dx+c/2))^3B}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{A}{ad}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/
2*d*x+1/2*c)^3*B-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+2/a/
d/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+3/a/d*A*arctan(tan(1/2*d*
x+1/2*c))-2/a/d*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.44804, size = 304, normalized size = 3.1

$$-\frac{A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(A*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.465594, size = 203, normalized size = 2.07

$$\frac{(3A-2B)dx\cos(dx+c)+(3A-2B)dx+\left(A\cos(dx+c)^2-(A-2B)\cos(dx+c)-4A+4B\right)\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((3*A - 2*B)*d*x*cos(d*x + c) + (3*A - 2*B)*d*x + (A*cos(d*x + c)^2 - (
A - 2*B)*cos(d*x + c) - 4*A + 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\cos^2(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{B\cos^2(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.29655, size = 166, normalized size = 1.69

$$\frac{\frac{(dx+c)(3A-2B)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((d*x + c)*(3*A - 2*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
1/2*c))/a - 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + A*
```

```
tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 +
 1)^2*a))/d
```

3.89 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(4A-3B)\sin^3(c+dx)}{3ad}+\frac{(4A-3B)\sin(c+dx)}{ad}-\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] (-3*(A - B)*x)/(2*a) + ((4*A - 3*B)*Sin[c + d*x])/(a*d) - (3*(A - B)*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x])) - ((4*A - 3*B)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.159081, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2633, 2635, 8}

$$-\frac{(4A-3B)\sin^3(c+dx)}{3ad}+\frac{(4A-3B)\sin(c+dx)}{ad}-\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*(A - B)*x)/(2*a) + ((4*A - 3*B)*Sin[c + d*x])/(a*d) - (3*(A - B)*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x])) - ((4*A - 3*B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \cos^3(c+dx)(a(4A-3B)-3a(A-B)}{a^2}$$
$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(4A-3B)\int \cos^3(c+dx)\,dx}{a} - \frac{(3(A-}{}$$
$$= -\frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3(}{}$$
$$= -\frac{3(A-B)x}{2a} + \frac{(4A-3B)\sin(c+dx)}{ad} - \frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad} - $$

Mathematica [B] time = 0.668246, size = 249, normalized size = 2.04

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(A-B)\cos\left(c+\frac{dx}{2}\right)-36dx(A-B)\cos\left(\frac{dx}{2}\right)+21A\sin\left(c+\frac{dx}{2}\right)+18A\sin\left(c+\frac{3dx}{2}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(A - B)*d*x*Cos[(d*x)/2] - 36*(A - B)*d*x*C
os[c + (d*x)/2] + 69*A*Sin[(d*x)/2] - 60*B*Sin[(d*x)/2] + 21*A*Sin[c + (d*x
)/2] - 12*B*Sin[c + (d*x)/2] + 18*A*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x
)/2] + 18*A*Sin[2*c + (3*d*x)/2] - 9*B*Sin[2*c + (3*d*x)/2] - 2*A*Sin[2*c +
 (5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] - 2*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin
[3*c + (5*d*x)/2] + A*Sin[3*c + (7*d*x)/2] + A*Sin[4*c + (7*d*x)/2]))/(24*a
*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.083, size = 281, normalized size = 2.3

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{(\tan(1/2\,dx+c/2))^5 B}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+5\,\frac{(\tan(1/2\,dx+c/2))^5 A}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-4\,\frac{(t}{ad\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B+5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)^5*A-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B+16/
3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A-1/a/d/(1+tan(1/2*d*
x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)+3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1
/2*d*x+1/2*c)-3/a/d*A*arctan(tan(1/2*d*x+1/2*c))+3/a/d*arctan(tan(1/2*d*x+1
/2*c))*B
```

Maxima [B] time = 1.66925, size = 419, normalized size = 3.43

$$\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{}{c}\right.}{a}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*B*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.471508, size = 243, normalized size = 1.99

$$-\frac{9(A-B)dx\cos(dx+c)+9(A-B)dx-\left(2A\cos(dx+c)^3-(A-3B)\cos(dx+c)^2+(7A-3B)\cos(dx+c)+16A-\right.}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(9*(A - B)*d*x*cos(d*x + c) + 9*(A - B)*d*x - (2*A*cos(d*x + c)^3 - (A
 - 3*B)*cos(d*x + c)^2 + (7*A - 3*B)*cos(d*x + c) + 16*A - 12*B)*sin(d*x +
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos^3(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\cos^3(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)**3*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.22082, size = 204, normalized size = 1.67

$$-\frac{\frac{9(dx+c)(A-B)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+16A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-12B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9A\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(9*(d*x + c)*(A - B)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1
/2*c))/a - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 + 16
*A*tan(1/2*d*x + 1/2*c)^3 - 12*B*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d*x +
 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.90 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$-\frac{4(2A-3B)\tan^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\tan(c+dx)}{a^2d}+\frac{(7A-10B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-10B)\tan(c+dx)\sec^3(}{3a^2d(\sec(c+dx)+1)}$$

```
[Out] ((7*A - 10*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(2*A - 3*B)*Tan[c + d*x
])/(a^2*d) + ((7*A - 10*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((7*A - 1
0*B)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*S
ec[c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(2*A - 3*B)*T
an[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.321115, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4019, 3787, 3768, 3770, 3767}

$$-\frac{4(2A-3B)\tan^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\tan(c+dx)}{a^2d}+\frac{(7A-10B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-10B)\tan(c+dx)\sec^3(}{3a^2d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((7*A - 10*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(2*A - 3*B)*Tan[c + d*x
])/(a^2*d) + ((7*A - 10*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((7*A - 1
0*B)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*S
ec[c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(2*A - 3*B)*T
an[c + d*x]^3)/(3*a^2*d)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-3a(A-2B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{(7A-10B)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \ldots$$

$$= \frac{(7A-10B)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \ldots$$

$$= \frac{(7A-10B)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{(7A-10B)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} \ldots$$

$$= \frac{(7A-10B)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{4(2A-3B)\tan(c+dx)}{a^2d} + \frac{(7A-10B)\sec \ldots}{}$$

Mathematica [B] time = 6.35213, size = 764, normalized size = 4.27

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4(c+dx)\left(195A\sin\left(c-\frac{dx}{2}\right)-51A\sin\left(c+\frac{dx}{2}\right)+189A\sin\left(2c+\frac{dx}{2}\right)-A\sin\left(c+\frac{3dx}{2}\right) \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*(-7*A + 10*B)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a + a*S
ec[c + d*x])^2) - (2*(-7*A + 10*B)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec
[c + d*x]^4*(A + B*Sec[c + d*x])*(45*A*Sin[(d*x)/2] - 6*B*Sin[(d*x)/2] - 20
1*A*Sin[(3*d*x)/2] + 310*B*Sin[(3*d*x)/2] + 195*A*Sin[c - (d*x)/2] - 306*B*
Sin[c - (d*x)/2] - 51*A*Sin[c + (d*x)/2] + 42*B*Sin[c + (d*x)/2] + 189*A*Si
n[2*c + (d*x)/2] - 270*B*Sin[2*c + (d*x)/2] - A*Sin[c + (3*d*x)/2] + 50*B*S
in[c + (3*d*x)/2] - 81*A*Sin[2*c + (3*d*x)/2] + 90*B*Sin[2*c + (3*d*x)/2] +
 119*A*Sin[3*c + (3*d*x)/2] - 170*B*Sin[3*c + (3*d*x)/2] - 129*A*Sin[c + (5
*d*x)/2] + 198*B*Sin[c + (5*d*x)/2] - 9*A*Sin[2*c + (5*d*x)/2] + 42*B*Sin[2
*c + (5*d*x)/2] - 57*A*Sin[3*c + (5*d*x)/2] + 66*B*Sin[3*c + (5*d*x)/2] + 6
3*A*Sin[4*c + (5*d*x)/2] - 90*B*Sin[4*c + (5*d*x)/2] - 75*A*Sin[2*c + (7*d*
x)/2] + 114*B*Sin[2*c + (7*d*x)/2] - 15*A*Sin[3*c + (7*d*x)/2] + 36*B*Sin[3
*c + (7*d*x)/2] - 39*A*Sin[4*c + (7*d*x)/2] + 48*B*Sin[4*c + (7*d*x)/2] + 2
1*A*Sin[5*c + (7*d*x)/2] - 30*B*Sin[5*c + (7*d*x)/2] - 32*A*Sin[3*c + (9*d*
x)/2] + 48*B*Sin[3*c + (9*d*x)/2] - 12*A*Sin[4*c + (9*d*x)/2] + 22*B*Sin[4*
c + (9*d*x)/2] - 20*A*Sin[5*c + (9*d*x)/2] + 26*B*Sin[5*c + (9*d*x)/2]))/(9
6*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 0.065, size = 382, normalized size = 2.1

$$-\frac{A}{6\,d a^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{B}{6\,d a^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{7\,A}{2\,d a^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{9\,B}{2\,d a^{2}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{7\,A}{2\,d a^{2}}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-7/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+9/2/d/a^2*B*tan(1/2*d*x+1/2*c)+7/2/d/a^2*ln(tan(1/2*
d*x+1/2*c)+1)*A-5/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B+3/2/d/a^2/(tan(1/2*d*x+1
/2*c)+1)^2*B-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2*A-5/d/a^2/(tan(1/2*d*x+1/2*
c)+1)*B+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A-1/3/d/a^2*B/(tan(1/2*d*x+1/2*c)+
1)^3+1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2*A-3/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^
2*B-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A+5/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B
-5/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*A-1/3/d/
a^2*B/(tan(1/2*d*x+1/2*c)-1)^3
```

Maxima [B] time = 1.01261, size = 574, normalized size = 3.21

$$\frac{B\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{2}-\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}\right)-A\left(\frac{6\left(\frac{3\sin(\ldots}{\cos(dx\ldots}\right.}{a^{2}-\frac{2a^{2}\sin\ldots}{(\cos(dx\ldots}}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Fricas [A] time = 0.506427, size = 616, normalized size = 3.44

$$3\left((7\,A-10\,B)\cos\left(dx+c\right)^{5}+2\,(7\,A-10\,B)\cos\left(dx+c\right)^{4}+(7\,A-10\,B)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left((7\,A-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((7*A - 10*B)*cos(d*x + c)^5 + 2*(7*A - 10*B)*cos(d*x + c)^4 + (7*A
 - 10*B)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((7*A - 10*B)*cos(d*x +
c)^5 + 2*(7*A - 10*B)*cos(d*x + c)^4 + (7*A - 10*B)*cos(d*x + c)^3)*log(-si
n(d*x + c) + 1) - 2*(16*(2*A - 3*B)*cos(d*x + c)^4 + (43*A - 66*B)*cos(d*x
+ c)^3 + 6*(A - 2*B)*cos(d*x + c)^2 - (3*A - 2*B)*cos(d*x + c) - 2*B)*sin(d
*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c
)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{B\sec^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.35096, size = 305, normalized size = 1.7

$$\frac{\frac{3(7A-10B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{3(7A-10B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-30B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-24A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cdots\right.}}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(7*A - 10*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A - 10*B)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 -
30*B*tan(1/2*d*x + 1/2*c)^5 - 24*A*tan(1/2*d*x + 1/2*c)^3 + 40*B*tan(1/2*d*
x + 1/2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 18*B*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(
1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/2*c) - 27*B*a^4*tan(1/2*d*x +
 1/2*c))/a^6)/d
```

3.91 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=156

$$\frac{2(5A-8B)\tan(c+dx)}{3a^2d}-\frac{(4A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(5A-8B)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(4A-7B)\tan(c+dx)}{2a^2d}$$

```
[Out] -((4*A - 7*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (2*(5*A - 8*B)*Tan[c + d*x
])/(3*a^2*d) - ((4*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((5*A -
8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*S
ec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.305533, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{2(5A-8B)\tan(c+dx)}{3a^2d}-\frac{(4A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(5A-8B)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(4A-7B)\tan(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((4*A - 7*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (2*(5*A - 8*B)*Tan[c + d*x
])/(3*a^2*d) - ((4*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((5*A -
8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*S
ec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)-a(2A-5B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{(5A-8B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \int$$

$$= \frac{(5A-8B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + (2$$

$$= -\frac{(4A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{(5A-8B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} +$$

$$= -\frac{(4A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{2(5A-8B)\tan(c+dx)}{3a^2d} - \frac{(4A-7B)\sec}{}$$

Mathematica [B] time = 4.05729, size = 496, normalized size = 3.18

$$\frac{96(4A-7B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (96*(4*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*S
ec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (64*A - 97*B)*Sin[(3*d*x)/
2] - 84*A*Sin[c - (d*x)/2] + 126*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (d*x)/2]
 - 42*B*Sin[c + (d*x)/2] - 56*A*Sin[2*c + (d*x)/2] + 98*B*Sin[2*c + (d*x)/2
] - 6*A*Sin[c + (3*d*x)/2] + 3*B*Sin[c + (3*d*x)/2] + 34*A*Sin[2*c + (3*d*x
)/2] - 37*B*Sin[2*c + (3*d*x)/2] - 36*A*Sin[3*c + (3*d*x)/2] + 63*B*Sin[3*c
 + (3*d*x)/2] + 48*A*Sin[c + (5*d*x)/2] - 75*B*Sin[c + (5*d*x)/2] + 6*A*Sin
[2*c + (5*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] -
 39*B*Sin[3*c + (5*d*x)/2] - 12*A*Sin[4*c + (5*d*x)/2] + 21*B*Sin[4*c + (5*
d*x)/2] + 20*A*Sin[2*c + (7*d*x)/2] - 32*B*Sin[2*c + (7*d*x)/2] + 6*A*Sin[3
*c + (7*d*x)/2] - 12*B*Sin[3*c + (7*d*x)/2] + 14*A*Sin[4*c + (7*d*x)/2] - 2
0*B*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.063, size = 294, normalized size = 1.9

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{5A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{7B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{A}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*d*x+1/2*c)-1/d/a^2/(tan(1/2*d*x+1
/2*c)+1)*A+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B-2/d/a^2*ln(tan(1/2*d*x+1/2*c)
+1)*A+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)
^2*B-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*A+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B+2/
d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B+1/2/d
/a^2/(tan(1/2*d*x+1/2*c)-1)^2*B
```

Maxima [B] time = 1.01654, size = 454, normalized size = 2.91

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+\ldots}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.495508, size = 566, normalized size = 3.63

$$-\frac{3\left((4A-7B)\cos(dx+c)^4+2(4A-7B)\cos(dx+c)^3+(4A-7B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((4A-7\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((4*A - 7*B)*cos(d*x + c)^4 + 2*(4*A - 7*B)*cos(d*x + c)^3 + (4*A
- 7*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((4*A - 7*B)*cos(d*x + c)^
4 + 2*(4*A - 7*B)*cos(d*x + c)^3 + (4*A - 7*B)*cos(d*x + c)^2)*log(-sin(d*x
 + c) + 1) - 2*(4*(5*A - 8*B)*cos(d*x + c)^3 + (28*A - 43*B)*cos(d*x + c)^2
 + 6*(A - B)*cos(d*x + c) + 3*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^
2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.41502, size = 267, normalized size = 1.71

$$-\frac{\frac{3\,(4\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\,(4\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{6\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,B\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(4*A - 7*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(4*A - 7*B)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(2*A*tan(1/2*d*x + 1/2*c)^3 - 5*
B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
 B*a^4*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

3.92 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{(A-4B)\tan(c+dx)}{3a^2d}+\frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-2B)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] ((A - 2*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((A - 4*B)*Tan[c + d*x])/(3*a^2
*d) - ((A - 2*B)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.257217, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(A-4B)\tan(c+dx)}{3a^2d}+\frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-2B)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((A - 2*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((A - 4*B)*Tan[c + d*x])/(3*a^2
*d) - ((A - 2*B)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^2(c+dx)(2a(A-B)-a(A-4B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec(c+dx)\,(}{}$$

$$= -\frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(A-4B)\int \sec}{3a^2}$$

$$= \frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^2(c+dx}{3d(a+a\sec(c}$$

$$= \frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-4B)\tan(c+dx)}{3a^2d} - \frac{(A-2B)\tan(c+d}{a^2d(1+\sec(c+dx)}$$

Mathematica [B] time = 1.68355, size = 292, normalized size = 2.7

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))\left(-(A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-A + B)*Sec[c/2]*Si
n[(d*x)/2] - 2*(4*A - 7*B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(
c + d*x)/2]^3*(-6*(A - 2*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (6*B*Sin[d*x])/((Cos[c/2] - Sin[c/
2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]))) - (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d
*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.053, size = 205, normalized size = 1.9

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*d*x+1/2*c)+1/d/a^2*ln(tan(1/2*d*
```

```
x+1/2*c)+1)*A-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^2/(tan(1/2*d*x+1/2*c
)+1)*B-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*
B-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [B] time = 1.02155, size = 329, normalized size = 3.05

$$\frac{B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - A*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

Fricas [A] time = 0.490716, size = 501, normalized size = 4.64

$$\frac{3\left((A-2B)\cos(dx+c)^3+2(A-2B)\cos(dx+c)^2+(A-2B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((A-2B)\cos(d\right.}{6\left(a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((A - 2*B)*cos(d*x + c)^3 + 2*(A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*c
os(d*x + c))*log(sin(d*x + c) + 1) - 3*((A - 2*B)*cos(d*x + c)^3 + 2*(A - 2
*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*
(2*A - 5*B)*cos(d*x + c)^2 + (5*A - 14*B)*cos(d*x + c) - 3*B)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.37016, size = 204, normalized size = 1.89

$$\frac{\frac{6\,(A-2\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,(A-2\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa^4}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(A - 2*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(A - 2*B)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x +
 1/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.93 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{(2A-5B)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((2*A - 5*B)*Tan[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.187242, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4008, 3998, 3770, 3794}

$$\frac{(2A-5B)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((2*A - 5*B)*Tan[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a(A-B)-3aB\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= -\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A-5B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a} + \frac{B\int \sec(c+dx)\,dx}{a^2}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A-5B)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

Mathematica [B] time = 0.53249, size = 169, normalized size = 2.14

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-(A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-2(A-4B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^2\left(\frac{1}{2}(\right.\right.}{3a^2d(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*B*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]
*Sin[(d*x)/2] - 2*(A - 4*B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - (A -
 B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.051, size = 119, normalized size = 1.5

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan(1/2*d*x
+1/2*c)-1)*B+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B
```

Maxima [A] time = 0.983702, size = 196, normalized size = 2.48

$$-\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
```

```
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.477731, size = 338, normalized size = 4.28

$$\frac{3\left(B\cos\left(dx+c\right)^{2}+2\,B\cos\left(dx+c\right)+B\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(B\cos\left(dx+c\right)^{2}+2\,B\cos\left(dx+c\right)+B\right)\log\left(-\sin\left(dx\right.\right.}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - 3*
(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*log(-sin(d*x + c) + 1) + 2*((A -
4*B)*cos(d*x + c) + 2*A - 5*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{2}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.30213, size = 151, normalized size = 1.91

$$\frac{\frac{6\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{6\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}+\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*B*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) - 9*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.94 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(A+2B)\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((A + 2*B)*Tan[c + d*
x])/(3*d*(a^2 + a^2*Sec[c + d*x]))
```

Rubi [A] time = 0.0797186, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4000, 3794}

$$\frac{(A+2B)\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((A + 2*B)*Tan[c + d*
x])/(3*d*(a^2 + a^2*Sec[c + d*x]))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx &= \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(A+2B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(A+2B)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.199152, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((2A+B)\sin\left(c+\frac{3dx}{2}\right)+3(A+B)\sin\left(\frac{dx}{2}\right)-3A\sin\left(c+\frac{dx}{2}\right)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(A + B)*Sin[(d*x)/2] - 3*A*Sin[c + (d*x)/2] +
 (2*A + B)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.049, size = 60, normalized size = 0.9

$$\frac{1}{2\,da^{2}}\left(-\frac{A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(-1/3*A*tan(1/2*d*x+1/2*c)^3+1/3*B*tan(1/2*d*x+1/2*c)^3+A*tan(1/2
*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.977688, size = 126, normalized size = 1.94

$$\frac{\frac{B\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}}+\frac{A\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(B*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + A*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.43814, size = 144, normalized size = 2.22

$$\frac{\left((2\,A+B)\cos\left(dx+c\right)+A+2\,B\right)\sin\left(dx+c\right)}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*((2*A + B)*cos(d*x + c) + A + 2*B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^{2}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(B*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.23922, size = 81, normalized size = 1.25

$$-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(A*tan(1/2*d*x + 1/2*c)^3 - B*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x
 + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.95 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{(4A-B)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (A*x)/a^2 - ((4*A - B)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B
)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.112471, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3922, 3919, 3794}

$$-\frac{(4A-B)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (A*x)/a^2 - ((4*A - B)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B
)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{-3aA+a(A-B)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= \frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(4A-B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(4A-B)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [B] time = 0.349109, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(12A\sin\left(c+\frac{dx}{2}\right)-10A\sin\left(c+\frac{3dx}{2}\right)+9Adx\cos\left(c+\frac{dx}{2}\right)+3Adx\cos\left(c+\frac{3dx}{2}\right)+3Adx\cos\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*A*d*x*Cos[(d*x)/2] + 9*A*d*x*Cos[c + (d*x)/
2] + 3*A*d*x*Cos[c + (3*d*x)/2] + 3*A*d*x*Cos[2*c + (3*d*x)/2] - 18*A*Sin[(
d*x)/2] + 6*B*Sin[(d*x)/2] + 12*A*Sin[c + (d*x)/2] - 6*B*Sin[c + (d*x)/2] -
 10*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.057, size = 97, normalized size = 1.4

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{A\arctan\left(\tan\left(1/2\right.\right.}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*A*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.47369, size = 162, normalized size = 2.31

$$-\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - B*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.451332, size = 228, normalized size = 3.26

$$\frac{3\,Adx\cos\left(dx+c\right)^2+6\,Adx\cos\left(dx+c\right)+3\,Adx-\left(\left(5\,A-2\,B\right)\cos\left(dx+c\right)+4\,A-B\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*d*x*cos(d*x + c)^2 + 6*A*d*x*cos(d*x + c) + 3*A*d*x - ((5*A - 2*B)
*cos(d*x + c) + 4*A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{B\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c +
 d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.24154, size = 115, normalized size = 1.64

$$\frac{\frac{6(dx+c)A}{a^2} + \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 9Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.96 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=98

$$\frac{2(5A-2B)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2A-B)}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -(((2*A - B)*x)/a^2) + (2*(5*A - 2*B)*Sin[c + d*x])/(3*a^2*d) - ((2*A - B)*
Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.23049, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4020, 3787, 2637, 8}

$$\frac{2(5A-2B)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2A-B)}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((2*A - B)*x)/a^2) + (2*(5*A - 2*B)*Sin[c + d*x])/(3*a^2*d) - ((2*A - B)*
Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(a(4A-B)-2a(A-B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= -\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \cos(c+dx)\left(2a^2(5A-2B)\right.}{3a}$$
$$= -\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2(5A-2B))\int \cos(c+dx)\,dx}{3a^2}$$
$$= -\frac{(2A-B)x}{a^2} + \frac{2(5A-2B)\sin(c+dx)}{3a^2d} - \frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sin(c}{3d(a+a\sec(c}$$

Mathematica [B] time = 0.586479, size = 245, normalized size = 2.5

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-18dx(2A-B)\cos\left(c+\frac{dx}{2}\right)-18dx(2A-B)\cos\left(\frac{dx}{2}\right)-30A\sin\left(c+\frac{dx}{2}\right)+41A\sin\left(c+\frac{3dx}{2}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-18*(2*A - B)*d*x*Cos[(d*x)/2] - 18*(2*A - B)*d
*x*Cos[c + (d*x)/2] - 12*A*d*x*Cos[c + (3*d*x)/2] + 6*B*d*x*Cos[c + (3*d*x)
/2] - 12*A*d*x*Cos[2*c + (3*d*x)/2] + 6*B*d*x*Cos[2*c + (3*d*x)/2] + 66*A*S
in[(d*x)/2] - 36*B*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2] + 24*B*Sin[c + (d*x
)/2] + 41*A*Sin[c + (3*d*x)/2] - 20*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3
*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2]))/(12*a^2*d*
(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.082, size = 149, normalized size = 1.5

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{A\tan\left(1/2\,dx\right.}{da^2\left(1+(\tan(1/2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*A*tan(1/2*d*x
+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a
^2*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.49117, size = 258, normalized size = 2.63

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d

---

Fricas [A] time = 0.464818, size = 296, normalized size = 3.02

$$-\frac{3\,(2\,A-B)dx\cos\left(dx+c\right)^{2}+6\,(2\,A-B)dx\cos\left(dx+c\right)+3\,(2\,A-B)dx-\left(3\,A\cos\left(dx+c\right)^{2}+(14\,A-5\,B)\cos\left(dx\cdots\right.\right.}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -1/3*(3*(2*A - B)*d*x*cos(d*x + c)^2 + 6*(2*A - B)*d*x*cos(d*x + c) + 3*(2*A - B)*d*x - (3*A*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 10*A - 4*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\cos\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)

[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2

Giac [A] time = 1.20842, size = 163, normalized size = 1.66

$$-\frac{\frac{6\,(dx+c)(2\,A-B)}{a^{2}}-\frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}+\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] -1/6*(6*(d*x + c)*(2*A - B)/a^2 - 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

3.97 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=143

$$-\frac{2(8A-5B)\sin(c+dx)}{3a^2d}+\frac{(7A-4B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7A-4B)}{2a^2}-\frac{(A-$$

```
[Out] ((7*A - 4*B)*x)/(2*a^2) - (2*(8*A - 5*B)*Sin[c + d*x])/(3*a^2*d) + ((7*A -
4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*A - 5*B)*Cos[c + d*x]*Sin[c
 + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.300401, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{2(8A-5B)\sin(c+dx)}{3a^2d}+\frac{(7A-4B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7A-4B)}{2a^2}-\frac{(A-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((7*A - 4*B)*x)/(2*a^2) - (2*(8*A - 5*B)*Sin[c + d*x])/(3*a^2*d) + ((7*A -
4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*A - 5*B)*Cos[c + d*x]*Sin[c
 + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(a(5A-2B)-3a(A-B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(8A-5B)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \mathrm{c}}{}$$

$$= -\frac{(8A-5B)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(2(8}{}$$

$$= -\frac{2(8A-5B)\sin(c+dx)}{3a^2d} + \frac{(7A-4B)\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{(8A-5B)\,\mathrm{c}}{3a^2d(}$$

$$= \frac{(7A-4B)x}{2a^2} - \frac{2(8A-5B)\sin(c+dx)}{3a^2d} + \frac{(7A-4B)\cos(c+dx)\sin(c+dx)}{2a^2d} -$$

Mathematica [B] time = 0.74134, size = 315, normalized size = 2.2

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(7A-4B)\cos\left(c+\frac{dx}{2}\right)+36dx(7A-4B)\cos\left(\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-239A\sin\left(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 4*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 4*B
)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 48*B*d*x*Cos[c + (3*
d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 48*B*d*x*Cos[2*c + (3*d*x)/2] - 3
81*A*Sin[(d*x)/2] + 264*B*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)/2] - 120*B*Sin
[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] + 164*B*Sin[c + (3*d*x)/2] - 63*A*
Sin[2*c + (3*d*x)/2] + 36*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d*x)/2
] + 12*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 12*B*Sin[3*c +
(5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] + 3*A*Sin[4*c + (7*d*x)/2]))/(48*a^2*
d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.091, size = 252, normalized size = 1.8

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 5\,\frac{A\,(\tan(1/2\,dx}{da^2\left(1+(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*B*ta
n(1/2*d*x+1/2*c)^3-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+
2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+7/d/a^2*A*arctan(ta
n(1/2*d*x+1/2*c))-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.51315, size = 382, normalized size = 2.67

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

Fricas [A] time = 0.471908, size = 342, normalized size = 2.39

$$\frac{3\,(7\,A-4\,B)dx\cos(dx+c)^2+6\,(7\,A-4\,B)dx\cos(dx+c)+3\,(7\,A-4\,B)dx+\left(3\,A\cos(dx+c)^3-6\,(A-B)\cos(dx+c\right.}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(7*A - 4*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 4*B)*d*x*cos(d*x + c) + 3*
(7*A - 4*B)*d*x + (3*A*cos(d*x + c)^3 - 6*(A - B)*cos(d*x + c)^2 - (43*A -
28*B)*cos(d*x + c) - 32*A + 20*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{B\cos^2(c+dx)\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1
), x))/a**2
```

Giac [A] time = 1.26574, size = 221, normalized size = 1.55

$$\frac{\frac{3\,(dx+c)(7\,A-4\,B)}{a^2} - \frac{6\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2} + \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - \cdots}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A - 4*B)/a^2 - 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(
1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4
*tan(1/2*d*x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*
d*x + 1/2*c))/a^6)/d
```

3.98 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{4(3A-2B)\sin^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\sin(c+dx)}{a^2d}-\frac{(10A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10A-7B)\sin(c+dx)\cos^2}{3a^2d(\sec(c+dx)+1}$$

```
[Out] -((10*A - 7*B)*x)/(2*a^2) + (4*(3*A - 2*B)*Sin[c + d*x])/(a^2*d) - ((10*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*A - 7*B)*Cos[c + d*x]^2*S
in[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(3*A - 2*B)*Sin[c + d*x]^3)/(3*a^2
*d)
```

Rubi [A] time = 0.319436, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2633, 2635, 8}

$$-\frac{4(3A-2B)\sin^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\sin(c+dx)}{a^2d}-\frac{(10A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10A-7B)\sin(c+dx)\cos^2}{3a^2d(\sec(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -((10*A - 7*B)*x)/(2*a^2) + (4*(3*A - 2*B)*Sin[c + d*x])/(a^2*d) - ((10*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*A - 7*B)*Cos[c + d*x]^2*S
in[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(3*A - 2*B)*Sin[c + d*x]^3)/(3*a^2
*d)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(3a(2A-B)-4a(A-B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(10A-7B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \cdots$$

$$= -\frac{(10A-7B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \cdots$$

$$= -\frac{(10A-7B)\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{(10A-7B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}$$

$$= -\frac{(10A-7B)x}{2a^2} + \frac{4(3A-2B)\sin(c+dx)}{a^2d} - \frac{(10A-7B)\cos(c+dx)\sin(c+d\cdots}{2a^2d}$$

Mathematica [B] time = 0.713892, size = 369, normalized size = 2.17

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(10A-7B)\cos\left(c+\frac{dx}{2}\right)-36dx(10A-7B)\cos\left(\frac{dx}{2}\right)-156A\sin\left(c+\frac{dx}{2}\right)+342A\sin\left(c\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(10*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(10*A -
7*B)*d*x*Cos[c + (d*x)/2] - 120*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c +
 (3*d*x)/2] - 120*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2
] + 516*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 156*A*Sin[c + (d*x)/2] + 147*
B*Sin[c + (d*x)/2] + 342*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] +
118*A*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 30*A*Sin[2*c + (5*
d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[
3*c + (5*d*x)/2] - 3*A*Sin[3*c + (7*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] - 3*
A*Sin[4*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2] + A*Sin[4*c + (9*d*x)/2]
+ A*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.11, size = 322, normalized size = 1.9

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+10\frac{(\tan(1/2\,dx\cdots}{da^2\left(1+(\tan(1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*A*tan(1/2*d*x+1/2*c)^3+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+9/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*d*x+1/2*c)+10/d/a^2/(1+tan(1/2*d
*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A-5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^5*B+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/
2*c)^3-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3+6/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)-3/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*B*tan(1/2*d*x+1/2*c)-10/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))+7/d/a^2*ar
ctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.53032, size = 502, normalized size = 2.95

$$\frac{A\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 0.480637, size = 389, normalized size = 2.29

$$-\frac{3(10A-7B)dx\cos(dx+c)^2+6(10A-7B)dx\cos(dx+c)+3(10A-7B)dx-\left(2A\cos(dx+c)^4-(2A-3B)\cos(\right.}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(10*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(10*A - 7*B)*d*x*cos(d*x + c) +
 3*(10*A - 7*B)*d*x - (2*A*cos(d*x + c)^4 - (2*A - 3*B)*cos(d*x + c)^3 + 6*
(2*A - B)*cos(d*x + c)^2 + (66*A - 43*B)*cos(d*x + c) + 48*A - 32*B)*sin(d*
x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29931, size = 259, normalized size = 1.52

$$-\frac{\frac{3\,(dx+c)(10\,A-7\,B)}{a^2}-\frac{2\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,B\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(10*A - 7*B)/a^2 - 2*(30*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*
tan(1/2*d*x + 1/2*c)^5 + 40*A*tan(1/2*d*x + 1/2*c)^3 - 24*B*tan(1/2*d*x + 1
/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d
*x + 1/2*c)^3 - 27*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2*d*x + 1/2*
c))/a^6)/d
```

3.99 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{8(9A-19B)\tan(c+dx)}{15a^3d}-\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{4(9A-19B)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(6A-13B)\tan(c}{2a}$$

```
[Out] -((6*A - 13*B)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (8*(9*A - 19*B)*Tan[c + d
*x])/(15*a^3*d) - ((6*A - 13*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) + ((A
- B)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*A - 11
*B)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (4*(9*A
- 19*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.474776, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{8(9A-19B)\tan(c+dx)}{15a^3d}-\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{4(9A-19B)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(6A-13B)\tan(c}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -((6*A - 13*B)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (8*(9*A - 19*B)*Tan[c + d
*x])/(15*a^3*d) - ((6*A - 13*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) + ((A
- B)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*A - 11
*B)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (4*(9*A
- 19*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-a(2A-7B)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \cdots$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \cdots$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \cdots$$

$$= -\frac{(6A-13B)\sec(c+dx)\tan(c+dx)}{2a^3d} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \cdots$$

$$= -\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d} + \frac{8(9A-19B)\tan(c+dx)}{15a^3d} - \frac{(6A-13B)\cdots}{\cdots}$$

Mathematica [B] time = 6.16131, size = 610, normalized size = 3.02

$$\frac{1920(6A-13B)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (1920*(6*A - 13*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2
]*Sec[c]*Sec[c + d*x]^2*((-870*A + 1235*B)*Sin[(d*x)/2] + 5*(366*A - 761*B)
*Sin[(3*d*x)/2] - 2094*A*Sin[c - (d*x)/2] + 4329*B*Sin[c - (d*x)/2] + 1314*
A*Sin[c + (d*x)/2] - 1989*B*Sin[c + (d*x)/2] - 1650*A*Sin[2*c + (d*x)/2] +
3575*B*Sin[2*c + (d*x)/2] - 450*A*Sin[c + (3*d*x)/2] + 475*B*Sin[c + (3*d*x
)/2] + 1230*A*Sin[2*c + (3*d*x)/2] - 2005*B*Sin[2*c + (3*d*x)/2] - 1050*A*S
in[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 1278*A*Sin[c + (5*d*x)/
2] - 2673*B*Sin[c + (5*d*x)/2] - 90*A*Sin[2*c + (5*d*x)/2] - 105*B*Sin[2*c
+ (5*d*x)/2] + 918*A*Sin[3*c + (5*d*x)/2] - 1593*B*Sin[3*c + (5*d*x)/2] - 4
50*A*Sin[4*c + (5*d*x)/2] + 975*B*Sin[4*c + (5*d*x)/2] + 630*A*Sin[2*c + (7
*d*x)/2] - 1325*B*Sin[2*c + (7*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 255*B*
Sin[3*c + (7*d*x)/2] + 480*A*Sin[4*c + (7*d*x)/2] - 875*B*Sin[4*c + (7*d*x)
```

```
/2] - 90*A*Sin[5*c + (7*d*x)/2] + 195*B*Sin[5*c + (7*d*x)/2] + 144*A*Sin[3*
c + (9*d*x)/2] - 304*B*Sin[3*c + (9*d*x)/2] + 30*A*Sin[4*c + (9*d*x)/2] - 9
0*B*Sin[4*c + (9*d*x)/2] + 114*A*Sin[5*c + (9*d*x)/2] - 214*B*Sin[5*c + (9*
d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.075, size = 334, normalized size = 1.7

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{2\,B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/2/d/a
^3*A*tan(1/2*d*x+1/2*c)^3-2/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)+7/2/d/a^3/(tan(1/2*d*x+1/2*
c)+1)*B-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*A
+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^3*B/(tan(1/2*d*x+1/2*c)+1)^2
+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B+7
/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*A+1/2/d/a^
3*B/(tan(1/2*d*x+1/2*c)-1)^2
```

Maxima [A] time = 1.07032, size = 509, normalized size = 2.52

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-3\,A\left(\cdots\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 0.505824, size = 757, normalized size = 3.75

$$-\frac{15\left((6\,A-13\,B)\cos(dx+c)^5+3\,(6\,A-13\,B)\cos(dx+c)^4+3\,(6\,A-13\,B)\cos(dx+c)^3+(6\,A-13\,B)\cos(dx+c)^2\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/60*(15*((6*A - 13*B)*cos(d*x + c)^5 + 3*(6*A - 13*B)*cos(d*x + c)^4 + 3*
(6*A - 13*B)*cos(d*x + c)^3 + (6*A - 13*B)*cos(d*x + c)^2)*log(sin(d*x + c)
 + 1) - 15*((6*A - 13*B)*cos(d*x + c)^5 + 3*(6*A - 13*B)*cos(d*x + c)^4 + 3
*(6*A - 13*B)*cos(d*x + c)^3 + (6*A - 13*B)*cos(d*x + c)^2)*log(-sin(d*x +
c) + 1) - 2*(16*(9*A - 19*B)*cos(d*x + c)^4 + 3*(114*A - 239*B)*cos(d*x + c
)^3 + (234*A - 479*B)*cos(d*x + c)^2 + 15*(2*A - 3*B)*cos(d*x + c) + 15*B)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(
d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.34511, size = 315, normalized size = 1.56

$$-\frac{\frac{30\,(6\,A-13\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(6\,A-13\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{60\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(6*A - 13*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(6*A - 1
3*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(2*A*tan(1/2*d*x + 1/2*c)^
3 - 7*B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + 5*B*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1
/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^
3 - 40*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 46
5*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.100 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=156

$$-\frac{(7A-27B)\tan(c+dx)}{15a^3d}+\frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-3B)\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] ((A - 3*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((7*A - 27*B)*Tan[c + d*x])/(15
*a^3*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3
) + ((4*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^
2) - ((A - 3*B)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.428503, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(7A-27B)\tan(c+dx)}{15a^3d}+\frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-3B)\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A - 3*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((7*A - 27*B)*Tan[c + d*x])/(15
*a^3*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3
) + ((4*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^
2) - ((A - 3*B)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)-a(A-6B)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \int$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(}{d(}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(}{d(}$$

$$= \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9}{15}$$

$$= \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(7A-27B)\tan(c+dx)}{15a^3d} + \frac{(A-B)\sec^3(c+}{5d(a+a\sec}$$

Mathematica [B] time = 3.97864, size = 480, normalized size = 3.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(5(32A-51B)\sin\left(\frac{dx}{2}\right)+(567B-167A)\sin\left(\frac{3dx}{2}\right)+170A\sin\left(c-\frac{dx}{2}\right)-17\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-960*(A - 3*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*S
ec[c]*Sec[c + d*x]*(5*(32*A - 51*B)*Sin[(d*x)/2] + (-167*A + 567*B)*Sin[(3*
d*x)/2] + 170*A*Sin[c - (d*x)/2] - 600*B*Sin[c - (d*x)/2] - 170*A*Sin[c + (
d*x)/2] + 375*B*Sin[c + (d*x)/2] + 160*A*Sin[2*c + (d*x)/2] - 480*B*Sin[2*c
 + (d*x)/2] + 75*A*Sin[c + (3*d*x)/2] - 60*B*Sin[c + (3*d*x)/2] - 167*A*Sin
[2*c + (3*d*x)/2] + 402*B*Sin[2*c + (3*d*x)/2] + 75*A*Sin[3*c + (3*d*x)/2]
- 225*B*Sin[3*c + (3*d*x)/2] - 95*A*Sin[c + (5*d*x)/2] + 315*B*Sin[c + (5*d
*x)/2] + 15*A*Sin[2*c + (5*d*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 95*A*Sin[3
*c + (5*d*x)/2] + 240*B*Sin[3*c + (5*d*x)/2] + 15*A*Sin[4*c + (5*d*x)/2] -
45*B*Sin[4*c + (5*d*x)/2] - 22*A*Sin[2*c + (7*d*x)/2] + 72*B*Sin[2*c + (7*d
*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] - 22*A*Sin[4*c + (7*d*x)/2] + 57*B*Sin[4
*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.057, size = 245, normalized size = 1.6

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/3/d/
a^3*A*tan(1/2*d*x+1/2*c)^3+1/2/d/a^3*B*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)+1/d/a^3*ln(tan(1/2*d*x+1/2*
c)+1)*A-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B
-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/
a^3/(tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [A] time = 1.02667, size = 386, normalized size = 2.47

$$\frac{3\,B\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-A\left(\frac{105\,\sin(\ldots}{\cos(dx+\ldots}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 0.498247, size = 668, normalized size = 4.28

$$\frac{15\left((A-3\,B)\cos(dx+c)^4+3\,(A-3\,B)\cos(dx+c)^3+3\,(A-3\,B)\cos(dx+c)^2+(A-3\,B)\cos(dx+c)\right)\log(\sin(dx+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*((A - 3*B)*cos(d*x + c)^4 + 3*(A - 3*B)*cos(d*x + c)^3 + 3*(A - 3*
B)*cos(d*x + c)^2 + (A - 3*B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((A
- 3*B)*cos(d*x + c)^4 + 3*(A - 3*B)*cos(d*x + c)^3 + 3*(A - 3*B)*cos(d*x +
c)^2 + (A - 3*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(11*A - 36*B)*
cos(d*x + c)^3 + 3*(17*A - 57*B)*cos(d*x + c)^2 + (32*A - 117*B)*cos(d*x +
```

```
c) - 15*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx + \int \frac{B\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.36327, size = 251, normalized size = 1.61

$$\frac{\frac{60(A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{60(A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} - \frac{120B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3} - \frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}{}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(A - 3*B)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan
(1/2*d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^12*tan(1/2*
d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x +
 1/2*c))/a^15)/d
```

3.101 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{(4A-29B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(2A-7B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((2*A - 7*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((4*A - 29*B)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.315356, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4019, 4008, 3998, 3770, 3794}

$$\frac{(4A-29B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(2A-7B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((2*A - 7*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((4*A - 29*B)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^2(c+dx)(2a(A-B)+5aB\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2A-7B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2\ldots}{}}{}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2A-7B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(4A-29B)\int\ldots}{1\ldots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2A-7B)\tan\ldots}{15ad(a+a\sec\ldots}$$

Mathematica [A] time = 0.893983, size = 197, normalized size = 1.58

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(5(4A-29B)\sin\left(\frac{dx}{2}\right)+10A\sin\left(c+\frac{3dx}{2}\right)+2A\sin\left(2c+\frac{5dx}{2}\right)+75B\sin\left(c+\frac{dx}{2}\right)-95B\sin\left(\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-240*B*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*A -
 29*B)*Sin[(d*x)/2] + 75*B*Sin[c + (d*x)/2] + 10*A*Sin[c + (3*d*x)/2] - 95*
B*Sin[c + (3*d*x)/2] + 15*B*Sin[2*c + (3*d*x)/2] + 2*A*Sin[2*c + (5*d*x)/2]
 - 22*B*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.054, size = 159, normalized size = 1.3

$$-\frac{B}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B+1/6/d/a^3*A*tan(1/2*d*x+1/2*c)^3-1/3/d/
a^3*B*tan(1/2*d*x+1/2*c)^3+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan
(1/2*d*x+1/2*c)^5*B+1/4/d/a^3*A*tan(1/2*d*x+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+
1/2*c)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B
```

Maxima [A] time = 1.03031, size = 252, normalized size = 2.02

$$-\frac{B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.486405, size = 481, normalized size = 3.85

$$\frac{15\left(B\cos(dx+c)^3+3\,B\cos(dx+c)^2+3\,B\cos(dx+c)+B\right)\log(\sin(dx+c)+1)-15\left(B\cos(dx+c)^3+3\,B\cos(dx+\right.}{30\left(a^3d\cos(dx+c)^3+3\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*log
(sin(d*x + c) + 1) - 15*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*
x + c) + B)*log(-sin(d*x + c) + 1) + 2*(2*(A - 11*B)*cos(d*x + c)^2 + 3*(2*
A - 17*B)*cos(d*x + c) + 7*A - 32*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.38604, size = 198, normalized size = 1.58

$$\frac{\frac{60\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,Ba^{12}\tan\left(\frac{1}{2}\,dx\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*B*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*B*a^12*tan(1/2*d*x +
 1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

3.102 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(3A+7B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-8B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Tan[c +
 d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + 7*B)*Tan[c + d*x])/(15*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.203119, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4008, 4000, 3794}

$$\frac{(3A+7B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-8B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Tan[c +
 d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + 7*B)*Tan[c + d*x])/(15*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a(A-B)-5aB\sec(c+dx))}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3A+7B)\int \frac{\sec(c+dx}{a+a\sec(c+}}{15a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3A+7B)\tan(c+d}{15d\left(a^3+a^3\sec(c+d\right.}$$

Mathematica [A] time = 0.290918, size = 96, normalized size = 0.94

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((3A+2B)\left(5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)+5(3A+4B)\sin\left(\frac{dx}{2}\right)-15A\sin\left(c+\frac{dx}{2}\right)\right)}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 4*B)*Sin[(d*x)/2] - 15*A*Sin[c + (d*x)
/2] + (3*A + 2*B)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2])))/(30*a^3*d
*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.052, size = 64, normalized size = 0.6

$$\frac{1}{4\,da^3}\left(\frac{-A+B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(-A+B)*tan(1/2*d*x+1/2*c)^5+2/3*B*tan(1/2*d*x+1/2*c)^3+A*tan
(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.062, size = 155, normalized size = 1.52

$$\frac{\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.439018, size = 227, normalized size = 2.23

$$\frac{\left((3\,A+2\,B)\cos\left(dx+c\right)^{2}+3\,(3\,A+2\,B)\cos\left(dx+c\right)+3\,A+7\,B\right)\sin\left(dx+c\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((3*A + 2*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 7*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^{2}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^{3}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.21226, size = 101, normalized size = 0.99

$$-\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 - 10*B*tan(1
/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

3.103 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2A+3B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2A+3B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Tan[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 3*B)*Tan[c + d*x])/(15*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.114332, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4000, 3796, 3794}

$$\frac{(2A+3B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2A+3B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Tan[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 3*B)*Tan[c + d*x])/(15*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}dx}{5a}$$
$$= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2A+3B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}dx}{15a^2}$$
$$= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2A+3B)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

Mathematica [A] time = 0.324102, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-15(2A+B)\sin\left(c+\frac{dx}{2}\right)+5(8A+3B)\sin\left(\frac{dx}{2}\right)+20A\sin\left(c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{3dx}{2}\right)+7A\right.}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(8*A + 3*B)*Sin[(d*x)/2] - 15*(2*A + B)*Sin[c
 + (d*x)/2] + 20*A*Sin[c + (3*d*x)/2] + 15*B*Sin[c + (3*d*x)/2] - 15*A*Sin[
2*c + (3*d*x)/2] + 7*A*Sin[2*c + (5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2]))/(3
0*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.061, size = 64, normalized size = 0.6

$$\frac{1}{4\,da^3}\left(\frac{A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(A-B)*tan(1/2*d*x+1/2*c)^5-2/3*A*tan(1/2*d*x+1/2*c)^3+A*tan(
1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.01165, size = 155, normalized size = 1.52

$$\frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.444781, size = 227, normalized size = 2.23

$$\frac{\left((7\,A+3\,B)\cos\left(dx+c\right)^{2}+3\,(2\,A+3\,B)\cos\left(dx+c\right)+2\,A+3\,B\right)\sin\left(dx+c\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((7*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 2*A + 3*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{2}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.32072, size = 101, normalized size = 0.99

$$\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 - 10*A*tan(1/
2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

3.104 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$-\frac{2(11A-B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-2B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] (A*x)/a^3 - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A - 2
*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (2*(11*A - B)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.186089, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3922, 3919, 3794}

$$-\frac{2(11A-B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-2B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (A*x)/a^3 - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A - 2
*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (2*(11*A - B)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5aA+2a(A-B)\sec(c+dx)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2A-a^2(7A-2B)\sec(c+dx)}{a+a\sec(c+dx)}dx}{15a^4}$$

$$= \frac{Ax}{a^3} - \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(2(11A-B))\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}dx}{15a^2}$$

$$= \frac{Ax}{a^3} - \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{2(11A-B)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

Mathematica [B] time = 0.566556, size = 241, normalized size = 2.23

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(270A\sin\left(c+\frac{dx}{2}\right)-230A\sin\left(c+\frac{3dx}{2}\right)+90A\sin\left(2c+\frac{3dx}{2}\right)-64A\sin\left(2c+\frac{5dx}{2}\right)+150Ad\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*A*d*x*Cos[(d*x)/2] + 150*A*d*x*Cos[c + (d
*x)/2] + 75*A*d*x*Cos[c + (3*d*x)/2] + 75*A*d*x*Cos[2*c + (3*d*x)/2] + 15*A
*d*x*Cos[2*c + (5*d*x)/2] + 15*A*d*x*Cos[3*c + (5*d*x)/2] - 370*A*Sin[(d*x)
/2] + 80*B*Sin[(d*x)/2] + 270*A*Sin[c + (d*x)/2] - 60*B*Sin[c + (d*x)/2] -
230*A*Sin[c + (3*d*x)/2] + 40*B*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c + (3*d*x)
/2] - 30*B*Sin[2*c + (3*d*x)/2] - 64*A*Sin[2*c + (5*d*x)/2] + 14*B*Sin[2*c
+ (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.065, size = 137, normalized size = 1.3

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/3/d/
a^3*A*tan(1/2*d*x+1/2*c)^3-1/6/d/a^3*B*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+1/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3*A*arctan(tan(1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.46819, size = 216, normalized size = 2.

$$-\frac{A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3)/d
```

Fricas [A] time = 0.460595, size = 351, normalized size = 3.25

$$\frac{15\,Adx\cos\left(dx+c\right)^{3}+45\,Adx\cos\left(dx+c\right)^{2}+45\,Adx\cos\left(dx+c\right)+15\,Adx-\left(\left(32\,A-7\,B\right)\cos\left(dx+c\right)^{2}+3\left(17\,A-2\,B\right.\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*A*d*x*cos(d*x + c)^3 + 45*A*d*x*cos(d*x + c)^2 + 45*A*d*x*cos(d*x
+ c) + 15*A*d*x - ((32*A - 7*B)*cos(d*x + c)^2 + 3*(17*A - 2*B)*cos(d*x + c
) + 22*A - 2*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)
+ Integral(B*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x))/a**3
```

Giac [A] time = 1.29854, size = 163, normalized size = 1.51

$$\frac{\frac{60\left(dx+c\right)A}{a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+10\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,Ba^{12}\tan\left(\frac{1}{2}\,dx\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*A/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 10*B*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 15*B*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

3.105 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=136

$$\frac{2(36A-11B)\sin(c+dx)}{15a^3d}-\frac{(3A-B)\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(3A-B)}{a^3}-\frac{(9A-4B)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -(((3*A - B)*x)/a^3) + (2*(36*A - 11*B)*Sin[c + d*x])/(15*a^3*d) - ((A - B)
*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - 4*B)*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((3*A - B)*Sin[c + d*x])/(d*(a^3 + a^3*Sec[
c + d*x]))
```

Rubi [A] time = 0.366747, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4020, 3787, 2637, 8}

$$\frac{2(36A-11B)\sin(c+dx)}{15a^3d}-\frac{(3A-B)\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(3A-B)}{a^3}-\frac{(9A-4B)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(((3*A - B)*x)/a^3) + (2*(36*A - 11*B)*Sin[c + d*x])/(15*a^3*d) - ((A - B)
*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - 4*B)*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((3*A - B)*Sin[c + d*x])/(d*(a^3 + a^3*Sec[
c + d*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(a(6A-B)-3a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)\left(a^2(27A-7B)-2a^2(9A\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3A-B)\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} + $$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3A-B)\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} + $$

$$= -\frac{(3A-B)x}{a^3} + \frac{2(36A-11B)\sin(c+dx)}{15a^3d} - \frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)}{15ad(a+a\,s}$$

Mathematica [B] time = 1.0293, size = 365, normalized size = 2.68

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-300dx(3A-B)\cos\left(c+\frac{dx}{2}\right)-300dx(3A-B)\cos\left(\frac{dx}{2}\right)-1125A\sin\left(c+\frac{dx}{2}\right)+1215A\sin\left(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-300*(3*A - B)*d*x*Cos[(d*x)/2] - 300*(3*A - B)
*d*x*Cos[c + (d*x)/2] - 450*A*d*x*Cos[c + (3*d*x)/2] + 150*B*d*x*Cos[c + (3
*d*x)/2] - 450*A*d*x*Cos[2*c + (3*d*x)/2] + 150*B*d*x*Cos[2*c + (3*d*x)/2]
- 90*A*d*x*Cos[2*c + (5*d*x)/2] + 30*B*d*x*Cos[2*c + (5*d*x)/2] - 90*A*d*x*
Cos[3*c + (5*d*x)/2] + 30*B*d*x*Cos[3*c + (5*d*x)/2] + 1755*A*Sin[(d*x)/2]
- 740*B*Sin[(d*x)/2] - 1125*A*Sin[c + (d*x)/2] + 540*B*Sin[c + (d*x)/2] + 1
215*A*Sin[c + (3*d*x)/2] - 460*B*Sin[c + (3*d*x)/2] - 225*A*Sin[2*c + (3*d*
x)/2] + 180*B*Sin[2*c + (3*d*x)/2] + 363*A*Sin[2*c + (5*d*x)/2] - 128*B*Sin
[2*c + (5*d*x)/2] + 75*A*Sin[3*c + (5*d*x)/2] + 15*A*Sin[3*c + (7*d*x)/2] +
 15*A*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.093, size = 189, normalized size = 1.4

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/2/d/a
^3*A*tan(1/2*d*x+1/2*c)^3+1/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3*A*tan(1/2*d*x+1/2*c)
/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arct
an(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.48971, size = 312, normalized size = 2.29

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-B\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.475016, size = 431, normalized size = 3.17

$$-\frac{15\,(3\,A-B)dx\cos(dx+c)^3+45\,(3\,A-B)dx\cos(dx+c)^2+45\,(3\,A-B)dx\cos(dx+c)+15\,(3\,A-B)dx-\left(15\,A\cos\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] -1/15*(15*(3*A - B)*d*x*cos(d*x + c)^3 + 45*(3*A - B)*d*x*cos(d*x + c)^2 +
45*(3*A - B)*d*x*cos(d*x + c) + 15*(3*A - B)*d*x - (15*A*cos(d*x + c)^3 + (
117*A - 32*B)*cos(d*x + c)^2 + 3*(57*A - 17*B)*cos(d*x + c) + 72*A - 22*B)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42345, size = 212, normalized size = 1.56

$$-\frac{\frac{60\,(dx+c)(3\,A-B)}{a^3}-\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+255}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -1/60*(60*(d*x + c)*(3*A - B)/a^3 - 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/
2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*B*a^12*tan(1/2*d*x
 + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/
2*c))/a^15)/d
```

3.106 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{8(19A-9B)\sin(c+dx)}{15a^3d}+\frac{(13A-6B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4(19A-9B)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13A-6B}{2a^3}$$

```
[Out] ((13*A - 6*B)*x)/(2*a^3) - (8*(19*A - 9*B)*Sin[c + d*x])/(15*a^3*d) + ((13*
A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]*Sin[c
 + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A - 6*B)*Cos[c + d*x]*Sin[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (4*(19*A - 9*B)*Cos[c + d*x]*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.469506, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{8(19A-9B)\sin(c+dx)}{15a^3d}+\frac{(13A-6B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4(19A-9B)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13A-6B}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((13*A - 6*B)*x)/(2*a^3) - (8*(19*A - 9*B)*Sin[c + d*x])/(15*a^3*d) + ((13*
A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]*Sin[c
 + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A - 6*B)*Cos[c + d*x]*Sin[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (4*(19*A - 9*B)*Cos[c + d*x]*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(a(7A-2B)-4a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B)\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \int \cos^2$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B)\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - 4(19A$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B)\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - 4(19A$$

$$= -\frac{8(19A-9B)\sin(c+dx)}{15a^3d} + \frac{(13A-6B)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A-B)\cos(}{5d(a+}$$

$$= \frac{(13A-6B)x}{2a^3} - \frac{8(19A-9B)\sin(c+dx)}{15a^3d} + \frac{(13A-6B)\cos(c+dx)\sin(c+dx)}{2a^3d}$$

Mathematica [B] time = 0.786978, size = 435, normalized size = 2.33

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(13A-6B)\cos\left(c+\frac{dx}{2}\right)+600dx(13A-6B)\cos\left(\frac{dx}{2}\right)+7560A\sin\left(c+\frac{dx}{2}\right)-9230A\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 6*B)*d*x*Cos[(d*x)/2] + 600*(13*A -
 6*B)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] - 1800*B*d*x*Cos
[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] - 1800*B*d*x*Cos[2*c + (3
*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 360*B*d*x*Cos[2*c + (5*d*x)/2]
+ 780*A*d*x*Cos[3*c + (5*d*x)/2] - 360*B*d*x*Cos[3*c + (5*d*x)/2] - 12760*A
*Sin[(d*x)/2] + 7020*B*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] - 4500*B*Sin[
c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] + 4860*B*Sin[c + (3*d*x)/2] + 930*
A*Sin[2*c + (3*d*x)/2] - 900*B*Sin[2*c + (3*d*x)/2] - 2782*A*Sin[2*c + (5*d
*x)/2] + 1452*B*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] + 300*B*S
in[3*c + (5*d*x)/2] - 105*A*Sin[3*c + (7*d*x)/2] + 60*B*Sin[3*c + (7*d*x)/2
] - 105*A*Sin[4*c + (7*d*x)/2] + 60*B*Sin[4*c + (7*d*x)/2] + 15*A*Sin[4*c +
 (9*d*x)/2] + 15*A*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.101, size = 292, normalized size = 1.6

$$-\frac{A}{20da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2A}{3da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{2da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31A}{4da^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+2/3/d/
a^3*A*tan(1/2*d*x+1/2*c)^3-1/2/d/a^3*B*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*ta
n(1/2*d*x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2
*c)^2)^2*A*tan(1/2*d*x+1/2*c)^3+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/
2*d*x+1/2*c)^3-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+2/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+13/d/a^3*A*arctan(tan(1
/2*d*x+1/2*c))-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.50664, size = 435, normalized size = 2.33

$$-\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3B\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(d}\right.}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*B*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.484462, size = 495, normalized size = 2.65

$$\frac{15(13A-6B)dx\cos(dx+c)^3+45(13A-6B)dx\cos(dx+c)^2+45(13A-6B)dx\cos(dx+c)+15(13A-6B)dx}{30\left(a^3d\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(13*A - 6*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 6*B)*d*x*cos(d*x + c)
^2 + 45*(13*A - 6*B)*d*x*cos(d*x + c) + 15*(13*A - 6*B)*d*x + (15*A*cos(d*x
 + c)^4 - 15*(3*A - 2*B)*cos(d*x + c)^3 - (479*A - 234*B)*cos(d*x + c)^2 -
3*(239*A - 114*B)*cos(d*x + c) - 304*A + 144*B)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\cos^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx + \int \frac{B\cos^2(c+dx)\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x)**3
+ 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.27541, size = 270, normalized size = 1.44

$$\frac{\frac{30\,(dx+c)(13\,A-6\,B)}{a^3} - \frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*A - 6*B)/a^3 - 60*(7*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*
tan(1/2*d*x + 1/2*c)^3 + 5*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5
- 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12
*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.107 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$-\frac{4(34A-19B)\sin^3(c+dx)}{15a^3d}+\frac{4(34A-19B)\sin(c+dx)}{5a^3d}-\frac{(23A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(23A-13B)\sin(c}{3d\left(a^3\sec(c\right.}$$

```
[Out] -((23*A - 13*B)*x)/(2*a^3) + (4*(34*A - 19*B)*Sin[c + d*x])/(5*a^3*d) - ((2
3*A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A - 8*B)*Cos[c + d*x]^2*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((23*A - 13*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d*x])) - (4*(34*A - 19*B)*Sin[c + d
*x]^3)/(15*a^3*d)
```

Rubi [A] time = 0.49492, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2633, 2635, 8}

$$-\frac{4(34A-19B)\sin^3(c+dx)}{15a^3d}+\frac{4(34A-19B)\sin(c+dx)}{5a^3d}-\frac{(23A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(23A-13B)\sin(c}{3d\left(a^3\sec(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -((23*A - 13*B)*x)/(2*a^3) + (4*(34*A - 19*B)*Sin[c + d*x])/(5*a^3*d) - ((2
3*A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A - 8*B)*Cos[c + d*x]^2*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((23*A - 13*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d*x])) - (4*(34*A - 19*B)*Sin[c + d
*x]^3)/(15*a^3*d)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(a(8A-3B)-5a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{c}{-}}{}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(23}{}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(23}{}$$

$$= -\frac{(23A-13B)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13}{}$$

$$= -\frac{(23A-13B)x}{2a^3} + \frac{4(34A-19B)\sin(c+dx)}{5a^3d} - \frac{(23A-13B)\cos(c+dx)\sin(c+}{2a^3d}$$

Mathematica [B] time = 1.04269, size = 491, normalized size = 2.25

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-600dx(23A-13B)\cos\left(c+\frac{dx}{2}\right)-600dx(23A-13B)\cos\left(\frac{dx}{2}\right)-11110A\sin\left(c+\frac{dx}{2}\right)+15380A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(23*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(23*A
 - 13*B)*d*x*Cos[c + (d*x)/2] - 6900*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*
Cos[c + (3*d*x)/2] - 6900*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c +
 (3*d*x)/2] - 1380*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)
/2] - 1380*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 20
410*A*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 11110*A*Sin[c + (d*x)/2] + 7560
*B*Sin[c + (d*x)/2] + 15380*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2
] - 380*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 4777*A*Sin[2*
c + (5*d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 1625*A*Sin[3*c + (5*d*x)/2]
- 750*B*Sin[3*c + (5*d*x)/2] + 230*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c +
 (7*d*x)/2] + 230*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] - 20*
A*Sin[4*c + (9*d*x)/2] + 15*B*Sin[4*c + (9*d*x)/2] - 20*A*Sin[5*c + (9*d*x)
/2] + 15*B*Sin[5*c + (9*d*x)/2] + 5*A*Sin[5*c + (11*d*x)/2] + 5*A*Sin[6*c +
 (11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.099, size = 362, normalized size = 1.7

$$\frac{A}{20\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{B}{20\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{5\,A}{6\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{2\,B}{3\,da^{3}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{49\,A}{4\,da^{3}}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-5/6/d/a
^3*A*tan(1/2*d*x+1/2*c)^3+2/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+49/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)+17/d/a^3/(1+tan(1/2*d*x+1/2
*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)^5*B+76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)^3
-12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3+11/d/a^3/(1+tan
(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^
3*B*tan(1/2*d*x+1/2*c)-23/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))+13/d/a^3*arcta
n(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.50998, size = 556, normalized size = 2.55

$$\frac{A\left(\frac{20\left(\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{51\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{3}+\frac{3a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{3}}-\frac{1380\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}\right)-B\left(\frac{60\left(\frac{5\sin(dx+}{\cos(dx+c)}\right.}{a^{3}+\frac{2a^{3}\sin(dx}{(\cos(dx+c)}}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(A*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.498422, size = 540, normalized size = 2.48

$$-\frac{15\,(23\,A-13\,B)dx\cos(dx+c)^{3}+45\,(23\,A-13\,B)dx\cos(dx+c)^{2}+45\,(23\,A-13\,B)dx\cos(dx+c)+15\,(23\,A-1}{30\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(23*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(23*A - 13*B)*d*x*cos(d*x +
 c)^2 + 45*(23*A - 13*B)*d*x*cos(d*x + c) + 15*(23*A - 13*B)*d*x - (10*A*co
s(d*x + c)^5 - 15*(A - B)*cos(d*x + c)^4 + 5*(19*A - 9*B)*cos(d*x + c)^3 +
(869*A - 479*B)*cos(d*x + c)^2 + 3*(429*A - 239*B)*cos(d*x + c) + 544*A - 3
04*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34235, size = 308, normalized size = 1.41

$$-\frac{\frac{30\,(dx+c)(23\,A-13\,B)}{a^3} - \frac{20\left(51\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 76\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 36\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 33\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 15\,B\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(d*x + c)*(23*A - 13*B)/a^3 - 20*(51*A*tan(1/2*d*x + 1/2*c)^5 - 2
1*B*tan(1/2*d*x + 1/2*c)^5 + 76*A*tan(1/2*d*x + 1/2*c)^3 - 36*B*tan(1/2*d*x
 + 1/2*c)^3 + 33*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^1
2*tan(1/2*d*x + 1/2*c)^5 - 50*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*B*a^12*tan
(1/2*d*x + 1/2*c)^3 + 735*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^12*tan(1/2*
d*x + 1/2*c))/a^15)/d
```

3.108 $\int \frac{\sec^6(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=238

$$\frac{8(83A-216B)\tan(c+dx)}{105a^4d}-\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(52A-129B)\tan(c+dx)\sec^3(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{4(83A-2}{10}$$

```
[Out] -((8*A - 21*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + (8*(83*A - 216*B)*Tan[c +
 d*x])/(105*a^4*d) - ((8*A - 21*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) + (
(52*A - 129*B)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2
) + (4*(83*A - 216*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c +
d*x])) + ((A - B)*Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4)
 + ((A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.655504, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 3787, 3767, 8, 3768, 3770}

$$\frac{8(83A-216B)\tan(c+dx)}{105a^4d}-\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(52A-129B)\tan(c+dx)\sec^3(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{4(83A-2}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -((8*A - 21*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + (8*(83*A - 216*B)*Tan[c +
 d*x])/(105*a^4*d) - ((8*A - 21*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) + (
(52*A - 129*B)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2
) + (4*(83*A - 216*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c +
d*x])) + ((A - B)*Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4)
 + ((A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^5(c+dx)(5a(A-B)-a(2A-9B)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(A-2B)\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} + \int \sec^4($$

$$= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + ($$

$$= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + ($$

$$= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + ($$

$$= -\frac{(8A-21B)\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}$$

$$= -\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{8(83A-216B)\tan(c+dx)}{105a^4d} - (8A-21B)\,\mathrm{s}$$

Mathematica [B] time = 6.47336, size = 880, normalized size = 3.7

$$-\frac{8(21B-8A)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^3(c+dx)(A+B\sec(c+dx))\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(B+A\cos(c+dx))(\sec(c+dx)a+a)^4} + 8(21B-8A)\log\left(\cos\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-8*(-8*A + 21*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (
d*x)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a +
a*Sec[c + d*x])^4) + (8*(-8*A + 21*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c
]*Sec[c + d*x]^5*(A + B*Sec[c + d*x])*(-38668*A*Sin[(d*x)/2] + 73206*B*Sin[
(d*x)/2] + 64384*A*Sin[(3*d*x)/2] - 166668*B*Sin[(3*d*x)/2] - 70896*A*Sin[c
 - (d*x)/2] + 183162*B*Sin[c - (d*x)/2] + 50316*A*Sin[c + (d*x)/2] - 100842
*B*Sin[c + (d*x)/2] - 59248*A*Sin[2*c + (d*x)/2] + 155526*B*Sin[2*c + (d*x)
/2] - 22820*A*Sin[c + (3*d*x)/2] + 37380*B*Sin[c + (3*d*x)/2] + 48004*A*Sin
```

```
[2*c + (3*d*x)/2] - 101148*B*Sin[2*c + (3*d*x)/2] - 39200*A*Sin[3*c + (3*d*
x)/2] + 102900*B*Sin[3*c + (3*d*x)/2] + 46032*A*Sin[c + (5*d*x)/2] - 119364
*B*Sin[c + (5*d*x)/2] - 8750*A*Sin[2*c + (5*d*x)/2] + 8820*B*Sin[2*c + (5*d
*x)/2] + 35742*A*Sin[3*c + (5*d*x)/2] - 78204*B*Sin[3*c + (5*d*x)/2] - 1904
0*A*Sin[4*c + (5*d*x)/2] + 49980*B*Sin[4*c + (5*d*x)/2] + 24664*A*Sin[2*c +
 (7*d*x)/2] - 64053*B*Sin[2*c + (7*d*x)/2] - 1050*A*Sin[3*c + (7*d*x)/2] -
3885*B*Sin[3*c + (7*d*x)/2] + 19834*A*Sin[4*c + (7*d*x)/2] - 44733*B*Sin[4*
c + (7*d*x)/2] - 5880*A*Sin[5*c + (7*d*x)/2] + 15435*B*Sin[5*c + (7*d*x)/2]
 + 8456*A*Sin[3*c + (9*d*x)/2] - 21987*B*Sin[3*c + (9*d*x)/2] + 630*A*Sin[4
*c + (9*d*x)/2] - 3675*B*Sin[4*c + (9*d*x)/2] + 6986*A*Sin[5*c + (9*d*x)/2]
 - 16107*B*Sin[5*c + (9*d*x)/2] - 840*A*Sin[6*c + (9*d*x)/2] + 2205*B*Sin[6
*c + (9*d*x)/2] + 1328*A*Sin[4*c + (11*d*x)/2] - 3456*B*Sin[4*c + (11*d*x)/
2] + 210*A*Sin[5*c + (11*d*x)/2] - 840*B*Sin[5*c + (11*d*x)/2] + 1118*A*Sin
[6*c + (11*d*x)/2] - 2616*B*Sin[6*c + (11*d*x)/2]))/(6720*d*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [A] time = 0.069, size = 374, normalized size = 1.6

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{9\,B}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{23\,A}{24\,da^4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+7/40/d/
a^4*tan(1/2*d*x+1/2*c)^5*A-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5*B+23/24/d/a^4*A*
tan(1/2*d*x+1/2*c)^3-13/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/2
*d*x+1/2*c)-111/8/d/a^4*B*tan(1/2*d*x+1/2*c)+9/2/d/a^4/(tan(1/2*d*x+1/2*c)+
1)*B-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*A-4/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A+21
/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^4*B/(tan(1/2*d*x+1/2*c)+1)^2+4/
d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A-21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B+9/2/
d/a^4/(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*A+1/2/d/a^4*B
/(tan(1/2*d*x+1/2*c)-1)^2
```

Maxima [A] time = 0.998842, size = 566, normalized size = 2.38

$$-\frac{3\,B\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\,\log\left(\frac{\sin}{\cos(}\right.}{a^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) - A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
```

```
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))
/d
```

Fricas [A] time = 0.516238, size = 940, normalized size = 3.95

$$-\frac{105\left((8A-21B)\cos(dx+c)^6+4(8A-21B)\cos(dx+c)^5+6(8A-21B)\cos(dx+c)^4+4(8A-21B)\cos(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/420*(105*((8*A - 21*B)*cos(d*x + c)^6 + 4*(8*A - 21*B)*cos(d*x + c)^5 +
6*(8*A - 21*B)*cos(d*x + c)^4 + 4*(8*A - 21*B)*cos(d*x + c)^3 + (8*A - 21*B
)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((8*A - 21*B)*cos(d*x + c)^6
+ 4*(8*A - 21*B)*cos(d*x + c)^5 + 6*(8*A - 21*B)*cos(d*x + c)^4 + 4*(8*A -
21*B)*cos(d*x + c)^3 + (8*A - 21*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)
- 2*(16*(83*A - 216*B)*cos(d*x + c)^5 + (4472*A - 11619*B)*cos(d*x + c)^4 +
 4*(1318*A - 3411*B)*cos(d*x + c)^3 + 4*(592*A - 1509*B)*cos(d*x + c)^2 + 2
10*(A - 2*B)*cos(d*x + c) + 105*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*
a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^
4*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**7/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.39089, size = 360, normalized size = 1.51

$$-\frac{\frac{420(8A-21B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{420(8A-21B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{840\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(8*A - 21*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(8*A
 - 21*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(2*A*tan(1/2*d*x + 1/2
*c)^3 - 9*B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + 7*B*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24*tan(1/2*d*
x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x +
1/2*c)^5 - 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2
*c)^3 - 1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*
c) - 11655*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.109 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=194

$$-\frac{(55A-244B)\tan(c+dx)}{105a^4d}+\frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{(25A-88B)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A-4B)\tan(c}{a^4d(\sec(c+dx)}$$

```
[Out] ((A - 4*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 244*B)*Tan[c + d*x])/(
105*a^4*d) + ((25*A - 88*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Se
c[c + d*x])^2) - ((A - 4*B)*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) + ((A
- B)*Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((5*A - 12
*B)*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.616241, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(55A-244B)\tan(c+dx)}{105a^4d}+\frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{(25A-88B)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A-4B)\tan(c}{a^4d(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ((A - 4*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 244*B)*Tan[c + d*x])/(
105*a^4*d) + ((25*A - 88*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Se
c[c + d*x])^2) - ((A - 4*B)*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) + ((A
- B)*Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((5*A - 12
*B)*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-a(A-8B)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(5A-12B)\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \cdots$$

$$= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \cdots$$

$$= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \cdots$$

$$= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \cdots$$

$$= \frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \cdots$$

$$= \frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-244B)\tan(c+dx)}{105a^4d} + \frac{(25A-88B)\,\mathrm{s}\cdots}{105a^4d(\cdots}$$

Mathematica [B] time = 6.38583, size = 754, normalized size = 3.89

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4(c+dx)\left(4795A\sin\left(c-\frac{dx}{2}\right)-4795A\sin\left(c+\frac{dx}{2}\right)+4165A\sin\left(2c+\frac{dx}{2}\right)+2275A\sin\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (16*(-A + 4*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x
)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a + a*S
ec[c + d*x])^4) - (16*(-A + 4*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec
[c + d*x]^4*(A + B*Sec[c + d*x])*(4165*A*Sin[(d*x)/2] - 10780*B*Sin[(d*x)/2
] - 4445*A*Sin[(3*d*x)/2] + 18788*B*Sin[(3*d*x)/2] + 4795*A*Sin[c - (d*x)/2
] - 20524*B*Sin[c - (d*x)/2] - 4795*A*Sin[c + (d*x)/2] + 14644*B*Sin[c + (d
*x)/2] + 4165*A*Sin[2*c + (d*x)/2] - 16660*B*Sin[2*c + (d*x)/2] + 2275*A*Si
n[c + (3*d*x)/2] - 4690*B*Sin[c + (3*d*x)/2] - 4445*A*Sin[2*c + (3*d*x)/2]
```

```
+ 14378*B*Sin[2*c + (3*d*x)/2] + 2275*A*Sin[3*c + (3*d*x)/2] - 9100*B*Sin[3
*c + (3*d*x)/2] - 2785*A*Sin[c + (5*d*x)/2] + 11668*B*Sin[c + (5*d*x)/2] +
735*A*Sin[2*c + (5*d*x)/2] - 630*B*Sin[2*c + (5*d*x)/2] - 2785*A*Sin[3*c +
(5*d*x)/2] + 9358*B*Sin[3*c + (5*d*x)/2] + 735*A*Sin[4*c + (5*d*x)/2] - 294
0*B*Sin[4*c + (5*d*x)/2] - 1015*A*Sin[2*c + (7*d*x)/2] + 4228*B*Sin[2*c + (
7*d*x)/2] + 105*A*Sin[3*c + (7*d*x)/2] + 315*B*Sin[3*c + (7*d*x)/2] - 1015*
A*Sin[4*c + (7*d*x)/2] + 3493*B*Sin[4*c + (7*d*x)/2] + 105*A*Sin[5*c + (7*d
*x)/2] - 420*B*Sin[5*c + (7*d*x)/2] - 160*A*Sin[3*c + (9*d*x)/2] + 664*B*Si
n[3*c + (9*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] - 160*A*Sin[5*c + (9*d*x)/2
] + 559*B*Sin[5*c + (9*d*x)/2]))/(1680*d*(B + A*Cos[c + d*x])*(a + a*Sec[c
+ d*x])^4)
```

Maple [A] time = 0.062, size = 285, normalized size = 1.5

$$-\frac{A}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,B}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{11\,A}{24\,d a^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/8/d/
a^4*tan(1/2*d*x+1/2*c)^5*A+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5*B-11/24/d/a^4*A*
tan(1/2*d*x+1/2*c)^3+23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/
2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+
1)*A-4/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B-1/
d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4
/(tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [A] time = 1.01956, size = 440, normalized size = 2.27

$$\frac{B\left(\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^4}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [A] time = 0.504279, size = 837, normalized size = 4.31

$$\frac{105\left((A-4B)\cos(dx+c)^5+4(A-4B)\cos(dx+c)^4+6(A-4B)\cos(dx+c)^3+4(A-4B)\cos(dx+c)^2+(A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*((A - 4*B)*cos(d*x + c)^5 + 4*(A - 4*B)*cos(d*x + c)^4 + 6*(A -
4*B)*cos(d*x + c)^3 + 4*(A - 4*B)*cos(d*x + c)^2 + (A - 4*B)*cos(d*x + c))*
log(sin(d*x + c) + 1) - 105*((A - 4*B)*cos(d*x + c)^5 + 4*(A - 4*B)*cos(d*x
 + c)^4 + 6*(A - 4*B)*cos(d*x + c)^3 + 4*(A - 4*B)*cos(d*x + c)^2 + (A - 4*
B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(20*A - 83*B)*cos(d*x + c)^4
 + (535*A - 2236*B)*cos(d*x + c)^3 + 4*(155*A - 659*B)*cos(d*x + c)^2 + 4*(
65*A - 296*B)*cos(d*x + c) - 105*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4
*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a
^4*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**6/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.33136, size = 297, normalized size = 1.53

$$\frac{\frac{840(A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{840(A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{1680B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}-\frac{15Aa^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-15Ba^{24}\tan\left(\frac{1}{2}dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(A - 4*B)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 1680*B*tan(1/2*d*x + 1/2*c)/((tan
(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^
24*tan(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*B*a^24*
tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan
(1/2*d*x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 5145*B*a^24*tan(1/
2*d*x + 1/2*c))/a^28)/d
```

3.110 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$\frac{(12A-215B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(6A-55B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(3}{}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((6*A - 55*B)*Tan[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) + ((12*A - 215*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c
 + d*x])) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + ((3*A - 10*B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x
])^3)
```

Rubi [A] time = 0.475118, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4019, 4008, 3998, 3770, 3794}

$$\frac{(12A-215B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(6A-55B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((6*A - 55*B)*Tan[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) + ((12*A - 215*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c
 + d*x])) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + ((3*A - 10*B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x
])^3)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)+7aB\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A-10B)\sec^2(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \cdots$$

$$= -\frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A-10B}{35ad}\cdots$$

$$= -\frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A-10B}{35ad}\cdots$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^3(c+dx)\tan\cdots}{7d(a+a\sec(c+\cdots}$$

Mathematica [A] time = 1.40435, size = 239, normalized size = 1.47

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(70(3A-49B)\sin\left(\frac{dx}{2}\right)+126A\sin\left(c+\frac{3dx}{2}\right)+42A\sin\left(2c+\frac{5dx}{2}\right)+6A\sin\left(3c+\frac{7dx}{2}\right)+217\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-6720*B*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(70*(3*A
 - 49*B)*Sin[(d*x)/2] + 2170*B*Sin[c + (d*x)/2] + 126*A*Sin[c + (3*d*x)/2]
- 2625*B*Sin[c + (3*d*x)/2] + 735*B*Sin[2*c + (3*d*x)/2] + 42*A*Sin[2*c + (
5*d*x)/2] - 1015*B*Sin[2*c + (5*d*x)/2] + 105*B*Sin[3*c + (5*d*x)/2] + 6*A*
Sin[3*c + (7*d*x)/2] - 160*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c +
 d*x])^4)
```

Maple [A] time = 0.06, size = 199, normalized size = 1.2

$$-\frac{B}{da^4}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{A}{8da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{11B}{24da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{3A}{40da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8da^4}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)^3-11/24/
d/a^4*B*tan(1/2*d*x+1/2*c)^3+3/40/d/a^4*tan(1/2*d*x+1/2*c)^5*A-1/8/d/a^4*ta
n(1/2*d*x+1/2*c)^5*B+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d
*x+1/2*c)^7*B+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c
)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B
```

Maxima [A] time = 1.02132, size = 308, normalized size = 1.89

$$-\frac{5\,B\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c}{(\cos(dx+c)+}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 3*A*(35*sin(d*x +
c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

Fricas [A] time = 0.49079, size = 624, normalized size = 3.83

$$105\left(B\cos\left(dx+c\right)^4+4\,B\cos\left(dx+c\right)^3+6\,B\cos\left(dx+c\right)^2+4\,B\cos\left(dx+c\right)+B\right)\log\left(\sin\left(dx+c\right)+1\right)-105\left(B\cos\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*
B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - 105*(B*cos(d*x + c)^4 + 4*B*cos
(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*log(-sin(d*x + c)
+ 1) + 2*(2*(3*A - 80*B)*cos(d*x + c)^3 + (24*A - 535*B)*cos(d*x + c)^2 + (
39*A - 620*B)*cos(d*x + c) + 36*A - 260*B)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.24066, size = 244, normalized size = 1.5

$$\frac{\frac{840\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,Ba^{24}}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*B*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
(1/2*d*x + 1/2*c)^7 + 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2*d*
x + 1/2*c)^3 + 105*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*B*a^24*tan(1/2*d*x +
1/2*c))/a^28)/d
```

3.111 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=146

$$\frac{(4A+3B)\tan(c+dx)}{15d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8(4A+3B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

```
[Out] -((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((4*A
 + 3*B)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*(4*A + 3*B)*Tan[
c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((4*A + 3*B)*Tan[c + d*x])/(
15*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.228662, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4012, 3799, 4000, 3794}

$$\frac{(4A+3B)\tan(c+dx)}{15d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8(4A+3B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((4*A
 + 3*B)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*(4*A + 3*B)*Tan[
c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((4*A + 3*B)*Tan[c + d*x])/(
15*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{(4A+3B)}{}$$

$$= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}-\frac{8(4A+3}{105d\left(a^2+\right.}$$

$$= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}-\frac{8(4A+3}{105d\left(a^2+\right.}$$

Mathematica [A] time = 0.340026, size = 109, normalized size = 0.75

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((4A+3B)\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35(2A+3B)\sin\left(\frac{dx}{2}\right)-70\right.}{210a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(35*(2*A + 3*B)*Sin[(d*x)/2] - 70*A*Sin[c + (d*x
)/2] + (4*A + 3*B)*(21*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*
c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.058, size = 88, normalized size = 0.6

$$\frac{1}{8\,da^4}\left(\frac{-A+B}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A+3\,B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A+3\,B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\,\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*B)*tan(1/2*d*x+1/2*c)^
5+1/3*(A+3*B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.00143, size = 236, normalized size = 1.62

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,B\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.444594, size = 311, normalized size = 2.13

$$\frac{\left(2\,(4\,A+3\,B)\cos\left(dx+c\right)^{3}+8\,(4\,A+3\,B)\cos\left(dx+c\right)^{2}+13\,(4\,A+3\,B)\cos\left(dx+c\right)+13\,A+36\,B\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(2*(4*A + 3*B)*cos(d*x + c)^3 + 8*(4*A + 3*B)*cos(d*x + c)^2 + 13*(4*
A + 3*B)*cos(d*x + c) + 13*A + 36*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{3}(c+dx)}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^{4}(c+dx)}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.38166, size = 158, normalized size = 1.08

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-63\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 21*A*ta
n(1/2*d*x + 1/2*c)^5 - 63*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2
*c)^3 - 105*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*t
an(1/2*d*x + 1/2*c))/(a^4*d)
```

3.112 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{(8A+13B)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{(8A+13B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(4A-11B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] -((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((4*A - 11*B)*Tan[c
+ d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((8*A + 13*B)*Tan[c + d*x])/(105*
d*(a^2 + a^2*Sec[c + d*x])^2) + ((8*A + 13*B)*Tan[c + d*x])/(105*d*(a^4 + a
^4*Sec[c + d*x]))
```

Rubi [A] time = 0.264638, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4008, 4000, 3796, 3794}

$$\frac{(8A+13B)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{(8A+13B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(4A-11B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((4*A - 11*B)*Tan[c
+ d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((8*A + 13*B)*Tan[c + d*x])/(105*
d*(a^2 + a^2*Sec[c + d*x])^2) + ((8*A + 13*B)*Tan[c + d*x])/(105*d*(a^4 + a
^4*Sec[c + d*x]))
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec(c+dx)(-4a(A-B)-7aB\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(8A+13B)\int \frac{\sec(c+}{(a+a\sec(}}{35a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(8A+13B)\tan(c+}{105d\left(a^2+a^2\sec(c+\right.}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(8A+13B)\tan(c+}{105d\left(a^2+a^2\sec(c+\right.}$$

Mathematica [A] time = 0.377154, size = 163, normalized size = 1.18

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(5A+4B)\sin\left(c+\frac{dx}{2}\right)+140(2A+B)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(2c+\frac{3dx}{2}\right.\right.}{420a^4d(\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(140*(2*A + B)*Sin[(d*x)/2] - 35*(5*A + 4*B)*Sin
[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105*A
*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*x)/2] + 56*B*Sin[2*c + (5*d*x)/
2] + 13*A*Sin[3*c + (7*d*x)/2] + 8*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 +
 Cos[c + d*x])^4)
```

Maple [A] time = 0.057, size = 88, normalized size = 0.6

$$\frac{1}{8\,da^4}\left(\frac{A-B}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-A+B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A-B)*tan(1/2*d*x+1/2*c)^5+1
/3*(-A+B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00691, size = 235, normalized size = 1.7

$$\frac{\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.450971, size = 309, normalized size = 2.24

$$\frac{\left((13\,A+8\,B)\cos\left(dx+c\right)^{3}+4\,(13\,A+8\,B)\cos\left(dx+c\right)^{2}+4\,(8\,A+13\,B)\cos\left(dx+c\right)+8\,A+13\,B\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*((13*A + 8*B)*cos(d*x + c)^3 + 4*(13*A + 8*B)*cos(d*x + c)^2 + 4*(8*A
 + 13*B)*cos(d*x + c) + 8*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{2}(c+dx)}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^{3}(c+dx)}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.36398, size = 158, normalized size = 1.14

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan
(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*
c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*tan
(1/2*d*x + 1/2*c))/(a^4*d)
```

3.113 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(3A+4B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] ((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((3*A + 4*B)*Tan[c +
d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d
*(a^2 + a^2*Sec[c + d*x])^2) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d*(a^4 + a
^4*Sec[c + d*x]))
```

Rubi [A] time = 0.150658, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4000, 3796, 3794}

$$\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(3A+4B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((3*A + 4*B)*Tan[c +
d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d
*(a^2 + a^2*Sec[c + d*x])^2) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d*(a^4 + a
^4*Sec[c + d*x]))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(2(3A+4B))\int \frac{\sec(c+}{(a+a\sec(c}}{35a^2}$$

$$= \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2(3A+4B)\tan(c+dx}{105d\left(a^2+a^2\sec(c+dx\right.}$$

$$= \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2(3A+4B)\tan(c+dx}{105d\left(a^2+a^2\sec(c+dx\right.}$$

Mathematica [A] time = 0.444291, size = 193, normalized size = 1.4

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(18A+5B)\sin\left(c+\frac{dx}{2}\right)+70(9A+4B)\sin\left(\frac{dx}{2}\right)+441A\sin\left(c+\frac{3dx}{2}\right)-315A\sin\left(2c+\frac{3}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(9*A + 4*B)*Sin[(d*x)/2] - 35*(18*A + 5*B)*S
in[c + (d*x)/2] + 441*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 315
*A*Sin[2*c + (3*d*x)/2] - 105*B*Sin[2*c + (3*d*x)/2] + 147*A*Sin[2*c + (5*d
*x)/2] + 91*B*Sin[2*c + (5*d*x)/2] - 105*A*Sin[3*c + (5*d*x)/2] + 36*A*Sin[
3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x]
)^4)
```

Maple [A] time = 0.059, size = 90, normalized size = 0.7

$$\frac{1}{8da^4}\left(\frac{-A+B}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3A-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-3A-B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\,\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-B)*tan(1/2*d*x+1/2*c)^5
+1/3*(-3*A-B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.00135, size = 236, normalized size = 1.71

$$\frac{\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.451195, size = 308, normalized size = 2.23

$$\frac{\left((36\,A+13\,B)\cos\left(dx+c\right)^3+13\,(3\,A+4\,B)\cos\left(dx+c\right)^2+8\,(3\,A+4\,B)\cos\left(dx+c\right)+6\,A+8\,B\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*((36*A + 13*B)*cos(d*x + c)^3 + 13*(3*A + 4*B)*cos(d*x + c)^2 + 8*(3*
A + 4*B)*cos(d*x + c) + 6*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^2\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)
**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.27616, size = 158, normalized size = 1.14

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,A\tan\left(\frac{1}{2}\,dx+\right.}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 63*A*ta
n(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/
2*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*t
an(1/2*d*x + 1/2*c))/(a^4*d)
```

3.114 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$-\frac{2(80A-3B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-6B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{(10A-3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] (A*x)/a^4 - ((55*A - 6*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) -
(2*(80*A - 3*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Tan
[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((10*A - 3*B)*Tan[c + d*x])/(35*a
*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.267809, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3922, 3919, 3794}

$$-\frac{2(80A-3B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-6B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{(10A-3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (A*x)/a^4 - ((55*A - 6*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) -
(2*(80*A - 3*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Tan
[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((10*A - 3*B)*Tan[c + d*x])/(35*a
*d*(a + a*Sec[c + d*x])^3)
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^4}dx = -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{-7aA+3a(A-B)\sec(c+dx)}{(a+a\sec(c+dx))^3}dx}{7a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{\int \frac{35a^2A-2a^2(10A-3B)\sec(c+dx)}{(a+a\sec(c+dx))^2}dx}{35a^4}$$

$$= -\frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{-105a^3A}{}}{}$$

$$= \frac{Ax}{a^4} - \frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{(2(8}{}$$

$$= \frac{Ax}{a^4} - \frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{2(8}{105a}$$

Mathematica [B] time = 0.755103, size = 329, normalized size = 2.38

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(8260A\sin\left(c+\frac{dx}{2}\right)-7140A\sin\left(c+\frac{3dx}{2}\right)+3780A\sin\left(2c+\frac{3dx}{2}\right)-2800A\sin\left(2c+\frac{5dx}{2}\right)+84\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*A*d*x*Cos[(d*x)/2] + 3675*A*d*x*Cos[c +
(d*x)/2] + 2205*A*d*x*Cos[c + (3*d*x)/2] + 2205*A*d*x*Cos[2*c + (3*d*x)/2]
+ 735*A*d*x*Cos[2*c + (5*d*x)/2] + 735*A*d*x*Cos[3*c + (5*d*x)/2] + 105*A*d
*x*Cos[3*c + (7*d*x)/2] + 105*A*d*x*Cos[4*c + (7*d*x)/2] - 9940*A*Sin[(d*x)
/2] + 1260*B*Sin[(d*x)/2] + 8260*A*Sin[c + (d*x)/2] - 1260*B*Sin[c + (d*x)/
2] - 7140*A*Sin[c + (3*d*x)/2] + 882*B*Sin[c + (3*d*x)/2] + 3780*A*Sin[2*c
+ (3*d*x)/2] - 630*B*Sin[2*c + (3*d*x)/2] - 2800*A*Sin[2*c + (5*d*x)/2] + 2
94*B*Sin[2*c + (5*d*x)/2] + 840*A*Sin[3*c + (5*d*x)/2] - 210*B*Sin[3*c + (5
*d*x)/2] - 520*A*Sin[3*c + (7*d*x)/2] + 72*B*Sin[3*c + (7*d*x)/2]))/(13440*
a^4*d)
```

Maple [A] time = 0.068, size = 177, normalized size = 1.3

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{3\,B}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{11\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/8/d/a
^4*tan(1/2*d*x+1/2*c)^5*A+3/40/d/a^4*tan(1/2*d*x+1/2*c)^5*B+11/24/d/a^4*A*t
an(1/2*d*x+1/2*c)^3-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2*d
*x+1/2*c)+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*A*arctan(tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.47748, size = 271, normalized size = 1.96

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{3\,B\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin}{(\cos}\right.}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.470773, size = 475, normalized size = 3.44

$$\frac{105\,Adx\cos\left(dx+c\right)^4+420\,Adx\cos\left(dx+c\right)^3+630\,Adx\cos\left(dx+c\right)^2+420\,Adx\cos\left(dx+c\right)+105\,Adx-\left(4\,(65\,A\right.}{105\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*A*d*x*cos(d*x + c)^4 + 420*A*d*x*cos(d*x + c)^3 + 630*A*d*x*cos(
d*x + c)^2 + 420*A*d*x*cos(d*x + c) + 105*A*d*x - (4*(65*A - 9*B)*cos(d*x +
 c)^3 + (620*A - 39*B)*cos(d*x + c)^2 + (535*A - 24*B)*cos(d*x + c) + 160*A
 - 6*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{B\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*se
c(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c +
d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.20594, size = 208, normalized size = 1.51

$$\frac{\frac{840\,(dx+c)A}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+63\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,Ba}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)*A/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 63*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2
*d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 105*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

3.115 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=166

$$\frac{8(83A-20B)\sin(c+dx)}{105a^4d}-\frac{(4A-B)\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{(88A-25B)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{x(4A-B)}{a^4}-\frac{(12A-5B)\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

```
[Out] -(((4*A - B)*x)/a^4) + (8*(83*A - 20*B)*Sin[c + d*x])/(105*a^4*d) - ((88*A
- 25*B)*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((4*A - B)*Sin[c +
 d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Sec[
c + d*x])^4) - ((12*A - 5*B)*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.572593, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4020, 3787, 2637, 8}

$$\frac{8(83A-20B)\sin(c+dx)}{105a^4d}-\frac{(4A-B)\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{(88A-25B)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{x(4A-B)}{a^4}-\frac{(12A-5B)\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(((4*A - B)*x)/a^4) + (8*(83*A - 20*B)*Sin[c + d*x])/(105*a^4*d) - ((88*A
- 25*B)*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((4*A - B)*Sin[c +
 d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Sec[
c + d*x])^4) - ((12*A - 5*B)*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(a(8A-B)-4a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^3}dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5B)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos(c+dx)\left(2a^2(26A-5B)-3a^2(1\right.}{(a+a\sec(c+dx}}{35a^4}$$

$$= -\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5B)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

$$= -\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5B)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

$$= -\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5B)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

$$= -\frac{(4A-B)x}{a^4} + \frac{8(83A-20B)\sin(c+dx)}{105a^4d} - \frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B}{7d(a+a}$$

Mathematica [B] time = 1.03939, size = 485, normalized size = 2.92

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-7350dx(4A-B)\cos\left(c+\frac{dx}{2}\right)-7350dx(4A-B)\cos\left(\frac{dx}{2}\right)-46130A\sin\left(c+\frac{dx}{2}\right)+46116A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-7350*(4*A - B)*d*x*Cos[(d*x)/2] - 7350*(4*A -
B)*d*x*Cos[c + (d*x)/2] - 17640*A*d*x*Cos[c + (3*d*x)/2] + 4410*B*d*x*Cos[c
 + (3*d*x)/2] - 17640*A*d*x*Cos[2*c + (3*d*x)/2] + 4410*B*d*x*Cos[2*c + (3*
d*x)/2] - 5880*A*d*x*Cos[2*c + (5*d*x)/2] + 1470*B*d*x*Cos[2*c + (5*d*x)/2]
 - 5880*A*d*x*Cos[3*c + (5*d*x)/2] + 1470*B*d*x*Cos[3*c + (5*d*x)/2] - 840*
A*d*x*Cos[3*c + (7*d*x)/2] + 210*B*d*x*Cos[3*c + (7*d*x)/2] - 840*A*d*x*Cos
[4*c + (7*d*x)/2] + 210*B*d*x*Cos[4*c + (7*d*x)/2] + 60830*A*Sin[(d*x)/2] -
 19880*B*Sin[(d*x)/2] - 46130*A*Sin[c + (d*x)/2] + 16520*B*Sin[c + (d*x)/2]
 + 46116*A*Sin[c + (3*d*x)/2] - 14280*B*Sin[c + (3*d*x)/2] - 18060*A*Sin[2*
c + (3*d*x)/2] + 7560*B*Sin[2*c + (3*d*x)/2] + 19292*A*Sin[2*c + (5*d*x)/2]
 - 5600*B*Sin[2*c + (5*d*x)/2] - 2100*A*Sin[3*c + (5*d*x)/2] + 1680*B*Sin[3
*c + (5*d*x)/2] + 3791*A*Sin[3*c + (7*d*x)/2] - 1040*B*Sin[3*c + (7*d*x)/2]
 + 735*A*Sin[4*c + (7*d*x)/2] + 105*A*Sin[4*c + (9*d*x)/2] + 105*A*Sin[5*c
+ (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.1, size = 229, normalized size = 1.4

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{23\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+7/40/d
/a^4*tan(1/2*d*x+1/2*c)^5*A-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5*B-23/24/d/a^4*A*
```

```
tan(1/2*d*x+1/2*c)^3+11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/
2*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*A*tan(1/2*d*x+1/2*c)/(
1+tan(1/2*d*x+1/2*c)^2)-8/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^4*arctan
(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.52736, size = 366, normalized size = 2.2

$$\frac{A\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5B\left(\frac{315\sin(dx}{\cos(dx+c)}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4))/d
```

Fricas [A] time = 0.486195, size = 574, normalized size = 3.46

$$-\frac{105\,(4\,A-B)dx\cos(dx+c)^4+420\,(4\,A-B)dx\cos(dx+c)^3+630\,(4\,A-B)dx\cos(dx+c)^2+420\,(4\,A-B)dx\cos}{105\left(a^4d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] -1/105*(105*(4*A - B)*d*x*cos(d*x + c)^4 + 420*(4*A - B)*d*x*cos(d*x + c)^3
 + 630*(4*A - B)*d*x*cos(d*x + c)^2 + 420*(4*A - B)*d*x*cos(d*x + c) + 105*
(4*A - B)*d*x - (105*A*cos(d*x + c)^4 + 4*(296*A - 65*B)*cos(d*x + c)^3 + 4
*(659*A - 155*B)*cos(d*x + c)^2 + (2236*A - 535*B)*cos(d*x + c) + 664*A - 1
60*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*
d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32156, size = 257, normalized size = 1.55

$$-\frac{\frac{840\,(dx+c)(4\,A-B)}{a^4}-\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+8\cdots}{a^{2\cdots}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(840*(d*x + c)*(4*A - B)/a^4 - 1680*A*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*t
an(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2
*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*B*a^24*tan(1/2*d*
x + 1/2*c))/a^28)/d
```

3.116 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=223

$$-\frac{8(216A-83B)\sin(c+dx)}{105a^4d}+\frac{(21A-8B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{4(216A-83B)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(129A}{}$$

```
[Out] ((21*A - 8*B)*x)/(2*a^4) - (8*(216*A - 83*B)*Sin[c + d*x])/(105*a^4*d) + ((
21*A - 8*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129*A - 52*B)*Cos[c +
d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*(216*A - 83*B)*Cos
[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d
*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((2*A - B)*Cos[c + d*x]*Si
n[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.648995, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2635, 8, 2637}

$$-\frac{8(216A-83B)\sin(c+dx)}{105a^4d}+\frac{(21A-8B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{4(216A-83B)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(129A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] ((21*A - 8*B)*x)/(2*a^4) - (8*(216*A - 83*B)*Sin[c + d*x])/(105*a^4*d) + ((
21*A - 8*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129*A - 52*B)*Cos[c +
d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*(216*A - 83*B)*Cos
[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d
*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((2*A - B)*Cos[c + d*x]*Si
n[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(a(9A-2B)-5a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(2A-B)\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} + \int \frac{\cos^2(c\cdots}{\cdots}$$

$$= -\frac{(129A-52B)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(2A\cdots}{\cdots}$$

$$= -\frac{(129A-52B)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(2A\cdots}{\cdots}$$

$$= -\frac{(129A-52B)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(2A\cdots}{\cdots}$$

$$= -\frac{8(216A-83B)\sin(c+dx)}{105a^4d} + \frac{(21A-8B)\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{(129A-5\cdots}{105\cdots}$$

$$= \frac{(21A-8B)x}{2a^4} - \frac{8(216A-83B)\sin(c+dx)}{105a^4d} + \frac{(21A-8B)\cos(c+dx)\sin(c+dx\cdots}{2a^4d}$$

Mathematica [B] time = 1.11128, size = 555, normalized size = 2.49

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(14700dx(21A-8B)\cos\left(c+\frac{dx}{2}\right)+14700dx(21A-8B)\cos\left(\frac{dx}{2}\right)+386190A\sin\left(c+\frac{dx}{2}\right)-4224\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(14700*(21*A - 8*B)*d*x*Cos[(d*x)/2] + 14700*(21
*A - 8*B)*d*x*Cos[c + (d*x)/2] + 185220*A*d*x*Cos[c + (3*d*x)/2] - 70560*B*
d*x*Cos[c + (3*d*x)/2] + 185220*A*d*x*Cos[2*c + (3*d*x)/2] - 70560*B*d*x*Co
s[2*c + (3*d*x)/2] + 61740*A*d*x*Cos[2*c + (5*d*x)/2] - 23520*B*d*x*Cos[2*c
 + (5*d*x)/2] + 61740*A*d*x*Cos[3*c + (5*d*x)/2] - 23520*B*d*x*Cos[3*c + (5
*d*x)/2] + 8820*A*d*x*Cos[3*c + (7*d*x)/2] - 3360*B*d*x*Cos[3*c + (7*d*x)/2
] + 8820*A*d*x*Cos[4*c + (7*d*x)/2] - 3360*B*d*x*Cos[4*c + (7*d*x)/2] - 539
490*A*Sin[(d*x)/2] + 243320*B*Sin[(d*x)/2] + 386190*A*Sin[c + (d*x)/2] - 18
4520*B*Sin[c + (d*x)/2] - 422478*A*Sin[c + (3*d*x)/2] + 184464*B*Sin[c + (3
*d*x)/2] + 132930*A*Sin[2*c + (3*d*x)/2] - 72240*B*Sin[2*c + (3*d*x)/2] - 1
81461*A*Sin[2*c + (5*d*x)/2] + 77168*B*Sin[2*c + (5*d*x)/2] + 3675*A*Sin[3*
c + (5*d*x)/2] - 8400*B*Sin[3*c + (5*d*x)/2] - 36003*A*Sin[3*c + (7*d*x)/2]
 + 15164*B*Sin[3*c + (7*d*x)/2] - 9555*A*Sin[4*c + (7*d*x)/2] + 2940*B*Sin[
4*c + (7*d*x)/2] - 945*A*Sin[4*c + (9*d*x)/2] + 420*B*Sin[4*c + (9*d*x)/2]
- 945*A*Sin[5*c + (9*d*x)/2] + 420*B*Sin[5*c + (9*d*x)/2] + 105*A*Sin[5*c +
 (11*d*x)/2] + 105*A*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x])
^4)
```

Maple [A] time = 0.1, size = 332, normalized size = 1.5

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,B}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{13\,A}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-9/40/d/
a^4*tan(1/2*d*x+1/2*c)^5*A+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5*B+13/8/d/a^4*A*t
an(1/2*d*x+1/2*c)^3-23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*A*tan(1/
2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^
2)^2*A*tan(1/2*d*x+1/2*c)^3+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*
x+1/2*c)^3-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+2/d/a^4/
(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+21/d/a^4*A*arctan(tan(1/2*d
*x+1/2*c))-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.52523, size = 491, normalized size = 2.2

$$-\frac{3\,A\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-B\left(\frac{\cdots}{\left(a^4+\frac{a^4\cdots}{(\cos\cdots}\right.}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 0.49927, size = 645, normalized size = 2.89

$$\frac{105\,(21\,A-8\,B)dx\cos(dx+c)^4+420\,(21\,A-8\,B)dx\cos(dx+c)^3+630\,(21\,A-8\,B)dx\cos(dx+c)^2+420\,(21\,A-8\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*(21*A - 8*B)*d*x*cos(d*x + c)^4 + 420*(21*A - 8*B)*d*x*cos(d*x +
 c)^3 + 630*(21*A - 8*B)*d*x*cos(d*x + c)^2 + 420*(21*A - 8*B)*d*x*cos(d*x
+ c) + 105*(21*A - 8*B)*d*x + (105*A*cos(d*x + c)^5 - 210*(2*A - B)*cos(d*x
 + c)^4 - 4*(1509*A - 592*B)*cos(d*x + c)^3 - 4*(3411*A - 1318*B)*cos(d*x +
 c)^2 - (11619*A - 4472*B)*cos(d*x + c) - 3456*A + 1328*B)*sin(d*x + c))/(a
^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a
^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47962, size = 315, normalized size = 1.41

$$\frac{\frac{420\,(dx+c)(21\,A-8\,B)}{a^4} - \frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(420*(d*x + c)*(21*A - 8*B)/a^4 - 840*(9*A*tan(1/2*d*x + 1/2*c)^3 - 2
*B*tan(1/2*d*x + 1/2*c)^3 + 7*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c
)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*A*a^24*tan(1/2*d*x + 1/2*c)^5
+ 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2*c)^3 -
805*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 11655*A*a^24*tan(1/2*d*x + 1/2*c) + 514
5*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.117 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4} \, dx$

Optimal. Leaf size=256

$$-\frac{8(227A-108B)\sin^3(c+dx)}{105a^4d}+\frac{8(227A-108B)\sin(c+dx)}{35a^4d}-\frac{(44A-21B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{(44A-21B)\,\text{s}}{3a^4d(\text{s}}$$

```
[Out] -((44*A - 21*B)*x)/(2*a^4) + (8*(227*A - 108*B)*Sin[c + d*x])/(35*a^4*d) -
((44*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((178*A - 87*B)*Cos[c
 + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((44*A - 21*B)*C
os[c + d*x]^2*Sin[c + d*x])/(3*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((16*A - 9*B)*Cos[c +
d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*(227*A - 108*B)*S
in[c + d*x]^3)/(105*a^4*d)
```

Rubi [A] time = 0.705455, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4020, 3787, 2633, 2635, 8}

$$-\frac{8(227A-108B)\sin^3(c+dx)}{105a^4d}+\frac{8(227A-108B)\sin(c+dx)}{35a^4d}-\frac{(44A-21B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{(44A-21B)\,\text{s}}{3a^4d(\text{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] -((44*A - 21*B)*x)/(2*a^4) + (8*(227*A - 108*B)*Sin[c + d*x])/(35*a^4*d) -
((44*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((178*A - 87*B)*Cos[c
 + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((44*A - 21*B)*C
os[c + d*x]^2*Sin[c + d*x])/(3*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((16*A - 9*B)*Cos[c +
d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*(227*A - 108*B)*S
in[c + d*x]^3)/(105*a^4*d)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(a(10A-3B)-6a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(16A-9B)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} + \ldots$$

$$= -\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \ldots$$

$$= -\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \ldots$$

$$= -\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \ldots$$

$$= -\frac{(44A-21B)\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} \ldots$$

$$= -\frac{(44A-21B)x}{2a^4} + \frac{8(227A-108B)\sin(c+dx)}{35a^4d} - \frac{(44A-21B)\cos(c+dx)\sin(c+dx)}{2a^4d} \ldots$$

Mathematica [B] time = 1.62759, size = 611, normalized size = 2.39

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-14700dx(44A-21B)\cos\left(c+\frac{dx}{2}\right)-14700dx(44A-21B)\cos\left(\frac{dx}{2}\right)-687260A\sin\left(c+\frac{dx}{2}\right)+8\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(44*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(
44*A - 21*B)*d*x*Cos[c + (d*x)/2] - 388080*A*d*x*Cos[c + (3*d*x)/2] + 18522
0*B*d*x*Cos[c + (3*d*x)/2] - 388080*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d
*x*Cos[2*c + (3*d*x)/2] - 129360*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*C
os[2*c + (5*d*x)/2] - 129360*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3
*c + (5*d*x)/2] - 18480*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (
7*d*x)/2] - 18480*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)
/2] + 1010660*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 687260*A*Sin[c + (d*
x)/2] + 386190*B*Sin[c + (d*x)/2] + 814107*A*Sin[c + (3*d*x)/2] - 422478*B*
Sin[c + (3*d*x)/2] - 204645*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*
d*x)/2] + 357609*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] + 1
8025*A*Sin[3*c + (5*d*x)/2] + 3675*B*Sin[3*c + (5*d*x)/2] + 72522*A*Sin[3*c
 + (7*d*x)/2] - 36003*B*Sin[3*c + (7*d*x)/2] + 24010*A*Sin[4*c + (7*d*x)/2]
 - 9555*B*Sin[4*c + (7*d*x)/2] + 2310*A*Sin[4*c + (9*d*x)/2] - 945*B*Sin[4*
```

```
c + (9*d*x)/2] + 2310*A*Sin[5*c + (9*d*x)/2] - 945*B*Sin[5*c + (9*d*x)/2] -
 175*A*Sin[5*c + (11*d*x)/2] + 105*B*Sin[5*c + (11*d*x)/2] - 175*A*Sin[6*c
+ (11*d*x)/2] + 105*B*Sin[6*c + (11*d*x)/2] + 35*A*Sin[6*c + (13*d*x)/2] +
35*A*Sin[7*c + (13*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.111, size = 402, normalized size = 1.6

$$-\frac{A}{56\,d a^{4}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{B}{56\,d a^{4}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{11\,A}{40\,d a^{4}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{9\,B}{40\,d a^{4}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{59\,A}{24\,d a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+11/40/
d/a^4*tan(1/2*d*x+1/2*c)^5*A-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5*B-59/24/d/a^4*
A*tan(1/2*d*x+1/2*c)^3+13/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3+209/8/d/a^4*A*tan(
1/2*d*x+1/2*c)-111/8/d/a^4*B*tan(1/2*d*x+1/2*c)+26/d/a^4/(1+tan(1/2*d*x+1/2
*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)^5*B+124/3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)^
3-16/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3+18/d/a^4/(1+ta
n(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)
^3*B*tan(1/2*d*x+1/2*c)-44/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))+21/d/a^4*arct
an(tan(1/2*d*x+1/2*c))*B
```

Maxima [A] time = 1.64083, size = 610, normalized size = 2.38

$$\frac{A\left(\frac{560\left(\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{62\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{39\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{4}+\frac{3a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{\frac{21945\sin(dx+c)}{\cos(dx+c)+1}-\frac{2065\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{231\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{36960\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(560*(27*sin(d*x + c)/(cos(d*x + c) + 1) + 62*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 39*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4 + 3*a^4*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (21945*sin(d*x + c)/(cos(d*x
+ c) + 1) - 2065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 231*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 36960*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4) - 3*B*(280*(7*sin(d*x + c)/(cos
(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) +
(3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 0.50868, size = 701, normalized size = 2.74

$$-\frac{105\,(44\,A-21\,B)dx\cos\left(dx+c\right)^{4}+420\,(44\,A-21\,B)dx\cos\left(dx+c\right)^{3}+630\,(44\,A-21\,B)dx\cos\left(dx+c\right)^{2}+420\,(44\,A-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*(44*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(44*A - 21*B)*d*x*cos(d*
x + c)^3 + 630*(44*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(44*A - 21*B)*d*x*cos
(d*x + c) + 105*(44*A - 21*B)*d*x - (70*A*cos(d*x + c)^6 - 35*(4*A - 3*B)*c
os(d*x + c)^5 + 140*(7*A - 3*B)*cos(d*x + c)^4 + 4*(3196*A - 1509*B)*cos(d*
x + c)^3 + 4*(7184*A - 3411*B)*cos(d*x + c)^2 + (24436*A - 11619*B)*cos(d*x
 + c) + 7264*A - 3456*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25121, size = 352, normalized size = 1.38

$$-\frac{\frac{420\,(dx+c)(44\,A-21\,B)}{a^{4}}-\frac{280\left(78\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-27\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+124\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+54\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-21\,B\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(d*x + c)*(44*A - 21*B)/a^4 - 280*(78*A*tan(1/2*d*x + 1/2*c)^5
- 27*B*tan(1/2*d*x + 1/2*c)^5 + 124*A*tan(1/2*d*x + 1/2*c)^3 - 48*B*tan(1/2
*d*x + 1/2*c)^3 + 54*A*tan(1/2*d*x + 1/2*c) - 21*B*tan(1/2*d*x + 1/2*c))/((
tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15
*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 231*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 189*B*
a^24*tan(1/2*d*x + 1/2*c)^5 + 2065*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 1365*B*a
^24*tan(1/2*d*x + 1/2*c)^3 - 21945*A*a^24*tan(1/2*d*x + 1/2*c) + 11655*B*a^
24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.118 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=187

$$\frac{2a(9A+8B)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4(9A+8B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}$$

```
[Out] (4*a*(9*A + 8*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9*A
+ 8*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
B*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.338126, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4016, 3803, 3800, 4001, 3792}

$$\frac{2a(9A+8B)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4(9A+8B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a*(9*A + 8*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9*A
+ 8*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
B*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2aB\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(9A+8B)\int \sec^4(c+dx)\sqrt{}$$
$$= \frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aB\sec^4(c+dx)\tan}{9d\sqrt{a+a\sec(c}}$$
$$= \frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aB\sec^4(c+dx)\tan}{9d\sqrt{a+a\sec(c}}$$
$$= \frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aB\sec^4(c+dx)\tan}{9d\sqrt{a+a\sec(c}}$$
$$= \frac{4a(9A+8B)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{2a(9A+8B)\sec^3(c+dx)\tan(c}{63d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.537751, size = 98, normalized size = 0.52

$$\frac{2a\tan(c+dx)\left(5(9A+8B)\sec^3(c+dx)+6(9A+8B)\sec^2(c+dx)+8(9A+8B)\sec(c+dx)+16(9A+8B)+35B\sec^4\right.}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(16*(9*A + 8*B) + 8*(9*A + 8*B)*Sec[c + d*x] + 6*(9*A + 8*B)*Sec[c + d
*x]^2 + 5*(9*A + 8*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c + d*x])/(
315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.324, size = 138, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(144A(\cos(dx+c))^4+128B(\cos(dx+c))^4+72A(\cos(dx+c))^3+64B(\cos(dx+c))^3+54A\right.}{315d(\cos(dx+c))^4\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(144*A*cos(d*x+c)^4+128*B*cos(d*x+c)^4+72*A*cos(d*
x+c)^3+64*B*cos(d*x+c)^3+54*A*cos(d*x+c)^2+48*B*cos(d*x+c)^2+45*A*cos(d*x+c
)+40*B*cos(d*x+c)+35*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/si
n(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.474214, size = 308, normalized size = 1.65

$$\frac{2\left(16\,(9\,A+8\,B)\cos(dx+c)^4+8\,(9\,A+8\,B)\cos(dx+c)^3+6\,(9\,A+8\,B)\cos(dx+c)^2+5\,(9\,A+8\,B)\cos(dx+c)+\right.}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(16*(9*A + 8*B)*cos(d*x + c)^4 + 8*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*
A + 8*B)*cos(d*x + c)^2 + 5*(9*A + 8*B)*cos(d*x + c) + 35*B)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\,(\sec(c+dx)+1)}\,(A+B\sec(c+dx))\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**4, x
)
```

Giac [A] time = 4.94975, size = 362, normalized size = 1.94

$$\frac{2\left(315\,\sqrt{2}Aa^5\mathrm{sgn}(\cos(dx+c))+315\,\sqrt{2}Ba^5\mathrm{sgn}(\cos(dx+c))-\left(630\,\sqrt{2}Aa^5\mathrm{sgn}(\cos(dx+c))+420\,\sqrt{2}Ba^5\mathrm{sgn}(\mathrm{co}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 2/315*(315*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^5*sgn(cos(d*x
+ c)) - (630*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 420*sqrt(2)*B*a^5*sgn(cos(d*
x + c)) - (756*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 882*sqrt(2)*B*a^5*sgn(cos(
d*x + c)) - (522*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 324*sqrt(2)*B*a^5*sgn(co
s(d*x + c)) - (81*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 107*sqrt(2)*B*a^5*sgn(c
os(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1
/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.119 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{2(7A+6B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7A+6B)\tan(c+dx}{15d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*(7*A + 6*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.276674, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4016, 3800, 4001, 3792}

$$\frac{2(7A+6B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7A+6B)\tan(c+dx}{15d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(7*A + 6*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(7A+6B)\int \sec^3(c+dx)\sqrt{}$$

$$= \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A+6B)(a+a\sec(c+dx)}{35ad}$$

$$= \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{4(7A+6B)\sqrt{a+a\sec(c+dx}}{105d}$$

$$= \frac{2a(7A+6B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{4}{}$$

Mathematica [A] time = 0.264548, size = 81, normalized size = 0.56

$$\frac{2a\tan(c+dx)\left(3(7A+6B)\sec^2(c+dx)+4(7A+6B)\sec(c+dx)+8(7A+6B)+15B\sec^3(c+dx)\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(8*(7*A + 6*B) + 4*(7*A + 6*B)*Sec[c + d*x] + 3*(7*A + 6*B)*Sec[c + d*
x]^2 + 15*B*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])]
)
```

Maple [A] time = 0.28, size = 116, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(56A(\cos(dx+c))^3+48B(\cos(dx+c))^3+28A(\cos(dx+c))^2+24B(\cos(dx+c))^2+21A\cos\right.}{105d(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(56*A*cos(d*x+c)^3+48*B*cos(d*x+c)^3+28*A*cos(d*x+
c)^2+24*B*cos(d*x+c)^2+21*A*cos(d*x+c)+18*B*cos(d*x+c)+15*B)*(a*(cos(d*x+c)
+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.470775, size = 265, normalized size = 1.84

$$\frac{2\left(8\,(7\,A+6\,B)\cos\left(dx+c\right)^{3}+4\,(7\,A+6\,B)\cos\left(dx+c\right)^{2}+3\,(7\,A+6\,B)\cos\left(dx+c\right)+15\,B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+\right.}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(8*(7*A + 6*B)*cos(d*x + c)^3 + 4*(7*A + 6*B)*cos(d*x + c)^2 + 3*(7*A
 + 6*B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**3, x
)
```

Giac [A] time = 4.88182, size = 300, normalized size = 2.08

$$-\frac{2\left(105\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(175\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ba^{4}\mathrm{sgn}\left(\mathrm{c}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -2/105*(105*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d*x
 + c)) - (175*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d
*x + c)) - (119*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 147*sqrt(2)*B*a^4*sgn(cos
(d*x + c)) - (49*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 27*sqrt(2)*B*a^4*sgn(cos
(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.120 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2(5A-2B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a(5A+7B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] (2*a*(5*A + 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*B*(a + a*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.227687, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4010, 4001, 3792}

$$\frac{2(5A-2B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a(5A+7B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*B*(a + a*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{2\int \sec(c+dx)\sqrt{a}$$
$$= \frac{2(5A-2B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2B(a+a\sec}{}$$
$$= \frac{2a(5A+7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A-2B)\sqrt{a+a\sec(c+dx}}{15d}$$

Mathematica [A] time = 0.277759, size = 80, normalized size = 0.79

$$\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}((5A+4B)\cos(c+dx)+(5A+4B)\cos(2(c+dx))+5A+7B)}{15d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(5*A + 7*B + (5*A + 4*B)*Cos[c + d*x] + (5*A + 4*B)*Cos[2*(c + d*x)])*Se
c[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(15*d*(1 + Cos[c + d*x]
))
```

Maple [A] time = 0.331, size = 94, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))\left(10A(\cos(dx+c))^2+8B(\cos(dx+c))^2+5A\cos(dx+c)+4B\cos(dx+c)+3B\right)}{15d(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos}{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(10*A*cos(d*x+c)^2+8*B*cos(d*x+c)^2+5*A*cos(d*x+c)+
4*B*cos(d*x+c)+3*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*
x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.464324, size = 217, normalized size = 2.15

$$\frac{2\left(2(5A+4B)\cos(dx+c)^2+(5A+4B)\cos(dx+c)+3B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/15*(2*(5*A + 4*B)*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 3*B)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*
x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**2, x
)
```

Giac [A] time = 4.81692, size = 238, normalized size = 2.36

$$\frac{2\left(15\sqrt{2}Aa^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\sqrt{2}Ba^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(20\sqrt{2}Aa^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+10\sqrt{2}Ba^{3}\mathrm{sgn}\left(\cos\left(dx+c\right.\right.\right.}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 2/15*(15*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 15*sqrt(2)*B*a^3*sgn(cos(d*x + c
)) - (20*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 10*sqrt(2)*B*a^3*sgn(cos(d*x + c
)) - (5*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 7*sqrt(2)*B*a^3*sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*
tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.121 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a(3A+B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*a*(3*A + B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*B*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0944036, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {4001, 3792}

$$\frac{2a(3A+B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(3*A + B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*B*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx=\frac{2B\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}(3A+B)\int \sec(c+$$
$$=\frac{2a(3A+B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2B\sqrt{a+a\sec(c+dx)}\tan(c+d}{3d}$$

Mathematica [A] time = 0.158314, size = 53, normalized size = 0.85

$$\frac{2\tan(c+dx)\sqrt{a(\sec(c+dx)+1)}((3A+2B)\cos(c+dx)+B)}{3d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(B + (3*A + 2*B)*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/
(3*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.273, size = 70, normalized size = 1.1

$$-\frac{\left(-2+2\,\cos\left(dx+c\right)\right)\left(3\,A\cos\left(dx+c\right)+2\,B\cos\left(dx+c\right)+B\right)}{3\,d\sin\left(dx+c\right)\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)+2*B*cos(d*x+c)+B)*(a*(cos(d*x+c)+1)/
cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.462124, size = 169, normalized size = 2.73

$$\frac{2\left((3\,A+2\,B)\cos\left(dx+c\right)+B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/3*((3*A + 2*B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 4.60645, size = 174, normalized size = 2.81

$$-\frac{2\left(3\sqrt{2}Aa^{2}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+3\sqrt{2}Ba^{2}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-\left(3\sqrt{2}Aa^{2}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+\sqrt{2}Ba^{2}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\right.\right.}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -2/3*(3*sqrt(2)*A*a^2*sgn(cos(d*x + c)) + 3*sqrt(2)*B*a^2*sgn(cos(d*x + c))
 - (3*sqrt(2)*A*a^2*sgn(cos(d*x + c)) + sqrt(2)*B*a^2*sgn(cos(d*x + c)))*ta
n(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.122 $\int \sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0882574, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3915, 3774, 203, 3792}

$$\frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = A\int \sqrt{a+a\sec(c+dx)}\,dx + B\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{2aB\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2aA)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aB\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.301342, size = 76, normalized size = 1.15

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}A\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*B*Sin[(c + d*x)/2]))/d
```

Maple [B] time = 0.253, size = 118, normalized size = 1.8

$$-\frac{1}{d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*sin(d*x+c)+2*B*cos(d*x+c)-2*B)/sin(d*x+c)
```

Maxima [B] time = 1.66367, size = 198, normalized size = 3.

$$A\sqrt{a}\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(
d*x + c))/d
```

Fricas [A] time = 0.511988, size = 620, normalized size = 9.39

$$\left[\frac{(A\cos(dx+c)+A)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c) + d), -2*((A*cos(d*x + c) + A)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\,(\sec{(c+dx)}+1)}\,(A+B\sec{(c+dx)})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.123 $\int \cos(c + dx)\sqrt{a + a\sec(c + dx)}(A + B\sec(c + dx))\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a}(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aA\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.106384, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4015, 3774, 203}

$$\frac{\sqrt{a}(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aA\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aA\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(A+2B)\int \sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{aA\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(a(A+2B))\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\,\mathrm{ta}}{\sqrt{a+a}}\right.}{d}$$

$$= \frac{\sqrt{a}(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{aA\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.235309, size = 93, normalized size = 1.37

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(A+2B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(A
 + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sqrt[Cos[c + d*x]]*Sin[(c +
d*x)/2]))/(2*d)
```

Maple [B] time = 0.292, size = 198, normalized size = 2.9

$$-\frac{1}{2d\sin(dx+c)}\left(A\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2B\sqrt{2}\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/2/d*(A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*B*
2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)
^2-2*A*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.00085, size = 1268, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/4*(4*B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
```

```
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A)/d
```

Fricas [A] time = 0.606879, size = 694, normalized size = 10.21

$$\left[\frac{2\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+\left((A+2\,B)\cos\left(dx+c\right)+A+2\,B\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{\cos(d}\right.}{2\,(d\cos\left(dx+c\right)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + ((A + 2*B)*cos(d*x + c) + A + 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
+ a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (A*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - ((A + 2*B)*cos
(d*x + c) + A + 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x), x)
```

Giac [B] time = 6.47288, size = 450, normalized size = 6.62

$$-\frac{\left(A\sqrt{-a}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+2\,B\sqrt{-a}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/2*((A*sqrt(-a)*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*sgn(cos(d*x + c)))*log(a
bs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
- a*(2*sqrt(2) + 3))) - (A*sqrt(-a)*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a*sgn(cos(d*
x + c)) - A*sqrt(-a)*a^2*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.124 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=117

$$\frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(3A+4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*(3*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.177077, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4015, 3805, 3774, 203}

$$\frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(3A+4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*(3*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(3A+4B)\int \cos(c+dx)\sqrt{a+$$

$$=\frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{8}(3A$$

$$=\frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-\frac{(a(3A}{}$$

$$=\frac{\sqrt{a}(3A+4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a(3A+4B)\sin(c+dx}{4d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.388255, size = 117, normalized size = 1.

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2A\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)+B\left(\cos(c+dx\right.\right.}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((B*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])
 + 2*A*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]]
)
```

Maple [B] time = 0.34, size = 398, normalized size = 3.4

$$\frac{1}{16\,d\cos(dx+c)\sin(dx+c)}\left(3\,A\sin(dx+c)\cos(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{c}{\cos}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/16/d*(3*A*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*2^(1/2)+4*B*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*2^(1/2)+3*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+
4*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*A*cos(d*
x+c)^4-4*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3+12*A*cos(d*x+c)^2+16*B*cos(d*x+c)
^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 2.33912, size = 2499, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - 1)))*A + 4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A] time = 0.617674, size = 801, normalized size = 6.85

$$\left[\frac{((3\,A+4\,B)\cos\left(dx+c\right)+3\,A+4\,B)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,A\cos\left(\right.\right.}{8\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/8*(((3*A + 4*B)*cos(d*x + c) + 3*A + 4*B)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^2 +
(3*A + 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c) + d), -1/4*(((3*A + 4*B)*cos(d*x + c) + 3*A + 4*B)*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - (2*A*cos(d*x + c)^2 + (3*A + 4*B)*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.80222, size = 851, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/8*((3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sgn(cos(d*x + c)))*log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sg
n(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(-a)*tan(1/
```

```
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a*sgn(co
s(d*x + c)) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^2*sgn(cos(d*x +
 c)) + 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^4*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 17*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^3*sgn(cos(d*x + c))
- 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 4*B
*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.125 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5A+6B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\,\mathrm{co}\ldots}{3d\sqrt{a\sec(c+\ldots}}$$

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*A + 6*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*(5*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (
a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.242449, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4015, 3805, 3774, 203}

$$\frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5A+6B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\,\mathrm{co}\ldots}{3d\sqrt{a\sec(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*A + 6*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*(5*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (
a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(5A+6B)\int \cos^2(c+dx$$

$$= \frac{a(5A+6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos^2(c+dx)\sin}{3d\sqrt{a+a\sec(c-}}$$

$$= \frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(5A+6B)\cos(c+dx)\sin(c+}{12d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(5A+6B)\cos(c+dx)\sin(c+}{12d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{a}(5A+6B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a(5A+6B)\sin(c+}{8d\sqrt{a+a\sec(c+\iota}}$$

Mathematica [C] time = 0.172956, size = 70, normalized size = 0.44

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(A\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)+B\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F
1[1/2, 4, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/
2])/d
```

Maple [B] time = 0.405, size = 580, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/192/d*(15*A*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*2^(1/2)+18*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*2^(1/2)+30*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*2^(1/2)+36*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*2^(1/2)+15*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)+18*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x
```

```
+c)+64*A*cos(d*x+c)^6+16*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+40*A*cos(d*x+c)^4
+48*B*cos(d*x+c)^4-120*A*cos(d*x+c)^3-144*B*cos(d*x+c)^3)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [B] time = 2.8863, size = 4024, normalized size = 25.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/96*((4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*ar
```

```
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arct
an2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + 6*(2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) -
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2
*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*
c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) +
 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A] time = 0.632572, size = 898, normalized size = 5.61

$$\left[\frac{3\left((5A+6B)\cos(dx+c)+5A+6B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8A\cos\right.}{48\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((5*A + 6*B)*cos(d*x + c) + 5*A + 6*B)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d*x + c)^3
 + 2*(5*A + 6*B)*cos(d*x + c)^2 + 3*(5*A + 6*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(3*((5
*A + 6*B)*cos(d*x + c) + 5*A + 6*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^3
+ 2*(5*A + 6*B)*cos(d*x + c)^2 + 3*(5*A + 6*B)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.13563, size = 1156, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 6*B*sqrt(-a)*sgn(cos(d*x + c)))*
log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 6*B*sqrt(-
a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(63*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a
*sgn(cos(d*x + c)) - 30*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a*sgn(cos(d*x + c)) - 369*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^2*sgn(
cos(d*x + c)) + 66*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1638*(sqrt(-a)*tan(1/2*d
```

```
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos
(d*x + c)) + 756*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 1074*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d
*x + c)) - 732*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x
+ c)) + 138*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 13*A*sqrt(-a)*a^6*sgn(cos(d*x +
 c)) - 6*B*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.126 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=203

$$\frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{5\sqrt{a}(7A+8B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a(7A+8B)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{5a(7A+8B)\,\mathrm{s}}{96d\sqrt{a}}$$

```
[Out] (5*Sqrt[a]*(7*A + 8*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (5*a*(7*A + 8*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]
) + (5*a*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*(7*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.298097, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4015, 3805, 3774, 203}

$$\frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{5\sqrt{a}(7A+8B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a(7A+8B)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{5a(7A+8B)\,\mathrm{s}}{96d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (5*Sqrt[a]*(7*A + 8*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (5*a*(7*A + 8*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]
) + (5*a*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*(7*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(7A+8B)\int \cos^3(c+dx$$

$$= \frac{a(7A+8B)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos^3(c+dx)\sin}{4d\sqrt{a+a\sec(c}$$

$$= \frac{5a(7A+8B)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a(7A+8B)\cos^2(c}{24d\sqrt{a+a}}$$

$$= \frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{5a(7A+8B)\cos(c+dx)\sin(c}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{5a(7A+8B)\cos(c+dx)\sin(c}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{5\sqrt{a}(7A+8B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{5a(7A+8B)\sin(c}{64d\sqrt{a+a\sec(c}}$$

Mathematica [C] time = 0.171044, size = 70, normalized size = 0.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(A\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)+B\text{Hypergeometric2F1}\left(\frac{}{}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(B*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F
1[1/2, 5, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/
2])/d
```

Maple [B] time = 0.343, size = 762, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3072/d*(105*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(7/2)*2^(1/2)+120*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(7/2)*2^(1/2)+315*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*2^(1/2)+360*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
```

```
(cos(d*x+c)+1))^(7/2)*2^(1/2)+315*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+360*B*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+105*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)*sin(d*x+c)+120*B*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)
*2^(1/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-128*A*cos(d*x+c)^7-1024*B*cos(d*x+c)
^7-224*A*cos(d*x+c)^6-256*B*cos(d*x+c)^6-560*A*cos(d*x+c)^5-640*B*cos(d*x+c
)^5+1680*A*cos(d*x+c)^4+1920*B*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [B] time = 4.10205, size = 11557, normalized size = 56.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/768*(8*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*si
n(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(
arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
```

```
))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B - (2*(cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x
 + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 9*sin(4*d*x + 4*c)^3 +
 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin
(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*(
2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2
*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*
x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (32*(cos(4
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d
*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*si
n(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*sin(3/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 36*(4*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2)*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 1)) - (9*cos(4*d*x + 4*c)^3 + 4*(9*cos(4*d*x +
4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - 10*cos(4*d*x + 4*c)^
2 - 7*cos(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 + 4*(9*cos(4*d*x + 4*
c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 + 26*cos(4*d*x + 4*c)^2
+ 25*cos(4*d*x + 4*c) + 8)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + 8*cos(4*d*x + 4*c)^2 - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^
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2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)
+ 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x +
 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x +
4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*
x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin
(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*
x + 4*c)^2 - cos(4*d*x + 4*c)^2 - 8*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*sin(3/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(9*cos(4*d*x + 4*c) + 8)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (9*cos(4*
d*x + 4*c) + 8)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a)
 - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((64*(cos(4*d*x + 4*c)^2 + sin(4*d*x +
4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^3 + 20*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x +
 4*c) + 1)*sin(4*d*x + 4*c) + 8*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 5*cos(4*d*x + 4
*c)^2*sin(4*d*x + 4*c) + 5*sin(4*d*x + 4*c)^3 + 4*(5*sin(4*d*x + 4*c)^3 + (
5*cos(4*d*x + 4*c)^2 + 10*cos(4*d*x + 4*c) - 11)*sin(4*d*x + 4*c) - 64*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 40*(cos
(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 10*(2*sin(4*d*x + 4*c)^3 + 2*(cos(4*d*x + 4*c)^2 -
cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + cos(1/4*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))*sin(4*d*x + 4*c) + (16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4
*c)^2 - 17*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5*cos(1/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(32*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 8*(4*c
os(4*d*x + 4*c)^2 - sin(4*d*x + 4*c)^2 - 40*sin(4*d*x + 4*c)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*cos(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*(cos(4*d*x + 4*c) + 1)*cos(1/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*sin(4*d*x + 4*c)^2 - 85*sin
(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5*(8*cos(4*d*x + 4*c)^2 + 8
*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) -
(64*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 5*cos(4*d*x + 4*c)^3 +
 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 18
*cos(4*d*x + 4*c)^2 + 8*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*
d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 37*c
os(4*d*x + 4*c) - 24)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c)^2 + 4*(5*cos(4*d*x + 4*c)^3
+ (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c)^2 - 14*cos(4*d*x + 4*c)^2 + 16
*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*(cos(4*d*x + 4*c)^2 + sin
(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c),
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cos(4*d*x + 4*c))) - 43*cos(4*d*x + 4*c) - 24)*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 - 24*cos(4*d*x + 4*c)^2 + 2*(10*cos(4*d*x + 4*c
)^3 + 10*(cos(4*d*x + 4*c) - 4)*sin(4*d*x + 4*c)^2 - 50*cos(4*d*x + 4*c)^2
+ (16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 21*cos(4*d*x + 4*c) + 5)
*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*sin(4*d*x + 4*c)*
sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 48*cos(4*d*x + 4*c))
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*cos(4*d*x + 4*c)
^2 + 8*sin(4*d*x + 4*c)^2 - 5*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) - 2*(128*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c)))^2*sin(4*d*x + 4*c) + 8*(5*(cos(4*d*x + 4*c) - 4)*sin(4*d*x + 4
*c) + 8*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*
c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(5*cos(4*d*x +
 4*c) - 24)*sin(4*d*x + 4*c) + 21*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(4*d*x + 4*c) - 5*(cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) - 5*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a)
 - 105*((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) +
1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c
)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1
/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1))) + 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c
)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)
 + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x +
4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*
(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) +
sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arc
tan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos
(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(si
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n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (4*(cos(4*d*x
 + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x +
4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c
)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arct
an2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*c
os(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (4*(cos(4*d*x +
 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*
c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^
2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos
(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*A/(4*(cos
(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*
d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x
 + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 0.714751, size = 995, normalized size = 4.9

$$\left[\frac{15\left((7A+8B)\cos\left(dx+c\right)+7A+8B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48A\,c\right.}{384\left(d\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/384*(15*((7*A + 8*B)*cos(d*x + c) + 7*A + 8*B)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*cos(d*x + c
)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 10*(7*A + 8*B)*cos(d*x + c)^2 + 15*(7*
A + 8*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d), -1/192*(15*((7*A + 8*B)*cos(d*x + c) + 7*A + 8*B)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) - (48*A*cos(d*x + c)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 10*
(7*A + 8*B)*cos(d*x + c)^2 + 15*(7*A + 8*B)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.29261, size = 1458, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/384*(15*(7*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*B*sqrt(-a)*sgn(cos(d*x + c))
)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3))) - 15*(7*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*B*sqr
t(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(279*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-
a)*a*sgn(cos(d*x + c)) - 504*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 285*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^
2*sgn(cos(d*x + c)) + 5976*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 4605*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a
^3*sgn(cos(d*x + c)) - 31320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 37281*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)
*a^4*sgn(cos(d*x + c)) + 90168*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 35643*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a
)*a^5*sgn(cos(d*x + c)) - 66024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 9175*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a
)*a^6*sgn(cos(d*x + c)) + 16904*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 1311*(sqrt(
```

```
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a
)*a^7*sgn(cos(d*x + c)) - 1992*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 43*A*sqrt(-a
)*a^8*sgn(cos(d*x + c)) + 104*B*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/
d
```

3.127 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{2a^2(9A+10B)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(39A+34B)\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{2(39A+34B)\tan(c+dx)(a\sec(c+dx)+}{105d}$$

```
[Out] (2*a^2*(39*A + 34*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*
B*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d)
```

Rubi [A] time = 0.461041, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4018, 4016, 3800, 4001, 3792}

$$\frac{2a^2(9A+10B)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(39A+34B)\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{2(39A+34B)\tan(c+dx)(a\sec(c+dx)+}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(39*A + 34*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*
B*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
```

```
1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{9d}+\frac{2}{9}\int \sec$$

$$= \frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sec^3(c+dx}{}$$

$$= \frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sec^3(c+dx}{}$$

$$= \frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}-\frac{4a(39A+34B)}{}$$

$$= \frac{2a^2(39A+34B)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(9A+10B)\sec^3(c+dx)}{63d\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 0.734687, size = 100, normalized size = 0.53

$$\frac{2a^2\tan(c+dx)\left(5(9A+17B)\sec^3(c+dx)+3(39A+34B)\sec^2(c+dx)+4(39A+34B)\sec(c+dx)+8(39A+34B)+3\right.}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(8*(39*A + 34*B) + 4*(39*A + 34*B)*Sec[c + d*x] + 3*(39*A + 34*B)*Se
c[c + d*x]^2 + 5*(9*A + 17*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c +
 d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.282, size = 139, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(312A(\cos(dx+c))^4+272B(\cos(dx+c))^4+156A(\cos(dx+c))^3+136B(\cos(dx+c))^3+11\right.}{315d(\cos(dx+c))^4\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(312*A*cos(d*x+c)^4+272*B*cos(d*x+c)^4+156*A*cos
(d*x+c)^3+136*B*cos(d*x+c)^3+117*A*cos(d*x+c)^2+102*B*cos(d*x+c)^2+45*A*cos
(d*x+c)+85*B*cos(d*x+c)+35*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c
)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.486077, size = 329, normalized size = 1.74

$$\frac{2\left(8\,(39\,A+34\,B)a\cos\left(dx+c\right)^{4}+4\,(39\,A+34\,B)a\cos\left(dx+c\right)^{3}+3\,(39\,A+34\,B)a\cos\left(dx+c\right)^{2}+5\,(9\,A+17\,B)a\,\mathrm{co}\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(8*(39*A + 34*B)*a*cos(d*x + c)^4 + 4*(39*A + 34*B)*a*cos(d*x + c)^3
+ 3*(39*A + 34*B)*a*cos(d*x + c)^2 + 5*(9*A + 17*B)*a*cos(d*x + c) + 35*B*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 +
d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.12644, size = 362, normalized size = 1.92

$$\frac{4\left(315\sqrt{2}Aa^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+315\sqrt{2}Ba^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(735\sqrt{2}Aa^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+525\sqrt{2}Ba^{6}\mathrm{sgn}\left(\mathrm{co}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 4/315*(315*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^6*sgn(cos(d*x
+ c)) - (735*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 525*sqrt(2)*B*a^6*sgn(cos(d*
x + c)) - (819*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 819*sqrt(2)*B*a^6*sgn(cos(
d*x + c)) - (513*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 423*sqrt(2)*B*a^6*sgn(co
s(d*x + c)) - 2*(57*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 47*sqrt(2)*B*a^6*sgn(
cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.128 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{8a^2(21A+19B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7A-2B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\tan(c+dx)\sqrt{a\sec(c+}}{105d}$$

```
[Out] (8*a^2*(21*A + 19*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a + a*Sec[c + d*
x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.297271, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4010, 4001, 3793, 3792}

$$\frac{8a^2(21A+19B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7A-2B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\tan(c+dx)\sqrt{a\sec(c+}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (8*a^2*(21*A + 19*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a + a*Sec[c + d*
x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{2\int \sec(c+dx)(a+}{}$$
$$= \frac{2(7A-2B)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2B(a+a\,\mathrm{se}}{}$$
$$= \frac{2a(21A+19B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} + \frac{2(7A-2}{}$$
$$= \frac{8a^2(21A+19B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(21A+19B)\sqrt{a+a\sec(c}}{105d}$$

Mathematica [A] time = 0.38045, size = 82, normalized size = 0.59

$$\frac{2a^2\tan(c+dx)\left(3(7A+13B)\sec^2(c+dx)+(63A+52B)\sec(c+dx)+2(63A+52B)+15B\sec^3(c+dx)\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(2*(63*A + 52*B) + (63*A + 52*B)*Sec[c + d*x] + 3*(7*A + 13*B)*Sec[c
 + d*x]^2 + 15*B*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [A] time = 0.241, size = 117, normalized size = 0.9

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(126\,A(\cos(dx+c))^3+104\,B(\cos(dx+c))^3+63\,A(\cos(dx+c))^2+52\,B(\cos(dx+c))^2+21\,A\right.}{105\,d\,(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(126*A*cos(d*x+c)^3+104*B*cos(d*x+c)^3+63*A*cos(
d*x+c)^2+52*B*cos(d*x+c)^2+21*A*cos(d*x+c)+39*B*cos(d*x+c)+15*B)*(a*(cos(d*
x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.47617, size = 279, normalized size = 2.02

$$\frac{2\left(2\,(63\,A+52\,B)a\cos\left(dx+c\right)^{3}+(63\,A+52\,B)a\cos\left(dx+c\right)^{2}+3\,(7\,A+13\,B)a\cos\left(dx+c\right)+15\,Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,s}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(2*(63*A + 52*B)*a*cos(d*x + c)^3 + (63*A + 52*B)*a*cos(d*x + c)^2 +
3*(7*A + 13*B)*a*cos(d*x + c) + 15*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.00943, size = 300, normalized size = 2.17

$$-\frac{4\left(105\,\sqrt{2}Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\,\sqrt{2}Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(210\,\sqrt{2}Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+140\,\sqrt{2}Ba^{5}\mathrm{sgn}\left(c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -4/105*(105*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^5*sgn(cos(d*x
 + c)) - (210*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 140*sqrt(2)*B*a^5*sgn(cos(d
*x + c)) - (147*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 133*sqrt(2)*B*a^5*sgn(cos
(d*x + c)) - 2*(21*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 19*sqrt(2)*B*a^5*sgn(c
os(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.129 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{8a^2(5A+3B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(5A+3B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (8*a^2*(5*A + 3*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*B*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.140036, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4001, 3793, 3792}

$$\frac{8a^2(5A+3B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(5A+3B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*B*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{1}{5}(5A+3B)\int \text{se}$$
$$= \frac{2a(5A+3B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2B(a+a\,\text{s}}{}$$
$$= \frac{8a^2(5A+3B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a(5A+3B)\sqrt{a+a\sec(c+}}{15d}$$

Mathematica [A] time = 0.284577, size = 70, normalized size = 0.69

$$\frac{2a\sqrt{a(\sec(c+dx)+1)}((25A+18B)\sin(c+dx)+\tan(c+dx)(5A+3B\sec(c+dx)+9B))}{15d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Sec[c + d*x])]*((25*A + 18*B)*Sin[c + d*x] + (5*A + 9*B +
3*B*Sec[c + d*x])*Tan[c + d*x]))/(15*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.227, size = 95, normalized size = 0.9

$$-\frac{2\,a\left(-1+\cos(dx+c)\right)\left(25\,A\left(\cos(dx+c)\right)^2+18\,B\left(\cos(dx+c)\right)^2+5\,A\cos(dx+c)+9\,B\cos(dx+c)+3\,B\right)}{15\,d\left(\cos(dx+c)\right)^2\sin(dx+c)}\sqrt{\frac{a\,(\text{c}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(25*A*cos(d*x+c)^2+18*B*cos(d*x+c)^2+5*A*cos(d*x+
c)+9*B*cos(d*x+c)+3*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin
(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.468321, size = 225, normalized size = 2.23

$$\frac{2\left((25\,A+18\,B)a\cos(dx+c)^2+(5\,A+9\,B)a\cos(dx+c)+3\,Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/15*((25*A + 18*B)*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + 3*B*a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*
cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}(A+B\sec(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))*sec(c + d*x), x
)
```

Giac [A] time = 5.31794, size = 238, normalized size = 2.36

$$\frac{4\left(15\sqrt{2}Aa^4\mathrm{sgn}(\cos(dx+c))+15\sqrt{2}Ba^4\mathrm{sgn}(\cos(dx+c))-\left(25\sqrt{2}Aa^4\mathrm{sgn}(\cos(dx+c))+15\sqrt{2}Ba^4\mathrm{sgn}(\cos(dx+c\right.\right.}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] 4/15*(15*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 15*sqrt(2)*B*a^4*sgn(cos(d*x + c
)) - (25*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 15*sqrt(2)*B*a^4*sgn(cos(d*x + c
)) - 2*(5*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 3*sqrt(2)*B*a^4*sgn(cos(d*x + c
)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((
a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.130 $\int (a + a\sec(c + dx))^{3/2}(A + B\sec(c + dx))\,dx$

Optimal. Leaf size=105

$$\frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(3*A + 4*B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.146275, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(3*A + 4*B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int\sqrt{a+a\sec(c+dx)}\left(\frac{3aA}{2}+\right.$$

$$=\frac{2aB\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+(aA)\int\sqrt{a+a\sec(c+dx)}\,dx+$$

$$=\frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}-$$

$$=\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a}}{}$$

Mathematica [A] time = 0.571275, size = 102, normalized size = 0.97

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((3A+5B)\cos(c+dx)+B)+3\sqrt{2}A\sin^{-1}\left(\sqrt{2}\sin\left(\right.\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*A*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(B + (3*A + 5*B)*Cos[
c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] time = 0.25, size = 237, normalized size = 2.3

$$\frac{a}{6\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\,A\sin(dx+c)\cos(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/6/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*A*sin(d*x+c)*cos(d*x+c)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+3*A*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-12*A*cos(d*x+c)^2-20*B*cos(d*x+c)^2+12*
A*cos(d*x+c)+16*B*cos(d*x+c)+4*B)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 1.95186, size = 1347, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*d)
```

Fricas [A] time = 0.52786, size = 814, normalized size = 7.75

$$\left[\frac{3\left(Aa\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,((3\,A+\cdots}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(A*a*cos(d*x + c)^2 + A*a*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((3*A + 5*B)*a*cos(
d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2 + d*cos(d*x + c)), -2/3*(3*(A*a*cos(d*x + c)^2 + A*a*cos(d*x +
 c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c))) - ((3*A + 5*B)*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}(A+B\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.131 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.241394, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4018, 4015, 3774, 203}

$$\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}+2\int \cos(c+dx)\sqrt{a+a}$$

$$= \frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}$$

$$= \frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}$$

$$= \frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2(A-2B)\sin(c+d}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.422956, size = 97, normalized size = 0.94

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(3A+2B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(A\cos(}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(3*A + 2*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*B + A*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.278, size = 212, normalized size = 2.1

$$-\frac{a}{2\,d\sin(dx+c)}\left(3\,A\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,B\sqrt{2}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/2/d*a*(3*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+
2*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*
x+c)^2-2*A*cos(d*x+c)+4*B*cos(d*x+c)-4*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)
```

Maxima [B] time = 2.30279, size = 2431, normalized size = 23.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A] time = 0.616499, size = 755, normalized size = 7.33

$$\left[\frac{((3A+2B)a\cos(dx+c)+(3A+2B)a)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2(Aa\cos\ldots}{2(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/2*(((3*A + 2*B)*a*cos(d*x + c) + (3*A + 2*B)*a)*sqrt(-a)*log((2*a*cos(d*
x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*
sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(A*a*cos(d*x + c
) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d), -(((3*A + 2*B)*a*cos(d*x + c) + (3*A + 2*B)*a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - (A*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.77829, size = 544, normalized size = 5.28

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}Ba^2\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+\left(3A\sqrt{-a}a\,\mathrm{sgn}(\cos(dx+c))+2B\sqrt{-a}a\,\mathrm{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{\ldots}\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a^2*sgn(cos(d*x + c))
*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (3*A*sqrt(-a)*a*sgn(
cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))
) - (3*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)))*l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
```

```
)^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) - A*
sqrt(-a)*a^3*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.132 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7A+12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(5*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a*A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.272987, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4017, 4015, 3774, 203}

$$\frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7A+12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(5*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a*A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}+\frac{1}{2}\int \text{co}$$

$$= \frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sqrt{a+a\sec(c}}{2d}$$

$$= \frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sqrt{a+a\sec(c}}{2d}$$

$$= \frac{a^{3/2}(7A+12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^2(5A+4B)\sin}{4d\sqrt{a+a\sec(}}$$

Mathematica [A] time = 0.597141, size = 111, normalized size = 0.93

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(7A+12B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\text{co}}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*
(7*A + 12*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(7*A +
 4*B + 2*A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.296, size = 399, normalized size = 3.4

$$\frac{a}{16\,d\cos(dx+c)\sin(dx+c)}\left(7\,A\sin(dx+c)\cos(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/16/d*a*(7*A*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*2^(1/2)+12*B*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*2^(1/2)+7*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+
c)+12*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*A*co
s(d*x+c)^4-20*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3+28*A*cos(d*x+c)^2+16*B*cos(d
*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.616904, size = 833, normalized size = 7.

$$\left[\frac{((7A+12B)a\cos(dx+c)+(7A+12B)a)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2(2A}{8(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/8*(((7*A + 12*B)*a*cos(d*x + c) + (7*A + 12*B)*a)*sqrt(-a)*log((2*a*cos(
d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a*cos(d*x
 + c)^2 + (7*A + 4*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((7*A + 12*B)*a*cos(d*x + c) +
 (7*A + 12*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a*cos(d*x + c)^2 + (7*A + 4*B)*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.10897, size = 863, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/8*((7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*B*sqrt(-a)*a*sgn(cos(d*x + c))
)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3))) - (7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*B*sqr
t(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a
)*a^2*sgn(cos(d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 95*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3
*sgn(cos(d*x + c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 53*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4*sgn(
cos(d*x + c)) + 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 5*A*sqrt(-a)*a^5*sgn(cos
(d*x + c)) - 4*B*sqrt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.133 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=164

$$\frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11A+14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(7A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+$$

```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(11*A + 14*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(7*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.365126, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11A+14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(7A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(11*A + 14*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(7*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\int c$$

$$= \frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos^2(c+dx)}{}$$

$$= \frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(7A+6B)\cos(c+dx)\mathrm{s}}{12d\sqrt{a+a\sec(c+}}$$

$$= \frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(7A+6B)\cos(c+dx)\mathrm{s}}{12d\sqrt{a+a\sec(c+}}$$

$$= \frac{a^{3/2}(11A+14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^2(11A+14B)}{8d\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 0.997973, size = 137, normalized size = 0.84

$$\frac{a\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sin(c+dx)\sqrt{1-\sec(c+dx)}(2(11A+6B)\cos(c+dx)+4A\cos(2(c+dx))+37A+\right.}{24d(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((37*A + 42*B + 2*(11*A + 6*B)*C
os[c + d*x] + 4*A*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 3
*(11*A + 14*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x]))/(24*d*(1 + Co
s[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.365, size = 581, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/192/d*a*(33*A*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
```

```
d*x+c))*2^(1/2)+42*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*2^(1/2)+66*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*2^(1/2)+84*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))*2^(1/2)+33*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*sin(d*x+c)+42*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d
*x+c)+64*A*cos(d*x+c)^6+112*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+88*A*cos(d*x+c
)^4+240*B*cos(d*x+c)^4-264*A*cos(d*x+c)^3-336*B*cos(d*x+c)^3)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.627092, size = 949, normalized size = 5.79

$$\left[\frac{3\,((11\,A+14\,B)a\cos{(dx+c)}+(11\,A+14\,B)a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(}{48\,(d\cos{(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((11*A + 14*B)*a*cos(d*x + c) + (11*A + 14*B)*a)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*a*co
s(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A + 14*B)*a*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
 + d), -1/24*(3*((11*A + 14*B)*a*cos(d*x + c) + (11*A + 14*B)*a)*sqrt(a)*ar
ctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - (8*A*a*cos(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A
+ 14*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.22666, size = 1166, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 14*B*sqrt(-a)*a*sgn(cos(d*x +
 c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 1
4*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3
3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*
A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 42*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 303*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*s
qrt(-a)*a^3*sgn(cos(d*x + c)) - 822*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqr
t(-a)*a^4*sgn(cos(d*x + c)) + 3780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt
(-a)*a^5*sgn(cos(d*x + c)) - 2508*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 309*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-
a)*a^6*sgn(cos(d*x + c)) + 498*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*A*sqrt(-a
)*a^7*sgn(cos(d*x + c)) - 30*B*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.134 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=209

$$\frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75A+88B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(9A+8B)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(75A+\cdots}{9\cdots}$$

```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a^2*(75*A + 88*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (a^2*(75*A + 88*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(9*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*S
ec[c + d*x]]) + (a*A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(4*d)
```

Rubi [A] time = 0.448674, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75A+88B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(9A+8B)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(75A+\cdots}{9\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a^2*(75*A + 88*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (a^2*(75*A + 88*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(9*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*S
ec[c + d*x]]) + (a*A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(4*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}\int \mathrm{c}$$

$$= \frac{a^2(9A+8B)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos^3(c+dx}{}$$

$$= \frac{a^2(75A+88B)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(9A+8B)\,\mathrm{c}}{24d\sqrt{a}}$$

$$= \frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(75A+88B)\cos(c+dx}{96d\sqrt{a+a\sec(c-}}$$

$$= \frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(75A+88B)\cos(c+dx}{96d\sqrt{a+a\sec(c-}}$$

$$= \frac{a^{3/2}(75A+88B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^2(75A+88B}{64d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 1.3009, size = 154, normalized size = 0.74

$$\frac{a\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sin(c+dx)\sqrt{1-\sec(c+dx)}(2(93A+88B)\cos(c+dx)+4(15A+8B)\cos(2(c+dx\right.}{192d(\cos(c+dx)+1)\sqrt{1-\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((285*A + 296*B + 2*(93*A + 88*B
)*Cos[c + d*x] + 4*(15*A + 8*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)])*S
qrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 3*(75*A + 88*B)*ArcTanh[Sqrt[1 - Sec[c
 + d*x]]]*Tan[c + d*x]))/(192*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.294, size = 763, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3072/d*a*(225*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)+264*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*2^(1/2)+675*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(7/2)*2^(1/2)+792*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+675*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+792*B*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+225*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(7/2)*2^(1/2)*sin(d*x+c)+264*B*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/
2)*2^(1/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-1152*A*cos(d*x+c)^7-1024*B*cos(d*x
+c)^7-480*A*cos(d*x+c)^6-1792*B*cos(d*x+c)^6-1200*A*cos(d*x+c)^5-1408*B*cos
(d*x+c)^5+3600*A*cos(d*x+c)^4+4224*B*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*
x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.717182, size = 1049, normalized size = 5.02

$$\left[\frac{3\left((75A+88B)a\cos(dx+c)+(75A+88B)a\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/384*(3*((75*A + 88*B)*a*cos(d*x + c) + (75*A + 88*B)*a)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*a*
cos(d*x + c)^4 + 8*(15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d*
```

```
x + c)^2 + 3*(75*A + 88*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((75*A + 88*B)*a*cos(
d*x + c) + (75*A + 88*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*a*cos(d*x + c)^4 + 8*(
15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d*x + c)^2 + 3*(75*A +
 88*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.80016, size = 1469, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/384*(3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*B*sqrt(-a)*a*sgn(cos(d*x
+ c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) +
88*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(
225*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
4*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 264*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 6
261*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 4008*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) +
35925*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^10*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 33960*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^4*sgn(cos(d*x + c))
 - 127449*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 131784*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^5*sgn(cos(d*x +
c)) + 101667*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 108312*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^6*sgn(cos(d*x
 + c)) - 26079*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 29432*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^7*sgn(cos(d*
x + c)) + 3303*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 3384*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^8*sgn(cos(d*x
 + c)) - 147*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 152*B*sqrt(-a)*a^9*sgn(cos(
```

```
d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*a + a^2)^4)/d
```

3.135 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{2a^3(209A+194B)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(11A+14B)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2a^3(803}{495d}$$

```
[Out] (2*a^3*(803*A + 710*B)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(1155*d) + (2*a*B*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(11*d)
```

Rubi [A] time = 0.656926, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4018, 4016, 3800, 4001, 3792}

$$\frac{2a^3(209A+194B)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(11A+14B)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2a^3(803}{495d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^3*(803*A + 710*B)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(1155*d) + (2*a*B*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(11*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{11d}+\frac{2}{11}\int$$

$$= \frac{2a^2(11A+14B)\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx}{99d}$$

$$= \frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(11A+1}{}$$

$$= \frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(11A+1}{}$$

$$= \frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}-\frac{4a^2(803A+}{}$$

$$= \frac{2a^3(803A+710B)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(209A+194B)\sec^3(c}{693d\sqrt{a+a\sec}}$$

Mathematica [B] time = 6.16646, size = 487, normalized size = 2.05

$$\frac{2A\tan(c+dx)\sec^3(c+dx)(a(\sec(c+dx)+1))^{5/2}}{9d(\sec(c+dx)+1)^2}+\frac{38A\tan(c+dx)\sec^3(c+dx)(a(\sec(c+dx)+1))^{5/2}}{63d(\sec(c+dx)+1)^3}+\frac{146A\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (1168*A*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(315*d*(1 + Sec[c + d*x]
)^3) + (2272*B*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(693*d*(1 + Sec[c
 + d*x])^3) + (584*A*Sec[c + d*x]*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]
)/(315*d*(1 + Sec[c + d*x])^3) + (1136*B*Sec[c + d*x]*(a*(1 + Sec[c + d*x])
)^(5/2)*Tan[c + d*x])/(693*d*(1 + Sec[c + d*x])^3) + (146*A*Sec[c + d*x]^2*
(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(105*d*(1 + Sec[c + d*x])^3) + (
284*B*Sec[c + d*x]^2*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(231*d*(1 +
```

```
Sec[c + d*x])^3) + (38*A*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c
 + d*x])/(63*d*(1 + Sec[c + d*x])^3) + (710*B*Sec[c + d*x]^3*(a*(1 + Sec[c
+ d*x]))^(5/2)*Tan[c + d*x])/(693*d*(1 + Sec[c + d*x])^3) + (46*B*Sec[c + d
*x]^4*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(99*d*(1 + Sec[c + d*x])^3
) + (2*A*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(9*d*(1
+ Sec[c + d*x])^2) + (2*B*Sec[c + d*x]^4*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c
 + d*x])/(11*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.276, size = 163, normalized size = 0.7

$$-\frac{2a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(6424A\left(\cos\left(dx+c\right)\right)^{5}+5680B\left(\cos\left(dx+c\right)\right)^{5}+3212A\left(\cos\left(dx+c\right)\right)^{4}+2840B\left(\cos\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(6424*A*cos(d*x+c)^5+5680*B*cos(d*x+c)^5+3212
*A*cos(d*x+c)^4+2840*B*cos(d*x+c)^4+2409*A*cos(d*x+c)^3+2130*B*cos(d*x+c)^3
+1430*A*cos(d*x+c)^2+1775*B*cos(d*x+c)^2+385*A*cos(d*x+c)+1120*B*cos(d*x+c)
+315*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.49293, size = 409, normalized size = 1.73

$$\frac{2\left(8\left(803A+710B\right)a^{2}\cos\left(dx+c\right)^{5}+4\left(803A+710B\right)a^{2}\cos\left(dx+c\right)^{4}+3\left(803A+710B\right)a^{2}\cos\left(dx+c\right)^{3}+5\left(286\right.\right.}{3465\left(d\cos\left(dx+c\right)^{6}+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(8*(803*A + 710*B)*a^2*cos(d*x + c)^5 + 4*(803*A + 710*B)*a^2*cos(d*
x + c)^4 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^3 + 5*(286*A + 355*B)*a^2*cos
(d*x + c)^2 + 35*(11*A + 32*B)*a^2*cos(d*x + c) + 315*B*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.7378, size = 424, normalized size = 1.79

$$-\frac{8\left(3465\sqrt{2}Aa^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+3465\sqrt{2}Ba^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(10395\sqrt{2}Aa^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+8085\sqrt{2}Ba^{8}\mathrm{sgn}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -8/3465*(3465*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 3465*sqrt(2)*B*a^8*sgn(cos(
d*x + c)) - (10395*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 8085*sqrt(2)*B*a^8*sgn
(cos(d*x + c)) - (15939*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 15015*sqrt(2)*B*a
^8*sgn(cos(d*x + c)) - (14157*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 12375*sqrt(
2)*B*a^8*sgn(cos(d*x + c)) - 4*(1573*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 1375
*sqrt(2)*B*a^8*sgn(cos(d*x + c)) - 2*(143*sqrt(2)*A*a^8*sgn(cos(d*x + c)) +
 125*sqrt(2)*B*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*d)
```

3.136 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{16a^2(15A+13B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(9A-2B)\tan(c+dx)(a\sec(c+}{63d}$$

```
[Out] (64*a^3*(15*A + 13*B)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*B*(a + a*Sec[c + d*x
])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.352722, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4010, 4001, 3793, 3792}

$$\frac{16a^2(15A+13B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(9A-2B)\tan(c+dx)(a\sec(c+}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (64*a^3*(15*A + 13*B)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*B*(a + a*Sec[c + d*x
])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
```

```
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad} + \frac{2\int \sec(c+dx)(a+\ldots}{\ldots}$$
$$= \frac{2(9A-2B)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d} + \frac{2B(a+a\,\mathrm{se}\ldots}{\ldots}$$
$$= \frac{2a(15A+13B)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d} + \frac{2(9A-\ldots}{\ldots}$$
$$= \frac{16a^2(15A+13B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{2a(15A\ldots}{\ldots}$$
$$= \frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(15A+13B)\sqrt{a+a\,\mathrm{se}\ldots}}{315d\ldots}$$

Mathematica [A] time = 0.642996, size = 96, normalized size = 0.55

$$\frac{2a^3\tan(c+dx)\left(5(9A+26B)\sec^3(c+dx)+3(60A+73B)\sec^2(c+dx)+(345A+292B)\sec(c+dx)+690A+35B\sec\ldots\right.}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^3*(690*A + 584*B + (345*A + 292*B)*Sec[c + d*x] + 3*(60*A + 73*B)*Sec[
c + d*x]^2 + 5*(9*A + 26*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c + d
*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.254, size = 141, normalized size = 0.8

$$-\frac{2\,a^2\left(-1+\cos(dx+c)\right)\left(690\,A\left(\cos(dx+c)\right)^4+584\,B\left(\cos(dx+c)\right)^4+345\,A\left(\cos(dx+c)\right)^3+292\,B\left(\cos(dx+c)\right)^3+\ldots\right.}{315\,d\left(\cos(dx+c)\right)^4\sin(dx+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(690*A*cos(d*x+c)^4+584*B*cos(d*x+c)^4+345*A*c
os(d*x+c)^3+292*B*cos(d*x+c)^3+180*A*cos(d*x+c)^2+219*B*cos(d*x+c)^2+45*A*c
os(d*x+c)+130*B*cos(d*x+c)+35*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*
x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.486768, size = 346, normalized size = 1.98

$$\frac{2\left(2\,(345\,A+292\,B)a^2\cos\left(dx+c\right)^4+(345\,A+292\,B)a^2\cos\left(dx+c\right)^3+3\,(60\,A+73\,B)a^2\cos\left(dx+c\right)^2+5\,(9\,A+26\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(2*(345*A + 292*B)*a^2*cos(d*x + c)^4 + (345*A + 292*B)*a^2*cos(d*x +
 c)^3 + 3*(60*A + 73*B)*a^2*cos(d*x + c)^2 + 5*(9*A + 26*B)*a^2*cos(d*x + c
) + 35*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.28357, size = 362, normalized size = 2.07

$$\frac{8\left(315\,\sqrt{2}Aa^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+315\,\sqrt{2}Ba^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(840\,\sqrt{2}Aa^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+630\,\sqrt{2}Ba^7\mathrm{sgn}\left(\mathrm{co}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 8/315*(315*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^7*sgn(cos(d*x
+ c)) - (840*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 630*sqrt(2)*B*a^7*sgn(cos(d*
x + c)) - (945*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 819*sqrt(2)*B*a^7*sgn(cos(
d*x + c)) - 4*(135*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 117*sqrt(2)*B*a^7*sgn(
cos(d*x + c)) - 2*(15*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 13*sqrt(2)*B*a^7*sg
n(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*
```

```
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x
+ 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.137 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{64a^3(7A+5B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(7A+5B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7A+5B)\tan(c+dx)(a\sec(c+dx}{35d}$$

```
[Out] (64*a^3*(7*A + 5*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a + a*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.183963, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4001, 3793, 3792}

$$\frac{64a^3(7A+5B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(7A+5B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7A+5B)\tan(c+dx)(a\sec(c+dx}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a + a*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{1}{7}(7A+5B)\int \sec(c$$
$$= \frac{2a(7A+5B)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2B(a+a\,\mathrm{se}}{}$$
$$= \frac{16a^2(7A+5B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} + \frac{2a(7A+5}{}$$
$$= \frac{64a^3(7A+5B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(7A+5B)\sqrt{a+a\sec(c+}}{105d}$$

Mathematica [A] time = 0.47565, size = 89, normalized size = 0.64

$$\frac{2a^2\sqrt{a(\sec(c+dx)+1)}\left((301A+230B)\sin(c+dx)+\tan(c+dx)\left(3(7A+20B)\sec(c+dx)+98A+15B\sec^2(c+dx)+\right.\right.}{105d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*Sqrt[a*(1 + Sec[c + d*x])]*((301*A + 230*B)*Sin[c + d*x] + (98*A + 1
15*B + 3*(7*A + 20*B)*Sec[c + d*x] + 15*B*Sec[c + d*x]^2)*Tan[c + d*x]))/(1
05*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.231, size = 119, normalized size = 0.9

$$-\frac{2a^2(-1+\cos(dx+c))\left(301A(\cos(dx+c))^3+230B(\cos(dx+c))^3+98A(\cos(dx+c))^2+115B(\cos(dx+c))^2+21\right.}{105d(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(301*A*cos(d*x+c)^3+230*B*cos(d*x+c)^3+98*A*co
s(d*x+c)^2+115*B*cos(d*x+c)^2+21*A*cos(d*x+c)+60*B*cos(d*x+c)+15*B)*(a*(cos
(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.479549, size = 292, normalized size = 2.12

$$\frac{2\left((301\,A+230\,B)a^2\cos\left(dx+c\right)^3+(98\,A+115\,B)a^2\cos\left(dx+c\right)^2+3\,(7\,A+20\,B)a^2\cos\left(dx+c\right)+15\,Ba^2\right)\sqrt{\frac{a\cos(dx}{\cos(d}}}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/105*((301*A + 230*B)*a^2*cos(d*x + c)^3 + (98*A + 115*B)*a^2*cos(d*x + c)
^2 + 3*(7*A + 20*B)*a^2*cos(d*x + c) + 15*B*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.07358, size = 300, normalized size = 2.17

$$-\frac{8\left(105\,\sqrt{2}Aa^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\,\sqrt{2}Ba^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(245\,\sqrt{2}Aa^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+175\,\sqrt{2}Ba^6\mathrm{sgn}\,(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -8/105*(105*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^6*sgn(cos(d*x
 + c)) - (245*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 175*sqrt(2)*B*a^6*sgn(cos(d
*x + c)) - 4*(49*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 35*sqrt(2)*B*a^6*sgn(cos
(d*x + c)) - 2*(7*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 5*sqrt(2)*B*a^6*sgn(cos
(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.138 $\int (a + a\sec(c + dx))^{5/2}(A + B\sec(c + dx))\,dx$

Optimal. Leaf size=142

$$\frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(5A+8B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(\cdots}{}$$

```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(35*A + 32*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(5*A + 8*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*B*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.222714, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(5A+8B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(35*A + 32*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(5*A + 8*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*B*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2aB(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int (a+a\sec(c+dx))^{3/2}\Bigg(\frac{5}{}$$

$$=\frac{2a^2(5A+8B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2aB(a+a\sec(c+dx}{5d}$$

$$=\frac{2a^2(5A+8B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2aB(a+a\sec(c+dx}{5d}$$

$$=\frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+a\sec(c+dx)}\tan}{15d}$$

$$=\frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(5}{}$$

Mathematica [A] time = 1.04268, size = 128, normalized size = 0.9

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(5A+14B)\cos(c+dx)+(40A+43B)\cos(2(}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(30*Sqrt[2]
*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(40*A + 49*B + 2
*(5*A + 14*B)*Cos[c + d*x] + (40*A + 43*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/
2]))/(30*d)
```

Maple [B] time = 0.262, size = 341, normalized size = 2.4

$$-\frac{a^2}{60\,d\sin(dx+c)(\cos(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\Bigg(15\,A(\cos(dx+c))^2\sin(dx+c)\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{5/2}\mathrm{Arta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/60/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*A*cos(d*x+c)^2*sin(d*x+
c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+30*A*cos(d*x+c)*sin(d*
x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+15*A*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+320*A*cos(d*x+c)^3+344*B*cos(
d*x+c)^3-280*A*cos(d*x+c)^2-232*B*cos(d*x+c)^2-40*A*cos(d*x+c)-88*B*cos(d*x
```

```
+c)-24*B)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [B] time = 2.08581, size = 1885, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 0.540248, size = 954, normalized size = 6.72

$$\left[\frac{15\left(Aa^2\cos\left(dx+c\right)^3+Aa^2\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left((4\right.}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(A*a^2*cos(d*x + c)^3 + A*a^2*cos(d*x + c)^2)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((40*A + 43
*B)*a^2*cos(d*x + c)^2 + (5*A + 14*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), -2/15*(15*(A*a^2*cos(d*x + c)^3 + A*a^2*cos(d*x + c)^2)*sqrt(a)*ar
ctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - ((40*A + 43*B)*a^2*cos(d*x + c)^2 + (5*A + 14*B)*a^2*cos(d*x + c)
+ 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.139 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(A+2B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\,\mathrm{s}}{}$$

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d - (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(A + 2*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*B*(a + a*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.410515, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4018, 4015, 3774, 203}

$$-\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(A+2B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d - (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(A + 2*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*B*(a + a*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{2}{3}\int \cos(c+dx)($$

$$= \frac{2a^2(A+2B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}+\frac{2aB(a+a}{}$$

$$= -\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(A+2B)\sqrt{a+a\sec(c}}{d}$$

$$= -\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(A+2B)\sqrt{a+a\sec(c}}{d}$$

$$= \frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{a^3(3A+14B)\sin(}{3d\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.836745, size = 126, normalized size = 0.88

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(5A+2B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+\sin\left(\frac{1}{2}(c\right.\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(5
*A + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (3*A + 4*B
+ 4*(3*A + 8*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*
d)
```

Maple [B] time = 0.292, size = 256, normalized size = 1.8

$$-\frac{a^2}{6d\cos(dx+c)\sin(dx+c)}\left(15A\sqrt{2}\sin(dx+c)\cos(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/6/d*a^2*(15*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))+6*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))+6*A*cos(d*x+c)^3+6*A*cos(d*x+c)^2+32*B*cos(d*x+c)^2-12*A*cos
(d*x+c)-28*B*cos(d*x+c)-4*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)
/sin(d*x+c)
```

Maxima [B] time = 2.52101, size = 3753, normalized size = 26.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/12*(3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x +
2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x
 + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) +
 4*a^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c
) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c)
 - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*
a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)
^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
 + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*c
os(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(
(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) +
 a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
```

```
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.62804, size = 968, normalized size = 6.77

$$\left[\frac{3\left((5A+2B)a^2\cos(dx+c)^2+(5A+2B)a^2\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos}{\cos(dx+c)+1}\right.}{6\left(d\cos(dx+c)^2+d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/6*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqr
t(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
 + 2*(3*A*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 2*B*a^2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*c
os(d*x + c)), -1/3*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos
(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c))) - (3*A*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*c
os(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.15887, size = 644, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/6*(3*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a^2*sgn(cos(d*x
+ c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) +
 2*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 12*sqrt(2
)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - A*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2) + 4*(3*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 9*sqrt(2)*B*a^4*sgn(cos(d*x +
c)) - (3*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 7*sqrt(2)*B*a^4*sgn(cos(d*x + c)
))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.140 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=154

$$\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-4B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19A+20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\,\mathrm{s}}{}$$

```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
- (a^2*(A - 4*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*A*Cos[c
+ d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.41877, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4017, 4018, 4015, 3774, 203}

$$\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-4B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19A+20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
- (a^2*(A - 4*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*A*Cos[c
+ d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d}+\frac{1}{2}\int \cos$$

$$= -\frac{a^2(A-4B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}+\frac{aA\cos(c+d}{}$$

$$= \frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}-\frac{a^2(A-4B)\sqrt{a+a\sec(c+dx)}\,\mathrm{si}}{2d}$$

$$= \frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}-\frac{a^2(A-4B)\sqrt{a+a\sec(c+dx)}\,\mathrm{si}}{2d}$$

$$= \frac{a^{5/2}(19A+20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^3(9A-4B)\sin(c}{4d\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.80738, size = 116, normalized size = 0.75

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(19A+20B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)((11\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(19*A + 20*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(A + 8*B + (11*A + 4*B
)*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.325, size = 410, normalized size = 2.7

$$\frac{a^2}{16\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(19\,A\sin(dx+c)\cos(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/16/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(19*A*sin(d*x+c)*cos(d*x+c)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+20*B*sin(d*x+c)*cos(d*x+c
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+19*A*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+20*B*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)^4-36*A*cos(d*x+c)^3-16*B*c
os(d*x+c)^3+44*A*cos(d*x+c)^2-16*B*cos(d*x+c)^2+32*B*cos(d*x+c))/cos(d*x+c)
/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.631955, size = 890, normalized size = 5.78

$$\left[\frac{\left((19\,A+20\,B)a^2\cos\left(dx+c\right)+(19\,A+20\,B)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{8\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/8*(((19*A + 20*B)*a^2*cos(d*x + c) + (19*A + 20*B)*a^2)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a^2
*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((19*A +
20*B)*a^2*cos(d*x + c) + (19*A + 20*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a^2*cos
(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.37288, size = 957, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/8*(16*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a^3*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (19*A*sqrt(-a)*a^2*
sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2
) + 3))) - (19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos
(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(19*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(
d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x
+ c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 89*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c))
 + 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 9*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) -
4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.141 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=164

$$\frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(25A+38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(3A+2B)\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+}}{4d}$$

```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*A + 54*B)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(3*A + 2*B)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
4*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.45647, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4017, 4015, 3774, 203}

$$\frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(25A+38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(3A+2B)\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*A + 54*B)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(3*A + 2*B)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
4*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{1}{3}\int \mathrm{co}$$

$$=\frac{a^2(3A+2B)\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}+$$

$$=\frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(3A+2B)\cos(c+dx)\sqrt{a+}}{4d}$$

$$=\frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(3A+2B)\cos(c+dx)\sqrt{a+}}{4d}$$

$$=\frac{a^{5/2}(25A+38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^3(49A+54B)\,\mathrm{si}}{24d\sqrt{a+a\sec(}}$$

Mathematica [C] time = 1.04461, size = 312, normalized size = 1.9

$$-\frac{a^2\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-192A\tan(c+dx)\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] -(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(-165*A*Sqrt[1 - Sec[c + d*x]
]*Sin[c + d*x] + 18*B*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 8*A*Cos[c + d*x
]^2*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] - 31*A*Sqrt[1 - Sec[c + d*x]]*Sin[2
*(c + d*x)] + 54*B*Sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)] - 165*A*ArcTanh[
Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 126*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]
]*Tan[c + d*x] - 576*B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]]*Tan[c + d*x] - 192*A*Hypergeometric2F1[1/2, 4, 3/2, 1 -
 Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]))/(72*d*(1 + Cos[c + d*x
])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.342, size = 583, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/192/d*a^2*(75*A*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*2^(1/2)+114*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
```

```
c)/cos(d*x+c))*2^(1/2)+150*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*2^(1/2)+228*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*2^(1/2)+75*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)+114*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*sin(d*x+c)+64*A*cos(d*x+c)^6+208*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+328*A*co
s(d*x+c)^4+432*B*cos(d*x+c)^4-600*A*cos(d*x+c)^3-528*B*cos(d*x+c)^3)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.635559, size = 976, normalized size = 5.95

$$\left[\frac{3\left((25\,A+38\,B)a^2\cos\left(dx+c\right)+(25\,A+38\,B)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{48\,(d\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((25*A + 38*B)*a^2*cos(d*x + c) + (25*A + 38*B)*a^2)*sqrt(-a)*log(
(2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*
a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x + c)^2 + 3*(25*A + 22*B)*a^
2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d), -1/24*(3*((25*A + 38*B)*a^2*cos(d*x + c) + (25*A + 38*B)*a
^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - (8*A*a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x
 + c)^2 + 3*(25*A + 22*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.64893, size = 1177, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(25*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 38*B*sqrt(-a)*a^2*sgn(cos(d
*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(25*A*sqrt(-a)*a^2*sgn(cos(d*x + c
)) + 38*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sq
rt(2)*(75*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^10*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 114*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^3*sgn(cos(d*x + c
)) - 1125*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1710*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^4*sgn(cos(d*x + c)
) + 6174*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 6804*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c))
 - 4314*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^4*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 4284*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^6*sgn(cos(d*x + c))
+ 807*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 858*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 4
9*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 54*B*sqrt(-a)*a^8*sgn(cos(d*x + c)))/(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
+ a^2)^3)/d
```

3.142 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=209

$$\frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163A+200B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(11A+8B)\sin(c+dx)\cos^2(c+dx)\sqrt{a}}{24d}$$

```
[Out] (a^(5/2)*(163*A + 200*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^3*(163*A + 200*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a^3*(95*A + 104*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*S
ec[c + d*x]]) + (a^2*(11*A + 8*B)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c
+ d*x])/(4*d)
```

Rubi [A] time = 0.580095, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163A+200B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(11A+8B)\sin(c+dx)\cos^2(c+dx)\sqrt{a}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(163*A + 200*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^3*(163*A + 200*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a^3*(95*A + 104*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*S
ec[c + d*x]]) + (a^2*(11*A + 8*B)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c
+ d*x])/(4*d)
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{1}{4}\int \text{co}$$

$$= \frac{a^2(11A+8B)\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{24d} +$$

$$= \frac{a^3(95A+104B)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(11A+8B)\text{co}}{}$$

$$= \frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(95A+104B)\cos(c+d}{96d\sqrt{a+a\sec(c+}}$$

$$= \frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(95A+104B)\cos(c+d}{96d\sqrt{a+a\sec(c+}}$$

$$= \frac{a^{5/2}(163A+200B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^3(163A+200}{64d\sqrt{a+a\text{s}}}$$

Mathematica [C] time = 1.292, size = 366, normalized size = 1.75

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4608A\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)+7680B\sqrt{1-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(6075*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 6600*B*ArcTanh[Sqrt[1 - Sec[
c + d*x]]] + 2079*A*Sqrt[1 - Sec[c + d*x]] + 1240*B*Sqrt[1 - Sec[c + d*x]]
+ 7641*A*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 6360*B*Cos[c + d*x]*Sqrt[1 -
 Sec[c + d*x]] + 2097*A*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1240*B*Co
s[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 522*A*Cos[3*(c + d*x)]*Sqrt[1 - Sec
[c + d*x]] - 80*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 18*A*Cos[4*(c +
 d*x)]*Sqrt[1 - Sec[c + d*x]] + 7680*B*Hypergeometric2F1[1/2, 4, 3/2, 1 - S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 4608*A*Hypergeometric2F1[1/2, 5, 3/2,
 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c
 + d*x])/(2880*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.285, size = 765, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3072/d*a^2*(489*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(7/2)*2^(1/2)+600*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)*2^(1/2)+1467*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)+1800*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1467*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1800*B*sin(d*x+c)*cos(d*x+c)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+489*A*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+600*B*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-2176*A*cos(d*x+c)^7-1024*B*c
os(d*x+c)^7-2272*A*cos(d*x+c)^6-3328*B*cos(d*x+c)^6-2608*A*cos(d*x+c)^5-524
8*B*cos(d*x+c)^5+7824*A*cos(d*x+c)^4+9600*B*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.719529, size = 1103, normalized size = 5.28

$$\left[\frac{3\left((163\,A + 200\,B)a^2\cos\left(dx + c\right) + (163\,A + 200\,B)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/384*(3*((163*A + 200*B)*a^2*cos(d*x + c) + (163*A + 200*B)*a^2)*sqrt(-a)
*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*
(48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 1
36*B)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*(
(163*A + 200*B)*a^2*cos(d*x + c) + (163*A + 200*B)*a^2)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
(48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 1
36*B)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.43719, size = 1480, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/384*(3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*B*sqrt(-a)*a^2*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x
 + c)) + 200*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) +
 4*sqrt(2)*(489*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^14*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 600*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^3*sgn(cos(d
*x + c)) - 10269*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^12*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 12600*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^4*sgn(co
s(d*x + c)) + 69885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^10*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 103992*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^5*sg
n(cos(d*x + c)) - 259233*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 339864*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^6
*sgn(cos(d*x + c)) + 209979*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 262920*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*
a^7*sgn(cos(d*x + c)) - 55511*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 73640*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)
```

```
*a^8*sgn(cos(d*x + c)) + 6687*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 8808*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*
a^9*sgn(cos(d*x + c)) - 299*A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 392*B*sqrt(
-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.143 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=254

$$\frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283A+326B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(13A+10B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec}}{40d}$$

```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*A + 326*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(283*A + 326*B)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^3*(157*A + 170*B)*Cos[c + d*x]^2*Sin[c + d*x])/(240*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(13*A + 10*B)*Cos[c + d*x]^3*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.653945, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4017, 4015, 3805, 3774, 203}

$$\frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283A+326B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(13A+10B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*A + 326*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(283*A + 326*B)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^3*(157*A + 170*B)*Cos[c + d*x]^2*Sin[c + d*x])/(240*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(13*A + 10*B)*Cos[c + d*x]^3*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\int$$

$$= \frac{a^2(13A+10B)\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+d}{40d}$$

$$= \frac{a^3(157A+170B)\cos^2(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(13A+1}{}$$

$$= \frac{a^3(283A+326B)\cos(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(157A+}{24}$$

$$= \frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(283A+326B)\cos(c}{192d\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(283A+326B)\cos(c}{192d\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{a^{5/2}(283A+326B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^3(283A+3}{128d\sqrt{a\,-}}$$

Mathematica [C] time = 1.78594, size = 416, normalized size = 1.64

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(15360A\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},6,\frac{3}{2},1-\sec(c+dx)\right)+21504B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(25935*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 28350*B*ArcTanh[Sqrt[1 - Se
c[c + d*x]]] + 11651*A*Sqrt[1 - Sec[c + d*x]] + 9702*B*Sqrt[1 - Sec[c + d*x
]] + 37029*A*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 35658*B*Cos[c + d*x]*Sqr
t[1 - Sec[c + d*x]] + 12653*A*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 978
6*B*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 3818*A*Cos[3*(c + d*x)]*Sqrt[
1 - Sec[c + d*x]] + 2436*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1002*A
```

```
*Cos[4*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 84*B*Cos[4*(c + d*x)]*Sqrt[1 - S
ec[c + d*x]] + 72*A*Cos[5*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 21504*B*Hyper
geometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 15360*
A*Hypergeometric2F1[1/2, 6, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*
Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(13440*d*(1 + Cos[c + d*x])*Sqrt[1
 - Sec[c + d*x]])
```

Maple [B] time = 0.323, size = 947, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/61440/d*a^2*(4245*A*cos(d*x+c)^4*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*2^(1/2)+4890*B*cos(d*x+c)^4*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*2^(1/2)+16980*A*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+19560*B*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+25470*A*cos(d*x+c)^2*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+29340*B*cos(d*x+c)^2*sin(d
*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+16980*A*cos(d*x+c)*
sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+19560*B*cos(d*
x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+4245*A*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+4890*B*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+12288*A*cos(d*x+c)^10+3
2256*A*cos(d*x+c)^9+15360*B*cos(d*x+c)^9+27904*A*cos(d*x+c)^8+43520*B*cos(d
*x+c)^8+18112*A*cos(d*x+c)^7+45440*B*cos(d*x+c)^7+45280*A*cos(d*x+c)^6+5216
0*B*cos(d*x+c)^6-135840*A*cos(d*x+c)^5-156480*B*cos(d*x+c)^5)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.737078, size = 1227, normalized size = 4.83

$$\left[\frac{15\left((283\,A+326\,B)a^2\cos\left(dx+c\right)+(283\,A+326\,B)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(\right.}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/3840*(15*((283*A + 326*B)*a^2*cos(d*x + c) + (283*A + 326*B)*a^2)*sqrt(-
a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) +
2*(384*A*a^2*cos(d*x + c)^5 + 48*(29*A + 10*B)*a^2*cos(d*x + c)^4 + 8*(283*
A + 230*B)*a^2*cos(d*x + c)^3 + 10*(283*A + 326*B)*a^2*cos(d*x + c)^2 + 15*
(283*A + 326*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c) + d), -1/1920*(15*((283*A + 326*B)*a^2*cos(d*x
 + c) + (283*A + 326*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (384*A*a^2*cos(d*x + c)^5 +
 48*(29*A + 10*B)*a^2*cos(d*x + c)^4 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^3
 + 10*(283*A + 326*B)*a^2*cos(d*x + c)^2 + 15*(283*A + 326*B)*a^2*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.60132, size = 1782, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/3840*(15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(
cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(283*A*sqrt(-a)*a^2*sgn(cos(
d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(4245*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^18*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 4890*(sqrt(-a)*tan(1/2
```

```
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*sqrt(-a)*a^3*sgn(
cos(d*x + c)) - 114615*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^16*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 132030*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*B*sqrt(-a)*a^4
*sgn(cos(d*x + c)) + 1298820*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 1319880*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(
-a)*a^5*sgn(cos(d*x + c)) - 6176700*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 688812
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*
B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 16394598*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^7*sgn(cos(d*x + c))
+ 18352620*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^10*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 14042770*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^8*sgn(cos(d*
x + c)) - 15746180*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 4791060*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^9*sgn(
cos(d*x + c)) + 5497320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 860300*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^10
*sgn(cos(d*x + c)) - 959320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^10*sgn(cos(d*x + c)) + 75885*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*
a^11*sgn(cos(d*x + c)) + 84810*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^11*sgn(cos(d*x + c)) - 2671*A*sqrt
(-a)*a^12*sgn(cos(d*x + c)) - 2990*B*sqrt(-a)*a^12*sgn(cos(d*x + c)))/((sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^
2)^5)/d
```

3.144 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=202

$$\frac{2(7A-B)\tan(c+dx)\sec^2(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(7A-31B)\tan(c+dx)\sqrt{a\sec(c+dx)+}}{105ad}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*
Sec[c + d*x]]) + (2*(7*A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*B*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.605622, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4021, 4010, 4001, 3795, 203}

$$\frac{2(7A-B)\tan(c+dx)\sec^2(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(7A-31B)\tan(c+dx)\sqrt{a\sec(c+dx)+}}{105ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*
Sec[c + d*x]]) + (2*(7*A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*B*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^3(c+dx)\left(3aB+\frac{1}{2}a(7A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{7a}$$

$$= \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{4\int \frac{\sec^2(c+\cdots}{\cdots}}{\cdots}$$

$$= \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{2(7A-31\cdots}{\cdots}$$

$$= \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+\cdots}{7d\sqrt{a+\cdots}}$$

$$= \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+\cdots}{7d\sqrt{a+\cdots}}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\cdots}{35d\cdots}$$

Mathematica [A] time = 0.52584, size = 140, normalized size = 0.69

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(3(7A-B)\sec^2(c+dx)+(31B-7A)\sec(c+dx)+91A+15B\sec^3(c+dx)-43B\right)-10\cdots\right)}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-105*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 -
 Sec[c + d*x]]*(91*A - 43*B + (-7*A + 31*B)*Sec[c + d*x] + 3*(7*A - B)*Sec[
c + d*x]^2 + 15*B*Sec[c + d*x]^3))*Tan[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*
x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.34, size = 785, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/840/d/a*(-105*A*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))+105*B*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))-315*A*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))+315*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
-315*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+315*
B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-105*A*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+105*B*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+1456*A*cos(d*x+c)^4-688*B*cos(d*x+c)^4-15
68*A*cos(d*x+c)^3+1184*B*cos(d*x+c)^3+448*A*cos(d*x+c)^2-544*B*cos(d*x+c)^2
-336*A*cos(d*x+c)+288*B*cos(d*x+c)-240*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.598308, size = 1116, normalized size = 5.52

$$\left[-\frac{105\sqrt{2}\left((A-B)a\cos\left(dx+c\right)^4+(A-B)a\cos\left(dx+c\right)^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\cdots\right.}{210\left(ad\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/210*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*
sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/
a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((91*A - 43*B)*cos(d*x + c)^3 - (7*A
```

```
- 31*B)*cos(d*x + c)^2 + 3*(7*A - B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)
^3), 1/105*(2*((91*A - 43*B)*cos(d*x + c)^3 - (7*A - 31*B)*cos(d*x + c)^2 +
 3*(7*A - B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c) + 105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c
)^3)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 9.47519, size = 387, normalized size = 1.92

$$-\frac{\frac{105\sqrt{2}(A-B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{2\left(\frac{105\sqrt{2}Aa^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \left(\left(\frac{\sqrt{2}(119Aa^3-92Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{7\sqrt{2}(37Aa^3-16Ba^3)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/105*(105*sqrt(2)*(A - B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
- 2*(105*sqrt(2)*A*a^3/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - ((sqrt(2)*(119*A*a
^3 - 92*B*a^3)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 7*s
qrt(2)*(37*A*a^3 - 16*B*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x +
 1/2*c)^2 + 35*sqrt(2)*(7*A*a^3 - 4*B*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 -
a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.145 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(5A-B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}-\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c}{5d\sqrt{a}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*B*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*(5*A - B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.419656, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4021, 4010, 4001, 3795, 203}

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(5A-B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}-\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c}{5d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*B*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*(5*A - B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
+ 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^2(c+dx)\left(2aB+\frac{1}{2}a(5A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad} + \frac{4\int$$

$$= -\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\,\mathrm{se}}}{15}$$

$$= -\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\,\mathrm{se}}}{15}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^2(c+dx}{5d\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 0.373977, size = 123, normalized size = 0.77

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((5A-B)\sec(c+dx)-5A+3B\sec^2(c+dx)+13B\right)+15\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx}}{\sqrt{2}}\right.\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((15*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - S
ec[c + d*x]]*(-5*A + 13*B + (5*A - B)*Sec[c + d*x] + 3*B*Sec[c + d*x]^2))*T
an[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.306, size = 595, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/60/d/a*(15*A*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)-15*B*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)+30*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-30*B
*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+15*A*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-15*B*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)*sin(d*x+c)+40*A*cos(d*x+c)^3-104*B*cos(d*x+c)^3-80*A*co
s(d*x+c)^2+112*B*cos(d*x+c)^2+40*A*cos(d*x+c)-32*B*cos(d*x+c)+24*B)*(a*(cos
(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.580259, size = 1019, normalized size = 6.41

$$\left[-\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\,c\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\cdots}{30\left(ad\cos(dx+c)^3+ad\cos\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/30*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*sq
rt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*
cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d*x + c)^2 - (5*A - B)
*cos(d*x + c) - 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/15*(2*((5*A - 13*B)*cos(d*x +
 c)^2 - (5*A - B)*cos(d*x + c) - 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c) + 15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x
+ c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 9.60731, size = 366, normalized size = 2.3

$$\frac{\frac{15\left(\sqrt{2}A-\sqrt{2}B\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\left(\left(10\sqrt{2}Aa^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-20\sqrt{2}Ba^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\left(10\sqrt{2}A\cdots\right.\right.\right.}{\cdots}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/15*(15*(sqrt(2)*A - sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - 2*((10*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 20*sqrt(2)*B*
a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (10*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1) - 17*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*B*a^2/sgn(tan(1/2*d*x + 1
/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.146 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*B*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.256772, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4010, 4001, 3795, 203}

$$-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*B*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} + \frac{2\int \frac{\sec(c+dx)\left(\frac{aB}{2}+\frac{1}{2}a(3A-2B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} + (-A+B)\int \frac{}{\sqrt{a}}$$

$$= \frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} + \frac{(2(A-B))\,\mathrm{Sub}}{}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sqrt{a+a\sec}}{}$$

Mathematica [A] time = 0.292916, size = 106, normalized size = 0.9

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}(3A+B\sec(c+dx)-B)-3\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{3d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-3*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - S
ec[c + d*x]]*(3*A - B + B*Sec[c + d*x]))*Tan[c + d*x])/(3*d*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.293, size = 405, normalized size = 3.4

$$-\frac{1}{6\,ad\sin(dx+c)\cos(dx+c)}\left(-3\,A\cos(dx+c)\sin(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\,\frac{\cos(dx}{\cos(dx+}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d/a*(-3*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))+3*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-3*A
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+3*B*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+12*A*cos(d*x+c)^2-4*B*cos(d*x+c)^2-12*A
*cos(d*x+c)+8*B*cos(d*x+c)-4*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x
+c)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.576071, size = 917, normalized size = 7.77

$$\left[-\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/6*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-
1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos
(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) - 4*((3*A - B)*cos(d*x + c) + B)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x +
 c)), 1/3*(2*((3*A - B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) + 3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d
*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 9.41653, size = 251, normalized size = 2.13

$$-\frac{\frac{3\sqrt{2}(A-B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(\frac{\sqrt{2}(3Aa-2Ba)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{3\sqrt{2}Aa}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/3*(3*sqrt(2)*(A - B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*
(sqrt(2)*(3*A*a - 2*B*a)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - 3*sqrt(2)*A*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.147 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (2*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.107447, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4001, 3795, 203}

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (2*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(A-B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{2B\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2(A-B))\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2B\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.165721, size = 88, normalized size = 1.13

$$\frac{\tan(c+dx)\left(\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2B\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*B*Sqrt[1 - Se
c[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [B] time = 0.24, size = 200, normalized size = 2.6

$$\frac{1}{ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+\cos(dx+c)-1\right)\right)\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-2*B*cos(d*x+c)+2*B)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.565507, size = 751, normalized size = 9.63

$$\left[-\frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4B\sqrt{}}{2(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
+ c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*
cos(d*x + c) + a*d), (2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a)
)/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 9.1963, size = 194, normalized size = 2.49

$$\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{(\sqrt{2}A-\sqrt{2}B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] (2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*tan(1/2*d*x + 1/2*c)/((a*t
an(1/2*d*x + 1/2*c)^2 - a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (sqrt(2)*A -
sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.148 $\int \frac{A+B\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.107178, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3920, 3774, 203, 3795}

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d)
```


Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{a} - (A-B)\int\frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= -\frac{(2A)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{(2(A-B))\,\text{Subst}\left(\int\frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+d}{\sqrt{a+a\sec(c}}\right.}{d}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.280624, size = 92, normalized size = 1.01

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-A)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}A\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (-A + B)*ArcTan[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [B] time = 0.236, size = 194, normalized size = 2.1

$$-\frac{1}{ad}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(A\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+\cos(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))-B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))+A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45452, size = 814, normalized size = 8.95

$$\left[-\frac{\sqrt{2}(A-B)a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+2A\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\cdots}}{\cdots}\right)\cdots}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(A - B)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 -
2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*A*sqrt(-a)*l
og((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d)
, (sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*A*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 11.2814, size = 302, normalized size = 3.32

$$-\frac{\frac{\sqrt{2}(A-B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2A\log\left(\left|\left(\sqrt{-\cdots}\right.\right.\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*A*lo
g(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*A*l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.149 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -(((A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x
]])
```

Rubi [A] time = 0.229464, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4022, 3920, 3774, 203, 3795}

$$-\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x
]])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-\frac{1}{2}a(A-2B)+\frac{1}{2}aA\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(A-2B)\int \sqrt{a+a\sec(c+dx)}\,dx}{2a} + (A-B)\int \frac{\sec(c}{\sqrt{a+a\,\text{se}}}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(A-2B)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{(2(A-B}{}$$

$$= -\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A\,\text{s}}{d\sqrt{a+}}$$

Mathematica [C] time = 26.457, size = 10104, normalized size = 84.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.299, size = 353, normalized size = 3.

$$\frac{1}{2\,ad\sin(dx+c)}\left(A\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\,\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c) - 2\,B\sqrt{2}\text{Artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*(A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-2*B
*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*B*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*A*cos(d*x+c))*(a*(cos(d*x+c)+1
)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 3.14261, size = 1214, normalized size = 10.2

$$\left[\frac{2\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(d}{\cos(dx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c)
 + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + ((A - 2*B)*cos(d*x + c) + A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2
 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d*cos(d*x + c) + a*d), (
A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + ((A -
 2*B)*cos(d*x + c) + A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B)*a*cos(d*x
 + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\cos(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 11.2844, size = 531, normalized size = 4.46

$$\frac{\frac{\sqrt{2}(A-B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{\sqrt{-a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{(A-2\,B)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{(A-2\,B)\log}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] 1/2*(sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (A - 2*
B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
(A - 2*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2*A*sqrt(-a) - A*sqrt(-a)*a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sgn(tan(1/2*d*x + 1/2*
c)^2 - 1)))/d
```

3.150 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{(7A-4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((7*A - 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*
Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]
])
```

Rubi [A] time = 0.368799, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4022, 3920, 3774, 203, 3795}

$$-\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{(7A-4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((7*A - 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*
Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]
])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}dx = \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{1}{2}a(A-4B)+\frac{3}{2}aA\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{2a}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\frac{1}{4}a^2(7A-4B)-\frac{1}{4}a^2(A-4B)\sec}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(7A-4B)\int\sqrt{a+a\sec(}}{8a}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(7A-4B)\,\mathrm{Subst}\left(\int\frac{1}{a+x^2}\right.}{4d}$$

$$= \frac{(7A-4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{(A-}{4d\sqrt{a}}$$

Mathematica [A] time = 0.41544, size = 135, normalized size = 0.82

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}(2A\cos(c+dx)-A+4B)+(7A-4B)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}(A-\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (((7*A - 4*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*(A - B)*ArcTanh[S
qrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-A + 4*B + 2*A*Cos[c + d*x])
*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(
1 + Sec[c + d*x])])
```

Maple [B] time = 0.374, size = 717, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d/a*(7*A*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
```

```
))*2^(1/2)-4*B*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*2^(1/2)+8*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))+7*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*B*c
os(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-4*B*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+8*A*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+co
s(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-8*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))*sin(d*x+c)-8*A*cos(d*x+c)^4+12*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3-4*A*c
os(d*x+c)^2+16*B*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+
c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 5.70298, size = 1323, normalized size = 8.02

$$\left[-\frac{4\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-(\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(4*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log(-(2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d
*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - ((7*A - 4*B)*cos(d*x + c) + 7*A - 4*B)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*A*cos(d*x
+ c)^2 - (A - 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(((7*A - 4*B)*cos(d*x + c) + 7*A
 - 4*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
```

```
/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c)^2 - (A - 4*B)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*((A - B)*a*c
os(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\cos^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [B] time = 11.5748, size = 876, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/8*(4*sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (7*A
 - 4*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)) - (7*A - 4*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)) + 4*sqrt(2)*(17*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a) - 57*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a + 76*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a
)*a + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*A*sqrt(-a)*a^2 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^2 - 3*A*sqrt(-a)*a^3 + 4*B*sqrt(-a)*a^3
)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4
 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*a + a^2)^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.151 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=206

$$\frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9A-14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-6B)\sin(c+d}{12d\sqrt{a\sec(c}$$

```
[Out] -((9*A - 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B)*Sin[c + d*x])/(8*d*Sqrt[a
+ a*Sec[c + d*x]]) - ((A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a
*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rubi [A] time = 0.554567, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9A-14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-6B)\sin(c+d}{12d\sqrt{a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((9*A - 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B)*Sin[c + d*x])/(8*d*Sqrt[a
+ a*Sec[c + d*x]]) - ((A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a
*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(-\frac{1}{2}a(A-6B)+\frac{5}{2}aA\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= -\frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(\frac{3}{4}\right.}{}}{}$$

$$= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin}{3d\sqrt{a+a\sec(c}}$$

$$= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin}{3d\sqrt{a+a\sec(c}}$$

$$= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin}{3d\sqrt{a+a\sec(c}}$$

$$= -\frac{(9A-14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{(7A}{8d}$$

Mathematica [A] time = 0.675438, size = 150, normalized size = 0.73

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left(-2(A-6B)\cos(c+dx)+8A\cos^2(c+dx)+21A-6B\right)+(42B-27A)\tanh^{-}\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (((-27*A + 42*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 24*Sqrt[2]*(A - B)*ArcTa
nh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(21*A - 6*B - 2*(A - 6*B)
*Cos[c + d*x] + 8*A*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(
24*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.312, size = 1067, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/192/d/a*(27*A*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*2^(1/2)-42*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*2^(1/2)+54*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*2^(1/2)+48*A*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(5/2)-84*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*2^(1/2)-48*B*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)+27*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d
*x+c)+96*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-
42*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-96*B*sin(
d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+48*A*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-48*B*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)*sin(d*x+c)-64*A*cos(d*x+c)^6+80*A*cos(d*x+c)^5-96*B*cos(d*x+
c)^5-184*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+168*A*cos(d*x+c)^3-48*B*cos(d*x+
c)^3)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 5.72163, size = 1426, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/48*(24*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 3*((9*A - 14*B)*cos(d*x + c) + 9*A - 14*B)*sqrt(-a)*log((2*
```

```
a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(8*A*cos
(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B)*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d),
 1/24*(3*((9*A - 14*B)*cos(d*x + c) + 9*A - 14*B)*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (8*A*c
os(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 24*sqrt(2)*((A - B)*a*
cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.9008, size = 1142, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/48*(24*sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*(
9*A - 14*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)) - 3*(9*A - 14*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(165*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a) - 102*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a) - 1323*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*
a + 954*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*B*sqrt(-a)*a + 3906*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2 - 2268*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^2 - 2118*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3
+ 1044*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^4*B*sqrt(-a)*a^3 + 393*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4 - 222*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^4 - 31*A*sqrt(-a)*a^5
+ 18*B*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*a + a^2)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.152 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(11A-15B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(35A-39B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((11*A - 15*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((65*A - 93*B)*Tan[c + d*x])/(15*a*d*Sqrt[a +
a*Sec[c + d*x]]) - ((5*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
 + a*Sec[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])
/(30*a^2*d)
```

Rubi [A] time = 0.632908, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 4021, 4010, 4001, 3795, 203}

$$\frac{(11A-15B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(35A-39B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 15*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((65*A - 93*B)*Tan[c + d*x])/(15*a*d*Sqrt[a +
a*Sec[c + d*x]]) - ((5*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
 + a*Sec[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])
/(30*a^2*d)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^3(c+dx)\left(3a(A-B)-\frac{1}{2}a(5A-9B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + \cdots$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + \frac{(35A \cdots}{\cdots}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} - \frac{(5A-9B)\sec^2(\cdots}{10ad\sqrt{a+\cdots}}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} - \frac{(5A-9B)\sec^2(\cdots}{10ad\sqrt{a+\cdots}}$$

$$= \frac{(11A-15B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65\cdots}{15a\cdots}$$

Mathematica [A] time = 2.37172, size = 160, normalized size = 0.74

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(4(5A-3B)\sec^2(c+dx)-12(5A-9B)\sec(c+dx)-95A+12B\sec^3(c+dx)+147B\right)+\cdots\right.}{30d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((15*Sqrt[2]*(11*A - 15*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
 d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-95*A + 147*B - 12*(5*A -
 9*B)*Sec[c + d*x] + 4*(5*A - 3*B)*Sec[c + d*x]^2 + 12*B*Sec[c + d*x]^3))*T
an[c + d*x])/(30*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.299, size = 793, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/240/d/a^2*(-1+cos(d*x+c))*(165*A*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(5/2)-225*B*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)+495*A*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(5/2)-675*B*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(5/2)+495*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(5/2)-675*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)+165*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-225*B*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+760*A*cos(d*x+c)^4-1176*
B*cos(d*x+c)^4-280*A*cos(d*x+c)^3+312*B*cos(d*x+c)^3-640*A*cos(d*x+c)^2+960
*B*cos(d*x+c)^2+160*A*cos(d*x+c)-192*B*cos(d*x+c)+96*B)*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.610318, size = 1315, normalized size = 6.09

$$\left[\frac{15\sqrt{2}\left((11A-15B)\cos(dx+c)^4+2(11A-15B)\cos(dx+c)^3+(11A-15B)\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(d\ldots}{\cos(d\ldots}}\ldots}{\ldots}\right)\ldots}{120\,(a\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^4 + 2*(11*A - 15*B)*cos(d*x
+ c)^3 + (11*A - 15*B)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d
*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
4*((95*A - 147*B)*cos(d*x + c)^3 + 12*(5*A - 9*B)*cos(d*x + c)^2 - 4*(5*A -
 3*B)*cos(d*x + c) - 12*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2
), -1/60*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^4 + 2*(11*A - 15*B)*cos(d*
x + c)^3 + (11*A - 15*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((95*
A - 147*B)*cos(d*x + c)^3 + 12*(5*A - 9*B)*cos(d*x + c)^2 - 4*(5*A - 3*B)*c
os(d*x + c) - 12*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.68822, size = 421, normalized size = 1.95

$$\frac{\frac{15\sqrt{2}(11A-15B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\left(\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}(245Aa^3-381Ba^3)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{5\ldots}{a^2\mathrm{sg}\ldots}\ldots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{\ldots}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/60*(15*sqrt(2)*(11*A - 15*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - (((15*sqrt(2)*(A*a^3 - B*a^3)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(245*A*a^3 - 381*B*a^3)/(a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*(73*A*a^3 - 105*B*a
^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 15*sqrt
(2)*(9*A*a^3 - 17*B*a^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x
 + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)))/d
```

3.153 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(7A-11B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-7B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+ \ldots$$

```
[Out] -((7*A - 11*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) + ((9*A - 13*B)*Tan[c + d*x])/(3*a*d*Sqrt[a + a*
Sec[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2
*d)
```

Rubi [A] time = 0.461061, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 4010, 4001, 3795, 203}

$$-\frac{(7A-11B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-7B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+ \ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((7*A - 11*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) + ((9*A - 13*B)*Tan[c + d*x])/(3*a*d*Sqrt[a + a*
Sec[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2
*d)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^2(c+dx)\left(2a(A-B)-\frac{1}{2}a(3A-7B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A-7B)\sqrt{a+a\sec(c+dx)}\tan(c+dx}{6a^2d}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A-7B)\sqrt{a}}{}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A-7B)\sqrt{a}}{}$$

$$= -\frac{(7A-11B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} +$$

Mathematica [A] time = 1.36585, size = 141, normalized size = 0.82

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(12(A-B)\sec(c+dx)+15A+4B\sec^2(c+dx)-19B\right)-3\sqrt{2}(7A-11B)\cos^2\left(\frac{1}{2}(c+\right.\right.}{6d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((-3*Sqrt[2]*(7*A - 11*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(15*A - 19*B + 12*(A - B)*S
ec[c + d*x] + 4*B*Sec[c + d*x]^2))*Tan[c + d*x])/(6*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.26, size = 603, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/24/d/a^2*(-1+cos(d*x+c))*(21*A*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))-33*B*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))+42*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))-66*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))+21*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-33*B*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-60*A*cos(d*x+c)^3+76*B*co
s(d*x+c)^3+12*A*cos(d*x+c)^2-28*B*cos(d*x+c)^2+48*A*cos(d*x+c)-64*B*cos(d*x
+c)+16*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.591592, size = 1189, normalized size = 6.95

$$\left[\frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^3+2(7A-11B)\cos(dx+c)^2+(7A-11B)\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\cdots}{\cos\cdots}\right.}{24\left(a^2d\cos(dx+c)^3+2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^3 + 2*(7*A - 11*B)*cos(d*x + c)
^2 + (7*A - 11*B)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^
2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((15*A
 - 19*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 4*B)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
 + c)^2 + a^2*d*cos(d*x + c)), 1/12*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^3
 + 2*(7*A - 11*B)*cos(d*x + c)^2 + (7*A - 11*B)*cos(d*x + c))*sqrt(a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) + 2*((15*A - 19*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 4
```

```
*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.4651, size = 400, normalized size = 2.34

$$\frac{\left(\left(\frac{3\left(\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)-\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a}-\frac{2\left(15\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)-23\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/12*(((3*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a - 2*(15*sqrt(2)*A*a*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1) - 23*sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))/a)*tan(1/2*d*x + 1/2*c)^2 + 27*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a)*tan(1/2*d*x + 1/2*c)
/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*(
7*sqrt(2)*A - 11*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
))/d
```

3.154 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3A-7B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{2B\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((3*A - 7*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + (2*B*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.258882, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4008, 4001, 3795, 203}

$$\frac{(3A-7B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{2B\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + (2*B*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{3}{2}a(A-B)-2aB\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2B\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{(3A-7B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+}}}{4a}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2B\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A-7B)\,\text{Subst}\left(\int \frac{1}{2a+}\right.}{2}$$

$$= \frac{(3A-7B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2B\tan(c}{ad\sqrt{a+a\,\text{se}}}$$

Mathematica [A] time = 0.750288, size = 125, normalized size = 1.06

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}(-A+4B\sec(c+dx)+5B)+\sqrt{2}(3A-7B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\text{se}}}{\sqrt{}}\right.\right.}{2d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((Sqrt[2]*(3*A - 7*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)
/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-A + 5*B + 4*B*Sec[c + d*x]))*
Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.241, size = 405, normalized size = 3.4

$$-\frac{-1+\cos(dx+c)}{4\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3A\sin(dx+c)\cos(dx+c)\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{si}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*A*sin(d*x
+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-7*B*sin(d*x+c)*cos
(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*A*ln(-(-(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)-7*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+2*A*cos(d*x+c)^2-10*B*cos(d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(d*x
+c)+8*B)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A] time = 0.575898, size = 1003, normalized size = 8.5

$$\left[\frac{\sqrt{2}\left((3A-7B)\cos(dx+c)^2+2(3A-7B)\cos(dx+c)+3A-7B\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos}{\cos(dx+c)^2+2\cos(dx+c)+}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*
A - 7*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((A - 5*B)*cos(d*x + c) - 4*
B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A - 7*B)*cos(d*x + c
)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*A - 7*B)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*
((A - 5*B)*cos(d*x + c) - 4*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.31143, size = 257, normalized size = 2.18

$$-\frac{\frac{\left(\frac{\sqrt{2}\left(Aa^2-Ba^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{\sqrt{2}\left(Aa^2-9\,Ba^2\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}-\frac{\sqrt{2}(3\,A-7\,B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/4*((sqrt(2)*(A*a^2 - B*a^2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)) - sqrt(2)*(A*a^2 - 9*B*a^2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(2)
*(3*A - 7*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.155 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2))
```

Rubi [A] time = 0.121754, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4000, 3795, 203}

$$\frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+3B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$
$$= \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A+3B)\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}$$
$$= \frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.79939, size = 127, normalized size = 1.46

$$\frac{2(A-B)\sin(c+dx)\sqrt{1-\sec(c+dx)} + 2\sqrt{2}(A+3B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)}{4ad(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(A - B)*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 2*Sqrt[2]*(A + 3*B)*ArcTan
h[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Tan[c + d*x])/(4*a*d*(
1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.19, size = 402, normalized size = 4.6

$$\frac{1}{4\,da^2(\cos(dx+c)+1)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\sin(dx+c)\cos(dx+c)\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\frac{\cos(}{\cos(d}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(A*sin(d*x+c)*cos(d*x+c)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+3*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos
(d*x+c)^2+2*B*cos(d*x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c))/(cos(d*x+c)+1)/si
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 0.57218, size = 957, normalized size = 11.

$$\left[\frac{4\,(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}\left((A+3\,B)\cos(dx+c)^2+2\,(A+3\,B)\cos(dx+c)+A+3\,B\right)\sqrt{-a}}{8\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A +
 3*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)
*cos(d*x + c) + A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x
)
```

Giac [B] time = 9.22598, size = 208, normalized size = 2.39

$$\frac{\frac{\left(\sqrt{2}A+3\sqrt{2}B\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{\left(\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)-\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/4*((sqrt(2)*A + 3*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - (sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1
/2*c)/a^3)/d
```

3.156 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(5A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2))
```

Rubi [A] time = 0.181414, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3922, 3920, 3774, 203, 3795}

$$-\frac{(5A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2))
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2aA+\frac{1}{2}a(A-B)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2} \\
&= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{(5A-B)\int\frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a} \\
&= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} + \frac{(5A-B)\,\mathrm{Subs}}{} \\
&= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 26.548, size = 10115, normalized size = 79.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.201, size = 554, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(4*A*2^(1/2)*sin(d*x+c)*cos(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+4*A*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+5*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+5*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^
```

```
2+2*B*cos(d*x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 7.62681, size = 1416, normalized size = 11.15

$$\left[-\frac{4(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}\left((5A-B)\cos(dx+c)^2+2(5A-B)\cos(dx+c)+5A-B\right)\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*
A - B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*(A*cos(d*x + c)^2 + 2*A*cos(d*x
 + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(
d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4
*(2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) - sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*A - B
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c))) + 8*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + dx))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 11.3232, size = 417, normalized size = 3.28

$$-\frac{\frac{\sqrt{2}(5A-B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}+\frac{8A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{8A\log}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2)*(5*A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 8*
A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
 8*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - 2*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/
2*c)/a^3)/d
```

3.157 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -(((3*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*A - 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a
*Sec[c + d*x])^(3/2)) + ((3*A - B)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c +
d*x]])
```

Rubi [A] time = 0.405304, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*A - 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a
*Sec[c + d*x])^(3/2)) + ((3*A - B)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c +
d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(a(3A-B)-\frac{3}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-a^2(3A-2B)+\frac{1}{2}a^2(3A-B)\mathrm{s}}{\sqrt{a+a\sec(c+dx)}}}{2a^3}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(9A-5B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+}}}{4a}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(9A-5B)\,\mathrm{Subst}\left(\int \frac{}{2a}\right.}{2}$$

$$= -\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(}{2d(}$$

Mathematica [C] time = 26.727, size = 10898, normalized size = 64.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.279, size = 713, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(-1+cos(d*x+c))*(6*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)/cos(d*x+c))-4*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))+9*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+6*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)-5*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)-4*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*A*
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*A*cos(d*x+c)^3-5*B*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*B*co
s(d*x+c)^2+6*A*cos(d*x+c)-2*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 9.97088, size = 1575, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(sqrt(2)*((9*A - 5*B)*cos(d*x + c)^2 + 2*(9*A - 5*B)*cos(d*x + c) + 9*
A - 5*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A - 2*B)*cos(d*x + c)^2
+ 2*(3*A - 2*B)*cos(d*x + c) + 3*A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2
+ 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(2*A*cos(d*x + c)^2 + (3*
A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((9*A
- 5*B)*cos(d*x + c)^2 + 2*(9*A - 5*B)*cos(d*x + c) + 9*A - 5*B)*sqrt(a)*arc
tan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - 4*((3*A - 2*B)*cos(d*x + c)^2 + 2*(3*A - 2*B)*cos(d*x + c)
```

```
+ 3*A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*A*cos(d*x + c)^2 + (3*A - B)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.158 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(19A-12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A-B)\sin(c+dx}{2ad\sqrt{a\sec(c+}$$

```
[Out] ((19*A - 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(3/2)*d) - ((13*A - 9*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A - 6*B)*Sin[c + d*x])/(4*a*d*Sq
rt[a + a*Sec[c + d*x]]) + ((2*A - B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt
[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.583225, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(19A-12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A-B)\sin(c+dx}{2ad\sqrt{a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(3/2)*d) - ((13*A - 9*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A - 6*B)*Sin[c + d*x])/(4*a*d*Sq
rt[a + a*Sec[c + d*x]]) + ((2*A - B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(2A-B)-\frac{5}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(2A-B)\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\int \text{co}\ldots}{\ldots}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)\cos(c\ldots}{2ad\sqrt{a+\ldots}}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)\cos(c\ldots}{2ad\sqrt{a+\ldots}}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)\cos(c\ldots}{2ad\sqrt{a+\ldots}}$$

$$= \frac{(19A-12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \ldots$$

Mathematica [C] time = 2.28628, size = 395, normalized size = 1.79

$$\frac{\sec(c+dx)\left(-40A\sqrt{1-\sec(c+dx)}(\sin(c+dx)+\tan(c+dx))\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)+(91\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] (Sec[c + d*x]*(-52*Sqrt[2]*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Sin[c
+ d*x] + 36*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Sin[c + d*x]
- 13*A*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 24*B*Sqrt[1 - Sec[c + d*x]]*Si
```

```
n[c + d*x] + 18*A*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + (13*
A*Sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)])/2 + 8*B*Sqrt[1 - Sec[c + d*x]]*S
in[2*(c + d*x)] - 52*Sqrt[2]*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[
c + d*x] + 36*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x
] + (91*A - 48*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + Tan[c + d
*x]) - 40*A*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]]*(Sin[c + d*x] + Tan[c + d*x])))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1
 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.333, size = 1075, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/16/d/a^2*(-1+cos(d*x+c))*(19*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))-12*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+26*A*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+38*A*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-18*B*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))-24*B*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+52*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))+19*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2
^(1/2)*sin(d*x+c)-36*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))-12*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+
c)+26*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-8*A*cos(d*x+c)
^5-18*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+20*A*cos(d*x+c
)^4-16*B*cos(d*x+c)^4+16*A*cos(d*x+c)^3-8*B*cos(d*x+c)^3-28*A*cos(d*x+c)^2+
24*B*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x
+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A] time = 14.6731, size = 1673, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(sqrt(2)*((13*A - 9*B)*cos(d*x + c)^2 + 2*(13*A - 9*B)*cos(d*x + c) +
13*A - 9*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c)
 - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^
2 + 2*(19*A - 12*B)*cos(d*x + c) + 19*A - 12*B)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^3
 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d), 1/4*(sqrt(2)*((13*A - 9*B)*cos(d*x + c)^2 + 2*(13*A - 9*B)
*cos(d*x + c) + 13*A - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((19*A - 12*B)*cos(d
*x + c)^2 + 2*(19*A - 12*B)*cos(d*x + c) + 19*A - 12*B)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(2*A*cos(d*x + c)^3 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B)*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.159 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{(47A-38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}+\frac{(5A-3B)\sin(c}{6ad\sqrt{a\sec}$$

```
[Out] -((47*A - 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8
*a^(3/2)*d) + ((17*A - 13*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (7*(3*A - 2*B)*Sin[c + d*x])/(8*
a*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A - 12*B)*Cos[c + d*x]*Sin[c + d*x])/(
12*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 3*B)*Cos[c + d*x]^2*Sin[c + d*x]
)/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.779661, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(47A-38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}+\frac{(5A-3B)\sin(c}{6ad\sqrt{a\sec}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((47*A - 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8
*a^(3/2)*d) + ((17*A - 13*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (7*(3*A - 2*B)*Sin[c + d*x])/(8*
a*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A - 12*B)*Cos[c + d*x]*Sin[c + d*x])/(
12*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 3*B)*Cos[c + d*x]^2*Sin[c + d*x]
)/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(a(5A-3B)-\frac{7}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-3B)\cos^2(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} + \int$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(13A-12B)\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A-12B)}{12ad\sqrt{}}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A-12B)}{12ad\sqrt{}}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A-12B)}{12ad\sqrt{}}$$

$$= -\frac{(47A-38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A-13B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 6.14309, size = 502, normalized size = 1.87

$$-\frac{A(\sec(c+dx)+1)^{3/2}\left(\frac{336\tan(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)}{d\sqrt{\sec(c+dx)+1}} + \frac{17\tan(c+dx)\left(-8\cos^3(c+dx)\sqrt{1-\sec(c+dx)}+2\cos^2(c+dx)\sqrt{1-}\right.}{}\right.}{96(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] -(B*Cos[c + d*x]*Sin[c + d*x])/(2*d*(a*(1 + Sec[c + d*x]))^(3/2)) - (A*Cos[
c + d*x]^2*Sin[c + d*x])/(2*d*(a*(1 + Sec[c + d*x]))^(3/2)) - (B*(1 + Sec[c
 + d*x])^(3/2)*((40*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Tan[c
+ d*x])/(d*Sqrt[1 + Sec[c + d*x]]) - (13*(7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]
 - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1
- Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d
*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/(16*(a*(1 + Sec[c + d*x])
)^(3/2)) - (A*(1 + Sec[c + d*x])^(3/2)*((336*Hypergeometric2F1[1/2, 4, 3/2,
 1 - Sec[c + d*x]]*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) + (17*(3*(9*Arc
Tanh[Sqrt[1 - Sec[c + d*x]]] - 8*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqr
t[2]] - 7*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]]) + 2*Cos[c + d*x]^2*Sqrt[1 -
Sec[c + d*x]] - 8*Cos[c + d*x]^3*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*S
qrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/(96*(a*(1 + Sec[c + d*x]))^
(3/2))
```

Maple [B] time = 0.292, size = 1425, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/192/d/a^2*(-1+cos(d*x+c))*(204*A*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(5/2)-156*B*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)+423*A*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*2^(1/2)-342*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-336*B*cos(d*x+c)^3+204*A*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-156*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)*sin(d*x+c)+612*A*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(5/2)-468*B*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)+612*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)-468*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)-208*A*cos(d*x+c)^4+423*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*2^(1/2)-342*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c))*2^(1/2)+112*A*cos(d*x+c)^6-344*A*cos(d*x+c)^5+240*B*c
os(d*x+c)^5+192*B*cos(d*x+c)^4+504*A*cos(d*x+c)^3-64*A*cos(d*x+c)^7-96*B*co
s(d*x+c)^6+141*A*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))-114*B*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))+141*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arc
```

```
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)-114*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*
x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A] time = 14.7118, size = 1789, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(6*sqrt(2)*((17*A - 13*B)*cos(d*x + c)^2 + 2*(17*A - 13*B)*cos(d*x +
c) + 17*A - 13*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d
*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*((47*A - 38*B)*cos(
d*x + c)^2 + 2*(47*A - 38*B)*cos(d*x + c) + 47*A - 38*B)*sqrt(-a)*log((2*a*
cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d
*x + c)^4 - 6*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B)*cos(d*x + c)^2 + 21*
(3*A - 2*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/24*(6*sqrt(2
)*((17*A - 13*B)*cos(d*x + c)^2 + 2*(17*A - 13*B)*cos(d*x + c) + 17*A - 13*
B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c))) - 3*((47*A - 38*B)*cos(d*x + c)^2 + 2*(47*A - 3
8*B)*cos(d*x + c) + 47*A - 38*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^4 - 6
*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B)*cos(d*x + c)^2 + 21*(3*A - 2*B)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.160 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{(75A-163B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(39A-95B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] -((75*A - 163*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) + ((9*A - 17*B)*Sec[c + d*x]^2*Tan[c + d*x])/
(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((93*A - 197*B)*Tan[c + d*x])/(24*a^2
*d*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[
c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.654887, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 4010, 4001, 3795, 203}

$$-\frac{(75A-163B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(39A-95B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((75*A - 163*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) + ((9*A - 17*B)*Sec[c + d*x]^2*Tan[c + d*x])/
(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((93*A - 197*B)*Tan[c + d*x])/(24*a^2
*d*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[
c + d*x])/(48*a^3*d)
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^3(c+dx)\left(3a(A-B)-\frac{1}{2}a(3A-11B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \int \cdots$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(39\cdots}{\cdots}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(93\cdots}{24a\cdots}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(93\cdots}{24a\cdots}$$

$$= -\frac{(75A-163B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \cdots$$

Mathematica [A] time = 2.57115, size = 161, normalized size = 0.75

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32(3A-5B)\sec^2(c+dx)+(255A-503B)\sec(c+dx)+147A+32B\sec^3(c+dx)-299\cdots\right.\right.}{48d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((-6*Sqrt[2]*(75*A - 163*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c
+ d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(147*A - 299*B + (255*A
 - 503*B)*Sec[c + d*x] + 32*(3*A - 5*B)*Sec[c + d*x]^2 + 32*B*Sec[c + d*x]^
3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)
```

)

Maple [B] time = 0.275, size = 795, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/192/d/a^3*(-1+cos(d*x+c))^2*(225*A*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))-489*B*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))+675*A*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))-1467*B*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))+675*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))-1467*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))+225*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-489*B
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-588*A*cos(d*x+c)^4+11
96*B*cos(d*x+c)^4-432*A*cos(d*x+c)^3+816*B*cos(d*x+c)^3+636*A*cos(d*x+c)^2-
1372*B*cos(d*x+c)^2+384*A*cos(d*x+c)-768*B*cos(d*x+c)+128*B)*(a*(cos(d*x+c)
+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.613782, size = 1474, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^4 + 3*(75*A - 163*B)*cos(d*x
 + c)^3 + 3*(75*A - 163*B)*cos(d*x + c)^2 + (75*A - 163*B)*cos(d*x + c))*sq
```

```
rt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*((147*A - 299*B)*cos(d*x + c)^3 + (255*A
 - 503*B)*cos(d*x + c)^2 + 32*(3*A - 5*B)*cos(d*x + c) + 32*B)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/96*(3*sqrt(
2)*((75*A - 163*B)*cos(d*x + c)^4 + 3*(75*A - 163*B)*cos(d*x + c)^3 + 3*(75
*A - 163*B)*cos(d*x + c)^2 + (75*A - 163*B)*cos(d*x + c))*sqrt(a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) + 2*((147*A - 299*B)*cos(d*x + c)^3 + (255*A - 503*B)*cos(d*x + c)^
2 + 32*(3*A - 5*B)*cos(d*x + c) + 32*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 10.1536, size = 420, normalized size = 1.94

$$\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}(Aa^5-Ba^5)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}(15Aa^5-23Ba^5)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}(75Aa^5-167Ba^5)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3\sqrt{2}(83Aa^5-155Ba^5)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^6*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(15*A*a^5 - 23*B*a^5)/(a^6*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(75*A*a^5 - 167*B*a^5
)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)
*(83*A*a^5 - 155*B*a^5)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)) - 3*sqrt(2)*(75*A - 163*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)))/d
```

3.161 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(19A-75B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-9B)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(5A-13B)\,\mathrm{ta}}{16ad(a\sec(c+}$$

```
[Out] ((19*A - 75*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*
(a + a*Sec[c + d*x])^(5/2)) - ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Se
c[c + d*x])^(3/2)) - ((A - 9*B)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rubi [A] time = 0.454957, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 4008, 4001, 3795, 203}

$$\frac{(19A-75B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-9B)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(5A-13B)\,\mathrm{ta}}{16ad(a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((19*A - 75*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*
(a + a*Sec[c + d*x])^(5/2)) - ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Se
c[c + d*x])^(3/2)) - ((A - 9*B)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d
*x]])
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^2(c+dx)\left(2a(A-B)-\frac{1}{2}a(A-9B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{3}{4}a^2\right.}{}}{}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(A-9B)\tan(}{4a^2d\sqrt{a+a\sec}}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(A-9B)\tan(}{4a^2d\sqrt{a+a\sec}}$$

$$= \frac{(19A-75B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5}{16a}$$

Mathematica [A] time = 1.50288, size = 144, normalized size = 0.85

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left((85B-13A)\sec(c+dx)-9A+32B\sec^2(c+dx)+49B\right)+2\sqrt{2}(19A-75B)\cos^4\left(\frac{1}{2}(c+\right.\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((2*Sqrt[2]*(19*A - 75*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-9*A + 49*B + (-13*A + 8
5*B)*Sec[c + d*x] + 32*B*Sec[c + d*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.259, size = 597, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(19*A*sin(
d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-75*B*sin(d*x+
c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+38*A*sin(d*x+c)*c
os(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-150*B*sin(d*x+c)*cos(d*
x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+19*A*ln(-(-(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+18*A*cos(d*x+c)^3-75*B*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-98*B*cos(d*x+c)^3+8*A*cos(d*x+c)^2-72*B*cos(
d*x+c)^2-26*A*cos(d*x+c)+106*B*cos(d*x+c)+64*B)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.591538, size = 1273, normalized size = 7.53

$$\left[\frac{\sqrt{2}\left((19\,A-75\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A-75\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A-75\,B)\cos\left(dx+c\right)+19\,A-75\,B\right)\sqrt{-a}\log\left(\cdots\right.}{64\left(a^{3}d\cos\left(dx\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)
^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(-a)*log(-(2*sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) -
 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((9*A - 49*B)*cos(d*x + c)^2 + (13*A - 85*B)*cos(d*x + c) - 32*
B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2
)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)^2 + 3*(19*A
- 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((9*A - 49
```

```
*B)*cos(d*x + c)^2 + (13*A - 85*B)*cos(d*x + c) - 32*B)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec{(c + dx)})\sec^3{(c + dx)}}{(a(\sec{(c + dx)} + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 9.93491, size = 390, normalized size = 2.31

$$-\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Aa^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\sqrt{2}Ba^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{9\sqrt{2}Aa^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-17\sqrt{2}Ba^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{32 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/32*(((2*(sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a^6*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^8 + (9*sqrt(2)*A*a
^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 17*sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1))/a^8)*tan(1/2*d*x + 1/2*c)^2 - (11*sqrt(2)*A*a^6*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1) - 83*sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*t
an(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - (19*sqrt(2)*A - 7
5*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.162 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5A+19B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((5*A + 19*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d
*x])^(5/2)) + ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2
))
```

Rubi [A] time = 0.275904, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4008, 4000, 3795, 203}

$$\frac{(5A+19B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((5*A + 19*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d
*x])^(5/2)) + ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2
))
```


Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{5}{2}a(A-B)-4aB\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(5A+19B)\int \frac{\sec(c+}{\sqrt{a+a\sec(}}}{32a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(5A+19B)\,\mathrm{Subst}\left(\int\right.}{1}$$

$$= \frac{(5A+19B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\,\mathrm{t}}{16ad(a+a\,\mathrm{sec}}$$

Mathematica [A] time = 1.57384, size = 131, normalized size = 1.04

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}((5A-13B)\sec(c+dx)+A-9B)+2\sqrt{2}(5A+19B)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((2*Sqrt[2]*(5*A + 19*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d
*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(A - 9*B + (5*A - 13*B)*Se
c[c + d*x]))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(5/2))
```

Maple [B] time = 0.244, size = 602, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(5*A*sin(d*
x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+19*B*sin(d*x+c)
*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+10*A*sin(d*x+c)*cos
(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+38*B*sin(d*x+c)*cos(d*x+c
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*A*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^3+19*B*ln(-(-(-2*cos(d*x+c)/(cos(d
```

```
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+18*B*cos(d*x+c)^3-8*A*cos(d*x+c)^2+8*B*cos(d*x+c)
^2+10*A*cos(d*x+c)-26*B*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.582109, size = 1233, normalized size = 9.79

$$\left[-\frac{\sqrt{2}\left((5\,A+19\,B)\cos\left(dx+c\right)^{3}+3\,(5\,A+19\,B)\cos\left(dx+c\right)^{2}+3\,(5\,A+19\,B)\cos\left(dx+c\right)+5\,A+19\,B\right)\sqrt{-a}\log\left(\frac{2\cdots}{\cdots}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*A + 19*B)*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^
2 + 3*(5*A + 19*B)*cos(d*x + c) + 5*A + 19*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) - 4*((A - 9*B)*cos(d*x + c)^2 + (5*A - 13*B)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A + 19*B)
*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^2 + 3*(5*A + 19*B)*cos(d*x +
c) + 5*A + 19*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((A - 9*B)*cos(d*x + c)^2 + (
5*A - 13*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 9.83643, size = 258, normalized size = 2.05

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{\sqrt{2}\left(3\,Aa^5-11\,Ba^5\right)}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}(5\,A+19\,B)\log\left(\left|-\sqrt{-a}\tan\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1
/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(3*A*a^5
- 11*B*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - s
qrt(2)*(5*A + 19*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.163 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A+5B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*
x])^(5/2)) + ((3*A + 5*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.164641, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4000, 3796, 3795, 203}

$$\frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3A+5B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*
x])^(5/2)) + ((3*A + 5*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(3A+5B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}}{32a^2}$$

$$= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(3A+5B)\,\mathrm{Subst}\left(\int \frac{1}{2a+}\right.}{16a}$$

$$= \frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan}{16ad(a+a\sec(}$$

Mathematica [C] time = 1.50567, size = 206, normalized size = 1.63

$$\frac{64A\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{1-\sec(c+dx)}\sec(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+B}{32a^2d(\cos(c+dx)+1)^2\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (40*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Se
c[c + d*x]*Sin[(c + d*x)/2] + 64*A*Cos[(c + d*x)/2]^5*Hypergeometric2F1[1/2
, 3, 3/2, (1 - Sec[c + d*x])/2]*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Sin[(c
+ d*x)/2] + B*Sqrt[1 - Sec[c + d*x]]*(10*Sin[c + d*x] + Sin[2*(c + d*x)]))/
(32*a^2*d*(1 + Cos[c + d*x])^2*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [B] time = 0.191, size = 594, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*A*sin(d*x+c)*cos(d*x+c)^2
*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*B*sin(d*x+c)*cos(d*x+c)^2*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+10*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+3*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-14*A*cos(d*x+c)^3+5*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-2*B*cos(d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+6*A*cos(d*x+c)+10
*B*cos(d*x+c))/(cos(d*x+c)+1)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.589359, size = 1219, normalized size = 9.67

$$\left[-\frac{\sqrt{2}\left((3A+5B)\cos(dx+c)^3+3(3A+5B)\cos(dx+c)^2+3(3A+5B)\cos(dx+c)+3A+5B\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-\cdots}}{\cdots}\right.\cdots}{64\left(a^3d\cos(dx+c)^3+3\cdots\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2
+ 3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos
(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
- 4*((7*A + B)*cos(d*x + c)^2 + (3*A + 5*B)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A + 5*B)*cos(d
*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 + 3*(3*A + 5*B)*cos(d*x + c) + 3*A
 + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((7*A + B)*cos(d*x + c)^2 + (3*A + 5*B
)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x
)
```

Giac [A] time = 9.39635, size = 258, normalized size = 2.05

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{\sqrt{2}\left(5\,Aa^5+3\,Ba^5\right)}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(3\,A+5\,B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\right.\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}\left(\mathrm{t}\right.}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/
2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(5*A*a^5 +
 3*B*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + sqr
t(2)*(3*A + 5*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.164 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{(43A-3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*A - 3*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rubi [A] time = 0.253715, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3922, 3920, 3774, 203, 3795}

$$-\frac{(43A-3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*A - 3*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4aA+\frac{3}{2}a(A-B)\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2A-\frac{1}{4}a^2(11A-3B)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\sec(c+dx)}\,dx}{a^3} - \frac{(43\ldots}{\ldots}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan\ldots}{\sqrt{a+a\,\text{s}\ldots}}\right.}{a^2d}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 26.6766, size = 10177, normalized size = 62.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.204, size = 824, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(32*A*sin(d*x+c)*cos(d*x+c)
^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+43*A*sin(d*x+c)*cos(d*
x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+64*A*2^(1/2)*sin(d*x+c)*co
s(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-3*B*sin(d*x+c)*cos(d*x+c
)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
```

```
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+86*A*sin(d*x+c)*cos(d*x+c)*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+32*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*sin(d*x+c)-6*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+43*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-30*A*cos(d*x+c)^3-3*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+14*B*cos(d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+22*A*cos(d*x+c)-
6*B*cos(d*x+c))/(cos(d*x+c)+1)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 16.1611, size = 1754, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2
 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*
cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt
(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
- 4*((15*A - 7*B)*cos(d*x + c)^2 + (11*A - 3*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*A - 3*B)*c
os(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2 + 3*(43*A - 3*B)*cos(d*x + c)
 + 43*A - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*
x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((15*A - 7*B)*cos(d*x
 + c)^2 + (11*A - 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.3761, size = 471, normalized size = 2.87

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\sqrt{2}\left(Aa^{5}-Ba^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(13\,Aa^{5}-5\,Ba^{5}\right)}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(43\,A-3\,B)\log\left(\left(\sqrt{-a}\tan\cdots\right.\right.}{\sqrt{-a}a^{2}\mathrm{s}\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan
(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(13*A*a
^5 - 5*B*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) +
 sqrt(2)*(43*A - 3*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 64
*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - 64*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))/d
```

3.165 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15A-7B)\,\mathrm{s}}{16ad(a\sec(c+}$$

```
[Out] -(((5*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*A - 43*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a
 + a*Sec[c + d*x])^(5/2)) - ((15*A - 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Sec[
c + d*x])^(3/2)) + ((35*A - 11*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```

Rubi [A] time = 0.557752, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15A-7B)\,\mathrm{s}}{16ad(a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] -(((5*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*A - 43*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a
 + a*Sec[c + d*x])^(5/2)) - ((15*A - 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Sec[
c + d*x])^(3/2)) + ((35*A - 11*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(a(5A-B)-\frac{5}{2}a(A-B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}a^2(35A-11B)-\right.}{\sqrt{a+a\sec}}}{8a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(}{4d(}$$

Mathematica [C] time = 26.9387, size = 10956, normalized size = 52.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.303, size = 1065, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(80*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-32*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+160*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+115*A*sin(d*x+c)*cos(d*x+c)^2*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-64*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-43*B*sin(d*x+c)*cos(d*x+c)^2*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+80*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))*sin(d*x+c)+230*A*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)-32*A*cos(d*x+c)^4-32*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-86*B*sin(d*x+c)*cos(d*x+c)*ln(-(-(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+115*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)-78*A*cos(d*x+c)^3-43*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+30*B*cos(d*x+c)^3+40*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+70*A*cos(d*x
+c)-22*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 21.2551, size = 1948, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] [1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^3 + 3*(115*A - 43*B)*cos(d*x +
c)^2 + 3*(115*A - 43*B)*cos(d*x + c) + 115*A - 43*B)*sqrt(-a)*log(-(2*sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1)) + 32*((5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2
 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2
 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(16*A*cos(d*x + c)^3 + 5
*(11*A - 3*B)*cos(d*x + c)^2 + (35*A - 11*B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((115*A - 43*B)*c
os(d*x + c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + 3*(115*A - 43*B)*cos(d*x
+ c) + 115*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 32*((5*A - 2*B)*cos(d*x + c
)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 2*(16*A*cos(d*x + c)^3 + 5*(11*A - 3*B)*cos(d*x + c)^2
 + (35*A - 11*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.166 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=264

$$-\frac{7(9A-5B)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(39A-20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A-1}{16a}$$

```
[Out] ((39*A - 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(5/2)*d) - ((219*A - 115*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c
+ d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A - 11*B)*Cos[c + d*x]*Sin[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (7*(9*A - 5*B)*Sin[c + d*x]
)/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*A - 15*B)*Cos[c + d*x]*Sin[c +
 d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.790338, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{7(9A-5B)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(39A-20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A-1}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((39*A - 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(5/2)*d) - ((219*A - 115*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c
+ d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A - 11*B)*Cos[c + d*x]*Sin[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (7*(9*A - 5*B)*Sin[c + d*x]
)/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*A - 15*B)*Cos[c + d*x]*Sin[c +
 d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(3A-B)-\frac{7}{2}a(A-B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \int \text{co}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + (31A$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{7(9A}{16a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{7(9A}{16a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{7(9A}{16a^2}$$

$$= \frac{(39A-20B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} -$$

Mathematica [C] time = 6.16367, size = 512, normalized size = 1.94

$$-\frac{A(\sec(c+dx)+1)^{5/2}\left(\frac{760\tan(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)}{d\sqrt{\sec(c+dx)+1}} + \frac{152\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^{3/2}} - \frac{219\tan(c+dx)\left(2\cos^2(c+dx)\sqrt{1-\sec(}\right.}{}\right.}{128(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] -(B*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (A*Cos[c + d*x]*Sin[
c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (5*B*(1 + Sec[c + d*x])^(5/2
)*((6*Sin[c + d*x])/(d*(1 + Sec[c + d*x])^(3/2)) + (9*(Cos[c + d*x] + ArcTa
nh[Sqrt[1 - Sec[c + d*x]]]/Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1
+ Sec[c + d*x]]) + (23*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh[S
qrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c
 + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/(32*(a*(1 + Se
c[c + d*x]))^(5/2)) - (A*(1 + Sec[c + d*x])^(5/2)*((152*Cos[c + d*x]*Sin[c
+ d*x])/(d*(1 + Sec[c + d*x])^(3/2)) + (760*Hypergeometric2F1[1/2, 3, 3/2,
1 - Sec[c + d*x]]*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) - (219*(7*ArcTan
h[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2
]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[c
+ d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/
(128*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.388, size = 1427, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/64/d/a^3*(-1+cos(d*x+c))^2*(468*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+468*A*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+219*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*sin(d*x+c)-115*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d
*x+c)-240*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))-240*B*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*2^(1/2)+80*B*cos(d*x+c)^3+300*A*cos(d*x+c)^4+657*A*sin(d*x+c)*cos(d*x
+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-345*B*sin(d*x+c)*cos(d*x+c)
^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-345*B*cos(d*x+c)*sin(d*x+c)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-32*A*cos(d*x+c)^6+112*A*cos(d*x+c)^
5-64*B*cos(d*x+c)^5-156*B*cos(d*x+c)^4-128*A*cos(d*x+c)^3+140*B*cos(d*x+c)^
2+219*A*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-1
15*B*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-252*
A*cos(d*x+c)^2+657*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))+156*A*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(3/2)-80*B*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)+156*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2
```

```
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*
sin(d*x+c)-80*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c
))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 28.2047, size = 2059, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [1/64*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^3 + 3*(219*A - 115*B)*cos(d*x
+ c)^2 + 3*(219*A - 115*B)*cos(d*x + c) + 219*A - 115*B)*sqrt(-a)*log((2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 8*((39*A - 20*B)*cos(d*x + c)^3 + 3*(39*A - 20*B)*cos(d*x
+ c)^2 + 3*(39*A - 20*B)*cos(d*x + c) + 39*A - 20*B)*sqrt(-a)*log((2*a*cos(
d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(8*A*cos(d*x +
 c)^4 - 4*(5*A - 4*B)*cos(d*x + c)^3 - 5*(19*A - 11*B)*cos(d*x + c)^2 - 7*(
9*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d), 1/32*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^3 + 3*(219*A - 115*B)*
cos(d*x + c)^2 + 3*(219*A - 115*B)*cos(d*x + c) + 219*A - 115*B)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - 8*((39*A - 20*B)*cos(d*x + c)^3 + 3*(39*A - 20*B)*cos(d*x
+ c)^2 + 3*(39*A - 20*B)*cos(d*x + c) + 39*A - 20*B)*sqrt(a)*arctan(sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*(
8*A*cos(d*x + c)^4 - 4*(5*A - 4*B)*cos(d*x + c)^3 - 5*(19*A - 11*B)*cos(d*x
 + c)^2 - 7*(9*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.167 $\int \frac{A+A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x
]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.146448, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3904, 3887, 481, 203}

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{A+A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx=-\left((aA)\int \frac{\tan^2(c+dx)}{(a-a\sec(c+dx))^{3/2}}\,dx\right)$$
$$=\frac{(2aA)\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$=-\frac{(2A)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}+\frac{(4A)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$=\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 0.5378, size = 140, normalized size = 1.57

$$-\frac{iA\left(-1+e^{i(c+dx)}\right)\left(\sqrt{2}\sinh^{-1}\left(e^{i(c+dx)}\right)-4\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] ((-I)*A*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*ArcSinh[E^(I*(c + d*x))] - 4*ArcTan
h[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*
ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c +
d*x))]*Sqrt[a - a*Sec[c + d*x]])
```

Maple [A] time = 0.243, size = 120, normalized size = 1.4

$$\frac{A\sqrt{2}\left(\cos(dx+c)+1\right)}{d\sin(dx+c)\,a}\left(\sqrt{2}\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)\sqrt{\frac{a\left(-1+\cos(dx+c)\right)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] A/d*2^(1/2)*(2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))*(a*(-1+cos(d*x+c))/cos(d
*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)/sin(d*x+c)
/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.505888, size = 780, normalized size = 8.76

$$\left[\frac{\sqrt{2}Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)-A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a}{}} \cdots}{\sin \cdots}\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sq
rt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin
(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - A*sqrt(-a)*log((2*(cos(d*x
+ c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (
2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), 2*(sqrt(2)*A*sqrt
(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - A*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\sec(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx + \int \frac{1}{\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] A*(Integral(sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x) + Integral(1/sqrt(-a
*sec(c + d*x) + a), x))
```

Giac [C] time = 1.88414, size = 225, normalized size = 2.53

$$-\frac{2\left(Aa\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)+\frac{\left(\sqrt{2}A\sqrt{a}\arctan(-i)-A\sqrt{a}\arctan\left(-\frac{1}{2}i \cdots\right.\right.}{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*(A*a*(sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - arctan(1/2*s
qrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)))) + (sqrt(2)*A*sqrt(a)*arctan(-I
) - A*sqrt(a)*arctan(-1/2*I*sqrt(2)))*sgn(tan(1/2*d*x + 1/2*c))/a)/d
```

3.168 $\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}+\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x
]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.220891, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}+\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x
]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{-\frac{3aA}{2}-\frac{1}{2}aA\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx}{a}$$
$$= \frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} + (2A)\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx + \frac{(3A)\int \sqrt{a-a\sec(c}{2a}$$
$$= \frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} + \frac{(3A)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d} - \frac{(4A)\,\mathrm{Subs}}{}$$
$$= \frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A\sin(c+dx}{d\sqrt{a-a\sec(c+}}$$

Mathematica [C] time = 1.49979, size = 269, normalized size = 2.34

$$\frac{Ae^{-\frac{1}{2}i(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(3e^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-\right.}{2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (A*((3*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/E^((I/2)*(c
+ d*x)) + (1 + E^((-I)*(c + d*x)) + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) -
 4*Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 3*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcT
anh[Sqrt[1 + E^((2*I)*(c + d*x))]])/E^((I/2)*(c + d*x)))*Sec[c + d*x]*(Cos[
(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2])/(2*d*E^((I/2)*(c + d*x
))*Sqrt[a - a*Sec[c + d*x]])
```

Maple [A] time = 0.299, size = 155, normalized size = 1.4

$$\frac{A\sqrt{2}\left(\cos(dx+c)+1\right)}{2d\sin(dx+c)a}\left(2\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}+3\arctan\left(1/2\sqrt{2}\sqrt{-2\frac{\cos(dx+c}{\cos(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2*A/d*2^(1/2)*(cos(d*x+c)+1)*(2*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)+3*arctan(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos
(d*x+c)*2^(1/2))*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.527598, size = 1112, normalized size = 9.67

$$\left[\frac{2\sqrt{2}Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)-3A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2\right.}{}\right.}{2ad\sin(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(2*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x +
c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) +
1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) - 3*A*sqrt
(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(
d*x + c) - 2*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/
cos(d*x + c)))/(a*d*sin(d*x + c)), (2*sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*si
n(d*x + c) - 3*A*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - (A*cos(d*x + c)^2 + A*cos(
d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\cos(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{\cos(c+dx)\sec(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c + d
*x)*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))
```

Giac [C] time = 1.92026, size = 346, normalized size = 3.01

$$-\frac{Aa\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] -(A*a*(2*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 3*arctan(1/2
*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 - a)/((a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)*sgn(tan(1/2*d*x + 1/2*c)))) + (2*I*sqrt(2)*A*sqrt(-a)*arctan(-I) - 3
*I*A*sqrt(-a)*arctan(-1/2*I*sqrt(2)) - sqrt(2)*A*sqrt(-a))*sgn(tan(1/2*d*x
+ 1/2*c))/a)/d
```

3.169 $\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=155

$$\frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}}+\frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos(c+dx)}{2d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*Sqrt[a]*d
) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (5*A*Sin[c + d*x])/(4*d*Sqrt[a - a*Sec[c + d*x]]) +
(A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.361976, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}}+\frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos(c+dx)}{2d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```


```
[Out] (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*Sqrt[a]*d
) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (5*A*Sin[c + d*x])/(4*d*Sqrt[a - a*Sec[c + d*x]]) +
(A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a - a*Sec[c + d*x]])
```


Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{5aA}{2}-\frac{3}{2}aA\sec(c+dx)\right)}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} + \frac{\int \frac{\frac{11a^2A}{4}+\frac{5}{4}a^2A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,d}{2a^2}$$

$$= \frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} + (2A)\int \frac{\sec(c+dx}{\sqrt{a-a\sec(c+}}$$

$$= \frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} + \frac{(11A)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,d\right.}{4d}$$

$$= \frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{5A\sin(c+}{4d\sqrt{a-a\sec(}}$$

Mathematica [C] time = 1.62897, size = 297, normalized size = 1.92

$$\frac{Ae^{-\frac{1}{2}i(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(11e^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (A*((11*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/E^((I/2)*(c
 + d*x)) + (7 + 6/E^(I*(c + d*x)) + 7*E^(I*(c + d*x)) + E^((-2*I)*(c + d*x)
) + 6*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)) - 16*Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))])] + 11*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])/E^((I/2)*(c + d*x)))*Sec[c + d*x]*(Cos[(c + d*x)/2] + I*Sin[(c
+ d*x)/2])*Sin[(c + d*x)/2])/(8*d*E^((I/2)*(c + d*x))*Sqrt[a - a*Sec[c + d*
x]])
```

Maple [B] time = 0.345, size = 367, normalized size = 2.4

$$\frac{A\sqrt{2}\left(-1+\cos(dx+c)\right)^2}{24\,d\left(\sin(dx+c)\right)^3}\left(6\left(\cos(dx+c)\right)^3\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}-16\cos(dx+c)\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/24*A/d*2^(1/2)*(-1+cos(d*x+c))^2*(6*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)-16*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
3/2)+27*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-16*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+4*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)+48*cos(d*x+c)*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+66*cos(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))+15*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+48*2^(1/2)*arct
an(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+66*arctan(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)))/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(a*(-1+co
s(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)^2}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.538782, size = 1188, normalized size = 7.66

$$\left[\frac{8\sqrt{2}Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)-11A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(8*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x +
c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) +
1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) - 11*A*sqr
t(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)
- a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin
(d*x + c) - 2*(2*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 + 5*A*cos(d*x + c))*
sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c)), 1/4*(8*sqrt(2)
*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 11*A*sqrt(a)*arctan(sqrt((a*cos(
d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x +
c) - (2*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 + 5*A*cos(d*x + c))*sqrt((a*c
os(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\cos^2(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx + \int \frac{\cos^2(c+dx)\sec(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)**2/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c
+ d*x)**2*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))
```

Giac [C] time = 2.01873, size = 379, normalized size = 2.45

$$-\frac{Aa\left(\frac{8\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{11\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}+10\sqrt{\cdots}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^2 a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\cdots\right.\right.}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/4*(A*a*(8*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^
(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 11*arcta
n(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(3*(a*tan(1/2*
d*x + 1/2*c)^2 - a)^(3/2) + 10*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a)/((a*ta
n(1/2*d*x + 1/2*c)^2 + a)^2*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d
*x + 1/2*c)))) + (8*I*sqrt(2)*A*sqrt(-a)*arctan(-I) - 11*I*A*sqrt(-a)*arcta
n(-1/2*I*sqrt(2)) - 7*sqrt(2)*A*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c))/a)/d
```

3.170 $\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=192

$$\frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}}+\frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a-a\sec(c+dx)}}+\frac{7A\,\mathrm{s}}{12}$$

```
[Out] (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*Sqrt[a]*d
) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (9*A*Sin[c + d*x])/(8*d*Sqrt[a - a*Sec[c + d*x]]) +
(7*A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a - a*Sec[c + d*x]]) + (A*Cos[c
+ d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.524277, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4022, 3920, 3774, 203, 3795}

$$\frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}}+\frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{a}d}-\frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a-a\sec(c+dx)}}+\frac{7A\,\mathrm{s}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```


```
[Out] (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*Sqrt[a]*d
) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (9*A*Sin[c + d*x])/(8*d*Sqrt[a - a*Sec[c + d*x]]) +
(7*A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a - a*Sec[c + d*x]]) + (A*Cos[c
+ d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a - a*Sec[c + d*x]])
```


Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)\left(-\frac{7aA}{2}-\frac{5}{2}aA\sec(c+dx)\right)}{\sqrt{a-a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a-a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(\frac{27a^2A}{4}\right.}{\sqrt{a-a\,\mathrm{se}}}}{6}$$

$$= \frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}} + \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c}{3d\sqrt{a-a\sec(c+a}}$$

$$= \frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}} + \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c}{3d\sqrt{a-a\sec(c+a}}$$

$$= \frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}} + \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c}{3d\sqrt{a-a\sec(c+a}}$$

$$= \frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{a}d} - \frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{9A\sin(c+}{8d\sqrt{a-a\sec}}$$

Mathematica [C] time = 1.88793, size = 330, normalized size = 1.72

$$\frac{Ae^{-4i(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(9e^{i(c+dx)}+40e^{2i(c+dx)}+47e^{3i(c+dx)}+47e^{4i}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (A*(2 + 9*E^(I*(c + d*x)) + 40*E^((2*I)*(c + d*x)) + 47*E^((3*I)*(c + d*x))
 + 47*E^((4*I)*(c + d*x)) + 40*E^((5*I)*(c + d*x)) + 9*E^((6*I)*(c + d*x))
+ 2*E^((7*I)*(c + d*x)) + 69*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*ArcSinh[E^(I*(c + d*x))] - 96*Sqrt[2]*E^((3*I)*(c + d*x))*Sqrt[1 + E^((
2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))])] + 69*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh
[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]*(Cos[(c + d*x)/2] + I*Sin[(c
+ d*x)/2])*Sin[(c + d*x)/2])/(48*d*E^((4*I)*(c + d*x))*Sqrt[a - a*Sec[c + d
*x]])
```

Maple [B] time = 0.404, size = 625, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/240*A/d*2^(1/2)*(-1+cos(d*x+c))^3*(96*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*cos(d*x+c)^2*2^(1/2)+192*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)
*2^(1/2)+40*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*2^(1/2)+96*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)-160*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(3/2)*cos(d*x+c)^2*2^(1/2)+190*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(
d*x+c)^4*2^(1/2)-320*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2
)+465*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-160*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-49*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+480*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)^2*2^(1/2)+155*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+960*cos(d*x+c)*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
+690*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+
135*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+480*2^(1/2)*arctan(1/(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+1380*cos(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))+690*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)))/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(a*(-1+cos(d*x+c))/c
os(d*x+c))^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)^3}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.548307, size = 1258, normalized size = 6.55

$$\left[\frac{48\sqrt{2}Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)-69A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+\cdots\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(48*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x
+ c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c)
+ 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) - 69*A*s
qrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c
) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*s
in(d*x + c) - 2*(8*A*cos(d*x + c)^4 + 22*A*cos(d*x + c)^3 + 41*A*cos(d*x +
```

```
c)^2 + 27*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin
(d*x + c)), 1/24*(48*sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
- a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 69*A
*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(
a)*sin(d*x + c)))*sin(d*x + c) - (8*A*cos(d*x + c)^4 + 22*A*cos(d*x + c)^3
+ 41*A*cos(d*x + c)^2 + 27*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*
x + c)))/(a*d*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 2.21682, size = 412, normalized size = 2.15

$$-\frac{Aa\left(\frac{48\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{69\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(21\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{5}{2}}+80\left(a\tan\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^3 a\mathrm{sgn}(t}\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/24*(A*a*(48*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(
a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 69*arc
tan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(21*(a*tan(1
/2*d*x + 1/2*c)^2 - a)^(5/2) + 80*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*a +
108*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)
^3*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)))) + (48*I*sq
rt(2)*A*sqrt(-a)*arctan(-I) - 69*I*A*sqrt(-a)*arctan(-1/2*I*sqrt(2)) - 49*s
qrt(2)*A*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c))/a)/d
```

3.171 $\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}-\frac{A\tan(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(3/2)*d) -
 (3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(S
qrt[2]*a^(3/2)*d) - (A*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.199528, antiderivative size = 133, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3904, 3887, 471, 522, 203}

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{A\sin(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{2ad\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(3/2)*d) -
 (3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(S
qrt[2]*a^(3/2)*d) + (A*Csc[(c + d*x)/2]^2*Sin[c + d*x])/(2*a*d*Sqrt[a - a*S
ec[c + d*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{3/2}}\,dx = -\left((aA)\int \frac{\tan^2(c+dx)}{(a-a\sec(c+dx))^{5/2}}\,dx\right)$$

$$= \frac{(2A)\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

$$= \frac{A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{A\,\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{ad}$$

$$= \frac{A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{(2A)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{ad} + \frac{(3A)\,\text{Sul}}{}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} + \frac{A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx}{2ad\sqrt{a-a\sec(c+dx)}}$$

Mathematica [C] time = 6.59493, size = 322, normalized size = 2.78

$$A\left(\frac{\sin^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-\frac{4\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{4\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}-\frac{2\cot\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{2\csc\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{(a-a\sec(c+dx))^{3/2}} - \frac{2\sqrt{2}e^{-}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] A*((-2*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] - (3*ArcTanh[(1 + E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] + ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(
c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((4*Cos[c/2]*Cos[(d
*x)/2])/d - (2*Cot[c/2]*Csc[c/2 + (d*x)/2])/d + (2*Csc[c/2]*Csc[c/2 + (d*x)
/2]^2*Sin[(d*x)/2])/d - (4*Sin[c/2]*Sin[(d*x)/2])/d)*Sin[c/2 + (d*x)/2]^3)/
(a - a*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.23, size = 298, normalized size = 2.6

$$-\frac{A\sqrt{2}\,(-1+\cos(dx+c))^2}{d\,(\sin(dx+c))^3}\left(\cos(dx+c)\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}+\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}+\cos(dx+c)\sqrt{2}\sqrt{-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x)
```

```
[Out] -A/d*2^(1/2)*(-1+cos(d*x+c))^2*(cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(3/2)+2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+cos(d*x+c)*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)*2^(1/2)*arctan(1/(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*cos(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))-2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*2
^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*arctan(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)
/sin(d*x+c)^3/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A\sec(dx+c)+A}{(-a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)/(-a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 0.53126, size = 1285, normalized size = 11.08

$$\left[-\frac{3\sqrt{2}(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4(A\cos}{4\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)
^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*
cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x
+ c) + 4*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c
))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a
)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(A*cos(d*x + c)^2 + A*cos(d*
x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2
*d)*sin(d*x + c)), 1/2*(3*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
```

```
c)))*sin(d*x + c) - 4*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x +
 c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) +
2*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c
)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\sec(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{1}{-a\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a\sqrt{\cdot}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(sec(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqr
t(-a*sec(c + d*x) + a)), x) + Integral(1/(-a*sqrt(-a*sec(c + d*x) + a)*sec(
c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A] time = 1.87604, size = 262, normalized size = 2.26

$$-\frac{A\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cdot}}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\cdot\right.\right.}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*A*(3*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/
2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 4*arctan(1/
2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 - a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2
*c))*tan(1/2*d*x + 1/2*c)^2))/d
```

3.172 $\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

```
[Out] (5*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(3/2)*d) -
 (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(S
qrt[2]*a^(3/2)*d) - (A*Sin[c + d*x])/(d*(a - a*Sec[c + d*x])^(3/2)) + (2*A*
Sin[c + d*x])/(a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.354706, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (5*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(3/2)*d) -
 (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(S
qrt[2]*a^(3/2)*d) - (A*Sin[c + d*x])/(d*(a - a*Sec[c + d*x])^(3/2)) + (2*A*
Sin[c + d*x])/(a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx &= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}+\frac{\int \frac{\cos(c+dx)(4aA+3aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^2}\\
&= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}+\frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}}-\frac{\int \frac{-5a^2A-2a^2A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^3}\\
&= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}+\frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}}+\frac{(5A)\int \sqrt{a-a\sec(c+dx}}{2a^2}\\
&= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}+\frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}}+\frac{(5A)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,\right.}{ad}\\
&= \frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}-\frac{A\sin(c+dx)}{d(a-a\sec(c+dx)}\end{aligned}$$

Mathematica [C] time = 6.61696, size = 361, normalized size = 2.47

$$A\left(\frac{\sin^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-\frac{2\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{2\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d}+\frac{2\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}-\frac{2\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d}-\frac{2\cot\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right.}{(a-a\sec(c+dx))^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*(-5*ArcSinh[E^(I*(c + d*x))] + 7*Sqrt[2]*ArcTanh[(1 + E^(I*(
c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 5*ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(
c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((2*Cos[c/2]*Cos[(d
*x)/2])/d - (2*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d - (2*Cot[c/2]*Csc[c/2 + (d*x)
/2])/d + (2*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/d - (2*Sin[c/2]*Sin
[(d*x)/2])/d + (2*Sin[(3*c)/2]*Sin[(3*d*x)/2])/d)*Sin[c/2 + (d*x)/2]^3)/(a
- a*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.283, size = 462, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3*A/d*2^(1/2)*(-1+cos(d*x+c))^3*(-3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*
cos(d*x+c)^2*2^(1/2)-6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*2^(1
/2)-7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*2^(1/2)-3*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)+3*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+21*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(
d*x+c)^2*2^(1/2)-2*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+7*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+30*arctan(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+5*cos(d*x+c)*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))-6*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-30*arctan(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))/(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(3/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)}{(-a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/(-a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0.541201, size = 1345, normalized size = 9.21

$$\left[-\frac{7\sqrt{2}(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+10(A c\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/4*(7*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)
^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*
cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x
+ c) + 10*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x +
c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) +
```

```
a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 4*(A*cos(d*x + c)^3 - A*cos(d
*x + c)^2 - 2*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^
2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/2*(7*sqrt(2)*(A*cos(d*x + c) - A
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 10*(A*cos(d*x + c) - A)*sqrt(a)*a
rctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c)))*sin(d*x + c) - 2*(A*cos(d*x + c)^3 - A*cos(d*x + c)^2 - 2*A*cos(d*x
 + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*
d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\cos(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx + \int \frac{\cos(c+dx)\sec(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a\sqrt{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqr
t(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)*sec(c + d*x)/(-a*sqrt(-
a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [B] time = 2.06882, size = 344, normalized size = 2.36

$$-\frac{A\left(\frac{7\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{\sqrt{2}\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}+4\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,a\right)}{\left(\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+2a^2\right)a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\cdots\right.\right.}\cdots\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/2*A*(7*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/
2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(3*
(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2) + 4*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)
*a)/(((a*tan(1/2*d*x + 1/2*c)^2 - a)^2 + 3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a
 + 2*a^2)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 10
*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))))/d
```

3.173 $\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{31A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d}-\frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}}+\frac{3A\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin}{d(a}$$

```
[Out] (31*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*a^(3/2)*d
) - (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(d*(a - a*Sec[c + d*x
])^(3/2)) + (13*A*Sin[c + d*x])/(4*a*d*Sqrt[a - a*Sec[c + d*x]]) + (3*A*Cos
[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.529145, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{31A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d}-\frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}}+\frac{3A\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin}{d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (31*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*a^(3/2)*d
) - (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(d*(a - a*Sec[c + d*x
])^(3/2)) + (13*A*Sin[c + d*x])/(4*a*d*Sqrt[a - a*Sec[c + d*x]]) + (3*A*Cos
[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a - a*Sec[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx = -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)(6aA+5aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{3A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(-13a^2\right.}{\sqrt{a-a\,\mathrm{s}}}}{4}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(c+dx)\sin(}{2ad\sqrt{a-a\sec(c+}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(c+dx)\sin(}{2ad\sqrt{a-a\sec(c+}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(c+dx)\sin(}{2ad\sqrt{a-a\sec(c+}}$$

$$= \frac{31A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} - \frac{A\cos(c+dx)\,\mathrm{si}}{d(a-a\sec(c+}$$

Mathematica [C] time = 6.66512, size = 408, normalized size = 2.1

$$A\left(\frac{\sin^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(\frac{3\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{2d}+\frac{5\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d}+\frac{\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{2d}-\frac{3\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{2d}-\frac{5\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d}-\frac{\mathrm{co}}{}\right.}{(a-a\sec(c+dx))^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] A*(-(Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*(31*ArcSinh[E^(I*(c + d*x))] - 44*Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*
```

```
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 31*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(2*Sqrt[2]*d*E^((I
/2)*(c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((-3*Cos[c/2]*
Cos[(d*x)/2])/(2*d) - (5*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d - (Cos[(5*c)/2]*Cos
[(5*d*x)/2])/(2*d) - (2*Cot[c/2]*Csc[c/2 + (d*x)/2])/d + (2*Csc[c/2]*Csc[c/
2 + (d*x)/2]^2*Sin[(d*x)/2])/d + (3*Sin[c/2]*Sin[(d*x)/2])/(2*d) + (5*Sin[(
3*c)/2]*Sin[(3*d*x)/2])/d + (Sin[(5*c)/2]*Sin[(5*d*x)/2])/(2*d))*Sin[c/2 +
(d*x)/2]^3)/(a - a*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.351, size = 883, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/60*A/d*2^(1/2)*(-1+cos(d*x+c))^4*(-278*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-930*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))+132*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*2^(1/2)+18
0*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*2^(1/2)+180*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*2^(1/2)+132*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*cos(d*x+c)^3*2^(1/2)-220*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*co
s(d*x+c)^3*2^(1/2)+660*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d
*x+c)^3*2^(1/2)+60*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^3*2^(1/2
)-132*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)+930*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-195*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)-660*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))-132*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*2^(1/2)+30*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*2^(1/2)-220*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*cos(d*x+c)^2*2^(1/2)+195*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)^4*2^(1/2)+660*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*cos(d*x+c)^2*2^(1/2)+220*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)+288*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-40*
cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-660*cos(d*x+c)*2^(1
/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+220*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(3/2)-930*cos(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+60*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+930*a
rctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3)/(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(3/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*
x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)^2}{(-a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/(-a*sec(d*x + c) + a)^(3/2),
x)
```

Fricas [A] time = 0.55599, size = 1415, normalized size = 7.29

$$\left[-\frac{22\sqrt{2}(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+31\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(22*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c
)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a
*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x
 + c) + 31*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x +
 c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) +
 a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*(2*A*cos(d*x + c)^4 + 9*A*
cos(d*x + c)^3 - 6*A*cos(d*x + c)^2 - 13*A*cos(d*x + c))*sqrt((a*cos(d*x +
c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/4*(22
*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 31*(A
*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*c
os(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - (2*A*cos(d*x + c)^4 + 9*
A*cos(d*x + c)^3 - 6*A*cos(d*x + c)^2 - 13*A*cos(d*x + c))*sqrt((a*cos(d*x
+ c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int\frac{\cos^2(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx+\int\frac{\cos^2(c+dx)\sec(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a\sqrt{\ldots}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)**2/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*
sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)**2*sec(c + d*x)/(-a*
sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A] time = 2.17359, size = 393, normalized size = 2.03

$$-\frac{A\left(\frac{22\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{31\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(7\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}+18\sqrt{a\ldots}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^2 a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\ldots\right.\right.}\ldots\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/4*A*(22*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3
/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 31*arctan(
1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(7*(a*tan(1/2*d*
x + 1/2*c)^2 - a)^(3/2) + 18*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a)/((a*tan(
1/2*d*x + 1/2*c)^2 + a)^2*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x
 + 1/2*c))) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/(a^2*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2))/d
```

3.174 $\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{85A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d}-\frac{15A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}}+\frac{4A\sin(c+dx)\cos^2(c+dx)}{3ad\sqrt{a-a\sec(c+dx)}}-$$

```
[Out] (85*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*a^(3/2)*d
) - (15*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a - a*Sec[c + d
*x])^(3/2)) + (35*A*Sin[c + d*x])/(8*a*d*Sqrt[a - a*Sec[c + d*x]]) + (25*A*
Cos[c + d*x]*Sin[c + d*x])/(12*a*d*Sqrt[a - a*Sec[c + d*x]]) + (4*A*Cos[c +
 d*x]^2*Sin[c + d*x])/(3*a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.701116, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{85A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d}-\frac{15A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}}+\frac{4A\sin(c+dx)\cos^2(c+dx)}{3ad\sqrt{a-a\sec(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (85*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*a^(3/2)*d
) - (15*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a - a*Sec[c + d
*x])^(3/2)) + (35*A*Sin[c + d*x])/(8*a*d*Sqrt[a - a*Sec[c + d*x]]) + (25*A*
Cos[c + d*x]*Sin[c + d*x])/(12*a*d*Sqrt[a - a*Sec[c + d*x]]) + (4*A*Cos[c +
 d*x]^2*Sin[c + d*x])/(3*a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx = -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)(8aA+7aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{4A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)\left(-25a^2\right.}{\sqrt{a-a\,\mathrm{se}}}}{6}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{25A\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}} + \frac{4A\cos^2(c+dx)}{3ad\sqrt{a-a\sec}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\cos(c+dx)\sin(c}{12ad\sqrt{a-a\sec(c+}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\cos(c+dx)\sin(c}{12ad\sqrt{a-a\sec(c+}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\cos(c+dx)\sin(c}{12ad\sqrt{a-a\sec(c+}}$$

$$= \frac{85A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d} - \frac{15A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} - \frac{A\cos^2(c+dx)\sin}{d(a-a\sec(c+d}$$

Mathematica [C] time = 6.68658, size = 452, normalized size = 1.92

$$A\left(\frac{\sin^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(\frac{65\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d}+\frac{25\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{3d}+\frac{5\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{4d}+\frac{\sin\left(\frac{7c}{2}\right)\sin\left(\frac{7dx}{2}\right)}{6d}-\frac{65\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{12d}-\frac{25\,\mathrm{c}}{}\right.}{(a-a\sec(c+dx))^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] A*((-5*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c
 + d*x))]*(17*ArcSinh[E^(I*(c + d*x))] - 24*Sqrt[2]*ArcTanh[(1 + E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 17*ArcTanh[Sqrt[1 + E^((2
*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sin[c/2 + (d*x)/2]^3)/(4*Sqrt[2]*d*E^(
(I/2)*(c + d*x))*(a - a*Sec[c + d*x])^(3/2)) + (Sec[c + d*x]^2*((-65*Cos[c/
2]*Cos[(d*x)/2])/(12*d) - (25*Cos[(3*c)/2]*Cos[(3*d*x)/2])/(3*d) - (5*Cos[(
5*c)/2]*Cos[(5*d*x)/2])/(4*d) - (Cos[(7*c)/2]*Cos[(7*d*x)/2])/(6*d) - (2*Co
t[c/2]*Csc[c/2 + (d*x)/2])/d + (2*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2
])/d + (65*Sin[c/2]*Sin[(d*x)/2])/(12*d) + (25*Sin[(3*c)/2]*Sin[(3*d*x)/2])
/(3*d) + (5*Sin[(5*c)/2]*Sin[(5*d*x)/2])/(4*d) + (Sin[(7*c)/2]*Sin[(7*d*x)/
2])/(6*d))*Sin[c/2 + (d*x)/2]^3)/(a - a*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.326, size = 1104, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/168*A/d*2^(1/2)*(-1+cos(d*x+c))^5*(1130*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-3570*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))+720*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*2^(1/2)+100
8*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^3*2^(1/2)-1680*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^3*2^(1/2)+5040*arctan(1/(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*2^(1/2)-720*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(7/2)*cos(d*x+c)^3*2^(1/2)-504*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2
^(1/2)-735*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2520*2^(1/2)*arctan
(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-1008*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*cos(d*x+c)*2^(1/2)+1225*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)^5*2^(1/2)-2103*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*2^(1/
2)-168*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+3570*cos(d*x+c)^4*arcta
n(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1680*cos(d*x+c)*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+952*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-875*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)-5040*cos(d*x+c)*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))+840*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-7140*cos(d*x+c)*arct
an(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+360*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(7/2)*2^(1/2)+7140*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)^3-672*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(9/2)+504*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-8
40*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+2520*2^(1/2)*c
os(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+56*2^(1/2)*cos(d
*x+c)^7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+350*2^(1/2)*cos(d*x+c)^6*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-168*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(9/2)-672*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(9/2)-360*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)-1008*2^
(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2))/(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)^3}{(-a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/(-a*sec(d*x + c) + a)^(3/2),
x)
```

Fricas [A] time = 0.572233, size = 1490, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/48*(180*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3
*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d
*x + c) + 255*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*
x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c
) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*(8*A*cos(d*x + c)^5 + 2
6*A*cos(d*x + c)^4 + 73*A*cos(d*x + c)^3 - 50*A*cos(d*x + c)^2 - 105*A*cos(
d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a
^2*d)*sin(d*x + c)), 1/24*(180*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(
sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c)))*sin(d*x + c) - 255*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x
+ c) - (8*A*cos(d*x + c)^5 + 26*A*cos(d*x + c)^4 + 73*A*cos(d*x + c)^3 - 50
*A*cos(d*x + c)^2 - 105*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x +
 c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.23083, size = 425, normalized size = 1.8

$$-\frac{A\left(\frac{180\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{255\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(63\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{\frac{5}{2}}+272\left(a\tan\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{3}a\operatorname{sgn}\left(\mathrm{ta}\right.}\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/24*A*(180*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^
(3/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 255*arct
an(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(3/2)*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(63*(a*tan(1/
2*d*x + 1/2*c)^2 - a)^(5/2) + 272*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*a +
324*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)
^3*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 12*sqrt(2
)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2))/d
```

3.175 $\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}-\frac{7A\tan(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}}-\frac{A\tan(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) -
 (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(
8*Sqrt[2]*a^(5/2)*d) - (A*Tan[c + d*x])/(2*d*(a - a*Sec[c + d*x])^(5/2)) -
(7*A*Tan[c + d*x])/(8*a*d*(a - a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.206017, antiderivative size = 185, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3904, 3887, 471, 527, 522, 203}

$$\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}+\frac{7A\sin(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{16a^2d\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)\cos(c+dx)\,c}{8a^2d\sqrt{a-a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) -
 (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(
8*Sqrt[2]*a^(5/2)*d) + (7*A*Csc[(c + d*x)/2]^2*Sin[c + d*x])/(16*a^2*d*Sqrt
[a - a*Sec[c + d*x]]) - (A*Cos[c + d*x]*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(8
*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{5/2}}\,dx = -\left((aA)\int \frac{\tan^2(c+dx)}{(a-a\sec(c+dx))^{7/2}}\,dx\right)$$

$$= \frac{(2A)\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{A\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\,\text{Subst}\left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c}{\sqrt{a-a\,\text{se}}}\right.}{2a^2d}$$

$$= \frac{7A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{16a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\,\text{Subs}}{}$$

$$= \frac{7A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{16a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} - \frac{(2A)\,\text{Su}}{}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} + \frac{7A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+}{16a^2d\sqrt{a-a\sec(c+dx)}}$$

Mathematica [C] time = 6.73887, size = 387, normalized size = 2.55

$$A\left(\frac{\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\left(\frac{11\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \frac{11\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d} - \frac{\cot\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{15\cot\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} + \frac{\csc\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\text{cs}}{d}\right.}{(a-a\sec(c+dx))^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] A*((Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
d*x))]*(8*ArcSinh[E^(I*(c + d*x))] - (23*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] + 8*ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*E^((I/2)
*(c + d*x))*(a - a*Sec[c + d*x])^(5/2)) + (Sec[c + d*x]^3*((-11*Cos[c/2]*Co
s[(d*x)/2])/d + (15*Cot[c/2]*Csc[c/2 + (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2
+ (d*x)/2]^3)/d - (15*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (
Csc[c/2]*Csc[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/d + (11*Sin[c/2]*Sin[(d*x)/2])/
d)*Sin[c/2 + (d*x)/2]^5)/(a - a*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.257, size = 695, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/12*A/d*2^(1/2)*(-1+cos(d*x+c))^4*(21*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*cos(d*x+c)^3*2^(1/2)+33*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2
*2^(1/2)+23*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^3*2^(1/2)+3*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*2^(1/2)-23*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(3/2)*cos(d*x+c)^2*2^(1/2)-9*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*2^(1/2)-5*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-69*a
rctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*2^(1/2)-23*cos(d
*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-96*arctan(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-11*cos(d*x+c)^2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+69*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)^2*2^(1/2)+23*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)+96*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^
2+37*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+69*cos(d*x+c)*
2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+96*cos(d*x+c)*arctan
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-21*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)-69*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))-96*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))/(a*(-1+
cos(d*x+c))/cos(d*x+c))^(5/2)/sin(d*x+c)^7/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A\sec(dx+c)+A}{(-a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)/(-a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 0.543397, size = 1544, normalized size = 10.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(23*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((
2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a
)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*
sin(d*x + c)))*sin(d*x + c) + 32*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*
sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x +
c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*
sin(d*x + c) - 4*(11*A*cos(d*x + c)^3 + 4*A*cos(d*x + c)^2 - 7*A*cos(d*x +
c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3
*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/16*(23*sqrt(2)*(A*cos(d*x + c)^2
- 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 32*(A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(11*A*c
os(d*x + c)^3 + 4*A*cos(d*x + c)^2 - 7*A*cos(d*x + c))*sqrt((a*cos(d*x + c)
 - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)
*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\sec(c+dx)}{a^2\sqrt{-a\sec(c+dx)+a}\sec^2(c+dx)-2a^2\sqrt{-a\sec(c+dx)+a}\sec(c+dx)+a^2\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] A*(Integral(sec(c + d*x)/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 -
2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) +
 a)), x) + Integral(1/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 - 2*a
**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) + a)
), x))
```

Giac [A] time = 2.03052, size = 296, normalized size = 1.95

$$-\frac{A\left(\frac{23\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{32\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(9\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}+7\sqrt{a\tan}\right.}{a^4\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\right.\right.}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*A*(23*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(
5/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 32*arctan
(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(5/2)*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(9*(a*tan(1/2*d
```

```
*x + 1/2*c)^2 - a)^(3/2) + 7*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a)/(a^4*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)
^4))/d
```

3.176 $\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}+\frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{79A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}-\frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}}-\frac{}{2d(a\dots}$$

```
[Out] (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) -
 (79*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(
8*Sqrt[2]*a^(5/2)*d) - (A*Sin[c + d*x])/(2*d*(a - a*Sec[c + d*x])^(5/2)) -
(11*A*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x])^(3/2)) + (23*A*Sin[c + d*x]
)/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.505332, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}+\frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{79A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}-\frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}}-\frac{}{2d(a\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) -
 (79*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(
8*Sqrt[2]*a^(5/2)*d) - (A*Sin[c + d*x])/(2*d*(a - a*Sec[c + d*x])^(5/2)) -
(11*A*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x])^(3/2)) + (23*A*Sin[c + d*x]
)/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx = -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)(6aA+5aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(23a^2A+\frac{33}{2}a^2A\,\mathrm{se}\right.}{\sqrt{a-a\sec(c+dx)}}}{8a^4}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}$$

$$= \frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{79A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} - \frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))}$$

Mathematica [C] time = 6.79081, size = 423, normalized size = 2.3

$$A\left(\frac{\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\left(\frac{15\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \frac{4\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d} - \frac{15\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d} + \frac{4\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d} - \frac{\cot\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{23}{}\right.}{(a-a\sec(c+dx))^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] A*((Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
d*x))]*(28*ArcSinh[E^(I*(c + d*x))] - (79*ArcTanh[(1 + E^(I*(c + d*x)))/(Sq
```

```
rt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])]/Sqrt[2] + 28*ArcTanh[Sqrt[1 + E^((2*
I)*(c + d*x))]])*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*E^((I/
2)*(c + d*x))*(a - a*Sec[c + d*x])^(5/2)) + (Sec[c + d*x]^3*((-15*Cos[c/2]*
Cos[(d*x)/2])/d + (4*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d + (23*Cot[c/2]*Csc[c/2
+ (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2 + (d*x)/2]^3)/d - (23*Csc[c/2]*Csc[c/
2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^4*Sin[(d*
x)/2])/d + (15*Sin[c/2]*Sin[(d*x)/2])/d - (4*Sin[(3*c)/2]*Sin[(3*d*x)/2])/d
)*Sin[c/2 + (d*x)/2]^5)/(a - a*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.342, size = 788, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/60*A/d*2^(1/2)*(-1+cos(d*x+c))^5*(195*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(7/2)+450*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)
^3*2^(1/2)+237*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+18
0*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*2^(1/2)-210*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*2^(1/2)-395*2^(1/2)*cos(d*x+c)^4*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(3/2)-474*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*co
s(d*x+c)^2*2^(1/2)-135*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+120*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*2^(1/2)-343*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*2^(1/2)+1185*2^(1/2)*cos(d*x+c)^4*arctan(
1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+790*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*cos(d*x+c)^2*2^(1/2)+237*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)
+1680*cos(d*x+c)^4*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
+736*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-578*cos(d*x+
c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2370*arctan(1/(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*2^(1/2)-395*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(3/2)-3360*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)^2-280*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+345*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1185*2^(1/2)*arctan
(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1680*arctan(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)))/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)/(a*(-1+co
s(d*x+c))/cos(d*x+c))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A\sec(dx+c)+A)\cos(dx+c)}{(-a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/(-a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 0.55588, size = 1609, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/32*(79*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((
2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a
)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*
sin(d*x + c)))*sin(d*x + c) + 112*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)
*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x +
 c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))
*sin(d*x + c) + 4*(8*A*cos(d*x + c)^4 - 27*A*cos(d*x + c)^3 - 12*A*cos(d*x
+ c)^2 + 23*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/16*(79*sq
rt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*si
n(d*x + c) - 112*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
*sin(d*x + c) - 2*(8*A*cos(d*x + c)^4 - 27*A*cos(d*x + c)^3 - 12*A*cos(d*x
+ c)^2 + 23*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.26015, size = 393, normalized size = 2.14

$$-\frac{A\left(\frac{79\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{112\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\right.}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/16*A*(79*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(
5/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 112*arcta
n(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(5/2)*sgn(tan(
```

```
1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 16*sqrt(2)*sqrt(a*tan(
1/2*d*x + 1/2*c)^2 - a)/((a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(17*(a*tan(1/2*d*x +
1/2*c)^2 - a)^(3/2) + 15*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a)/(a^4*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^4))
/d
```

3.177 $\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$\frac{49A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}+\frac{59A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d}-\frac{167A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}+\frac{23A\sin(c+dx)\cos(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}-15$$

```
[Out] (59*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*a^(5/2)*d
) - (167*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*(a - a*Sec[c
+ d*x])^(5/2)) - (15*A*Cos[c + d*x]*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x
])^(3/2)) + (49*A*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (23*A*
Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.730891, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{49A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}+\frac{59A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d}-\frac{167A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}+\frac{23A\sin(c+dx)\cos(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}-15$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (59*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*a^(5/2)*d
) - (167*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*(a - a*Sec[c
+ d*x])^(5/2)) - (15*A*Cos[c + d*x]*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x
])^(3/2)) + (49*A*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (23*A*
Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx = -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)(8aA+7aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(46a^{\cdots}\right.}{\sqrt{a-\cdots}}}{\cdots}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A\cos(c+d\cdots}{8a^2d\sqrt{a-a\cdots}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{49A\sin(c\cdots}{8a^2d\sqrt{a-a\sec\cdots}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{49A\sin(c\cdots}{8a^2d\sqrt{a-a\sec\cdots}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{49A\sin(c\cdots}{8a^2d\sqrt{a-a\sec\cdots}}$$

$$= \frac{59A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{167A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} - \frac{A\cos(c+dx)\cdots}{2d(a-a\sec(\cdots}$$

Mathematica [C] time = 6.81463, size = 458, normalized size = 1.94

$$A\left(\frac{\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\left(\frac{12\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}-\frac{14\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d}-\frac{\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{d}-\frac{12\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}+\frac{14\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d}+\cdots\right.}{(a-a\sec(c+\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] A*((Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
d*x))]*(59*ArcSinh[E^(I*(c + d*x))] - (167*ArcTanh[(1 + E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])])/Sqrt[2] + 59*ArcTanh[Sqrt[1 + E^((2
*I)*(c + d*x))]])*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*E^((I
/2)*(c + d*x))*(a - a*Sec[c + d*x])^(5/2)) + (Sec[c + d*x]^3*((-12*Cos[c/2]
*Cos[(d*x)/2])/d + (14*Cos[(3*c)/2]*Cos[(3*d*x)/2])/d + (Cos[(5*c)/2]*Cos[(
5*d*x)/2])/d + (31*Cot[c/2]*Csc[c/2 + (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2 +
 (d*x)/2]^3)/d - (31*Csc[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (C
sc[c/2]*Csc[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/d + (12*Sin[c/2]*Sin[(d*x)/2])/d
 - (14*Sin[(3*c)/2]*Sin[(3*d*x)/2])/d - (Sin[(5*c)/2]*Sin[(5*d*x)/2])/d)*Si
n[c/2 + (d*x)/2]^5)/(a - a*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.34, size = 1475, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/420*A/d*2^(1/2)*(-1+cos(d*x+c))^6*(-1322*cos(d*x+c)^3*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)+24780*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))-7014*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*2^(1/2
)+5010*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*2^(1/2)-2505*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*2^(1/2)-7014*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(5/2)*cos(d*x+c)^3*2^(1/2)+11690*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(3/2)*cos(d*x+c)^3*2^(1/2)-35070*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)^3*2^(1/2)+5010*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*
x+c)^3*2^(1/2)+3507*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)-49560*arct
an(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+5145*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+17535*2^(1/2)*arctan(1/(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))+3507*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*
x+c)*2^(1/2)-11633*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*2^(1/2
)+11690*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*2^(1/2)+15573*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*2^(1/2)-35070*arctan(1/(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*2^(1/2)+1575*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(9/2)+24780*cos(d*x+c)^4*arctan(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))-5845*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)-12768*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+1015*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+17535*cos
(d*x+c)*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+24780*cos(d*
x+c)^5*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-1995*2^(1/2
)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-2505*2^(1/2)*cos(d*x+c)
^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)+3507*2^(1/2)*cos(d*x+c)^5*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)-5845*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)+17535*2^(1/2)*cos(d*x+c)^5*arctan(1/(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))-5845*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+24780*cos(
d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2505*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-49560*arctan(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+4305*2^(1/2)*cos(d*x+c)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(9/2)+3507*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(5/2)-5845*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2
)+17535*2^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
+420*2^(1/2)*cos(d*x+c)^7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3570*2^(1/2)
```

```
*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6405*2^(1/2)*cos(d*x+c)^
4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-5670*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(9/2)-2505*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(7/2)+1470*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/
2))/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(5/
2)/sin(d*x+c)^11
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.574989, size = 1671, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/32*(167*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log(
(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) -
a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)
*sin(d*x + c)))*sin(d*x + c) + 236*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A
)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x
+ c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)
)*sin(d*x + c) + 4*(4*A*cos(d*x + c)^5 + 22*A*cos(d*x + c)^4 - 57*A*cos(d*x
 + c)^3 - 26*A*cos(d*x + c)^2 + 49*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a
)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin
(d*x + c)), 1/16*(167*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))*sin(d*x + c) - 236*(A*cos(d*x + c)^2 - 2*A*cos(d*x +
c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(4*A*cos(d*x + c)^5 + 22*A*cos(d*
x + c)^4 - 57*A*cos(d*x + c)^3 - 26*A*cos(d*x + c)^2 + 49*A*cos(d*x + c))*s
qrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*co
s(d*x + c) + a^3*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.15859, size = 409, normalized size = 1.73

$$-\frac{A\left(\frac{167\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{236\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\left(69\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{\frac{7}{2}}+315\left(a\tan\left(\frac{1}{2}d\right.\right.\right.}{\left(\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}+3\left(a\tan\left(\frac{1}{2}c\right.\right.\right.}\right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/16*A*(167*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^
(5/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 236*arct
an(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(5/2)*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - sqrt(2)*(69*(a*tan(1/
2*d*x + 1/2*c)^2 - a)^(7/2) + 315*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(5/2)*a +
444*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*a^2 + 196*sqrt(a*tan(1/2*d*x + 1/2
*c)^2 - a)*a^3)/(((a*tan(1/2*d*x + 1/2*c)^2 - a)^2 + 3*(a*tan(1/2*d*x + 1/2
*c)^2 - a)*a + 2*a^2)^2*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x
 + 1/2*c))))/d
```

3.178 $\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$\frac{21A\sin(c+dx)}{2a^2d\sqrt{a-a\sec(c+dx)}}+\frac{203A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d}-\frac{287A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}+\frac{77A\sin(c+dx)\cos^2(c+dx}{24a^2d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] (203*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*a^(5/2)*
d) - (287*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]]
)])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a - a*Sec
[c + d*x])^(5/2)) - (19*A*Cos[c + d*x]^2*Sin[c + d*x])/(8*a*d*(a - a*Sec[c
+ d*x])^(3/2)) + (21*A*Sin[c + d*x])/(2*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (
119*A*Cos[c + d*x]*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (77*
A*Cos[c + d*x]^2*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.906291, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4020, 4022, 3920, 3774, 203, 3795}

$$\frac{21A\sin(c+dx)}{2a^2d\sqrt{a-a\sec(c+dx)}}+\frac{203A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d}-\frac{287A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}+\frac{77A\sin(c+dx)\cos^2(c+dx}{24a^2d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```



```
[Out] (203*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*a^(5/2)*
d) - (287*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]]
)])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a - a*Sec
[c + d*x])^(5/2)) - (19*A*Cos[c + d*x]^2*Sin[c + d*x])/(8*a*d*(a - a*Sec[c
+ d*x])^(3/2)) + (21*A*Sin[c + d*x])/(2*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (
119*A*Cos[c + d*x]*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (77*
A*Cos[c + d*x]^2*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx = -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)(10aA+9aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(77a^2 \ldots\right.}{\sqrt{a-a \ldots}}}{\ldots}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{77A\cos^2(c+dx \ldots}{24a^2d\sqrt{a-a \ldots}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{119A\cos(c+dx \ldots}{24a^2d\sqrt{a-a \ldots}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{21A\sin(c+ \ldots}{2a^2d\sqrt{a-a\sec(\ldots}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{21A\sin(c+ \ldots}{2a^2d\sqrt{a-a\sec(\ldots}}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{21A\sin(c+ \ldots}{2a^2d\sqrt{a-a\sec(\ldots}}$$

$$= \frac{203A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d} - \frac{287A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d} - \frac{A\cos^2(c+dx)\,\mathrm{s} \ldots}{2d(a-a\sec(c \ldots}$$

Mathematica [C] time = 6.8404, size = 514, normalized size = 1.84

$$A\left(\frac{\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\left(\frac{7\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{6d} - \frac{92\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{3d} - \frac{7\sin\left(\frac{5c}{2}\right)\sin\left(\frac{5dx}{2}\right)}{2d} - \frac{\sin\left(\frac{7c}{2}\right)\sin\left(\frac{7dx}{2}\right)}{3d} - \frac{7\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{6d} + \frac{92\cos \ldots}{\ldots}\right.}{(\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] A*((7*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c
+ d*x))]*(29*ArcSinh[E^(I*(c + d*x))] - 41*Sqrt[2]*ArcTanh[(1 + E^(I*(c + d
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 29*ArcTanh[Sqrt[1 + E^((2*
I)*(c + d*x))]])*Sec[c + d*x]^(5/2)*Sin[c/2 + (d*x)/2]^5)/(2*Sqrt[2]*d*E^((
I/2)*(c + d*x))*(a - a*Sec[c + d*x])^(5/2)) + (Sec[c + d*x]^3*((-7*Cos[c/2]
*Cos[(d*x)/2])/(6*d) + (92*Cos[(3*c)/2]*Cos[(3*d*x)/2])/(3*d) + (7*Cos[(5*c
)/2]*Cos[(5*d*x)/2])/(2*d) + (Cos[(7*c)/2]*Cos[(7*d*x)/2])/(3*d) + (39*Cot[
c/2]*Csc[c/2 + (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2 + (d*x)/2]^3)/d - (39*Cs
c[c/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (Csc[c/2]*Csc[c/2 + (d*x)
/2]^4*Sin[(d*x)/2])/d + (7*Sin[c/2]*Sin[(d*x)/2])/(6*d) - (92*Sin[(3*c)/2]*
Sin[(3*d*x)/2])/(3*d) - (7*Sin[(5*c)/2]*Sin[(5*d*x)/2])/(2*d) - (Sin[(7*c)/
2]*Sin[(7*d*x)/2])/(3*d))*Sin[c/2 + (d*x)/2]^5)/(a - a*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.44, size = 1964, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/180*A/d*2^(1/2)*(-1+cos(d*x+c))^7*(-10335*cos(d*x+c)^3*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)-945*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/
2)+18270*cos(d*x+c)^6*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))+18270*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2583*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*2^(1/2)+1845*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*2^(1/2)-3690*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(7/2)*cos(d*x+c)*2^(1/2)-10332*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos
(d*x+c)^3*2^(1/2)+17220*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^3*2
^(1/2)-51660*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*2^
(1/2)+7380*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^3*2^(1/2)+2583*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)-18270*arctan(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+3780*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+12915*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))+5166*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*2^(1/2)+14285*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*2^(1/2)+4305*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*2^(1/2)+6254*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)^4*2^(1/2)-12915*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)^2*2^(1/2)+1435*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(9/2)-18270*cos(d*x+c)^4*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))-8610*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-8652*co
s(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4515*cos(d*x+c)*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+25830*cos(d*x+c)*2^(1/2)*arctan(1
/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+36540*cos(d*x+c)^5*arctan(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2870*2^(1/2)*cos(d*x+c)^5*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(9/2)-3690*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)+5166*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)-8610*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+25830*2^
(1/2)*cos(d*x+c)^5*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+120*2^(1/
2)*cos(d*x+c)^9*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+930*2^(1/2)*cos(d*x+c)
^8*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1125*2^(1/2)*cos(d*x+c)^6*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(11/2)+4680*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(cos(
```

```
d*x+c)+1))^(11/2)+1435*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(9/2)+6525*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)+1800*
2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)-1845*2^(1/2)*cos
(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)-3825*2^(1/2)*cos(d*x+c)^2*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)-3600*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(11/2)+2583*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)-4305*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+1
2915*2^(1/2)*cos(d*x+c)^6*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-43
05*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+36540*cos(d*x+c)*arctan(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-1845*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*2^(1/2)-73080*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)^3+2870*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(9/2)-2583*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+430
5*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-12915*2^(1/2)*c
os(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4215*2^(1/2)*cos
(d*x+c)^7*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-15112*2^(1/2)*cos(d*x+c)^6*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1435*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(9/2)-5740*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(9/2)+1845*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)-
1435*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2))/(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(5/2)/sin(d*x+c)^1
3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.612642, size = 1744, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/96*(861*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log(
(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) -
a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)
*sin(d*x + c)))*sin(d*x + c) + 1218*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) +
A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x
 + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c
))*sin(d*x + c) + 4*(8*A*cos(d*x + c)^6 + 30*A*cos(d*x + c)^5 + 113*A*cos(d
*x + c)^4 - 294*A*cos(d*x + c)^3 - 133*A*cos(d*x + c)^2 + 252*A*cos(d*x + c
))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*
d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/48*(861*sqrt(2)*(A*cos(d*x + c)^2
- 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 1218*(A*cos
```

```
(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(8*A*
cos(d*x + c)^6 + 30*A*cos(d*x + c)^5 + 113*A*cos(d*x + c)^4 - 294*A*cos(d*x
 + c)^3 - 133*A*cos(d*x + c)^2 + 252*A*cos(d*x + c))*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*s
in(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.04878, size = 459, normalized size = 1.64

$$-\frac{A\left(\frac{861\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{1218\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(129\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{\frac{5}{2}}+560\left(a\,\mathrm{t}\cdots\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{3}a^{2}\mathrm{sgn}\left(\cdots\right.}\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/48*A*(861*sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^
(5/2)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 1218*arc
tan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(a^(5/2)*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 2*sqrt(2)*(129*(a*ta
n(1/2*d*x + 1/2*c)^2 - a)^(5/2) + 560*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*
a + 636*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2)/((a*tan(1/2*d*x + 1/2*c)^2
+ a)^3*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))) - 3*s
qrt(2)*(33*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2) + 31*sqrt(a*tan(1/2*d*x + 1
/2*c)^2 - a)*a)/(a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*
c))*tan(1/2*d*x + 1/2*c)^4))/d
```

3.179 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=199

$$\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(A+B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin}{}$$

```
[Out] (-6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(21*d) + (6*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
5*d) + (2*a*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*(A +
 B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*B*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d)
```

Rubi [A] time = 0.179355, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3997, 3787, 3768, 3771, 2641, 2639}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{6a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(21*d) + (6*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
5*d) + (2*a*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*(A +
 B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*B*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d)
```


Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(7A\right.$$

$$= \frac{2aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B))\int \sec^{\frac{7}{2}}(c+dx$$

$$= \frac{2a(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{2a(A+B)\sec^{\frac{5}{2}}(c}{5}$$

$$= \frac{6a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a(7A+5B)\sec^{\frac{3}{2}}(c}{21}$$

$$= \frac{2a(7A+5B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} +$$

$$= -\frac{6a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +$$

Mathematica [A] time = 0.739832, size = 200, normalized size = 1.01

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)(A+B\sec(c+dx))\left(5(7A+5B)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-63(A+B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(A + B*Sec[c + d*x])*(-63*(A + B)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(7*A + 5*B)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 63*A*Sin[c + d*x] + 63*B*Sin[c + d*x] + 35
*A*Tan[c + d*x] + 25*B*Tan[c + d*x] + 21*A*Sec[c + d*x]*Tan[c + d*x] + 21*B
*Sec[c + d*x]*Tan[c + d*x] + 15*B*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d*(B +
 A*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [B] time = 5.554, size = 691, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*(1/2*A+1/2*B)/(8*sin(1/2*d*x+1/2*c)^6-1
2*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)(a\sec (dx+c)+a)\sec (dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\sec (dx+c)^4+(A+B)a\sec (dx+c)^3+Aa\sec (dx+c)^2\right)\sqrt{\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^4 + (A + B)*a*sec(d*x + c)^3 + A*a*sec(d*x + c)^
2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)(a\sec (dx+c)+a)\sec (dx+c)^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.180 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=172

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(5A+3B)\sin(c}{5}$$

```
[Out] (-2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*B*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.164512, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*B*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(5A\right.$$
$$= \frac{2aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \sec^{\frac{5}{2}}(c+dx$$
$$= \frac{2a(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a(A+B)\sec^{\frac{3}{2}}(c}{3}$$
$$= \frac{2a(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a(A+B)\sec^{\frac{3}{2}}(c}{3}$$
$$= -\frac{2a(5A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A] time = 0.643106, size = 168, normalized size = 0.98

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)(A+B\sec(c+dx))\left(5(A+B)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-3(5A+3B)\right.}{15d\sec^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(A + B*Sec[c + d*x])*(-3*(5*A + 3*
B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(A + B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 15*A*Sin[c + d*x] + 9*B*Sin[c + d*x] + 5*A*
Tan[c + d*x] + 5*B*Tan[c + d*x] + 3*B*Sec[c + d*x]*Tan[c + d*x]))/(15*d*(B
+ A*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [B] time = 5.197, size = 662, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*(1/2*A+1/2*
B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*B/(8*sin(1/2*d*x+1/2*c)^6-1
2*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Ba\sec\left(dx+c\right)^{3}+(A+B)a\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^3 + (A + B)*a*sec(d*x + c)^2 + A*a*sec(d*x + c))
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.181 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a(A+B)\sqrt{\cos(c}{}$$

```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.14369, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a(3\right.$$
$$= \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + (a(A+B))\int \sec^{\frac{3}{2}}(c+dx$$
$$= \frac{2a(A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin}{3d}$$
$$= \frac{2a(3A+B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} +$$
$$= -\frac{2a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [A] time = 0.515784, size = 94, normalized size = 0.7

$$\frac{a\sec^{\frac{3}{2}}(c+dx)\left(2(3A+B)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-6(A+B)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+d\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(-6*(A + B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2,
 2] + 2*(3*A + B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*(
A + B)*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 4.205, size = 427, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(
-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*(1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)*sqrt(sec(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.182 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

```
[Out] (2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.134114, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3997, 3787, 3771, 2639, 2641}

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2aB\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + 2\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c+d}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aB\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + (a(A-B))\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + (a(A$$

$$= \frac{2aB\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{2a(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a(A+B)\sqrt{c}}{}$$

Mathematica [A] time = 0.298746, size = 77, normalized size = 0.73

$$\frac{2a\sqrt{\sec(c+dx)}\left((A+B)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sin(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + (A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x])
)/d
```

Maple [A] time = 1.761, size = 240, normalized size = 2.3

$$-2\,\frac{a\left(A\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-A\sqrt{2\,(\sin(1/2\,dx+c/2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*a*(A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2 + (A+B)a\sec(dx+c) + Aa}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sqrt(sec(d*x +
 c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int A\sqrt{\sec(c+dx)}\,dx + \int B\sqrt{\sec(c+dx)}\,dx + \int B\sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)), x) +
Integral(B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.183 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.12956, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3996, 3787, 3771, 2639, 2641}

$$\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}a(A+B) - \frac{1}{2}a(A+3B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + (a(A+B))\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}(a(A+3B))\int \sqrt{\mathrm{se}$$
$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \left(a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}$$
$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a(A+3B)\sqrt{\cos}}{}$$

Mathematica [A] time = 0.307421, size = 83, normalized size = 0.75

$$\frac{a\sqrt{\sec(c+dx)}\left(2(A+3B)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(2(c+d}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + 2*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[2*(c
+ d*x)]))/(3*d)
```

Maple [B] time = 1.844, size = 321, normalized size = 2.9

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+A\sqrt{2\left(\sin(1/2\,dx+c/2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-2*A*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*B*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

MaximA [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sec(d*x + c)^(
3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx+\int \frac{B}{\sqrt{\sec(c+dx)}}\,dx+\int B\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) +
 Integral(B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.184 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{5d}$$

```
[Out] (2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.153239, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{1}{2}a(3A+5B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(a(3A+5B))\int$$
$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(a(A+B))\int \sqrt{\sec(c+$$
$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2aA\sin(c}{5d\sec^{\frac{3}{2}}(c}$$
$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+B)}{}$$

Mathematica [A] time = 0.562546, size = 99, normalized size = 0.7

$$\frac{a\sqrt{\sec(c+dx)}\left(10(A+B)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))(5(A+B)+3A\cos(c+dx))+6(3A}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*(A +
B) + 3*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 1.652, size = 355, normalized size = 2.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(44\,A+20\,B)\left(s\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-16*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-15*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sec(d*x + c)^(
5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(A/sec(c + d*x)**(3/2), x)
+ Integral(B/sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.185 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a(5A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{6}{}$$

```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.161307, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B)-\frac{1}{2}a(5A+7B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{7}(a(5A+7B))\int$$
$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$
$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$
$$= \frac{6a(A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(5A+7B)}{}$$

Mathematica [A] time = 0.969435, size = 113, normalized size = 0.66

$$\frac{a\sqrt{\sec(c+dx)}\left(20(5A+7B)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))(42(A+B)\cos(c+dx)+15A\cos(\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(252*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 20*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*A
+ 70*B + 42*(A + B)*Cos[c + d*x] + 15*A*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

Maple [A] time = 1.702, size = 383, normalized size = 2.2

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+(-528\,A-168\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(448*A+308*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-12
2*A-112*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sec(d*x + c)^(
7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=234

$$\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(7A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{4a^2(7A+6$$

```
[Out] (-4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (4*a^2*(7*A + 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
1*d) + (2*a^2*(7*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*B*Se
c[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.342083, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a^2(7A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{4a^2(7A+6B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{4a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (4*a^2*(7*A + 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
1*d) + (2*a^2*(7*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*B*Se
c[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{2B\sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d}+\frac{2}{7}\int$$

$$= \frac{2a^2(7A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2B\sec^{\frac{5}{2}}(c+dx}{}$$

$$= \frac{2a^2(7A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2B\sec^{\frac{5}{2}}(c+dx}{}$$

$$= \frac{4a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{4a^2(7A+6B)\,\mathrm{s}}{}$$

$$= \frac{4a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{4a^2(7A+6B)\,\mathrm{s}}{}$$

$$= -\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [C] time = 4.45944, size = 463, normalized size = 1.98

$$\frac{a^2\csc(c)e^{-idx}\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(7\sqrt{2}\left(-1+e^{2ic}\right)(4A+3B)e^{2idx}\sqrt{\frac{e}{1+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Csc[c]*Sec[(c + d*x)/2]^4*(7*Sqrt[2]*(4*A + 3*B)*E^((2*
I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] - ((-1 + E^((2*I)*c))*(7*A*(-5 + 9*E^(I*(c + d*x)) - 5*E^((2*I)*(
c + d*x)) + 36*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c
 + d*x)) + 5*E^((6*I)*(c + d*x)) + 12*E^((7*I)*(c + d*x))) + 3*B*(-10 + 7*E
^(I*(c + d*x)) - 20*E^((2*I)*(c + d*x)) + 63*E^((3*I)*(c + d*x)) + 20*E^((4
*I)*(c + d*x)) + 77*E^((5*I)*(c + d*x)) + 10*E^((6*I)*(c + d*x)) + 21*E^((7
*I)*(c + d*x))) + (5*I)*(7*A + 6*B)*(1 + E^((2*I)*(c + d*x)))^3*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(E^(I*(c - d*x))*(1
+ E^((2*I)*(c + d*x)))^3))*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/(210*
d*E^(I*d*x)*(B + A*Cos[c + d*x]))
```

Maple [B] time = 6.524, size = 852, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*(1/2*A+1/
4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-8/5*(1/4*A+1/2*B)/(8*sin(1/2*
d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+
1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1
/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2
+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)
^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*s
in(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\sec(dx+c)^4+(A+2B)a^2\sec(dx+c)^3+(2A+B)a^2\sec(dx+c)^2+Aa^2\sec(dx+c)\right)\sqrt{\sec(dx+c)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^4 + (A + 2*B)*a^2*sec(d*x + c)^3 + (2*A + B)*a
^2*sec(d*x + c)^2 + A*a^2*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

3.187 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=199

$$\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(5A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(5A+4B}{}$$

```
[Out] (-4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(5*A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(5*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*B*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.299393, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a^2(5A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(5A+4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(5*A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(5*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*B*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{2B\sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2a^2(5A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2B\sec^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2a^2(5A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2B\sec^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{4a^2(5A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a^2(5A+7B)}{}$$

$$= \frac{4a^2(2A+B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}$$

$$= -\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [C] time = 6.2851, size = 321, normalized size = 1.61

$$\frac{a^2e^{-ic}\left(-1+e^{2ic}\right)\csc(c)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(2(5A+4B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hy}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(-1 + E^((2*I)*c))*Csc[c]*(5*A + 10*B - 30*A*E^(I*(c + d*x)) - 18*B*E^
(I*(c + d*x)) - 60*A*E^((3*I)*(c + d*x)) - 54*B*E^((3*I)*(c + d*x)) - 5*A*E
```

```
^((4*I)*(c + d*x)) - 10*B*E^((4*I)*(c + d*x)) - 30*A*E^((5*I)*(c + d*x)) -
24*B*E^((5*I)*(c + d*x)) - (10*I)*(2*A + B)*(1 + E^((2*I)*(c + d*x)))^2*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5*A + 4*B)*E^(I*(c + d*x))*(
1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))])*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/(
60*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2*(B + A*Cos[c + d*x])*Sec[c + d*x]^
(5/2))
```

Maple [B] time = 5.715, size = 743, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*(
1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*B/(8*sin(1/2*d*x+
1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*
c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+8*(1/2*A+1/4*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\sec(dx+c)^3+(A+2B)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+Aa^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

3.188 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=160

$$\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2B\sin(c+\ldots}{}$$

```
[Out] (-4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*B*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.278166, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*B*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2B\sqrt{\sec(c+dx)}\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{(a+a\sec(c$$

$$= \frac{2a^2(3A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2B\sqrt{\sec(c+dx)}\left(a^2+a^2\,\mathrm{se}\right.}{3d}$$

$$= \frac{2a^2(3A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2B\sqrt{\sec(c+dx)}\left(a^2+a^2\,\mathrm{se}\right.}{3d}$$

$$= \frac{2a^2(3A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2B\sqrt{\sec(c+dx)}\left(a^2+a^2\,\mathrm{se}\right.}{3d}$$

$$= -\frac{4a^2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2(3A+2B)\sqrt{}}{}$$

Mathematica [C] time = 3.07969, size = 310, normalized size = 1.94

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(\frac{-3\csc(c)\cos(dx)(A\cos(2c)-A-4B)+6A\cos(c)\sin(dx)+2B\tan(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}-\frac{4i\sqrt{2}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(((-4*I)*
Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*Cos[c + d*x]^3*(3*B*E^(I*c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))] + E^(I*d*x)*((3*A + 2*B)*(-1 + E^((2*I)*c))*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + B*E^(I*(c + d*x))*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c)
)) + (-3*(-A - 4*B + A*Cos[2*c])*Cos[d*x]*Csc[c] + 6*A*Cos[c]*Sin[d*x] + 2*
B*Tan[c + d*x])/Sec[c + d*x]^(5/2)))/(12*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 2.067, size = 513, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/3*a^2*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+2*B)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(3*A+7*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*sin(1/2*d*x+1/2*c)^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ba^2\sec(dx+c)^3+(A+2B)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+Aa^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

3.189 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a^2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2A\sin(c+d}{3d}$$

```
[Out] (4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) - (2*a^2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.255765, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{2a^2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|2\right)}{3d}+\frac{2A\sin(c+dx)\left(a^2\right.}{3d\sqrt{\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) - (2*a^2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int\frac{(a+a\sec(c+dx))\left(\frac{1}{2}a(5$$

$$= -\frac{2a^2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2a^2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2a^2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{4a^2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2(2A+3B)\sqrt{\cos}}{}$$

Mathematica [C] time = 2.40561, size = 320, normalized size = 2.03

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2(A+B\sec(c+dx))\left(\frac{-3(2A-B)\csc(c)\cos(dx)-3(2A+B)\csc(c)\cos(2c+dx)+A\sin(2(c+dx))}{4d\sec^{\frac{5}{2}}(c+dx)}+\frac{i\sqrt{2}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((I*Sqrt[
2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*Cos[c + d*x]^3*(3*A*E^(I*c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))] + E^(I*d*x)*(-((2*A + 3*B)*(-1 + E^((2*I)*c))*Hypergeometric
2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]) + A*E^(I*(c + d*x))*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])))/(d*E^(I*d*x)*(-1 + E^((2*I)*c)
)) + (-3*(2*A - B)*Cos[d*x]*Csc[c] - 3*(2*A + B)*Cos[2*c + d*x]*Csc[c] + A*
Sin[2*(c + d*x)])/(4*d*Sec[c + d*x]^(5/2))))/(3*(B + A*Cos[c + d*x]))
```

Maple [B] time = 1.832, size = 388, normalized size = 2.5

$$-\frac{4a^2}{3d}\left(2A\sqrt{-2\left(\sin\left(1/2dx+c/2\right)\right)^4+\left(\sin\left(1/2dx+c/2\right)\right)^2}\cos\left(1/2dx+c/2\right)\left(\sin\left(1/2dx+c/2\right)\right)^4-\sqrt{-2\left(\sin\left(1/2dx+c/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^2*(2*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*A*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2\sec(dx+c)^3+(A+2B)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+Aa^2}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{2A}{\sqrt{\sec(c+dx)}}\,dx+\int A\sqrt{\sec(c+dx)}\,dx+\int\frac{B}{\sqrt{\sec(c+dx)}}\,dx+\int 2B\sqrt{\sec(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(2*A/sqrt(sec(c + d*x)),
 x) + Integral(A*sqrt(sec(c + d*x)), x) + Integral(B/sqrt(sec(c + d*x)), x)
 + Integral(2*B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

3.190 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}}{}$$

```
[Out] (4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```

Rubi [A] time = 0.260026, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{(a+a\sec(c+dx))\left(\frac{1}{2}a(7\right.}{\mathrm{s}}$$
$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{4}{1}$$
$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{3}$$
$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{3}$$
$$= \frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+}{}$$

Mathematica [C] time = 1.78556, size = 153, normalized size = 0.92

$$\frac{a^2\sqrt{\sec(c+dx)}\left(-4i(4A+5B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+20(A+2B)\sqrt{\cos(c+d}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(20*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] - (4*I)*(4*A + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((48*I
)*A + (60*I)*B + 10*(2*A + B)*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)])))/(15*d)
```

Maple [A] time = 1.663, size = 357, normalized size = 2.2

$$-\frac{4a^2}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(32\,A+10\,B)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(32*A+10*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-13*A-5*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-12*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-15*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2 \sec(dx+c)^3 + (A+2B)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{2A}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{2B}{\sqrt{\sec(c+dx)}}\,dx + \int B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(2*A/sec(c + d*x)**(3/2)
, x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(B/sec(c + d*x)**(3/2),
```

```
x) + Integral(2*B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x
)
```

3.191 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{4a^2(6A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(6A+7B)\sin(c+dx}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (4*a^2*(3*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.289876, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(6A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a^2*(3*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2}{7}\int\frac{(a+a\sec(c+dx))\left(\frac{1}{2}a(9\right.}{}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{4}{3}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{1}{5}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2A\left(a^2+a^2\right.}{7}$$

$$= \frac{4a^2(3A+4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a^2(9A+}{35d}$$

$$= \frac{4a^2(3A+4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(6A+}{}$$

Mathematica [C] time = 2.35129, size = 193, normalized size = 0.96

$$a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(3A+4B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(6*A + 7*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(3*A + 4*B)*E^(I*(c + d*x))*Sqrt
```

```
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((504*I)*A + (672*I)*B + 5*(51*A + 56*B)*Sin[c + d*x]
+ 42*(2*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x)
)
```

Maple [A] time = 1.915, size = 385, normalized size = 1.9

$$-\frac{4a^2}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-348\,A-84\,B\right)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(378*A+224*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
17*A-91*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-84*B*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ba^2\sec\left(dx+c\right)^3+(A+2\,B)a^2\sec\left(dx+c\right)^2+(2\,A+B)a^2\sec\left(dx+c\right)+Aa^2}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

3.192 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(11A+9B)\sin(c+d\cdots}{63d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] (4*a^2*(8*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (4*a^2*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(11*A + 9*B)*Sin[c + d*x])/(63*d
*Sec[c + d*x]^(5/2)) + (4*a^2*(8*A + 9*B)*Sin[c + d*x])/(45*d*Sec[c + d*x]^
(3/2)) + (4*a^2*(5*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*
(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.320409, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^2(5A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(\cdots}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (4*a^2*(8*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (4*a^2*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(11*A + 9*B)*Sin[c + d*x])/(63*d
*Sec[c + d*x]^(5/2)) + (4*a^2*(8*A + 9*B)*Sin[c + d*x])/(45*d*Sec[c + d*x]^
(3/2)) + (4*a^2*(5*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*
(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+dx))\left(\frac{1}{2}a(1\right.}{}$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} -$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} +$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A+}{21d\sqrt{s}}$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A+}{21d\sqrt{s}}$$

$$= \frac{4a^2(8A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^2(5A+}{}$$

Mathematica [C] time = 2.9442, size = 217, normalized size = 0.93

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(8A+9B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(5*A + 6*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(8*A + 9*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((2688*I)*A + (3024*I)*B + 30*(46*A + 51*B)*Sin[c +
d*x] + 14*(37*A + 36*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 90*B*Si
n[3*(c + d*x)] + 35*A*Sin[4*(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 1.755, size = 413, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1568*A+1134*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-351*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*A*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+90*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2 \sec\left(dx+c\right)^3 + (A+2B)a^2 \sec\left(dx+c\right)^2 + (2A+B)a^2 \sec\left(dx+c\right) + Aa^2}{\sec\left(dx+c\right)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x
)
```

3.193 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=277

$$\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(24A+23B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^3(13$$

```
[Out] (-4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(21*A + 17*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(13*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105
*d) + (2*a*B*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
+ (2*(9*A + 13*B)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(63*d)
```

Rubi [A] time = 0.540878, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{4a^3(24A+23B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^3(13A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(9A+13B)\sin(c+dx)\sec^{\frac{5}{2}}(c-}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(21*A + 17*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(13*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105
*d) + (2*a*B*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
+ (2*(9*A + 13*B)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(63*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
```

```
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{9d}+\frac{2}{9}\int$$

$$= \frac{2aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{9d}+\frac{2(9A}{}$$

$$= \frac{4a^3(24A+23B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2aB\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{4a^3(24A+23B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2aB\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{4a^3(21A+17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{4a^3(13A+1}{}$$

$$= \frac{4a^3(21A+17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{4a^3(13A+1}{}$$

$$= -\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{15d}$$

Mathematica [C] time = 6.80617, size = 793, normalized size = 2.86

$$\frac{7A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+d}\right.\right.}{30\sqrt{2}d(A\cos(c+dx)+B)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(30*
Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])) + (17*B*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*
(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6*(a +
 a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(90*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos
[c + d*x])) + (13*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(42*d*(B + A*Cos[c
+ d*x])*Sec[c + d*x]^(7/2)) + (11*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(42
*d*(B + A*Cos[c + d*x])*Sec[c + d*x]^(7/2)) + (Sec[c/2 + (d*x)/2]^6*(a + a*
Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(((21*A + 17*B)*Cos[d*x]*Csc[c])/(30*d
) + (B*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(36*d) + (Sec[c]*Sec[c + d*x]^3*(7*B
*Sin[c] + 9*A*Sin[d*x] + 27*B*Sin[d*x]))/(252*d) + (Sec[c]*Sec[c + d*x]^2*(
45*A*Sin[c] + 135*B*Sin[c] + 189*A*Sin[d*x] + 238*B*Sin[d*x]))/(1260*d) + (
Sec[c]*Sec[c + d*x]*(189*A*Sin[c] + 238*B*Sin[c] + 390*A*Sin[d*x] + 330*B*S
in[d*x]))/(1260*d) + ((13*A + 11*B)*Tan[c])/(42*d)))/((B + A*Cos[c + d*x])*
Sec[c + d*x]^(7/2))
```

Maple [B] time = 8.211, size = 1180, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(3/8*A+1
/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-16/5*(3/8*A+3/8*B)/(8*sin(1/
2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*
x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)+16*(1/8*A+3/8*B)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
```

```
(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/
180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^3 \sec(dx+c)^5 + (A+3B)a^3 \sec(dx+c)^4 + 3(A+B)a^3 \sec(dx+c)^3 + (3A+B)a^3 \sec(dx+c)^2 + Aa^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^5 + (A + 3*B)*a^3*sec(d*x + c)^4 + 3*(A + B)*a
^3*sec(d*x + c)^3 + (3*A + B)*a^3*sec(d*x + c)^2 + A*a^3*sec(d*x + c))*sqrt
(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

3.194 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(42A+41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(7\ldots}{}$$

```
[Out] (-4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(9*A + 7*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(42*A + 41*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(7*A + 11*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```

Rubi [A] time = 0.438735, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{4a^3(42A+41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(7A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(9A+7\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(9*A + 7*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(42*A + 41*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(7*A + 11*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d}+\frac{2}{7}\int\sqrt{\text{s}$$

$$= \frac{2aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d}+\frac{2(7A+}{}$$

$$= \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2aB\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2aB\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{4a^3(9A+7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{4a^3(42A+41B)\text{s}}{}$$

$$= \frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}$$

$$= -\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 5.13715, size = 465, normalized size = 1.91

$$\frac{a^3\csc(c)e^{-idx}\cos^4(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3(A+B\sec(c+dx))\left(7\sqrt{2}\left(-1+e^{2ic}\right)(9A+7B)e^{2idx}\sqrt{\frac{e^{i(c+d}}{1+e^{2i(c}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(9*A + 7*B)*E^((2*
I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] - ((-1 + E^((2*I)*c))*(21*A*(-5 + 16*E^(I*(c + d*x)) - 5*E^((2*I)
*(c + d*x)) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 56*E^((5*I)*
(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) + 2*B*(-65 + 8
4*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c + d*x)) + 95*E
^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c + d*x)) + 147
*E^((7*I)*(c + d*x))) + (10*I)*(21*A + 13*B)*(1 + E^((2*I)*(c + d*x)))^3*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(2*E^(I*(c
- d*x))*(1 + E^((2*I)*(c + d*x)))^3))*(1 + Sec[c + d*x])^3*(A + B*Sec[c + d
*x]))/(420*d*E^(I*d*x)*(B + A*Cos[c + d*x]))
```

Maple [B] time = 6.857, size = 931, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*
(3/8*A+3/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-16/5*(1/8*A+3/8*B)/(
8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/si
n(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+16*(3/8*A+1/8*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ba^3\sec\left(dx+c\right)^4+(A+3\,B)a^3\sec\left(dx+c\right)^3+3\,(A+B)a^3\sec\left(dx+c\right)^2+(3\,A+B)a^3\sec\left(dx+c\right)+Aa^3\right)\sqrt{\mathrm{se}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)*sqrt(sec(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^3\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

3.195 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(20A+21B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A}{}$$

```
[Out] (-4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(20*A + 21*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(5*A + 9*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.415908, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(20A+21B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(5A+3B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (-4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(20*A + 21*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(5*A + 9*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2aB\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a+a\sec(c+d}{}$$

$$= \frac{2aB\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2(5A+9B)\sqrt{\sec(c+}}{}$$

$$= \frac{4a^3(20A+21B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB\sqrt{\sec(c+dx)}(a+a\sec(}{5d}$$

$$= \frac{4a^3(20A+21B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB\sqrt{\sec(c+dx)}(a+a\sec(}{5d}$$

$$= \frac{4a^3(20A+21B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aB\sqrt{\sec(c+dx)}(a+a\sec(}{5d}$$

$$= -\frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(5A+}{}$$

Mathematica [C] time = 2.31908, size = 244, normalized size = 1.16

$$\frac{a^3e^{-idx}\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(2i(5A+9B)e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^3*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-90*I)*A*Cos[c + d*x] - (
162*I)*B*Cos[c + d*x] - (30*I)*A*Cos[3*(c + d*x)] - (54*I)*B*Cos[3*(c + d*x
)] + 40*(5*A + 3*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + ((2*I)*(
5*A + 9*B)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 45*A*Sin[c + d*x] + 66*B*Sin[c +
d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 45*A*Sin[3*(c + d*x)
] + 54*B*Sin[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Maple [B] time = 5.977, size = 916, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 4/15*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+100*A*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-180*A*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6+108*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+60*B*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-216*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-100*A*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+190*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-108*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*B*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+246*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+15*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-50*A*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+27*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-72*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ba^3\sec(dx+c)^4+(A+3B)a^3\sec(dx+c)^3+3(A+B)a^3\sec(dx+c)^2+(3A+B)a^3\sec(dx+c)+Aa^3}{\sqrt{\sec(dx+c)}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sqrt(sec(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

3.196 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{20a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(A+4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}-\frac{2(A-B)\,\text{si}}{}$$

```
[Out] (4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (4*a^3*(A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
- (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*d
)
```

Rubi [A] time = 0.40942, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4017, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(A+4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}-\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{3d}+\frac{20a^3(A+B)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (4*a^3*(A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
- (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*d
)
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(c+dx))^2\left(\frac{1}{2}a(7A \cdots\right.}{\sqrt{\sec \cdots}}$$

$$= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(A-B)\sqrt{\sec(c+dx)}\left(a^3+a^3\sec \cdots\right.}{3d}$$

$$= \frac{4a^3(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+ \cdots}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{4a^3(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+ \cdots}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{4a^3(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+ \cdots}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{4a^3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3(A+B)\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 1.94521, size = 202, normalized size = 1.02

$$\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-4i(A-B)\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+40(A+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((12*I)*A - (12*I)*B + (12*
I)*A*Cos[2*(c + d*x)] - (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d*x]
^(3/2)*EllipticF[(c + d*x)/2, 2] - (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x)))^
(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + A*Sin[c + d*
x] + 4*B*Sin[c + d*x] + 6*A*Sin[2*(c + d*x)] + 18*B*Sin[2*(c + d*x)] + A*Si
n[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

Maple [B] time = 2.202, size = 654, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(-4*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(5*A+9*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+5*B)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\sec(dx+c)^4+(A+3B)a^3\sec(dx+c)^3+3(A+B)a^3\sec(dx+c)^2+(3A+B)a^3\sec(dx+c)+Aa^3}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(3/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

3.197 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3(6A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5}{}$$

```
[Out] (4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(6*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.413033, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{4a^3(6A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(3A+5B)\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(6*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\sec(c+dx))^2\left(\frac{1}{2}a(9A\right.}{\sec^{\frac{3}{2}}}$$

$$= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(9A+5B)\left(a^3+a^3\sec(c+dx)\right)}{15d\sqrt{\sec(c+dx)}}$$

$$= -\frac{4a^3(6A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a+a\sec(c+dx))^2\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(6A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a+a\sec(c+dx))^2\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(6A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a+a\sec(c+dx))^2\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(3A+5I}{}$$

Mathematica [C] time = 1.6948, size = 207, normalized size = 0.98

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-8i(9A+5B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((216*I)*A*Cos[c + d*x] + (
120*I)*B*Cos[c + d*x] + 40*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] - (8*I)*(9*A + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x] +
```

```
 60*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] + 10*B*Sin[2*(c + d*x)] + 3*A*Si
n[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Maple [B] time = 1.976, size = 519, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*a^3*(-12*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(21*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A+10*B)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-27*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\sec\left(dx+c\right)^4+(A+3B)a^3\sec\left(dx+c\right)^3+3\left(A+B\right)a^3\sec\left(dx+c\right)^2+(3A+B)a^3\sec\left(dx+c\right)+Aa^3}{\sec\left(dx+c\right)^{\frac{5}{2}}},$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(5/2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{3A}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3A}{\sqrt{\sec(c+dx)}}\,dx + \int A\sqrt{\sec(c+dx)}\,dx + \int \frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a**3*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(3*A/sec(c + d*x)**(3/2)
, x) + Integral(3*A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)),
 x) + Integral(B/sec(c + d*x)**(3/2), x) + Integral(3*B/sqrt(sec(c + d*x)),
 x) + Integral(3*B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2),
 x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x
)
```

3.198 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(11A+7B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.437065, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(11A+7B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\sec(c+dx))^2\left(\frac{1}{2}a(11A\right.}{\sec}$$

$$= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(11A+7B)\left(a^3+a^3\sec(c+dx)\right)}{35d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(13A+2}{}$$

Mathematica [C] time = 2.51958, size = 194, normalized size = 0.92

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(7A+9B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(13*A + 21*B)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(7*A + 9*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((168*I)*(7*A + 9*B) + 5*(107*A + 84*B)*Sin[c + d*x]
 + 42*(3*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x
))
```

Maple [A] time = 1.741, size = 385, normalized size = 1.8

$$-\frac{4a^3}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-432\,A-84\,B\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-432*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(602*A+294*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2
08*A-126*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*A*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-147*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+105*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\sec\left(dx+c\right)^4+\left(A+3\,B\right)a^3\sec\left(dx+c\right)^3+3\left(A+B\right)a^3\sec\left(dx+c\right)^2+\left(3\,A+B\right)a^3\sec\left(dx+c\right)+Aa^3}{\sec\left(dx+c\right)^{\frac{7}{2}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(7/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x
)
```

3.199 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(11A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(13A+9B)\sin(\ldots}{63\ldots}$$

```
[Out] (4*a^3*(17*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(13*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```

Rubi [A] time = 0.476664, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(13A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(11\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (4*a^3*(17*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(13*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+dx))^2\left(\frac{1}{2}a(13A\right.}{\sec}$$

$$= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+a^3\sec(c+dx)\right)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + 2($$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + 2($$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(11A+13B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + 2aA(a+a$$

$$= \frac{4a^3(17A+21B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^3(23A}{105d}$$

$$= \frac{4a^3(17A+21B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + 4a^3(11A$$

Mathematica [C] time = 2.78518, size = 196, normalized size = 0.8

$$a^3\sqrt{\sec(c+dx)}\left(-112i(17A+21B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+240(11A+13B)\sqrt{\cos}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(240*(11*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(17*A + 21*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*
x]*((5712*I)*A + (7056*I)*B + 30*(97*A + 107*B)*Sin[c + d*x] + 14*(73*A + 5
4*B)*Sin[2*(c + d*x)] + 270*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 35
*A*Sin[4*(c + d*x)])))/(1260*d)
```

Maple [A] time = 1.717, size = 413, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2200*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-3412*A-1296*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(2702*A+1806*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-738*A-624*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+195*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\sec(dx+c)^4 + (A+3B)a^3\sec(dx+c)^3 + 3(A+B)a^3\sec(dx+c)^2 + (3A+B)a^3\sec(dx+c) + Aa^3}{\sec(dx+c)^{\frac{9}{2}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(9/2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x
)
```

3.200 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{4a^3(105A+121B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(15A+17B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{20a^3(21A+22\ldots}{693d\sec\ldots}$$

```
[Out] (4*a^3*(15*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(21*A + 22*B)*Sin[c + d*
x])/(693*d*Sec[c + d*x]^(5/2)) + (4*a^3*(15*A + 17*B)*Sin[c + d*x])/(45*d*S
ec[c + d*x]^(3/2)) + (4*a^3*(105*A + 121*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c
 + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2)) + (2*(15*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(99*d*Sec[
c + d*x]^(7/2))
```

Rubi [A] time = 0.510461, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^3(15A+17B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(15A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{4a^3(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x]
```


```
[Out] (4*a^3*(15*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(21*A + 22*B)*Sin[c + d*
x])/(693*d*Sec[c + d*x]^(5/2)) + (4*a^3*(15*A + 17*B)*Sin[c + d*x])/(45*d*S
ec[c + d*x]^(3/2)) + (4*a^3*(105*A + 121*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c
 + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2)) + (2*(15*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(99*d*Sec[
c + d*x]^(7/2))
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\sec(c+dx))^2\left(\frac{1}{2}a(15\right.}{s}$$

$$= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(15A+11B)\left(a^3+a^3\sec(c+dx\right.}{99d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(15A+17B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(105A}{231}$$

$$= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(15A+17B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(105A}{231}$$

$$= \frac{4a^3(15A+17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^3(105A}{}$$

Mathematica [C] time = 3.49177, size = 239, normalized size = 0.86

$$a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2464i(15A+17B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+d}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(480*(105*A + 121*B)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(15*A + 17*B)*E^(I*(c + d
*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))] + Cos[c + d*x]*((110880*I)*A + (125664*I)*B + 30*(1953*A + 2
134*B)*Sin[c + d*x] + 308*(75*A + 73*B)*Sin[2*(c + d*x)] + 8505*A*Sin[3*(c
+ d*x)] + 5940*B*Sin[3*(c + d*x)] + 2310*A*Sin[4*(c + d*x)] + 770*B*Sin[4*(
c + d*x)] + 315*A*Sin[5*(c + d*x)])))/(27720*d*E^(I*d*x))
```

Maple [A] time = 1.659, size = 441, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-43680*A-6160*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(77280*A+24200*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-72240*A-37532*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(39270
*A+29722*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8820*A-8118*B)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1575*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3465*A*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+1815*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3927*B*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^3\sec(dx+c)^4 + (A+3B)a^3\sec(dx+c)^3 + 3(A+B)a^3\sec(dx+c)^2 + (3A+B)a^3\sec(dx+c) + Aa^3}{\sec(dx+c)^{\frac{11}{2}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] integral((B*a^3*sec(d*x + c)^4 + (A + 3*B)*a^3*sec(d*x + c)^3 + 3*(A + B)*a
^3*sec(d*x + c)^2 + (3*A + B)*a^3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(11/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2),
x)
```

3.201 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=229

$$\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+d}{5ad}$$

```
[Out] (3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a*d) - (3*(5*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
5*a*d) + (5*(A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((5*A - 7*B)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.24767, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3768, 3771, 2641, 2639}

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}+\frac{5(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(5A-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```


```
[Out] (3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a*d) - (3*(5*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
5*a*d) + (5*(A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((5*A - 7*B)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(5A-7B)\text{s}\right.}{a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(5A-7B)\int \sec^{\frac{7}{2}}(c+dx)\,dx}{2a} + \frac{(5(A-B))\int}{}$$

$$= \frac{5(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{(5A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{(A-}{}$$

$$= -\frac{3(5A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{5(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{(5}{}$$

$$= \frac{5(A-B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad} - \frac{3(5A-7B)\sqrt{\sec(c+dx}}{5ad}$$

$$= \frac{3(5A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(A-B)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 7.51226, size = 814, normalized size = 3.55

$$-\frac{Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec}{\sqrt{2}d(B+A\cos(c+dx))(\sec(c+dx)a+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x]))/(Sqrt[2]*d*E^(I*d*x)*(B + A*Cos
[c + d*x])*(a + a*Sec[c + d*x]))) + (7*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(
-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*
```

```
x]))/(5*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (5
*A*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos
[c + d*x])*(a + a*Sec[c + d*x])) - (5*B*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d
*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*
Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (Co
s[c/2 + (d*x)/2]^2*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x])*((3*(-5*A + 7*B)
*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - ((-A + B)*Sec[c/2]*Sec[c]*(-Sin[c/2] +
 5*Sin[(3*c)/2]))/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2])
 + B*Sin[(d*x)/2]))/d + (4*B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(5*d) + (4*Sec
[c]*Sec[c + d*x]*(3*B*Sin[c] + 5*A*Sin[d*x] - 5*B*Sin[d*x]))/(15*d)))/((B +
 A*Cos[c + d*x])*(a + a*Sec[c + d*x]))
```

Maple [B] time = 6.423, size = 806, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((2*A-2*B)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(A-
B)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*A+2*B)*(-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

3.202 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{(3A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(3A-5B)\sin(c\cdots}{3\cdots}$$

```
[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a*d) + (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) -
((3*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((A - B)*Sec[c + d*
x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.227129, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(3A-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```


```
[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a*d) + (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) -
((3*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((A - B)*Sec[c + d*
x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(3A-5B)\text{s}\right.}{a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-5B)\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a} + \frac{(3(A-B))\int}{}$$

$$= \frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(3A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{(A-}{}$$

$$= \frac{3(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(3A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{(A-}{}$$

$$= -\frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 3.29854, size = 372, normalized size = 1.94

$$\frac{e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(A+B\sec(c+dx))\left(i\left(3(A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-((3*A - 5*B)*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x))
 + E^((3*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]) + I*(
-3*A + 5*B - 6*A*E^(I*(c + d*x)) + 8*B*E^(I*(c + d*x)) - 12*A*E^((2*I)*(c +
 d*x)) + 10*B*E^((2*I)*(c + d*x)) - 6*A*E^((3*I)*(c + d*x)) + 4*B*E^((3*I)*
(c + d*x)) - 9*A*E^((4*I)*(c + d*x)) + 9*B*E^((4*I)*(c + d*x)) + 3*(A - B)*
E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*(1 + E^(I*(c + d*x)) + E^((2*
I)*(c + d*x)) + E^((3*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))]))*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(3*a*d*E^((I/2)
*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x
]))
```

Maple [B] time = 5.281, size = 493, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*B*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-A+B)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+(2*A-2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec(dx+c)^3+A\sec(dx+c)^2\right)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.203 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(A-3B)\sin(c+dx)}{ad}$$

```
[Out] ((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + ((A - B
)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.186229, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + ((A - B
)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A-B)-\frac{1}{2}a(A-3B)\,\mathrm{se}\right.}{a^2}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-3B)\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{(A-B)\int \sqrt{\mathrm{s}}}{2}$$

$$= -\frac{(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A-}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-3B)\sqrt{\sec(c+dx)}\,\mathrm{sir}}{ad}$$

$$= \frac{(A-3B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-B)\sqrt{\cos(c+dx)}F\Big(}{}$$

Mathematica [C] time = 4.37199, size = 420, normalized size = 2.75

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(-2\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2F1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x)
)/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[
1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2
F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(
I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c
]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 12*A*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 12*B*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 6*Sqrt[Sec[c + d*x
]]*(2*(A - 3*B)*Cos[d*x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2])))/(6*a*d*(B
+ A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A] time = 3.819, size = 318, normalized size = 2.1

$$-\frac{1}{ad}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+
1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-3*B)*sin(1/2*d*x+1/2*c)^4-(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-5*B)*sin(1/2*d*x+1/2*c)^
2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sec(dx+c)^2+A\sec(dx+c)\right)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a*sec(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.204 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{(A-B)\sqrt{\cos(c+dx}}{}$$

```
[Out] -(((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d)) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c
+ d*x]))
```

Rubi [A] time = 0.170815, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 3787, 3771, 2639, 2641}

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d)) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c
+ d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{(A+B)\int \sqrt{\sec(c}}{2a}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\mathrm{co}}}{2a}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+B)\sqrt{\cos(c+dx)}F}{}$$

Mathematica [C] time = 1.12843, size = 200, normalized size = 1.63

$$\frac{\left(-1+e^{2ic}\right)e^{-\frac{1}{2}i(4c+dx)}\left(\csc\left(\frac{c}{2}\right)+i\sec\left(\frac{c}{2}\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((A-B)\left(e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\mathrm{Hype}\right.\right.}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-1 + E^((2*I)*c))*((-3*I)*(A + B)*(1 + E^(I*(c + d*x)))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + (A - B)*(-3*(1 + E^((2*I)*(c + d*x))) + E^(I*
(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*(Csc[c/2] + I*Sec[c/2])*Sec[(c
 + d*x)/2]*Sqrt[Sec[c + d*x]])/(24*a*d*E^((I/2)*(4*c + d*x)))
```

Maple [A] time = 1.793, size = 243, normalized size = 2.

$$-\frac{1}{ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\mathrm{Elli}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\sec(c+dx)+1}\,dx + \int \frac{B\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sec(c +
d*x)**(3/2)/(sec(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.205 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=128

$$-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(3A-B)\sqrt{\cos(c+dx)}}{}$$

```
[Out] ((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c +
 d*x]))
```

Rubi [A] time = 0.177518, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4020, 3787, 3771, 2639, 2641}

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c +
 d*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A-B)-\frac{1}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int\sqrt{\sec(c+dx)}\,dx}{2a} + \frac{(3A-B)}{}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= \frac{(3A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-B)\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 2.63117, size = 445, normalized size = 3.48

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(-6\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (6*((2*A - B)*Cos[(c - d*x)/2]
+ A*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*
x]] - 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
+ 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(A
+ B*Sec[c + d*x]))/(6*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.739, size = 244, normalized size = 1.9

$$\frac{1}{ad}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF
```

```
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^2 + a*sec(
d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec
(c + d*x)/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

3.206 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=164

$$\frac{(5A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a}$$

```
[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]
) - ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.194242, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}+\frac{(5A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]
) - ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B)-\frac{3}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{(5A-3B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} - \frac{(3(A-B))}{}$$

$$= \frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{(5A-3B)\int \sqrt{s}}{6a}$$

$$= -\frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-3B)\sin(c+d}{3ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{3(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-3B)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 2.36257, size = 232, normalized size = 1.41

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(3i(A-B)e^{\frac{1}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*(2*(5*A - 3*B)
*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*I)*(A -
 B)*E^((I/2)*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2*Cos[c + d*x]*((
-9*I)*(A - B)*Cos[(c + d*x)/2] + (5*A - 3*B + 2*A*Cos[c + d*x])*Sin[(c + d*
x)/2])))/(3*a*d*E^(I*d*x)*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.688, size = 262, normalized size = 1.6

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\,\right(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B)*sin(1/2*d*x+1/2*c)^4+(-7*A
+3*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a\sec(dx+c)^3+a\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^3 + a*sec(
d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(B*se
c(c + d*x)/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

3.207 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=197

$$-\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (3*(7*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a*d) + ((7*A - 5*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3
/2)) - (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c
 + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.212539, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (3*(7*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a*d) + ((7*A - 5*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3
/2)) - (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c
 + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(7A-5B)-\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{(7A-5B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a} - \frac{(5(A-B))}{}$$

$$= \frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{3(7A-5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{5(A-B)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 3.75535, size = 540, normalized size = 2.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(-84\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((-84*Sqrt[2]*A*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt
[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (60*Sqrt[2]*B*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc
[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 200*A*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 200*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d
*x]]*(-3*(51*A - 40*B + (33*A - 20*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2]
```

```
- 40*(A - B)*Cos[2*d*x]*Sin[2*c] + 12*A*Cos[3*d*x]*Sin[3*c] + 120*(A - B)*S
ec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 12*(33*A - 20*B)*Cos[c]*Sin[d*x] -
40*(A - B)*Cos[2*c]*Sin[2*d*x] + 12*A*Cos[3*c]*Sin[3*d*x] + 120*(A - B)*Tan
[c/2])))/(60*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.807, size = 282, normalized size = 1.4

$$-\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/2*d*x+1/2*c
)^6+(-30*A+90*B)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B)*sin(1/2*d*x+1/2*c)^2)/a/c
os(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{4}+a\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^4 + a*sec(
d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

3.208 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=230

$$\frac{5(9A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21ad}-\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{7(A-B)\sin(c+dx}{5ad\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-21*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*a*d) + (5*(9*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*a*d) + ((9*A - 7*B)*Sin[c + d*x])/(7*a*d*Sec[c + d*x]
^(5/2)) - (7*(A - B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (5*(9*A - 7
*B)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(d*S
ec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.230189, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{7(A-B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(9A-7B)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(9A-7B)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}}+\frac{5(9A-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-21*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*a*d) + (5*(9*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*a*d) + ((9*A - 7*B)*Sin[c + d*x])/(7*a*d*Sec[c + d*x]
^(5/2)) - (7*(A - B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (5*(9*A - 7
*B)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(d*S
ec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(9A-7B)-\frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{(9A-7B)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx}{2a} - \frac{(7(A-B))}{}$$

$$= \frac{(9A-7B)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{7(A-B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{(9A-7B)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{7(A-B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(9A-7B)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}} - \frac{}{d}$$

$$= -\frac{21(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{(9A-7B)\sin(c+}{7ad\sec^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{21(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(9A-7B)\sqrt{\cos(c}{}$$

Mathematica [C] time = 3.93781, size = 568, normalized size = 2.47

$$\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(588\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((588*Sqrt[2]*A*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt
[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (588*Sqrt[2]*B*Sqrt[
E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cs
```

```
c[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 1800*A*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 1400*B*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c
+ d*x]]*(63*(A - B)*(17 + 11*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 20*(27*
A - 14*B)*Cos[2*d*x]*Sin[2*c] - 84*(A - B)*Cos[3*d*x]*Sin[3*c] + 30*A*Cos[4
*d*x]*Sin[4*c] - 840*(A - B)*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2772*
(A - B)*Cos[c]*Sin[d*x] + 20*(27*A - 14*B)*Cos[2*c]*Sin[2*d*x] - 84*(A - B)
*Cos[3*c]*Sin[3*d*x] + 30*A*Cos[4*c]*Sin[4*d*x] - 840*(A - B)*Tan[c/2])))/(
420*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.687, size = 300, normalized size = 1.3

$$-\frac{1}{105\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(225*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-175*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))-480*A*sin(1/2*d*x+1/2*c)^10+(864*A+336*B)*sin(1/2*d*
x+1/2*c)^8+(-888*A-392*B)*sin(1/2*d*x+1/2*c)^6+(930*A-210*B)*sin(1/2*d*x+1/
2*c)^4+(-321*A+161*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{5}+a\sec\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^5 + a*sec(
d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x
)
```

3.209 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=237

$$-\frac{5(A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(4A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5(A-2B)\sin(c+}{3a}$$

```
[Out] -(((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d)) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + ((4*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) - (5*(A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + ((4*A
 - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A
 - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.371134, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{(4A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5(A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}+\frac{(4A-7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5(A-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d)) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + ((4*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) - (5*(A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + ((4*A
 - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A
 - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{3}{2}a(A-3B)\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{(4A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \int$$

$$= \frac{(4A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + (4$$

$$= \frac{(4A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} +$$

$$= \frac{(4A-7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} +$$

$$= -\frac{(4A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - 5(A-2B)\sqrt{\cos(c}$$

Mathematica [C] time = 7.80913, size = 865, normalized size = 3.65

$$\frac{4\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+d}}\right.}{3d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*
d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (7*Sqrt[2]*B*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
```

```
 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x
])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*
Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
 + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (20*B*
Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*
(A + B*Sec[c + d*x])*((-2*(-4*A + 7*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-5*A*Sin[(d*x)/2] + 8*B*Sin[(d*x)/2]))/(3*d
) + (8*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(2*B - 5*A*Cos[c] + 10*B*
Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])
/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 6.082, size = 750, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(4*B*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(-A+B)*(2*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7
*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+(-2*A+4*B)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(4*A-8*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a^2*sec(
d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x
)
```

3.210 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=204

$$\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(2A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(A-4B)\sin(c+d\ldots}{a^2d}$$

```
[Out] ((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((A - 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + ((2*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*
x])) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^
2)
```

Rubi [A] time = 0.345241, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(2A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(A-4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\ldots\right.}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((A - 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + ((2*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*
x])) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^
2)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(A-7B)\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \int$$

$$= \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + (2$$

$$= -\frac{(A-4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} +$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{(A-4B)\sqrt{\sec(c+d}}{a^2d}$$

$$= \frac{(A-4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-5B)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.6399, size = 455, normalized size = 2.23

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))\left(-2\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hyperg}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*Sqrt[2]*A*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc
[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (8*Sqrt[
2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((
```

```
2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x)
 + 8*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 20
*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 2*Sqrt
[Sec[c + d*x]]*(6*(A - 4*B)*Cos[d*x]*Csc[c] - (3*(A - 2*B) + (2*A - 5*B)*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/(3*a^2*d*(B + A*Cos[c +
d*x])*(1 + Sec[c + d*x])^2)
```

Maple [B] time = 2.295, size = 492, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A-4*B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-37*B)*sin(1/2*d*x+1/2*c)^2)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a^2*sec(
d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x
)
```

3.211 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2d}$$

```
[Out] (B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```

Rubi [A] time = 0.302637, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 3787, 3771, 2639, 2641}

$$\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A-B)+\frac{1}{2}a(A+5B)\sec(c+dx)\right)}{a+a\sec(c+dx)}dx}{3a^2}$$

$$= -\frac{B\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{3a^2B}{2}+\frac{1}{2}\ldots}{\ldots}}{\ldots}$$

$$= -\frac{B\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{B\int \frac{1}{\sqrt{\sec(\ldots}}}{2a\ldots}$$

$$= -\frac{B\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(B\sqrt{\cos(\ldots}\right.}{\ldots}$$

$$= \frac{B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\ldots\right.}{3a^2\ldots}$$

Mathematica [C] time = 2.60395, size = 256, normalized size = 1.59

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(8(A+2B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\mathrm{E}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*B*(1 + E^(I*(c + d*x)))^3*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*Co
s[c + d*x]*(-A + 7*B + (A + 5*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x]))*(C
os[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c +
d*x])^2)
```

Maple [A] time = 1.937, size = 350, normalized size = 2.2

$$-\frac{1}{6a^2d}\sqrt{\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2A(\cos(1/2dx+c/2))^3\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{-2(\cos(1/2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6+4*B*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+16*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a^2*sec(d*
x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x
)
```

3.212 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$\frac{(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(2A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{A\sqrt{\cos(c+dx)}\sqrt{\sec\cdots}}{\cdots}$$

```
[Out] -((A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((2*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rubi [A] time = 0.309636, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(2A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\cdots}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((2*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{3}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= \frac{(2A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(2A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{A}{}$$

$$= \frac{(2A+B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(A}{}$$

$$= -\frac{A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A+B)\sqrt{\cos(c+dx)}F\left(\right.}{3a}$$

Mathematica [C] time = 3.24178, size = 256, normalized size = 1.52

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(Ae^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\text{Hy}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/
2]) + I*((A*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - 2*Cos[c +
d*x]*(5*A + B + (7*A - B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x])))*(Cos[(c
+ 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^
2)
```

Maple [A] time = 1.894, size = 350, normalized size = 2.1

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^3\sqrt{\left(\sin\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+9*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sqrt{\sec\left(c+dx\right)}}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^{\frac{3}{2}}\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) +
 Integral(B*sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))
/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

3.213 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=177

$$-\frac{(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(5A-2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)\sqrt{\cos(c+dx)}$$

```
[Out] ((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*
(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.325614, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4020, 3787, 3771, 2639, 2641}

$$-\frac{(5A-2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)\sqrt{\cos(c+dx)}\sqrt{s}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*
(a + a*Sec[c + d*x])^2)
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A-B)-\frac{3}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(5A-2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} +$$

$$= -\frac{(5A-2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} -$$

$$= -\frac{(5A-2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} -$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(5A-2B)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.75206, size = 854, normalized size = 4.82

$$-\frac{4\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+}}\right.}{3d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I
*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (Sqrt[2]*B*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
+ (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x]
)*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*C
sc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
+ d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (4*B*Co
s[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*(A
 + B*Sec[c + d*x])*((-2*(3*A - B + A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/
d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/
(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-7*A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2
]))/(3*d) + (8*A*Cos[c]*Sin[d*x])/d - (4*(-7*A + 4*B)*Tan[c/2])/(3*d) + (2*
```

```
(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((B + A*Cos[c + d*x])*(a +
a*Sec[c + d*x])^2)
```

Maple [A] time = 2.047, size = 421, normalized size = 2.4

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+10\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^3\sqrt{\left(\sin\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1
/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+24*A*co
s(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*
cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4+
15*A*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2-A+B)/cos(1/2*d*x+1/2*c)^
3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^3+2\,a^2\sec(dx+c)^2+a^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^3 + 2*a^
2*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x
))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3
/2) + sqrt(sec(c + d*x))), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

3.214 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{5(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+$$

```
[Out] -(((7*A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d)) + (5*(2*A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec
[c + d*x]]) - ((7*A - 4*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + S
ec[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
 + d*x])^2)
```

Rubi [A] time = 0.356715, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{5(2A-B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}+\frac{5(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-(7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] -(((7*A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d)) + (5*(2*A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec
[c + d*x]]) - ((7*A - 4*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + S
ec[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
 + d*x])^2)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A-B)-\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{5(2A-B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A-}{3d\sqrt{\sec(c+}}$$

$$= -\frac{(7A-4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{5(2A-B)\sin(c+}{3a^2d\sqrt{\sec(c+d}}$$

$$= -\frac{(7A-4B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{5(2A-B)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 6.80139, size = 899, normalized size = 4.26

$$\frac{7\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+d}}\right.}{3d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] (7*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*
```

```
d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (4*Sqrt[2]*B*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x
])*(a + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*
Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
 + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (10*B*
Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*
(A + B*Sec[c + d*x])*((-2*(-5*A + 3*B - 2*A*Cos[2*c] + B*Cos[2*c])*Cos[d*x]
*Csc[c/2]*Sec[c/2])/d + (4*A*Cos[2*d*x]*Sin[2*c])/(3*d) - (2*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*S
ec[c/2 + (d*x)/2]*(-10*A*Sin[(d*x)/2] + 7*B*Sin[(d*x)/2]))/(3*d) + (8*(-2*A
 + B)*Cos[c]*Sin[d*x])/d + (4*A*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*(-10*A + 7*
B)*Tan[c/2])/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((B
 + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 1.951, size = 435, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*cos(
1/2*d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+20*A*cos(1/2*d*x+1/2*c)^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+42*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*
B*cos(1/2*d*x+1/2*c)^6-10*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
24*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*A*cos(1/2*d*x+1/2*c)
^4+38*B*cos(1/2*d*x+1/2*c)^4+21*A*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2
*c)^2-A+B)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^4+2\,a^2\sec(dx+c)^3+a^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^4 + 2*a^
2*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

3.215 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$-\frac{5(3A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{7(8A-5B)\sin(c}{15a^2d\sec^{\frac{3}{2}}(c+}$$

```
[Out] (7*(8*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a^2*d) - (5*(3*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (7*(8*A - 5*B)*Sin[c + d*x])/(15*a^2*d*Se
c[c + d*x]^(3/2)) - (5*(3*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]
]) - ((3*A - 2*B)*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x]
)) - ((A - B)*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.381588, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{7(8A-5B)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(3A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{5(3A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (7*(8*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a^2*d) - (5*(3*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (7*(8*A - 5*B)*Sin[c + d*x])/(15*a^2*d*Se
c[c + d*x]^(3/2)) - (5*(3*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]
]) - ((3*A - 2*B)*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x]
)) - ((A - B)*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(11A-5B)-\frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} +$$

$$= -\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} +$$

$$= \frac{7(8A-5B)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A-2B)\sin(c+}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\mathrm{se}}$$

$$= \frac{7(8A-5B)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A-2B)\sin(c+}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\mathrm{se}}$$

$$= \frac{7(8A-5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5(3A-2B)\sqrt{\cos(}}{}$$

Mathematica [C] time = 6.90719, size = 946, normalized size = 3.88

$$-\frac{56\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c}}\right.}{15d(B+A\cos(c+dx))(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] (-56*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
```

```
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(15*d*E^
(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (7*Sqrt[2]*B*Sqrt[E^
(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[
c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)
*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x
]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Se
c[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (20*B
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
 + d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)
*(A + B*Sec[c + d*x])*(((-151*A + 100*B - 73*A*Cos[2*c] + 40*B*Cos[2*c])*Co
s[d*x]*Csc[c/2]*Sec[c/2])/(10*d) + (4*(-2*A + B)*Cos[2*d*x]*Sin[2*c])/(3*d)
 + (2*A*Cos[3*d*x]*Sin[3*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*
Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-1
3*A*Sin[(d*x)/2] + 10*B*Sin[(d*x)/2]))/(3*d) - (2*(-73*A + 40*B)*Cos[c]*Sin
[d*x])/(5*d) + (4*(-2*A + B)*Cos[2*c]*Sin[2*d*x])/(3*d) + (2*A*Cos[3*c]*Sin
[3*d*x])/(5*d) - (4*(-13*A + 10*B)*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 2.023, size = 465, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*A*cos
(1/2*d*x+1/2*c)^10-352*A*cos(1/2*d*x+1/2*c)^8+80*B*cos(1/2*d*x+1/2*c)^8+120
*A*cos(1/2*d*x+1/2*c)^6-150*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-336*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+60*B*cos(1/2*d*x+1/2
*c)^6+100*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*B*cos(1/2*d*
x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+266*A*cos(1/2*d*x+1/2*c)^4-240*B*cos(1
/2*d*x+1/2*c)^4-135*A*cos(1/2*d*x+1/2*c)^2+105*B*cos(1/2*d*x+1/2*c)^2+5*A-5
*B)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^5+2a^2\sec(dx+c)^4+a^2\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^5 + 2*a^
2*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

3.216 $\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=292

$$-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{7(7A-17B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)}{}$$

```
[Out] (-7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(7*A - 17*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(10*a^3*d) - ((13*A - 33*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*
a^3*d) + ((A - B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x]
)^3) + ((A - 2*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*
x])^2) + (7*(7*A - 17*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*
Sec[c + d*x]))
```

Rubi [A] time = 0.559542, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{7(7A-17B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6a^3d}+\frac{7(7A-17B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(7*A - 17*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(10*a^3*d) - ((13*A - 33*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*
a^3*d) + ((A - B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x]
)^3) + ((A - 2*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*
x])^2) + (7*(7*A - 17*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*
Sec[c + d*x]))
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(3A-13B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,d}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} + \int$$

$$= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} + \frac{7(7}{}$$

$$= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} + \frac{7(7}{}$$

$$= \frac{7(7A-17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{6a^3d}$$

$$= \frac{7(7A-17B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{6a^3d}$$

$$= -\frac{7(7A-17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-33B)\sqrt{c}}{}$$

Mathematica [C] time = 7.96317, size = 953, normalized size = 3.26

$$\frac{49\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+}}\right.}{15d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (49*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E
^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (119*Sqrt[2]*B*Sqrt
[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*C
os[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d
*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^3) - (26*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
 + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A
+ B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
 + (22*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*
x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]
^(5/2)*(A + B*Sec[c + d*x])*((-14*(-7*A + 17*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])
/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/
2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-8*A*Sin[(d*x)/2] + 13*B*Sin
[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-13*A*Sin[(d*x)/2] + 2
9*B*Sin[(d*x)/2]))/(3*d) + (16*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(
4*B - 13*A*Cos[c] + 33*B*Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (4*(-8*A + 13*B)*
Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Ta
n[c/2])/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.949, size = 876, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*(4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-165*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+357*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-165*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+357*B*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-165*B*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+357*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-165*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+357*B*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-168*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-17*B)*sin(1/2*d*x+1/2*c)^10+8*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(482*A-1167*B)*sin(1/2*d*x+1/2*c
)^8-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(461*A-1111*B)*
sin(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*(169*A-404*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(439*A-1029*B)*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(3/2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{4}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(sec(d*x + c))/(a^3*sec(
d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x
)
```

3.217 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(3A-13B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(9A-49B)\sin(c$$

```
[Out] ((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((9*A - 49*B)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec
[c + d*x])^3) + ((3*A - 8*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a +
a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.536257, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(3A-13B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(9A-49B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((9*A - 49*B)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec
[c + d*x])^3) + ((3*A - 8*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a +
a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(A-11B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3}{}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3}{}$$

$$= -\frac{(9A-49B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} +$$

$$= \frac{(3A-13B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{(9A-49B)\sqrt{\sec(c}}{10a}$$

$$= \frac{(9A-49B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A-13B)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 7.24829, size = 924, normalized size = 3.54

$$-\frac{3\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+}}\right.}{5d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^
(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (49*Sqrt[2]*B*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x
)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
)*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c +
d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B
*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2
6*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5
/2)*(A + B*Sec[c + d*x])*((2*(-9*A + 49*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d
) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/
(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-3*A*Sin[(d*x)/2] + 8*B*Sin[(d*x)
/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-3*A*Sin[(d*x)/2] + 13*B*Sin
[(d*x)/2]))/(3*d) - (4*(-3*A + 13*B)*Tan[c/2])/(3*d) - (4*(-3*A + 8*B)*Sec[
c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/
2])/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.524, size = 685, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A-49*B)*s
in(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(147*A-817*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(43*A-248*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(69*A-439*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d
*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a^3*sec(
d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x
)
```

3.218 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=220

$$\frac{(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+9B)\sqrt{\cos(c+dx)}}{}$$

```
[Out] ((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) + ((A - 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.48983, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4019, 3787, 3771, 2639, 2641}

$$-\frac{(A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{10a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) + ((A - 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)+\frac{1}{2}a(A+9B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A+9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+3B)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 6.88495, size = 919, normalized size = 4.18

$$-\frac{\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+d}}\right.}{15d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] -(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(
I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (3*Sqrt[2]*B*Sqrt[E^(
I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c
/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*
Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(I*d*x)*(B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x
```

```
]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Se
c[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (2*
B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2
]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*
(A + B*Sec[c + d*x])*((-2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) +
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/(3*d)
+ (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/(
15*d) + (4*(A + 3*B)*Tan[c/2])/(3*d) + (4*(2*A + 3*B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((B +
 A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.099, size = 451, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8-30*B*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6-138*B*cos(1/2*d*x+1/2*c)^6+
6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2
+3*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a^3*sec(
d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x
)
```

3.219 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=216

$$\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\text{s}}}{1}$$

```
[Out] -((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a +
 a*Sec[c + d*x])^3) - ((A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(
a + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^
3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.483609, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a +
 a*Sec[c + d*x])^3) - ((A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(
a + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^
3 + a^3*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A-B)+\frac{1}{2}a(3A+7B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+B)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 6.86815, size = 918, normalized size = 4.25

$$\frac{\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right.}{15d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
```

```
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I
*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (Sqrt[2]*B*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
+ (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c + d
*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]
*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[
c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (2*B*
Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*
(A + B*Sec[c + d*x])*((-2*(-A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*d) + (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-7*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2]))/(15
*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(-7*A + 2*B)*Sec[c/2 + (d*x)/2]^2*Tan
[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((B + A*
Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.077, size = 451, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8+10*B*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+22*B*cos(1/2*d*x+1/2*c)^6-24*
A*cos(1/2*d*x+1/2*c)^4-6*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-7
*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a^3*sec(d*
x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x
)
```

3.220 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(3A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(9A+B)\sqrt{\cos(c+d}\cdots}{\cdots}$$

```
[Out] -((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + ((3*A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.489448, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(3A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(\cdots}}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + ((3*A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{5}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \ldots$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \ldots$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \ldots$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \ldots$$

$$= -\frac{(9A+B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+B)\sqrt{\cos(c+\ldots}}{\ldots}$$

Mathematica [C] time = 6.96037, size = 919, normalized size = 4.14

$$\frac{3\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+d\ldots}}\right.}{5d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
```

```
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(
I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Sqrt[2]*B*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]
]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec
[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (2*B*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*(
A + B*Sec[c + d*x])*((2*(9*A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (4*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(-9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*d) + (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-12*A*Sin[(d*x)/2] + 7*B*Sin[(d*x)/2]))/
(15*d) + (4*(-9*A + B)*Tan[c/2])/(3*d) - (4*(-12*A + 7*B)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(
(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 1.961, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*co
s(1/2*d*x+1/2*c)^8+30*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*A
*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8+1
0*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6-2*B*cos(1/2*d*x+1/2*c
)^6+114*A*cos(1/2*d*x+1/2*c)^4-24*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1
/2*c)^2+17*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^3+3a^3\sec(dx+c)^2+3a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^
3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x
)
```

3.221 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=228

$$-\frac{(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(13A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-9B)\sqrt{\mathrm{co}}}{}$$

```
[Out] ((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(10*a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d*(a + a*Sec[c + d*x])^3) - ((8*A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(15*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.498514, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4020, 3787, 3771, 2639, 2641}

$$-\frac{(13A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B)\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] ((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(10*a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d*(a + a*Sec[c + d*x])^3) - ((8*A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(15*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A-B)-\frac{5}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= \frac{(49A-9B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - (13A-3B)\sqrt{\cos(}$$

Mathematica [C] time = 6.5179, size = 364, normalized size = 1.6

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))(A+B\sec(c+dx))\left(i(49A-9B)e^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*(Cos[d*x] + I*Si
n[d*x])*(160*(13*A - 3*B)*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2] + (I*(49*A - 9*B)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*
(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(((3*
I)/2)*(c + d*x)) + 2*Cos[c + d*x]*((-30*I)*(49*A - 9*B)*Cos[(c + d*x)/2] -
(15*I)*(49*A - 9*B)*Cos[(3*(c + d*x))/2] - (147*I)*A*Cos[(5*(c + d*x))/2] +
 (27*I)*B*Cos[(5*(c + d*x))/2] + 142*A*Sin[(c + d*x)/2] - 42*B*Sin[(c + d*x
)/2] + 205*A*Sin[(3*(c + d*x))/2] - 45*B*Sin[(3*(c + d*x))/2] + 87*A*Sin[(5
*(c + d*x))/2] - 27*B*Sin[(5*(c + d*x))/2])))/(120*a^3*d*E^(I*d*x)*(B + A*C
os[c + d*x])*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.895, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos
(1/2*d*x+1/2*c)^8+130*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+294*
A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8
-30*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-54*B*cos(1/2*d*x+1/2*c
)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+
1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2
*d*x+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/cos(1/2*d*x+1/2*c)^5/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^4+3\,a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+a^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^4 + 3*a^
3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

3.222 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$\frac{(33A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(33A-13B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{7(17A-7B)\sin(c+}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c\right.}$$

```
[Out] (-7*(17*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*
Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a
*Sec[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
 a*Sec[c + d*x])^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]
]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.553005, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(33A-13B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{7(17A-7B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{(33A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (-7*(17*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*
Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a
*Sec[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
 a*Sec[c + d*x])^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]
]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B)-\frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{(33A-13B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2}{3ad\sqrt{\sec(}}$$

$$= -\frac{7(17A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(33A-13B)\sin}{6a^3d\sqrt{\sec(c}}$$

$$= -\frac{7(17A-7B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(33A-13B)\sqrt{c}}{}$$

Mathematica [C] time = 6.89894, size = 377, normalized size = 1.44

$$e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))(A+B\sec(c+dx))\left(7i(17A-7B)e^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*(Cos[d*x] + I*Sin
[d*x])*(160*(33*A - 13*B)*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2] + ((7*I)*(17*A - 7*B)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(
((3*I)/2)*(c + d*x)) + 2*Cos[c + d*x]*((-210*I)*(17*A - 7*B)*Cos[(c + d*x)/
2] - (105*I)*(17*A - 7*B)*Cos[(3*(c + d*x))/2] - (357*I)*A*Cos[(5*(c + d*x)
)/2] + (147*I)*B*Cos[(5*(c + d*x))/2] + 352*A*Sin[(c + d*x)/2] - 142*B*Sin[
(c + d*x)/2] + 545*A*Sin[(3*(c + d*x))/2] - 205*B*Sin[(3*(c + d*x))/2] + 22
7*A*Sin[(5*(c + d*x))/2] - 87*B*Sin[(5*(c + d*x))/2] + 10*A*Sin[(7*(c + d*x
))/2])))/(120*a^3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 2.198, size = 465, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*co
s(1/2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-348*B*cos(1/2*d*x+1/2*c)^8-130*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2
*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-264*B*c
os(1/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A
-3*B)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^5+3\,a^3\sec(dx+c)^4+3\,a^3\sec(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^5 + 3*a^
3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

3.223 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=294

$$-\frac{(21A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{3(21A-11B)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33A-17B}{30a^3d\,\mathrm{se}}$$

```
[Out] (7*(33*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) - ((21*A - 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) + (7*(33*A - 17*B)*Sin[c + d*x])/(30*a^3
*d*Sec[c + d*x]^(3/2)) - ((21*A - 11*B)*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c +
 d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^3) - ((12*A - 7*B)*Sin[c + d*x])/(15*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c
 + d*x])^2) - (3*(21*A - 11*B)*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3
+ a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.570189, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{3(21A-11B)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33A-17B)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(21A-11B)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{(21A-11B)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (7*(33*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) - ((21*A - 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) + (7*(33*A - 17*B)*Sin[c + d*x])/(30*a^3
*d*Sec[c + d*x]^(3/2)) - ((21*A - 11*B)*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c +
 d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^3) - ((12*A - 7*B)*Sin[c + d*x])/(15*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c
 + d*x])^2) - (3*(21*A - 11*B)*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3
+ a^3*Sec[c + d*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{5}{2}a(3A-B)-\frac{9}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= \frac{7(33A-17B)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(21A-11B)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(}{5d\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{7(33A-17B)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(21A-11B)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(}{5d\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{7(33A-17B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(21A-11B)\sqrt{\mathrm{co}}}{}$$

Mathematica [C] time = 7.36201, size = 1032, normalized size = 3.51

$$-\frac{77\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(}}\right.}{5d(B+A\cos(c+dx))(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (-77*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E
^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (119*Sqrt[2]*B*Sqrt
[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*C
os[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d
*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^3) - (42*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
 + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A
+ B*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) +
 (22*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A*C
os[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(
5/2)*(A + B*Sec[c + d*x])*(((-329*A + 178*B - 133*A*Cos[2*c] + 60*B*Cos[2*c
])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (8*(-3*A + B)*Cos[2*d*x]*Sin[2*c])/(
3*d) + (4*A*Cos[3*d*x]*Sin[3*c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(
-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]
^3*(-27*A*Sin[(d*x)/2] + 22*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 +
 (d*x)/2]*(-69*A*Sin[(d*x)/2] + 43*B*Sin[(d*x)/2]))/(3*d) - (4*(-133*A + 60
*B)*Cos[c]*Sin[d*x])/(5*d) + (8*(-3*A + B)*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*
A*Cos[3*c]*Sin[3*d*x])/(5*d) - (4*(-69*A + 43*B)*Tan[c/2])/(3*d) + (4*(-27*
A + 22*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*
x)/2]^4*Tan[c/2])/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.108, size = 493, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*co
s(1/2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10+160*B*cos(1/2*d*x+1/2*c)^10
-228*A*cos(1/2*d*x+1/2*c)^8-630*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-1386*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*B*cos(1/2*d
*x+1/2*c)^8+330*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+714*B*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6-1058
*B*cos(1/2*d*x+1/2*c)^6-744*A*cos(1/2*d*x+1/2*c)^4+474*B*cos(1/2*d*x+1/2*c)
^4+57*A*cos(1/2*d*x+1/2*c)^2-47*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/cos(1/2*d*x
+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^6+3\,a^3\sec(dx+c)^5+3\,a^3\sec(dx+c)^4+a^3\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^6 + 3*a^
3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

3.224 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{a(6A+5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aB}{}$$

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*d) + (a*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqr
t[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rubi [A] time = 0.288429, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4016, 3803, 3801, 215}

$$\frac{a(6A+5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*d) + (a*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqr
t[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(6A+5B)\int \sec^{\frac{5}{2}}(c+dx$$

$$= \frac{a(6A+5B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aB\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}}{3d\sqrt{a+a\sec(c}}$$

$$= \frac{a(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(6A+5B)\sec^{\frac{5}{2}}(c}{12d\sqrt{a+a}}$$

$$= \frac{a(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(6A+5B)\sec^{\frac{5}{2}}(c}{12d\sqrt{a+a}}$$

$$= \frac{\sqrt{a}(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a(6A+5B)\sec^{\frac{3}{2}}(c}{8d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 1.38198, size = 131, normalized size = 0.74

$$\frac{\sqrt{a(\sec(c+dx)+1)}\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)(4(6A+5B)\cos(c+dx)+3(6A+5B)\cos(2(c+dx))+18A+31B)}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(6*A + 5*B)*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]]*Sec[(c + d*x)/2] + (18*A + 31*B + 4*(6*A + 5*B)*Cos[c + d*x] + 3*
(6*A + 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Tan[(c + d*x)/2]))/(48*d*Sqrt[
Sec[c + d*x]])
```

Maple [B] time = 0.355, size = 408, normalized size = 2.3

$$\frac{(\cos(dx+c))^2-1}{96\,d\,(\sin(dx+c))^2}\left(18\,A\,(\cos(dx+c))^3\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/96/d*(18*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))*2^(1/2)+18*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+15*B*cos(d*x+c)^3*ar
```

```
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/
2)+15*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x
+c)-1+sin(d*x+c)))*2^(1/2)+36*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2)+30*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+24*A*cos(d*x+
c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+20*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2)+16*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))
^(5/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*
x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.73419, size = 4512, normalized size = 25.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sq
rt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d
*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqr
t(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1
2*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1
)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
+ 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*c
os(2*d*x + 2*c) + 1) + (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x
+ 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x
```

```
 + c))) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqr
t(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(
sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x
 + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*
d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5
/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*
sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*
cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 2) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c
) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*co
s(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
 + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2
) - 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) -
20*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(
sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt
(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*c
os(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*
d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(3*cos(4*d*
x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 +
6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(
2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.760887, size = 1152, normalized size = 6.55

$$\left[\frac{3\left((6A+5B)\cos(dx+c)^3+(6A+5B)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{si}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((6*A + 5*B)*cos(d*x + c)^3 + (6*A + 5*B)*cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(6*A + 5*B)*cos
(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x
 + c)^2), 1/48*(3*((6*A + 5*B)*cos(d*x + c)^3 + (6*A + 5*B)*cos(d*x + c)^2)
*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*
(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

3.225 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{a(4A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a*(4*A + 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec
[c + d*x]])
```

Rubi [A] time = 0.237136, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4016, 3803, 3801, 215}

$$\frac{a(4A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a*(4*A + 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec
[c + d*x]])
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(4A+3B)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{}$$

$$=\frac{a(4A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+}}$$

$$=\frac{a(4A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+}}$$

$$=\frac{\sqrt{a}(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a(4A+3B)\sec^{\frac{3}{2}}(c+}{4d\sqrt{a+a\sec(}}$$

Mathematica [A] time = 0.488002, size = 106, normalized size = 0.81

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(4A+3B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(4A+2B\right.}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(4*A + 3*B)*ArcTanh[S
qrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 3*B + 2*B*Sec[c + d*x])*Si
n[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.363, size = 344, normalized size = 2.6

$$-\frac{-1+\cos(dx+c)}{8\,d\,(\sin(dx+c))^2}\left(4\,A\,(\cos(dx+c))^2\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{2}+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(4*A*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+4*A*cos(d*x+c)^2*arctan(1/4*2
^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+3*B*co
s(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d
*x+c)))*2^(1/2)+3*B*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2)+6*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*B*(-2
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(3/2)*(a*(cos(d*x+c)+1)/c
```

```
os(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.45047, size = 2601, normalized size = 19.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*s
qrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) +
 (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*si
n(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos
(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log
(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 3*(
2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*
d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
```

```
s(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*cos
(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sqrt(
2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1))/d
```

Fricas [A] time = 0.755814, size = 1045, normalized size = 7.98

$$\left[\frac{\left((4A+3B)\cos\left(dx+c\right)^2+(4A+3B)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(d}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(((4*A + 3*B)*cos(d*x + c)^2 + (4*A + 3*B)*cos(d*x + c))*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((4*A + 3*B)*cos(d*x +
 c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((4*A + 3*B)*cos(d*x + c
)^2 + (4*A + 3*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)) + 2*((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

3.226 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a}(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (a*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.16025, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4016, 3801, 215}

$$\frac{\sqrt{a}(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (a*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(2A+B)\int \sqrt{\sec(c+dx}$$

$$= \frac{aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2A+B)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\right.}{d}$$

$$= \frac{\sqrt{a}(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{aB\sec^{\frac{3}{2}}(c+dx)\sin}{d\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 0.262547, size = 89, normalized size = 1.14

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(2A+B)\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2
*A + B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*B*Sin[(c + d*x)/
2]))/(2*d)
```

Maple [B] time = 0.328, size = 278, normalized size = 3.6

$$\frac{(\cos(dx+c))^2-1}{4\,d\,(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2\,A\cos(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*A*cos(d*x
+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))+2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(-cos(d*x+c)-1+sin(d*x+c)))+B*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+B*cos(d*x+c)*2^(1/2)*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+2*B*(-2/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(c
os(d*x+c)^2-1)
```

Maxima [B] time = 2.35899, size = 1222, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(2*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2)) - (4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))
/d
```

Fricas [B] time = 0.743637, size = 863, normalized size = 11.06

$$\left[\frac{((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4B\sqrt{\frac{a\cos\cdots}{\cos\cdots}}}{\sqrt{\cdots}}}{4(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(((2*A + B)*cos(d*x + c) + 2*A + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(((2*A + B)*cos(d*
x + c) + 2*A + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c
) - 2*a)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{a\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

3.227 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.157209, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4015, 3801, 215}

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx = \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + B\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2B)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}dx, x, -\frac{a\tan(c+}{\sqrt{a+a\sec(}}\right.}{d}$$

$$= \frac{2\sqrt{a}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.378419, size = 83, normalized size = 1.09

$$\frac{2a\left(A\sin(c+dx)\sqrt{-(\sec(c+dx)-1)\sec(c+dx)} - B\tan(c+dx)\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (2*a*(A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - B*ArcSin[S
qrt[Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec
[c + d*x])])
```

Maple [B] time = 0.286, size = 177, normalized size = 2.3

$$-\frac{1}{2d\sin(dx+c)}\left(B\sqrt{2}\arctan\left(\frac{\sqrt{2}(-\cos(dx+c)-1+\sin(dx+c))}{4}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/2/d*(B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)
-1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+B*2^(1/2)*arctan(1/4*2
^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+4*A*cos(d*x+c)-4*A)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)/(1/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.06103, size = 354, normalized size = 4.66

$$4\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right) + B\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 2\sqrt{2}\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + B*sqrt(a)*(log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [B] time = 0.569846, size = 819, normalized size = 10.78

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(B\cos(dx+c)+B)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\cdots}}{\sqrt{\cos(dx+c}\cdots}}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right.}{2\,(d\cos(dx+c)+d)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*A*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) +
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\,(\sec(c+dx)+1)}\,(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sqrt(sec(c + d*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{a\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

3.228 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.158257, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {4013, 3804}

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}(A+3B)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a(A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.218533, size = 56, normalized size = 0.68

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(A\cos(c+dx)+2A+3B)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*(2*A + 3*B + A*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2]
)/(3*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.307, size = 75, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))(A\cos(dx+c)+2A+3B)(\cos(dx+c))^2}{3d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(A*cos(d*x+c)+2*A+3*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))
^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [A] time = 2.00967, size = 181, normalized size = 2.21

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*
sqrt(a)*sin(1/2*d*x + 1/2*c))/d
```

Fricas [A] time = 0.460212, size = 197, normalized size = 2.4

$$\frac{2\left(A\cos(dx+c)^2+(2A+3B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{3(d\cos(dx+c)+d)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*(A*cos(d*x + c)^2 + (2*A + 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

3.229 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]])
```

Rubi [A] time = 0.222103, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4015, 3805, 3804}

$$\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]])
```


Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}(4A+5B)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}$$
$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(4A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(4A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.285668, size = 71, normalized size = 0.55

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}(2(4A+5B)\cos(c+dx)+3A\cos(2(c+dx))+19A+20B)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a*(19*A + 20*B + 2*(4*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.314, size = 96, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(3A(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+c)+8A+10B\right)(\cos(dx+c))^3}{15d\sin(dx+c)}\sqrt{\frac{a(\cos(d}{\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+8*A
+10*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2
)/sin(d*x+c)
```

Maxima [B] time = 2.12817, size = 428, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
```

```
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 + 10*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.460691, size = 243, normalized size = 1.87

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{3}+\left(4\,A+5\,B\right)\cos\left(dx+c\right)^{2}+2\left(4\,A+5\,B\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^3 + (4*A + 5*B)*cos(d*x + c)^2 + 2*(4*A + 5*B)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x +
 c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
 x)
```

3.230 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) + (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.287717, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4015, 3805, 3804}

$$\frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) + (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(6A+7B)\int \frac{\sqrt{a+a\sec(c+d}}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 0.311128, size = 91, normalized size = 0.52

$$\frac{2a\sin(c+dx)\left(8(6A+7B)\sec^3(c+dx)+4(6A+7B)\sec^2(c+dx)+3(6A+7B)\sec(c+dx)+15A\right)}{105d\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2
),x]
```

```
[Out] (2*a*(15*A + 3*(6*A + 7*B)*Sec[c + d*x] + 4*(6*A + 7*B)*Sec[c + d*x]^2 + 8*
(6*A + 7*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(5/2)*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [A] time = 0.349, size = 118, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^3+18A(\cos(dx+c))^2+21B(\cos(dx+c))^2+24A\cos(dx+c)+28B\cos\right.}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+48*A+56*B)*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.15824, size = 672, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
```

```
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) + 14*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x
+ 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*c
os(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))) + 6*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.466919, size = 290, normalized size = 1.66

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{4}+3\,(6\,A+7\,B)\cos\left(dx+c\right)^{3}+4\,(6\,A+7\,B)\cos\left(dx+c\right)^{2}+8\,(6\,A+7\,B)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{105\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^4 + 3*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*c
os(d*x + c)^2 + 8*(6*A + 7*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
 x)
```

3.231 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^2(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.547072, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^2(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}+\frac{1}{4}\int \mathrm{s}$$

$$=\frac{a^2(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{7}{2}}(c+dx}{}$$

$$=\frac{a^2(88A+75B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(8A+9B)}{24d\sqrt{}}$$

$$=\frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(88A+75}{96d}$$

$$=\frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(88A+75}{96d}$$

$$=\frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{a^2(88A+75}{64d}$$

Mathematica [A] time = 1.37033, size = 153, normalized size = 0.67

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(6\sqrt{2}(88A+75B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)((}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(88*A + 75*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]] + (352*A + 492*B + (1048*A + 1155*B)*Cos[c +
 d*x] + 4*(88*A + 75*B)*Cos[2*(c + d*x)] + 264*A*Cos[3*(c + d*x)] + 225*B*C
os[3*(c + d*x)])*Sec[c + d*x]^4*Sin[(c + d*x)/2]))/(768*d*Sqrt[Sec[c + d*x]
])
```

Maple [B] time = 0.314, size = 479, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/768/d*a*(264*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+264*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+225*B*cos(d*x+c
)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
*2^(1/2)+225*B*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-
cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+528*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*
x+c)+1))^(1/2)+450*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+352*
A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+300*B*cos(d*x+c)^2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+128*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c
)+1))^(1/2)+240*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+96*B*(-2/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/co
s(d*x+c))^(5/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+
c)^2-1)
```

Maxima [B] time = 3.92637, size = 7937, normalized size = 34.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] -1/768*(8*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x
 + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c)
+ 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
```

```
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9
*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a
*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*
x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
+ 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*
c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)
^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2
 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a
*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos
(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x
 + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3
*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*
x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2
)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c
) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(
6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c)
+ sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sq
rt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2
*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*A*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9
*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*
x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2
*c) + 1) + 3*(300*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c
) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(15/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*a*sin(8*d*x + 8*
c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2
)*a*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1140*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt
(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqr
t(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*
d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqr
t(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*
d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6
*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*
cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*a*sin(8
*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) +
 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 300*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) +
```

```
 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*c
os(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*s
in(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*
a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(
6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x +
 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c
) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) +
 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a
*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a
*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 4
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*co
s(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x
 + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6
*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c)
 + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16
*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 +
 a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 +
 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*
cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d
*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x +
 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*
c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*
d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 +
16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2
 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2
 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*
a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8
*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
 + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x +
2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x
+ 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sq
rt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(15/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x
+ 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt
(2)*a)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2
)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x
 + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2
)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x
```

```
 + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)
*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c
) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)
*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
 + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)
*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(
8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c)
+ 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*co
s(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x +
6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)
^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*
sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x +
2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.00373, size = 1297, normalized size = 5.71

$$\left[\frac{3\left((88\,A+75\,B)a\cos(dx+c)^4+(88\,A+75\,B)a\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\cdots}}{\sqrt{\cos(dx+c)}}\cdots}{\cos(dx+c)^3+\cos(dx+c)^2}\right)\cdots}{768\left(d\cos(dx+c)^4+\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/768*(3*((88*A + 75*B)*a*cos(d*x + c)^4 + (88*A + 75*B)*a*cos(d*x + c)^3)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(88*A
+ 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B
)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/384*(3*((
88*A + 75*B)*a*cos(d*x + c)^4 + (88*A + 75*B)*a*cos(d*x + c)^3)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(88*A + 75*B
)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*co
s(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

3.232 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^2(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(14A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}$$

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^2*(14*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt
[a + a*Sec[c + d*x]]) + (a^2*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
 d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.418528, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^2(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(14A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^2*(14*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt
[a + a*Sec[c + d*x]]) + (a^2*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
 d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
```

```
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\int \sec^{\frac{3}{2}}$$

$$= \frac{a^2(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{5}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(6A+7B)\sec}{12d\sqrt{a+}}$$

$$= \frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(6A+7B)\sec}{12d\sqrt{a+}}$$

$$= \frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^2(14A+11B)\sec}{8d\sqrt{a+}}$$

Mathematica [A] time = 1.18738, size = 134, normalized size = 0.74

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(14A+11B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)(4(6A\right.}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]] + (7*(6*A + 7*B) + 4*(6*A + 11*B)*Cos[c + d*
x] + (42*A + 33*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]))/(48*
d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.309, size = 415, normalized size = 2.3

$$-\frac{a(-1+\cos(dx+c))}{48d\cos(dx+c)(\sin(dx+c))^2}\left(42A(\cos(dx+c))^3\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/48/d*a*(-1+cos(d*x+c))*(42*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+42*A*cos(d*x+c)^3*arctan(
1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+3
3*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
+sin(d*x+c)))*2^(1/2)+33*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+84*A*cos(d*x+c)^2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)+66*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2)+24*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+44*B*cos(d*x+c)*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/cos(d*x+c)/si
n(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.96634, size = 6218, normalized size = 34.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*
sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin
(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
```

```
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)
*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(2*(2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (132*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x +
6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x +
6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 +
```

```
9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*
a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d
*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
 + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2
*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log
(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c
)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^
2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*
a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*co
s(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*
x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x
 + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c)
+ 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6
*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)
*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*c
os(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*
(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*
sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x
+ 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*
a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(3*cos(4*d*x + 4*c)
+ 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2
*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.765067, size = 1197, normalized size = 6.65

$$\left[\frac{3\left((14\,A+11\,B)a\cos\left(dx+c\right)^{3}+(14\,A+11\,B)a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(\cdots}{\cos(\cdots}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\cdots\right.}{96\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((14*A + 11*B)*a*cos(d*x + c)^3 + (14*A + 11*B)*a*cos(d*x + c)^2)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(14*A +
 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 8*B*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2), 1/48*(3*((14*A + 11*B)*a*cos(d*x + c)^3 + (14*A
+ 11*B)*a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos
(d*x + c) - 2*a)) + 2*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*
cos(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

3.233 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=133

$$\frac{a^2(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^2*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(2*d)
```

Rubi [A] time = 0.336036, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4018, 4016, 3801, 215}

$$\frac{a^2(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^2*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(2*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}+\frac{1}{2}\int \sqrt{\mathrm{se}$$

$$= \frac{a^2(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{a^2(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^2(4A+5B)\sec^{\frac{3}{2}}}{4d\sqrt{a+a}}$$

Mathematica [A] time = 0.669928, size = 107, normalized size = 0.8

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(12A+7B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(4A+}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(12*A + 7*B)*ArcTan
h[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 7*B + 2*B*Sec[c + d*x])
*Sin[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.332, size = 355, normalized size = 2.7

$$\frac{a\left((\cos(dx+c))^2-1\right)}{16\,d\,(\sin(dx+c))^2\cos(dx+c)}\left(12\,A\,(\cos(dx+c))^2\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] 1/16/d*a*(12*A*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*2^(1/2)+12*A*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+7*B*cos(d*x+c)^2*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1
/2)+7*B*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x
+c)-1+sin(d*x+c)))*2^(1/2)+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
```

```
/2)+14*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*B*(-2/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^
(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.61047, size = 4575, normalized size = 34.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(
2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(
2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(
2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3
/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
```

```
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*
a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) -
 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*
cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7
*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sq
rt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt
(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(
3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)
*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*ar
```

```
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a)/(2*(2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d
```

Fricas [A] time = 0.757715, size = 1072, normalized size = 8.06

$$\left[\frac{\left((12\,A+7\,B)a\cos\left(dx+c\right)^{2}+(12\,A+7\,B)a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx\cdots}{\cos(dx\cdots}}}{\sqrt{\cos(dx+c)}}\cdots}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\cdots\right.\cdots}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(((12*A + 7*B)*a*cos(d*x + c)^2 + (12*A + 7*B)*a*cos(d*x + c))*sqrt(a
)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*
x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((4*A + 7*B)*a*c
os(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((12*A + 7*B)*
a*cos(d*x + c)^2 + (12*A + 7*B)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*((4*A + 7*B)*a*cos(d*x + c) + 2
*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

3.234 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=124

$$\frac{a^2(2A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+}}{d}$$

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^2*(2*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c
 + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
d
```

Rubi [A] time = 0.313531, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4018, 4015, 3801, 215}

$$\frac{a^2(2A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^2*(2*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c
 + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
d
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{aB\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}+\int\frac{\sqrt{a+a\sec(c+dx}}{}$$

$$=\frac{a^2(2A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}{d}$$

$$=\frac{a^2(2A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}{d}$$

$$=\frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2(2A-B)\sqrt{\sec(c+dx)}\sin(c}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 1.52081, size = 107, normalized size = 0.86

$$\frac{a^2\tan(c+dx)\left(\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}(2A\cos(c+dx)+B)+2A\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-3B\sin^{-1}\left(\sqrt{\sec(c+}\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^2*(2*A*ArcSin[Sqrt[1 - Sec[c + d*x]]] - 3*B*ArcSin[Sqrt[Sec[c + d*x]]] +
 (B + 2*A*Cos[c + d*x])*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Tan[c + d
*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.34, size = 346, normalized size = 2.8

$$-\frac{a}{4d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2A\sin(dx+c)\cos(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(-\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*A*sin(d*x+c)*cos(d*x+c)*2^(
1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)
))*(-2/(cos(d*x+c)+1))^(1/2)+2*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2
^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)
+1))^(1/2)+3*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+3*B*si
n(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+8*A*cos(d*x+c)^2-8*A*cos
(d*x+c)+4*B*cos(d*x+c)-4*B)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.27494, size = 1913, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.760473, size = 957, normalized size = 7.72

$$\left[\frac{\left((2A+3B)a\cos\left(dx+c\right)+(2A+3B)a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+}{4\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a
)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*A*a*cos(d*x + c) + B*a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c) + d), 1/2*(((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*a*cos(d*x +
 c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

3.235 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a^(3/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]])
```

Rubi [A] time = 0.339215, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4017, 4015, 3801, 215}

$$\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{1}{2}a(4A\right.}{\sqrt{\sec(c}}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.586402, size = 109, normalized size = 0.87

$$\frac{2a^2\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}(A\cos(c+dx)+5A+3B)+3B\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (2*a^2*((5*A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*B*ArcSin[Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(3*d*Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.311, size = 211, normalized size = 1.7

$$-\frac{a(\cos(dx+c))^2}{6d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3B\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(-\cos(dx+c)-1+\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*B*2^(1/2)*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+3*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*A*cos(d*x
+c)^2+16*A*cos(d*x+c)+12*B*cos(d*x+c)-20*A-12*B)*cos(d*x+c)^2*(1/cos(d*x+c)
)^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.08744, size = 424, normalized size = 3.39

$$\frac{3\sqrt{2}\left(\sqrt{2}a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}a\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*B
*sqrt(a) + 4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/
2*c))*A*sqrt(a))/d
```

Fricas [A] time = 0.576282, size = 971, normalized size = 7.77

$$\left[\frac{3\left(Ba\cos\left(dx+c\right)+Ba\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(Aa\cos(dx+c)^2+(5\right.}{}}{6\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/6*(3*(B*a*cos(d*x + c) + B*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(A*a*cos(d*x + c)^2 + (5*A + 3*B)*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c) + d), 1/3*(3*(B*a*cos(d*x + c) + B*a)*sqrt(-a)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(A*a*cos(d*x + c)^2 + (5*A +
3*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

3.236 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{8a^2(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)(a\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (8*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[
c + d*x]^(3/2))
```

Rubi [A] time = 0.256374, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4013, 3809, 3804}

$$\frac{8a^2(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)(a\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (8*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[
c + d*x]^(3/2))
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}(3A+5B)\int \frac{(a+a\sec(c+dx}{\sec^{\frac{3}{2}}(c+dx}$$
$$=\frac{2a(3A+5B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A(a+a\sec(c+dx))^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx}$$
$$=\frac{8a^2(3A+5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a(3A+5B)\sqrt{a+a\sec(c+}}{15d\sqrt{\sec(c+dx}}$$

Mathematica [A] time = 0.473275, size = 73, normalized size = 0.56

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}(2(9A+5B)\cos(c+dx)+3A\cos(2(c+dx))+39A+50B)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*(39*A + 50*B + 2*(9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.302, size = 97, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(3A(\cos(dx+c))^2+9A\cos(dx+c)+5B\cos(dx+c)+18A+25B\right)(\cos(dx+c))^3}{15d\sin(dx+c)}\sqrt{\frac{a(\cos(}{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*x+c)+1
8*A+25*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(
5/2)/sin(d*x+c)
```

Maxima [B] time = 2.0118, size = 338, normalized size = 2.58

$$3\sqrt{2}\left(20a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x +
 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*
```

```
sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))))*A*sqrt(a) + 20*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2
*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 0.464131, size = 251, normalized size = 1.92

$$\frac{2\left(3\,Aa\cos\left(dx+c\right)^{3}+(9\,A+5\,B)a\cos\left(dx+c\right)^{2}+(18\,A+25\,B)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*a*cos(d*x + c)^3 + (9*A + 5*B)*a*cos(d*x + c)^2 + (18*A + 25*B)*a
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos
(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2
), x)
```

3.237 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqr
t[a + a*Sec[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[
c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.437968, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqr
t[a + a*Sec[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[
c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
```

```
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{1}{2}a(8\right.}{}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+63B)\sin(c+}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.456874, size = 92, normalized size = 0.51

$$\frac{2a^2\sin(c+dx)\left(2(52A+63B)\sec^3(c+dx)+(52A+63B)\sec^2(c+dx)+3(13A+7B)\sec(c+dx)+15A\right)}{105d\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] (2*a^2*(15*A + 3*(13*A + 7*B)*Sec[c + d*x] + (52*A + 63*B)*Sec[c + d*x]^2 +
 2*(52*A + 63*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(5/2)*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.304, size = 119, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+39A(\cos(dx+c))^2+21B(\cos(dx+c))^2+52A\cos(dx+c)+63B\,c\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*
x+c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+104*A+126*B)*(a*(cos(d*x+c)+1)/cos(d
*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.17754, size = 694, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 42*sqrt(2)*(20*a*cos(4/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) +
5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*
x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin(5/2*d*x + 5/2*c) + 5*a*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*a*sin(1/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.469683, size = 305, normalized size = 1.69

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{4}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B)a\cos\left(dx+c\right)^{2}+2\,(52\,A+63\,B)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^4 + 3*(13*A + 7*B)*a*cos(d*x + c)^3 + (52*A + 63
*B)*a*cos(d*x + c)^2 + 2*(52*A + 63*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2
), x)
```

3.238 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(34A+39B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(34A+39B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Se
c[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Se
c[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.507969, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^2(34A+39B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(34A+39B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Se
c[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Se
c[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{1}{2}a(1\right.}{}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(34A+39B)\sin(c+}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.658926, size = 110, normalized size = 0.48

$$\frac{2a^2\sin(c+dx)\left(8(34A+39B)\sec^4(c+dx)+4(34A+39B)\sec^3(c+dx)+3(34A+39B)\sec^2(c+dx)+5(17A+9B\right.}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] (2*a^2*(35*A + 5*(17*A + 9*B)*Sec[c + d*x] + 3*(34*A + 39*B)*Sec[c + d*x]^2
 + 4*(34*A + 39*B)*Sec[c + d*x]^3 + 8*(34*A + 39*B)*Sec[c + d*x]^4)*Sin[c +
 d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.316, size = 141, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(35A(\cos(dx+c))^4+85A(\cos(dx+c))^3+45B(\cos(dx+c))^3+102A(\cos(dx+c))^2+1\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*
x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+136*A*cos(d*x+c)+156*B*cos(d*x
```

```
+c)+272*A+312*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*
x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.23489, size = 945, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) + 6*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7
/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))
 - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.476827, size = 355, normalized size = 1.56

$$\frac{2\left(35\,Aa\cos\left(dx+c\right)^{5}+5\,(17\,A+9\,B)a\cos\left(dx+c\right)^{4}+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^{3}+4\,(34\,A+39\,B)a\cos\left(dx+c\right)^{2}+\right.}{315\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^5 + 5*(17*A + 9*B)*a*cos(d*x + c)^4 + 3*(34*A +
39*B)*a*cos(d*x + c)^3 + 4*(34*A + 39*B)*a*cos(d*x + c)^2 + 8*(34*A + 39*B)
*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*c
os(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

3.239 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a^2(10A+13B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{40d}+\frac{a^3(170A+157B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+28}{192d}$$

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(170*A + 157*B)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(10*A + 13
*B)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B
*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.693094, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^2(10A+13B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{40d}+\frac{a^3(170A+157B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+28}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(170*A + 157*B)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(10*A + 13
*B)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B
*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
```

```
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\int$$

$$= \frac{a^2(10A+13B)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{40d}$$

$$= \frac{a^3(170A+157B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(10A+}{}$$

$$= \frac{a^3(326A+283B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(170A-}{2}$$

$$= \frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A-}{1}$$

$$= \frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A-}{1}$$

$$= \frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^3(326A+}{1}$$

Mathematica [A] time = 2.09831, size = 178, normalized size = 0.65

$$a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(60\sqrt{2}(326A+283B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^5(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(326*A + 283*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (22030*A + 24863*B + 36*(650*A + 781*
B)*Cos[c + d*x] + 4*(6730*A + 6509*B)*Cos[2*(c + d*x)] + 6520*A*Cos[3*(c +
d*x)] + 5660*B*Cos[3*(c + d*x)] + 4890*A*Cos[4*(c + d*x)] + 4245*B*Cos[4*(c
 + d*x)])*Sec[c + d*x]^5*Sin[(c + d*x)/2]))/(15360*d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.333, size = 543, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/7680/d*a^2*(4890*A*cos(d*x+c)^5*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+4890*A*cos(d*x+c)^5*2^(1/2)*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+4245*B*cos
(d*x+c)^5*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))+4245*B*cos(d*x+c)^5*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+
c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+9780*A*sin(d*x+c)*cos(d*x+c)^4*(-2
/(cos(d*x+c)+1))^(1/2)+8490*B*sin(d*x+c)*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(
1/2)+6520*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+5660*B*sin(d*
x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+3680*A*cos(d*x+c)^2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)+4528*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2)+960*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2784*B*cos(d*
x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+768*B*(-2/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(co
s(d*x+c)+1))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 6.93489, size = 12477, normalized size = 45.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] -1/7680*(10*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*co
s(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(
8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6
*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2
*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*
x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt
(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*s
in(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)
```

```
*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(
sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a
^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(
6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^
2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^
2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 +
8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x
+ 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*
x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2
*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin
(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*
d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^
2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 +
 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c)
 + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d
*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2
*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos
(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*
sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*c
os(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 +
 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*
c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(
6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8
*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos
(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a
^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16
*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*
c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*
x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x +
2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*c
os(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2
*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*
a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) +
3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a
^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos
(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x +
2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqr
```

```
t(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*s
in(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*co
s(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x +
 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*s
qrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^
2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) +
sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(s
qrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^
2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x +
 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4
*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2
*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*
x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*A*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1) + (16980*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8
*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x +
4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(19/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*si
n(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x
 + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(17/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 81504*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^
2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4
*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 8320*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)
*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 86440*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqr
t(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^
2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5
*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2
)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^2*sin(10*d*x + 10*c) +
 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 81504*(sqrt(2)*a^2*sin(10*d*x + 10*c
) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*s
qrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 5660*(sqrt(2)*a^2*sin(10*d*x + 10
*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10
*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16980*(sqrt(2)*a^2*sin(10*d*x +
 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) +
 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4245*(a^2*cos(10*d*x + 10*c)
^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4
*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a
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^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 4245*(a^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*c)

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 + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(
2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16980*(sqrt(2)*a^2*cos(10*d*x +
10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) +
10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2)*sin(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 5660*(sqrt(2)*
a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*co
s(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(17/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) +
 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*c
os(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*a^2*cos(10*d*x + 10*c)
 + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 86440*(sqrt(2)*a^2*
cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*
d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*
c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 83
20*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sq
rt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^
2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d
*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*
c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqr
t(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^
2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16980*(sqrt(2)*a^2*cos(10*d*
x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c
) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt
(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2
*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*
d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 10*(10*cos(6*d*
x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) +
 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*
cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)^2 + 10*(sin(8*
d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*si
n(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x + 6*c) + 2*sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d*x + 8*c)^2 + 1
00*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 100*sin(6*d*x
 + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.02346, size = 1476, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/7680*(15*((326*A + 283*B)*a^2*cos(d*x + c)^5 + (326*A + 283*B)*a^2*cos(d
*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
+ c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*(15*(326*A + 283*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x +
c)^3 + 8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x
+ c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/3840*(15*((326*A +
 283*B)*a^2*cos(d*x + c)^5 + (326*A + 283*B)*a^2*cos(d*x + c)^4)*sqrt(-a)*a
rctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(15*(326*A + 2
83*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^3 + 8*(230*A
 + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x + c) + 384*B*a^
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2
), x)
```

3.240 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^3(104A+95B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(200A+163B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sec^{\frac{5}{2}}(}{24d}$$

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(104*A + 95*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 11*B)*Sec[c + d*x]^(5
/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Sec[c + d*x]^(5/2)
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.59461, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4018, 4016, 3803, 3801, 215}

$$\frac{a^3(104A+95B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(200A+163B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sec^{\frac{5}{2}}(}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(104*A + 95*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 11*B)*Sec[c + d*x]^(5
/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Sec[c + d*x]^(5/2)
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{1}{4}\int$$

$$= \frac{a^2(8A+11B)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+d}{24d}$$

$$= \frac{a^3(104A+95B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(8A+11}{}$$

$$= \frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(104A+}{9}$$

$$= \frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(104A+}{9}$$

$$= \frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^3(200A+}{6}$$

Mathematica [A] time = 1.40905, size = 154, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(6\sqrt{2}(200A+163B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^4(c+d\right.}{768}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(200*A + 163*B)
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (544*A + 844*B + (2056*A + 2203*B)*Cos
[c + d*x] + (544*A + 652*B)*Cos[2*(c + d*x)] + 600*A*Cos[3*(c + d*x)] + 489
*B*Cos[3*(c + d*x)])*Sec[c + d*x]^4*Sin[(c + d*x)/2]))/(768*d*Sqrt[Sec[c +
d*x]])
```

Maple [B] time = 0.329, size = 479, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/384/d*a^2*(-1+cos(d*x+c))*(600*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+600*A*cos(d*x+c)^4*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1
/2)+489*B*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))*2^(1/2)+489*B*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+1200*A*sin(d*x+c)*cos(
d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+978*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d
*x+c)+1))^(1/2)+544*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+652
*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+128*A*cos(d*x+c)*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+368*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2)+96*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos
(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/cos(d*x+c)^2/sin(d*x+c)^2/(-2/(cos(d*x+
c)+1))^(1/2)
```

Maxima [B] time = 4.42439, size = 9897, normalized size = 43.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt
(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
- 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a
^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(
9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 +
9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^
2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(
6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
```

os(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c)
+ 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(
6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
c))) + 6(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c)
+ a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c
)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c
) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(
a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*
cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(
a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*
cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c)

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+ 3*sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x
+ 3/2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x
+ 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*
d*x + 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(cos(6*d*x + 6*c)^2 + 6*
(cos(6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 9*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(
6*d*x + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + si
n(6*d*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*cos(6*d*x + 6*c) + 1) - (1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*
sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a
^2*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c
) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqr
t(2)*a^2*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*co
s(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8
*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d
*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c)
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x
 + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*
cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 3
6*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^
2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x +
6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8
*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x +
 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6
*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c
))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
```

*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos
(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a
^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)
^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 +
 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x
 + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d
*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(
2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*si
n(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4
*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) + 2) - 489(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a
^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2
+ 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c
) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(
2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*co
s(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*
cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2
+ 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6
*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos
(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(
8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*co
s(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*
a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*
d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8
*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*s
qrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*c
os(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*
sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2
)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2
*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4
*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*
a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(
sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a
^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4

```
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) +
 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x +
4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6
*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x
 + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4
*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 +
36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x
 + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.01437, size = 1351, normalized size = 5.95

$$\left[\frac{3\left((200A+163B)a^2\cos(dx+c)^4+(200A+163B)a^2\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\cdots}{\sqrt{\cos\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2\cdots}\right.}{768\left(d\cos(dx+c\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/768*(3*((200*A + 163*B)*a^2*cos(d*x + c)^4 + (200*A + 163*B)*a^2*cos(d*x
 + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x +
c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(
3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*cos(d*x + c)^2
 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x
+ c)^3), 1/384*(3*((200*A + 163*B)*a^2*cos(d*x + c)^4 + (200*A + 163*B)*a^2
*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
 - 2*a)) + 2*(3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*
cos(d*x + c)^2 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

3.241 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^3(54A+49B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{a^{5/2}(38A+25B)\,\mathrm{si}}{}$$

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^3*(54*A + 49*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a^2*(2*A + 3*B)*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.513432, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4018, 4016, 3801, 215}

$$\frac{a^3(54A+49B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{a^{5/2}(38A+25B)\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^3*(54*A + 49*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a^2*(2*A + 3*B)*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int$$

$$= \frac{a^2(2A+3B)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{a^3(54A+49B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(2A+3B}{}$$

$$= \frac{a^3(54A+49B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(2A+3B}{}$$

$$= \frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a^3(54A+49}{24d}$$

Mathematica [A] time = 1.24843, size = 133, normalized size = 0.74

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(38A+25B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)(\right.}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(38*A + 25*B)*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (66*A + 91*B + 4*(6*A + 17*B)*Cos[c + d*
x] + (66*A + 75*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]))/(48*
d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.306, size = 419, normalized size = 2.3

$$\frac{a^2\left((\cos(dx+c))^2-1\right)}{96\,d\,(\sin(dx+c))^2(\cos(dx+c))^2}\left(114\,A\,(\cos(dx+c))^3\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] 1/96/d*a^2*(114*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+114*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(
```

```
-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)+75*B*cos(d*x+c
)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
*2^(1/2)+75*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-c
os(d*x+c)-1+sin(d*x+c)))*2^(1/2)+132*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2)+150*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+24*A*
cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+68*B*cos(d*x+c)*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)+16*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)/sin(d*x+c)^2/cos(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 22.0953, size = 8501, normalized size = 47.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)
*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2
- 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4
*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
```

n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*
c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co
s(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) +
 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c
) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*
d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*
x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x
 + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt(a
)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos
(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) - (300*sqrt(2)*a^2*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(6*d*x + 6*c) - 28
*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 2
8*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos
(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x +
 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(
sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3

/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*
sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*
c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d
*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) +
 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9
*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6
*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) +
 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*
cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6
*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) +
 a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)
^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c)
 + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a

```
^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 28*(sqrt(
2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(3/2*d*x + 3/2*c))*sin(6*d*x +
 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(11/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2
*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*cos(7/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*co
s(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*cos(6*d
*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(5/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos
(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 300*(sqrt(2)*a^2*cos(6*d*x +
 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*B*sqrt(a)/(cos(6*d*x + 6*c)^2 + 6*(cos(6*d*x + 6*c) + 3*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(8/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x + 6*c) + 1)*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d*x + 6*c)^2 + 6*(sin(6*d*x +
 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(6*d*x + 6*c) + 1))
/d
```

Fricas [A] time = 0.772432, size = 1224, normalized size = 6.8

$$\left[\frac{3\left((38A+25B)a^2\cos(dx+c)^3+(38A+25B)a^2\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\cdots}{\sqrt{\cos(d\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right.}{96\left(d\cos(dx+c)^3+d\cos(d\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((38*A + 25*B)*a^2*cos(d*x + c)^3 + (38*A + 25*B)*a^2*cos(d*x + c)
^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
- 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
```

```
 c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(22
*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(3*((38*A + 25*B)*a^2*cos(d*x + c
)^3 + (38*A + 25*B)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2
*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^
2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(4A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{4d}+\frac{a^{5/2}(20A+19B)\,\mathrm{s}}{}$$

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(4*d) + (a^3*(4*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
 + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.504055, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4018, 4015, 3801, 215}

$$\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(4A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{4d}+\frac{a^{5/2}(20A+19B)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(4*d) + (a^3*(4*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
 + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(2*d)
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx = \frac{aB\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{(a+a\sec(c+$$

$$= \frac{a^2(4A+7B)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{aB\sqrt{\sec(c+}$$

$$= \frac{a^3(4A-9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(4A+7B)\sqrt{\sec(c+dx)}\sqrt{a+}}{4d}$$

$$= \frac{a^3(4A-9B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(4A+7B)\sqrt{\sec(c+dx)}\sqrt{a+}}{4d}$$

$$= \frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^3(4A-9B)\sqrt{\sec(c+dx)}\sin}{4d\sqrt{a+a\sec(c+dx}}$$

Mathematica [A] time = 1.81061, size = 137, normalized size = 0.76

$$\frac{a^3\left(\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}(\tan(c+dx)(4A+2B\sec(c+dx)+11B)+8A\sin(c+dx))+20A\tan(c+dx)\sin^{-1}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^3*(20*A*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 19*B*ArcSin[Sqrt[S
ec[c + d*x]]]*Tan[c + d*x] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(8*A
*Sin[c + d*x] + (4*A + 11*B + 2*B*Sec[c + d*x])*Tan[c + d*x])))/(4*d*Sqrt[1
 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.367, size = 386, normalized size = 2.1

$$-\frac{a^2}{16\,d\sin(dx+c)\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(20\,A(\cos(dx+c))^2\sin(dx+c)\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/16/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(20*A*cos(d*x+c)^2*sin(d*x+
c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(
d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+20*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2
```

```
/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))+19*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^
(1/2)+19*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+32*A*cos(
d*x+c)^3-16*A*cos(d*x+c)^2+44*B*cos(d*x+c)^2-16*A*cos(d*x+c)-36*B*cos(d*x+c
)-8*B)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.768991, size = 1169, normalized size = 6.49

$$\left[\frac{\left((20\,A+19\,B)a^2\cos\left(dx+c\right)^2+(20\,A+19\,B)a^2\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a}{}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(((20*A + 19*B)*a^2*cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(8*A*a^2*c
os(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
d*cos(d*x + c)), 1/8*(((20*A + 19*B)*a^2*cos(d*x + c)^2 + (20*A + 19*B)*a^2
*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
 2*a)) + 2*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

3.243 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(2A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{a^{5/2}(2A+5B)\sin\cdots}{}$$

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^3*(14*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*
Sec[c + d*x]]) - (a^2*(2*A - 3*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.505445, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4017, 4018, 4015, 3801, 215}

$$\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(2A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{a^{5/2}(2A+5B)\sin\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^3*(14*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*
Sec[c + d*x]]) - (a^2*(2*A - 3*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{3}{2}a\right.}{}$$

$$= -\frac{a^2(2A-3B)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(a+a}{}$$

$$= \frac{a^3(14A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B)\sqrt{\sec(c+dx)}\sqrt{a}}{3d}$$

$$= \frac{a^3(14A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B)\sqrt{\sec(c+dx)}\sqrt{a}}{3d}$$

$$= \frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3(14A+3B)\sqrt{\sec(c+dx)}\sin(}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.966409, size = 133, normalized size = 0.75

$$\frac{a^3\left(3(2A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+\sqrt{1-\sec(c+dx)}(\tan(c+dx)(16A+3B\sec(c+dx)\right.}{3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a^3*(3*(2*A + 5*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2)*Sin[c
 + d*x] + Sqrt[1 - Sec[c + d*x]]*(2*A*Sin[c + d*x] + (16*A + 6*B + 3*B*Sec[
c + d*x])*Tan[c + d*x])))/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.348, size = 376, normalized size = 2.1

$$-\frac{a^2\cos(dx+c)}{12\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(6\,A\sin(dx+c)\cos(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(-\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/12/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(6*A*sin(d*x+c)*cos(d*x+c)*
2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x
+c)))*(-2/(cos(d*x+c)+1))^(1/2)+6*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x
+c)+1))^(1/2)+15*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+15
*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+8*A*cos(d*x+c)^3+56
*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-64*A*cos(d*x+c)-12*B*cos(d*x+c)-12*B)*cos
(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.77292, size = 1085, normalized size = 6.13

$$\left[\frac{3\left((2A+5B)a^2\cos(dx+c)+(2A+5B)a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{12\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/12*(3*((2*A + 5*B)*a^2*cos(d*x + c) + (2*A + 5*B)*a^2)*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*A*a^2*cos(d*x + c)^2 + 2
*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/6*(3*((2*A +
 5*B)*a^2*cos(d*x + c) + (2*A + 5*B)*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*
B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

3.244 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)}}\right.}{d}$$

```
[Out] (2*a^(5/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*
d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.488931, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4017, 4015, 3801, 215}

$$\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*a^(5/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*
d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2))
```


Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{1}{2}a\right.}{\sec^{\frac{3}{2}}}$$

$$= \frac{2a^2(8A+5B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^3}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a\sec(c}}{15d\sqrt{\sec(c+}}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a\sec(c}}{15d\sqrt{\sec(c+}}$$

$$= \frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\sin(c+a}{15d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 1.86389, size = 127, normalized size = 0.74

$$\frac{a^3\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}(2(14A+5B)\cos(c+dx)+3A\cos(2(c+dx))+89A+80B)+30B\sqrt{\sec(c+dx)}\sin^{-1}\right(}{15d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] (a^3*((89*A + 80*B + 2*(14*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sq
rt[1 - Sec[c + d*x]] + 30*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x
]])*Tan[c + d*x])/(15*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [A] time = 0.333, size = 235, normalized size = 1.4

$$-\frac{a^2(\cos(dx+c))^3}{30\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\,B\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(-\cos(dx+c)-1+\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/30/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*B*2^(1/2)*arctan(1/4*2^
(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+15*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+12*A*c
os(d*x+c)^3+44*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+116*A*cos(d*x+c)+140*B*cos(
d*x+c)-172*A-160*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.22638, size = 884, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(5*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*
d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*B*sqrt(a) + 2*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2
*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a))/d
```

Fricas [A] time = 0.587795, size = 1100, normalized size = 6.4

$$\left[\frac{15\left(Ba^2\cos\left(dx+c\right)+Ba^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3\,Aa^2\cos(dx\right.}{}}{30\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/30*(15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(3*A*a^2*cos(d*x + c)^3 + (14*A + 5*B)*a^2*cos
(d*x + c)^2 + (43*A + 40*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/15*(15*
(B*a^2*cos(d*x + c) + B*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)) + 2*(3*A*a^2*cos(d*x + c)^3 + (14*A + 5*B)*a^2*cos(d*x
 + c)^2 + (43*A + 40*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

3.245 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{64a^3(5A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(5A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7B)\sin(c+dx)(a\ldots}{35d\sec^{\frac{3}{2}}(c\ldots}$$

```
[Out] (64*a^3*(5*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^2*(5*A + 7*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*(5*A + 7*B)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.316518, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4013, 3809, 3804}

$$\frac{64a^3(5A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(5A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7B)\sin(c+dx)(a\ldots}{35d\sec^{\frac{3}{2}}(c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (64*a^3*(5*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^2*(5*A + 7*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*(5*A + 7*B)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(5A+7B)\int \frac{(a+a\sec(c+dx}{\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2a(5A+7B)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^{5}}{7d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{16a^2(5A+7B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2a(5A+7B)(a+a\sec}{35d\sec}$$

$$= \frac{64a^3(5A+7B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(5A+7B)\sqrt{a+a\sec(c}}{105d\sqrt{\sec(c+}}$$

Mathematica [A] time = 0.551393, size = 91, normalized size = 0.51

$$\frac{2a^3\sin(c+dx)\left((230A+301B)\sec^3(c+dx)+(115A+98B)\sec^2(c+dx)+3(20A+7B)\sec(c+dx)+15A\right)}{105d\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] (2*a^3*(15*A + 3*(20*A + 7*B)*Sec[c + d*x] + (115*A + 98*B)*Sec[c + d*x]^2
+ (230*A + 301*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(5/2)*S
qrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.309, size = 121, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+60A(\cos(dx+c))^2+21B(\cos(dx+c))^2+115A\cos(dx+c)+98B\cos\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+60*A*cos(d*x+c)^2+21*B*cos(
d*x+c)^2+115*A*cos(d*x+c)+98*B*cos(d*x+c)+230*A+301*B)*(a*(cos(d*x+c)+1)/co
s(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.11634, size = 520, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*
cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^
2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin
(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 28*(3*sq
rt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*
sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 0.46815, size = 317, normalized size = 1.78

$$\frac{2\left(15\,Aa^2\cos\left(dx+c\right)^4+3\,(20\,A+7\,B)a^2\cos\left(dx+c\right)^3+(115\,A+98\,B)a^2\cos\left(dx+c\right)^2+(230\,A+301\,B)a^2\cos\left(dx\right.\right.}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a^2*cos(d*x + c)^4 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^3 + (115*A
 + 98*B)*a^2*cos(d*x + c)^2 + (230*A + 301*B)*a^2*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*
x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2
), x)
```

3.246 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(292A+345B)\sin(c+dx)\sqrt{s}}{315d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*a*A*(a + a*Se
c[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.631508, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(292A+345B)\sin(c+dx)\sqrt{s}}{315d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*a*A*(a + a*Se
c[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+dx))^{3/2}\left(\right.}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+d}{9d\sec^{\frac{7}{2}}(c}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(4A+3B)\sqrt{a+a\sec(c}}{21d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A+345B)\sin(c}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A+345B)\sin(c}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.763781, size = 108, normalized size = 0.47

$$\frac{2a^3\sin(c+dx)\left((584A+690B)\sec^4(c+dx)+(292A+345B)\sec^3(c+dx)+3(73A+60B)\sec^2(c+dx)+5(26A+9\right.}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] (2*a^3*(35*A + 5*(26*A + 9*B)*Sec[c + d*x] + 3*(73*A + 60*B)*Sec[c + d*x]^2
 + (292*A + 345*B)*Sec[c + d*x]^3 + (584*A + 690*B)*Sec[c + d*x]^4)*Sin[c +
 d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.306, size = 143, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(35A(\cos(dx+c))^4+130A(\cos(dx+c))^3+45B(\cos(dx+c))^3+219A(\cos(dx+c))^2+\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+45*B*cos
(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+292*A*cos(d*x+c)+345*B*cos(
```

```
d*x+c)+584*A+690*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos
(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.20182, size = 1007, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) + 30*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7
/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4
/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c
) + 21*a^2*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin
(7/2*d*x + 7/2*c) - 315*a^2*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7
/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*
sin(7/2*d*x + 7/2*c) + 21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) + 77*a^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 315*a^2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.478615, size = 371, normalized size = 1.63

$$\frac{2\left(35\,Aa^2\cos\left(dx+c\right)^5+5\,(26\,A+9\,B)a^2\cos\left(dx+c\right)^4+3\,(73\,A+60\,B)a^2\cos\left(dx+c\right)^3+(292\,A+345\,B)a^2\cos\left(dx+\right.\right.}{315\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^5 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^4 + 3*(73*
A + 60*B)*a^2*cos(d*x + c)^3 + (292*A + 345*B)*a^2*cos(d*x + c)^2 + 2*(292*
A + 345*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2
), x)
```

3.247 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=275

$$\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(14A+11B)\sin(c+dx)\sqrt{a\sec(c+a}}{99d\sec^{\frac{7}{2}}(c+dx)}$$

```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(
3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/
2)) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9
/2))
```

Rubi [A] time = 0.699486, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4017, 4015, 3805, 3804}

$$\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(14A+11B)\sin(c+dx)\sqrt{a\sec(c+a}}{99d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(
3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/
2)) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9
/2))
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\sec(c+dx))^{3/2}}{}$$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+}{11d\sec^{\frac{9}{2}}}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(14A+11B)\sqrt{a+a\sec(}}{99d\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A+803B)\sin(}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A+803B)\sin(}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A+803B)\sin(}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 4.27278, size = 127, normalized size = 0.46

$$\frac{2a^3\sin(c+dx)\left(8(710A+803B)\sec^5(c+dx)+4(710A+803B)\sec^4(c+dx)+3(710A+803B)\sec^3(c+dx)+5(355\right.}{3465d\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(1
1/2),x]
```

```
[Out] (2*a^3*(315*A + 35*(32*A + 11*B)*Sec[c + d*x] + 5*(355*A + 286*B)*Sec[c + d
*x]^2 + 3*(710*A + 803*B)*Sec[c + d*x]^3 + 4*(710*A + 803*B)*Sec[c + d*x]^4
 + 8*(710*A + 803*B)*Sec[c + d*x]^5)*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(9/
2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.329, size = 165, normalized size = 0.6

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(315\,A\left(\cos\left(dx+c\right)\right)^{5}+1120\,A\left(\cos\left(dx+c\right)\right)^{4}+385\,B\left(\cos\left(dx+c\right)\right)^{4}+1775\,A\left(\cos\left(dx+c\right)\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B
*cos(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+2130*A*cos(d*x+c)^2+2
409*B*cos(d*x+c)^2+2840*A*cos(d*x+c)+3212*B*cos(d*x+c)+5680*A+6424*B)*(a*(c
os(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c
)
```

Maxima [B] time = 2.26537, size = 1276, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 34
65*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 318
78*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11
/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1
287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2
*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 1
1/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*
x + 11/2*c))))*A*sqrt(a) + 22*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) +
756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/
2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arc
tan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x +
9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a
^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2
```

```
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(si
n(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.4878, size = 435, normalized size = 1.58

$$\frac{2\left(315\,Aa^{2}\cos\left(dx+c\right)^{6}+35\,(32\,A+11\,B)a^{2}\cos\left(dx+c\right)^{5}+5\,(355\,A+286\,B)a^{2}\cos\left(dx+c\right)^{4}+3\,(710\,A+803\,B)a^{2}\right.}{3465\,(d\cos\left(dx+c\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(315*A*a^2*cos(d*x + c)^6 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^5 + 5*
(355*A + 286*B)*a^2*cos(d*x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 +
 4*(710*A + 803*B)*a^2*cos(d*x + c)^2 + 8*(710*A + 803*B)*a^2*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)
*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/
2), x)
```

3.248 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=190

$$\frac{(4A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{B\sin}{2}$$

```
[Out] -((4*A - 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (B*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.572924, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4021, 4023, 3808, 206, 3801, 215}

$$\frac{(4A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{B\sin}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] -((4*A - 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (B*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aB}{2}+\frac{1}{2}a(4A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \int \frac{\sqrt{\sec(c+}}{}$$

$$= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(4A-7B}{}$$

$$= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(4A-7B}{}$$

$$= -\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{ad}} + \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+d}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right.}{\sqrt{ad}}$$

Mathematica [A] time = 0.867175, size = 125, normalized size = 0.66

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(4A-7B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\right.}{4d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] -
 Sqrt[2]*(4*A - 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*
A - B + 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Sec[c + d*x])
])
```

Maple [B] time = 0.385, size = 423, normalized size = 2.2

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{16\,ad\,(\sin(dx+c))^2}\left(-4\,A\,(\cos(dx+c))^2\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(-\cos(dx+c)-1+\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d/a*(-4*A*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-
cos(d*x+c)-1+sin(d*x+c)))*2^(1/2)-4*A*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+7*B*cos(d*x+c)^2*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*2^(1
/2)+7*B*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1+sin(d*x+c)))*2^(1/2)+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)+16*A*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-2*B*c
os(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-16*B*cos(d*x+c)^2*arctan(1/2
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+4*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.52875, size = 3407, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2
*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
```

```
sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - (4*(sqrt(2)*
sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c)
, cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c
))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*sin(4*d*x + 4
*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 7*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr
t(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*
x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(sqrt(2)*cos(4*d*x
 + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*s
qrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2
*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqr
t(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arcta
n2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) +
4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))))*B/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*
x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.862214, size = 1620, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(((4*A - 7*B)*cos(d*x + c)^2 + (4*A - 7*B)*cos(d*x + c))*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 8*sqrt(2)*((A - B)*a*co
s(d*x + c)^2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*
((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/8*
(8*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-1/a)*a
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d
*x + c))/sin(d*x + c)) + ((4*A - 7*B)*cos(d*x + c)^2 + (4*A - 7*B)*cos(d*x
+ c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) -
2*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

3.249 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[
a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (B*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.387128, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[
a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (B*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}+\frac{1}{2}a(2A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}\\ &= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{(2A-B)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{2a}+\\ &= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{(2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sqrt{\sec(c+dx)}\sin(c+}{\sqrt{a+a\sec(c+dx)}}\right.}{d}\\ &= \frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{ad}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{ad}}+\end{aligned}$$

Mathematica [A] time = 0.409229, size = 106, normalized size = 0.75

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(2A-B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\right.\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(-2*(A - B)*ArcTanh[Sin[(c + d*x)/2]]
+ Sqrt[2]*(2*A - B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sec[c + d*x]*Si
n[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.36, size = 353, normalized size = 2.5

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{4\,ad\,(\sin(dx+c))^2}\left(2\,A\cos(dx+c)\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a*(2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))+2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))-B*cos(d*x+c)*2^(1/2)*arct
an(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-B*cos(d
*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+s
in(d*x+c)))-4*A*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))
+2*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*B*cos(d*x+c)*arctan(1/2*sin(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(cos(d*x
+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)
^2-1)
```

Maxima [B] time = 2.42403, size = 1827, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*(2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) + (4*sqrt(2)*cos(3/2*arctan2(
sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
```

```
(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) -
4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.845835, size = 1401, normalized size = 9.94

$$\left[-\frac{((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}((A-\cdots}{\cdots}}{4(ad\cos(dx+c)+ad\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(((2*A - B)*cos(d*x + c) + 2*A - B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)
^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/2*(2*sqrt
(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))
 + ((2*A - B)*cos(d*x + c) + 2*A - B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B \sec(dx + c) + A) \sec(dx + c)^{\frac{3}{2}}}{\sqrt{a \sec(dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

3.250 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
+ (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.23246, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
+ (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = (A-B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx + \frac{B\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{a}$$

$$= -\frac{(2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{(2B)\,\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.191266, size = 95, normalized size = 0.95

$$\frac{\tan(c+dx)\left(\sqrt{2}(B-A)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) - 2B\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] ((-2*B*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*(-A + B)*ArcTan[(Sqrt[2]*Sqrt[S
ec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*
x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.322, size = 210, normalized size = 2.1

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{2\,ad\,(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(B\sqrt{2}\arctan\left(\frac{\sqrt{2}(-\cos(dx+c)-1+\sin(}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)
*(B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin
(d*x+c)))+B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))+2*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-2*B*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*(-2/(cos(d*x+c)+1))^(1/2)/
sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.39485, size = 765, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (sqrt(2)*log(cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(2)*l
og(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2))*B/sqrt(a))/d
```

Fricas [A] time = 0.616092, size = 965, normalized size = 9.65

$$\left[-\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2\cdots\right.}{\cdots}}{\cos(dx+c)^3\cdots}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*co
s(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - B*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), -(sqrt(2)*(A - B)*a*sq
rt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*
sqrt(cos(d*x + c))/sin(d*x + c)) - B*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
)^2 - a*cos(d*x + c) - 2*a)))/(a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a),
 x)
```

3.251 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.185057, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4013, 3808, 206}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```


```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (-A+B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,s}{\sqrt{a+a\sec(}}\right.}{d}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.275908, size = 114, normalized size = 1.15

$$\frac{\tan(c+dx)\left(\sqrt{2}(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2A\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((2*A*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
 d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-
((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.271, size = 150, normalized size = 1.5

$$\frac{1}{ad\sin(dx+c)}\left(\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}A\sin(dx+c)-\arctan\left(\frac{\sin}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))
^(1/2)*A*sin(d*x+c)-arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(c
os(d*x+c)+1))^(1/2)*B*sin(d*x+c)-2*A*cos(d*x+c)+2*A)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.02725, size = 263, normalized size = 2.66

$$-\frac{\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-4\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A}{\sqrt{a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a))/d
```

Fricas [A] time = 0.51165, size = 813, normalized size = 8.21

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+ }{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2
- 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B)*a*cos(d*x + c) + (A
 - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

3.252 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.331171, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4022, 4013, 3808, 206}

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-3B)+aA\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + ($$

$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - ($$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+d}}$$

Mathematica [A] time = 0.37362, size = 132, normalized size = 0.93

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}(A\cos(c+dx)-A+3B)-3\sqrt{2}(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((2*(-A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*(A - B)*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*
x]])*Tan[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [A] time = 0.33, size = 183, normalized size = 1.3

$$-\frac{(\cos(dx+c))^2}{3\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-3*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)
+2*A*cos(d*x+c)^2-4*A*cos(d*x+c)+6*B*cos(d*x+c)+2*A-6*B)*cos(d*x+c)^2*(1/co
s(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.15046, size = 522, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*B/sqrt(a))/d
```

Fricas [A] time = 0.51814, size = 940, normalized size = 6.62

$$\left[-\frac{\frac{3\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left(A\cos(dx+c)^2-(A-3B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx\cdots}{\cos(dx\cdots}}}{\sqrt{\cos(dx+c)}}}{6\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2
+ 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/sqrt(a) - 4*(A*cos(d*x + c)^2 - (A - 3*B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d), -1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arc
tan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
 + c))/sin(d*x + c)) - 2*(A*cos(d*x + c)^2 - (A - 3*B)*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos
(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c) + A}{\sqrt{a \sec(dx+c) + a} \sec(dx+c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

3.253 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=187

$$\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A - 5*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.506952, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4022, 4013, 3808, 206}

$$\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A - 5*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+2aA\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 1.09654, size = 133, normalized size = 0.71

$$\frac{\frac{15\sqrt{2}(A-B)\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \sin(c+dx)\sqrt{\sec(c+dx)}(-2(A-5B)\cos(c+dx)+3A\cos(2(c+dx))+29A-1}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((29*A - 10*B - 2*(A - 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x] + (15*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*
x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.347, size = 205, normalized size = 1.1

$$\frac{(\cos(dx+c))^3}{15\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*arctan(1/2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-6*A*cos(d*x+c
)^3-15*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))
^(1/2)*B*sin(d*x+c)+8*A*cos(d*x+c)^2-10*B*cos(d*x+c)^2-28*A*cos(d*x+c)+20*B
*cos(d*x+c)+26*A-10*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.2602, size = 864, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) - 10*(3*sqrt(2)*cos(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*
cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*
sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin
(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*B/sqrt(a))/d
```

Fricas [A] time = 0.521899, size = 1030, normalized size = 5.51

$$\left[-\frac{\frac{15\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left(3A\cos(dx+c)^3-(A-5B)\cos(dx+c)^2+(13A-\cdots}{\sqrt{\cos(dx\cdots}}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^
2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/sqrt(a) - 4*(3*A*cos(d*x + c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5
*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B)*a*cos(d
*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x +
c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c
) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

3.254 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=230

$$-\frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d*Sec
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B)*Sin[c + d*x])/(
105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A - 91*B)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.68914, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4022, 4013, 3808, 206}

$$-\frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```


```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d*Sec
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B)*Sin[c + d*x])/(
105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A - 91*B)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
```

```
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-7B)+3aA\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+d}$$

Mathematica [A] time = 1.60039, size = 152, normalized size = 0.66

$$\frac{-\frac{2\sin(c+dx)\left((43A-91B)\sec^3(c+dx)+(7B-31A)\sec^2(c+dx)+3(A-7B)\sec(c+dx)-15A\right)}{\sec^{\frac{5}{2}}(c+dx)} - \frac{105\sqrt{2}(A-B)\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}}}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((-2*(-15*A + 3*(A - 7*B)*Sec[c + d*x] + (-31*A + 7*B)*Sec[c + d*x]^2 + (43
*A - 91*B)*Sec[c + d*x]^3)*Sin[c + d*x])/Sec[c + d*x]^(5/2) - (105*Sqrt[2]*
(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c +
 d*x])/Sqrt[1 - Sec[c + d*x]])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.38, size = 227, normalized size = 1.

$$-\frac{(\cos(dx+c))^4}{105\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(30\,A(\cos(dx+c))^4+105\,\arctan\left(1/2\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/105/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(30*A*cos(d*x+c)^4+105*arcta
n(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin
(d*x+c)-36*A*cos(d*x+c)^3-105*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+42*B*cos(d*x+c)^3+68*A*cos(d*x+c
)^2-56*B*cos(d*x+c)^2-148*A*cos(d*x+c)+196*B*cos(d*x+c)+86*A-182*B)*cos(d*x
+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.30024, size = 1087, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) - 14*s
qrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin
(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin(2/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))))*B/sqrt(a))/d
```

Fricas [A] time = 0.532089, size = 1129, normalized size = 4.91

$$\left[-\frac{\frac{105\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left(15A\cos(dx+c)^4-3(A-7B)\cos(dx+c)^3\cdots\right.}{}}{210\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/210*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c
)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1))/sqrt(a) - 4*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3 + (31*
A - 7*B)*cos(d*x + c)^2 - (43*A - 91*B)*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d
), -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arct
an(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
+ c))/sin(d*x + c)) - 2*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3 +
 (31*A - 7*B)*cos(d*x + c)^2 - (43*A - 91*B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)
), x)
```

3.255 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(A}{}$$

```
[Out] -((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(
4*a^(3/2)*d) + ((9*A - 13*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((6*A -
7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((
A - 2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.782686, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] -((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(
4*a^(3/2)*d) + ((9*A - 13*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((6*A -
7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((
A - 2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-2a(A-2B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \ldots$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} - \ldots$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} - \ldots$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} - \ldots$$

$$= -\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} + \frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c\ldots}{\sqrt{2}\sqrt{a+a\sec(c+d\ldots}}\right)}{2\sqrt{2}a^{3/2}d}\ldots$$

Mathematica [B] time = 4.44593, size = 497, normalized size = 2.01

$$\frac{4(6A-7B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+8(9A-13B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (4*(6*A - 7*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*
x]^2*Sin[(c + d*x)/2] + 8*(9*A - 13*B)*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c +
d*x)/2]^3*Sec[c + d*x]^2*Sin[(c + d*x)/2] + 6*A*Sqrt[1 - Sec[c + d*x]]*Sec[
c + d*x]^(3/2)*Sin[c + d*x] - 7*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)
*Sin[c + d*x] + 4*A*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x]
- 3*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*B*Sqrt[1 -
 Sec[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x] - 9*Sqrt[2]*A*ArcTan[(Sqrt[2
]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 13*Sqrt[2]*B*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9
*Sqrt[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[
c + d*x]*Tan[c + d*x] + 13*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.308, size = 541, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d/a^2*(-1+cos(d*x+c))*(12*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-12*A*cos(d*x+c
)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))-19*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+19*B*cos(d*x+c)^2*sin
(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))+12*A*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-36*A*cos(d*x+c)^2*
sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-14*B*cos(d*x+c)
^3*(-2/(cos(d*x+c)+1))^(1/2)+52*B*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+
8*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-8*A*cos(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2)+10*B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-4*B*(-2/(cos(d*x+c)+1))^
(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(7/2
)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.00176, size = 1993, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(2)*((9*A - 13*B)*cos(d*x + c)^3 + 2*(9*A - 13*B)*cos(d*x + c
)^2 + (9*A - 13*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((
12*A - 19*B)*cos(d*x + c)^3 + 2*(12*A - 19*B)*cos(d*x + c)^2 + (12*A - 19*B
)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos
(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) - 4*((6*A - 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*co
s(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/8*(2*sqrt(2
)*((9*A - 13*B)*cos(d*x + c)^3 + 2*(9*A - 13*B)*cos(d*x + c)^2 + (9*A - 13*
B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + ((12*A - 19*B)*cos(d*
x + c)^3 + 2*(12*A - 19*B)*cos(d*x + c)^2 + (12*A - 19*B)*cos(d*x + c))*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((6*A -
 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3
+ 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.256 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(\ldots}{}$$

```
[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.600548, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4019, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```



```
[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-a(A-3B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec}}{}}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(5A-}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(5A-}{}$$

$$= \frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} +$$

Mathematica [A] time = 1.74952, size = 132, normalized size = 0.67

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((9B-5A)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{2}(2A-3B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\tan\left(\right.\right.}{2ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*((-5*A + 9*B)*ArcTanh[Sin[(c + d*x)/2]
] + 2*Sqrt[2]*(2*A - 3*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + Sec[(c + d*x)
/2]*(-A + 3*B + 2*B*Sec[c + d*x])*Tan[(c + d*x)/2]))/(2*a*d*Sqrt[a*(1 + Sec
[c + d*x])])
```

Maple [B] time = 0.328, size = 477, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-2*A*sin(d*x+c)*cos(d*
x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin
(d*x+c)))-3*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+3*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*
arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-5*A
*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+A*c
os(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+9*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^
(1/2)-A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+B*cos(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2)+2*B*(-2/(cos(d*x+c)+1))^(1/2))/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c
)^3
```

Maxima [B] time = 3.60667, size = 9527, normalized size = 48.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*
cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqr
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
 + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2
```

```
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*
a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)) - (12*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4*d*x + 4*c)
 + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x +
```

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4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 +
4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4
*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sq
rt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(2)*cos(4*d*
x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d
*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*
x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqr
t(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*s
qrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)
*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqr
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t(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*s
qrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sq
rt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x +
4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 9*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4*d*x + 4*c)
 + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4*c) + 2*cos
(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*x + 4*c) +
2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 9*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4*d*x + 4*c)
 + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4*c) + 2*cos
(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*x + 4*c) +
2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(sin(2*d*x +

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2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((sqrt
(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*x + 4*c)^
2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x +
 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*c
os(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2
*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sq
rt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*
a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) +
2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)))/d
```

Fricas [A] time = 0.987151, size = 1750, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5
*A - 9*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*((2*A - 3*B)*cos(d*x +
c)^2 + 2*(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(c
os(d*x + c)^3 + cos(d*x + c)^2)) + 4*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*co
s(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((5*A - 9*B)*cos
(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A - 9*B)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) + 2*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c)
+ 2*A - 3*B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2
*a)) - 2*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.257 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.394869, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4019, 4023, 3808, 206, 3801, 215}

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(A-B)+2aB\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-5B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a} + \frac{B\int \sqrt{\sec(c+}}{}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,\mathrm{s}}{\sqrt{a+a\sec(}}\right.}{2ad}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)}{2d(}$$

Mathematica [A] time = 0.799141, size = 113, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left((A-B)\tan\left(\frac{1}{2}(c+dx)\right)+(A-5B)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\sqrt{2}B\cos\left(\frac{1}{2}(c+dx)\right)\tan\right.}{2ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((A - 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] +
 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B)*T
an[(c + d*x)/2]))/(2*a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.302, size = 316, normalized size = 2.2

$$-\frac{(\cos(dx+c))^2\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\left(2B\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(2*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))*sin(d*x+c)-2*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+A*cos(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2)-A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+
c)-B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+5*B*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))*sin(d*x+c)-A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c
)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c)
)^(3/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.668694, size = 1597, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5
*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 +
2*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(
2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) - 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c
) + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.258 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((3*A + B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.194627, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {4012, 3808, 206}

$$\frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((3*A + B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}dx}{4a}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A+B)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}dx, x, -\frac{a\sqrt{\sec(c+d}}{\sqrt{a+a\,se}}\right.}{2ad}$$

$$= \frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.21888, size = 84, normalized size = 0.79

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left((B-A)\sin\left(\frac{1}{2}(c+dx)\right)+(3A+B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*((3*A + B)*ArcTanh[Sin[(c + d*x)/2]]*C
os[(c + d*x)/2]^2 + (-A + B)*Sin[(c + d*x)/2]))/(d*(a*(1 + Sec[c + d*x]))^(
3/2))
```

Maple [B] time = 0.292, size = 219, normalized size = 2.1

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\cos(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+3\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+
c)*(A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*A*arctan(1/2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*sin(d*x+c)-B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+B*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-A*(-2/(cos(d*x+c)
+1))^(1/2)+B*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c
)^3*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.513246, size = 995, normalized size = 9.3

$$\left[\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(d}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*
(3*A + B)*cos(d*x + c) + 3*A + B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A
 - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.259 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -((7*A - 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.36181, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4020, 4013, 3808, 206}

$$-\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```


```
[Out] -((7*A - 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int\frac{\frac{1}{2}a(5A-B)-a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}-$$

$$=-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}+$$

$$=-\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 1.42876, size = 174, normalized size = 1.12

$$\frac{\sin(c+dx)\left((5A-B)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+4A\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\right)+2\sqrt{2}(7A-3B)\sin\left(\frac{1}{2}(c-\right.}{2d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (2*Sqrt[2]*(7*A - 3*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*Sin[(c + d*x)/2] + ((5*A - B)*Sqr
t[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*A*Sqrt[-((-1 + Sec[c + d*x])*Sec
[c + d*x])])*Sin[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(3/2))
```

Maple [B] time = 0.3, size = 287, normalized size = 1.8

$$-\frac{-1+\cos(dx+c)}{4da^2(\sin(dx+c))^3}\left(7A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/a^2*(-1+cos(d*x+c))*(7*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*x+c)*cos(d*
x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
1/2)+7*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))
^(1/2)*A*sin(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-8*A*cos(d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(
```

```
d*x+c)+10*A-2*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1/cos(d*
x+c))^(1/2)
```

Maxima [B] time = 2.44526, size = 11081, normalized size = 71.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c
)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
- 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*
x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d
*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/
2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/
2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin
(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3
/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin
(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))
*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
```

2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*
sin(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/
2*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*
c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 +
(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x
+ 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x +
 1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + (8*(7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 259*(log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 91*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
^2 - 104*sin(1/2*d*x + 1/2*c)^3 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2
*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 8
*(37*cos(1/2*d*x + 1/2*c)^2 + 21)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c
)^2 + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
c)^3 + 63(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 7*(log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*c
os(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 8*cos(1/2*d*x + 1/2*c)*sin(1/2
*d*x + 1/2*c)^3 + 13*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*
d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + (2*(7*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(
3/2*d*x + 3/2*c)^2 + 2*(84*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 7*(lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)

+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 - 16*(6*co
s(1/2*d*x + 1/2*c)^2 + 1)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7
*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*
d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*
x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/
2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*cos(3/2*d*x + 3/2*c) - 8*cos(1/2*d*x +
 1/2*c))*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^3 + 2*cos(1/2*d*x
 + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(147*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(1/2*d*x + 1/2*c)^3 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*si
n(1/2*d*x + 1/2*c)^2 - 40*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 - 56*
(3*cos(1/2*d*x + 1/2*c)^3 + cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos
(3/2*d*x + 3/2*c) + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin
(3/2*d*x + 3/2*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d
*x + 1/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 8*sin(1/2*d*x +
1/2*c)^4 + (7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*
c)^2 - 4)*cos(3/2*d*x + 3/2*c)^2 + (35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*
c) - 40*sin(1/2*d*x + 1/2*c)^2 - 36)*sin(3/2*d*x + 3/2*c)^2 - 4*(18*cos(1/2
*d*x + 1/2*c)^2 + 5)*sin(1/2*d*x + 1/2*c)^2 + 6*(7*(log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/
2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
1/2*c)^2 - 4*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 36*cos(1/2*d*x +
1/2*c)^2 + 2*((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x +
3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 14*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(1/2*d*x + 1/2*c)^2 - 16*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos
(3/2*d*x + 3/2*c) - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c))
*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(133*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 21*(log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x

+ 1/2*c)^3 - 24*sin(1/2*d*x + 1/2*c)^4 + 2*(21*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*
x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*cos(3/2*d*x + 3/2*c)^2 - 8*(19
*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c)^2 + 16*(7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1
/2*d*x + 1/2*c)^2 - 5*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 80*cos(1
/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^4 + 11*
cos(1/2*d*x + 1/2*c)^2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a)/(4*sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)^4 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3*cos(1/2*d*x + 1/
2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^4 + 4*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c)^4 + 12*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3*sin(1/2*d*x + 1/2*c) + 10*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c)^4 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*co
s(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(5/2*d*x +
5/2*c)^2 + (61*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^
2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2
*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^
2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2
*c)^2)*sin(5/2*d*x + 5/2*c)^2 + (8*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 28*
sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 37*sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + 13*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3
/2*c)^2 + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3 + 13*sqrt(2)*a^2*cos(3/2*
d*x + 3/2*c)^2*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3
+ sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + (2*sqrt(2)*a^2*
cos(3/2*d*x + 3/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*
c)^2 + 2*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x +
1/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin
(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*
sin(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c) + 2*(21*sqrt(2)*a^2*cos(1/2*d*x
+ 1/2*c)^3 + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2)*cos
(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3 + sqrt(2)*a^2*c
os(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x
+ 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2*sin(
1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 + 2*(sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c
) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c)^2)*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(6*sqrt(2)*a^2*cos(3/2
*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 16*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*
cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*a^2*cos(1/2*d*x + 1/
2*c)^2*sin(1/2*d*x + 1/2*c) + 3*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3)*sin(3/2
*d*x + 3/2*c)) - (3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*cos(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1

```
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c
) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2
*d*x + 3/2*c)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) + 4*sin(1/2*d*x + 1/2*c))*B/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*
sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2
*d*x + 2*c)*sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x
 + c) + 2*(2*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)
*a)*sqrt(a)))/d
```

Fricas [A] time = 0.527197, size = 1114, normalized size = 7.14

$$\left[-\frac{\sqrt{2}\left((7A-3B)\cos(dx+c)^2+2(7A-3B)\cos(dx+c)+7A-3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\mathrm{si}}{\cos(dx+c)^2+2\cos(dx+c)+}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7
*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)^2 + (
5*A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d),
 1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*
A - 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c)^2 + (5*A
- B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec (dx + c) + A}{(a \sec (dx + c) + a)^{\frac{3}{2}} \sqrt{\sec (dx + c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

3.260 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(19A-15B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A-3B)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((11*A - 7*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2
*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*
x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((19*A - 15*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.552987, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4020, 4022, 4013, 3808, 206}

$$\frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(19A-15B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A-3B)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((11*A - 7*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2
*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*
x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((19*A - 15*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
```

```
- b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B)-2a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}$$

$$= \frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+}$$

Mathematica [A] time = 1.72503, size = 173, normalized size = 0.85

$$\frac{\tan(c+dx)\sqrt{1-\sec(c+dx)}(\sec(c+dx)(2A\cos(2(c+dx))-17A+15B)+12(B-A))-6\sqrt{2}(11A-7B)\sin\left(\frac{1}{2}(c+\right.}{6d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (-6*Sqrt[2]*(11*A - 7*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2] + Sqrt[1 -
 Sec[c + d*x]]*(12*(-A + B) + (-17*A + 15*B + 2*A*Cos[2*(c + d*x)])*Sec[c +
 d*x])*Tan[c + d*x])/(6*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 +
 Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.312, size = 317, normalized size = 1.6

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^2}{12\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(33\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(33*A*sin(d*
x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d
*x+c)+1))^(1/2)-21*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+8*A*cos(d*x+c)^3+33*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)
-21*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1
/2)*B*sin(d*x+c)-32*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-14*A*cos(d*x+c)+6*B*co
s(d*x+c)+38*A-30*B)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.529207, size = 1218, normalized size = 6.

$$\left[-\frac{3\sqrt{2}\left((11\,A-7\,B)\cos(dx+c)^2+2\,(11\,A-7\,B)\cos(dx+c)+11\,A-7\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(d\ldots}}{\cos(dx+c)^2+2\cos(\ldots}\right.}{24\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c
) + 11*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d
*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)
^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d
*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) - 2*(4*A*cos(d*x + c)^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A -
```

```
 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{3}{2}} \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

3.261 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] -((15*A - 11*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/
(2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A - 5*B)*Sin[c +
d*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B)
*Sin[c + d*x])/(30*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((147
*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Sec[c + d*x]
])
```

Rubi [A] time = 0.734469, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4020, 4022, 4013, 3808, 206}

$$-\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```


```
[Out] -((15*A - 11*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/
(2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A - 5*B)*Sin[c +
d*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B)
*Sin[c + d*x])/(30*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((147
*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Sec[c + d*x]
])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(9A-5B)-3a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\sin(c+dx}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}$$

Mathematica [A] time = 1.40641, size = 171, normalized size = 0.68

$$\frac{\sec(c+dx)\left(\frac{15\sqrt{2}(15A-11B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \sin(c+dx)\sqrt{\sec(c+dx)}(3(39A-20B)\cos(c+dx)+\right.}{30d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (Sec[c + d*x]*((141*A - 85*B + 3*(39*A - 20*B)*Cos[c + d*x] + (-6*A + 10*B)
*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] +
 (15*Sqrt[2]*(15*A - 11*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec
[c + d*x]]]*Cos[(c + d*x)/2]^2*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]]))/(30*d
*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.324, size = 339, normalized size = 1.4

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^3}{60\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(24\,A(\cos(dx+c))^4-225\,A\sin(dx+c)\cos(dx+c)\arctan\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/60/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(24*A*cos(d*
x+c)^4-225*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+165*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-48*A*cos(d*x
+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+
1))^(1/2)*A*sin(d*x+c)+40*B*cos(d*x+c)^3+165*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+240*A*cos(d*x+c)^
2-160*B*cos(d*x+c)^2+78*A*cos(d*x+c)-70*B*cos(d*x+c)-294*A+190*B)*cos(d*x+c
)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.543179, size = 1328, normalized size = 5.31

$$\left[-\frac{15\sqrt{2}\left((15\,A-11\,B)\cos(dx+c)^2+2\,(15\,A-11\,B)\cos(dx+c)+15\,A-11\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cdots}}{\cos(dx+c)^2+2\cdots}\right.}{120\left(a^2d\cos(dx+c)^{2}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/120*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x
 + c) + 15*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqr
```

```
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(12*A*cos(d*
x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 + (
147*A - 95*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^
2*d), 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(
d*x + c) + 15*A - 11*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*
x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 + (
147*A - 95*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^
2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2)), x)
```

3.262 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{(11A-35B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}$$

```
[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 15
*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) -
((11*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```

Rubi [A] time = 0.820014, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4019, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(11A-35B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 15
*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) -
((11*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
```

```
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-a(A-5B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \int \text{se}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - (11A$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - (11A$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - (11A$$

$$= \frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}$$

Mathematica [B] time = 6.16511, size = 941, normalized size = 3.83

$$\frac{7B(\sec(c+dx)+1)\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)}{16d(a(\sec(c+dx)+1))^{5/2}} - \frac{B\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)}{4d(a(\sec(c+dx)+1))^{5/2}} - \frac{7B(\sec(c+dx)+1)^2\sin(c+dx)\sec^{\frac{9}{2}}(c+}{16d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] -(A*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (
B*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (3
*A*Sec[c + d*x]^(9/2)*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Sec[c
+ d*x]))^(5/2)) + (7*B*Sec[c + d*x]^(11/2)*(1 + Sec[c + d*x])*Sin[c + d*x])
/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*Sec[c + d*x]^(3/2)*(1 + Sec[c
+ d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (35*B*Sec[c +
 d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))
^(5/2)) + (7*A*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*
(a*(1 + Sec[c + d*x]))^(5/2)) - (15*B*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])
^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*A*Sec[c + d*x]^(7
/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2))
+ (11*B*Sec[c + d*x]^(7/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 +
 Sec[c + d*x]))^(5/2)) - (7*B*Sec[c + d*x]^(9/2)*(1 + Sec[c + d*x])^2*Sin[c
 + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*ArcSin[Sqrt[1 - Sec[c
+ d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a
*(1 + Sec[c + d*x]))^(5/2)) + (35*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec
[c + d*x])^2*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(5/2)) - (43*A*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c +
d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (115*B*A
rcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/(16*d*Sqrt[1 -
 Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (43*A*ArcTan[(Sqrt[2]*Sqrt[S
ec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/(1
6*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) - (115*B*A
```

```
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x
])^2*Tan[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(5/2))
```

Maple [B] time = 0.335, size = 831, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(-1+cos(d*x+c))^2*(16*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-16*A*cos(d*
x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1-sin(d*x+c)))-40*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+40*B*cos(d*x+c)^2*
sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1-sin(d*x+c)))+11*A*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-43*A*cos(d*x+c)
^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+16*A*sin(d*x
+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))-16*A*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-35*B*cos(d*x+c)^3*(-2/(
cos(d*x+c)+1))^(1/2)+115*B*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))-40*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+40*B*sin(d*x+c)*co
s(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))+4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-43*A*sin(d*x+c)*co
s(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-20*B*cos(d*x+c)^2
*(-2/(cos(d*x+c)+1))^(1/2)+115*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c
)*(-2/(cos(d*x+c)+1))^(1/2))-15*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+39*B
*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*B*(-2/(cos(d*x+c)+1))^(1/2))*(a*(c
os(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(7/2)/(-2/(cos(d
*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.11442, size = 2122, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x +
 c)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 16*((2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x +
 c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B)*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*
A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c
)^3 + 3*(43*A - 115*B)*cos(d*x + c)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*
A - 115*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 16*((2*A - 5*B)*cos(d*x + c
)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((
11*A - 35*B)*cos(d*x + c)^2 + 5*(3*A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.263 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(3A-11}{16a}$$

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.587663, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4019, 4023, 3808, 206, 3801, 215}

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(3A-11}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)+4aB\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \int \frac{\sqrt{s}}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(3A}{}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(3A}{}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-}{4}$$

Mathematica [B] time = 5.66836, size = 570, normalized size = 2.94

$$\frac{16(3A-11B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+16(3A-43B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (16*(3*A - 11*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^5*Sec[c +
d*x]^3*Sin[(c + d*x)/2] + 16*(3*A - 43*B)*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c
 + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + 6*A*Sqrt[1 - Sec[c + d*x]]*S
ec[c + d*x]^(3/2)*Sin[c + d*x] - 22*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(
3/2)*Sin[c + d*x] + 14*A*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
```

```
d*x] - 30*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt
[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d
*x] + 43*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x
]]]*Tan[c + d*x] - 6*Sqrt[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
 Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 86*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 3*Sqr
t[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c +
d*x]^2*Tan[c + d*x] + 43*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt
[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x
]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.309, size = 550, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(-1+cos(d*x+c))^2*(-16*B*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+16*B*sin(d*x
+c)*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))+3*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))-3*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-43*B*sin(d*x
+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-16*B*2^(1/2
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*s
in(d*x+c)+16*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))*sin(d*x+c)+11*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+
3*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-4*A*cos(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2)-43*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2))*sin(d*x+c)+4*B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+7*A*(-2/(cos(d
*x+c)+1))^(1/2)-15*B*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))
^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.714004, size = 1972, normalized size = 10.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^
2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(d*x + c)^
2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 32*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B
)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 -
2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((3*A -
11*B)*cos(d*x + c)^2 + (7*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A -
 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2 + 3*(3*A - 43*B)*cos(
d*x + c) + 3*A - 43*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 32*(B*cos(d*x +
 c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((3*A - 11*B)*cos(d*x + c)^2
 + (7*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.264 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((5*A + 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*B)*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.271553, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4012, 3810, 3808, 206}

$$\frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((5*A + 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*B)*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(5A}{}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(5A}{}$$

$$= \frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} +$$

Mathematica [A] time = 0.678115, size = 106, normalized size = 0.68

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)((5A+3B)\cos(c+dx)+A+7B)+(5A+3B)\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\right.\right.}{4d(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*((5*A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]
*Cos[(c + d*x)/2]^4 + ((A + 7*B + (5*A + 3*B)*Cos[c + d*x])*Sin[(c + d*x)/2
])/2))/(4*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.302, size = 349, normalized size = 2.2

$$\frac{(-1+\cos(dx+c))^2(\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(5\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)-5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(-1+cos(d*x+c))^2*(5*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))-5*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+3*
B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*
B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+5*A*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))*sin(d*x+c)+4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*B*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-4*B*cos(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)+A*(-2/(cos(d*x+c)+1))^(1/2)+7*B*(-2/(cos(d*x+c)+1
))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(
```

3/2)/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.516858, size = 1289, normalized size = 8.26

$$\left[\frac{\sqrt{2}\left((5\,A+3\,B)\cos\left(dx+c\right)^{3}+3\,(5\,A+3\,B)\cos\left(dx+c\right)^{2}+3\,(5\,A+3\,B)\cos\left(dx+c\right)+5\,A+3\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+\cdots}{\cdots}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) + 4*((5*A + 3*B)*cos(d*x + c)^2 + (A + 7*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqr
t(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A +
3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((5*A
 + 3*B)*cos(d*x + c)^2 + (A + 7*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.265 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((9*A - B)*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.572176, antiderivative size = 203, normalized size of antiderivative = 1.3, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4019, 4020, 4013, 3808, 206}

$$-\frac{(9A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*B)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((9*A - B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{-\frac{1}{2}a(A-B)+2a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \int -\frac{1}{4}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - (9A$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - (9A$$

$$= \frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 1.05339, size = 103, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\tan\left(\frac{1}{2}(c+dx)\right)((B-9A)\sec(c+dx)-13A+5B)+2(19A+5B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tanh^{-1}\left(\text{si}\right.\right.}{16ad(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(19*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)
/2]^3*Sec[c + d*x] + (-13*A + 5*B + (-9*A + B)*Sec[c + d*x])*Tan[(c + d*x)/
2]))/(16*a*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.309, size = 347, normalized size = 2.2

$$\frac{\cos(dx+c)(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(13\,A(\cos(dx+c))^2\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x
+c)*(-1+cos(d*x+c))^2*(13*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+19*A*sin
(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*B*cos
(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+5*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+19*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*B*cos(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+5*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2))*sin(d*x+c)-9*A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c)+1))^(
1/2))/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 5.0641, size = 7997, normalized size = 51.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2
+ 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x
+ 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
 + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x +
4*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7
/2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
```

```
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10
*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*s
in(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*
sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(
1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
 57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)
+ 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
 - 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*c
os(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2
*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
 + 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
 + 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(
1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x +
3/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*si
n(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a
^2*cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*
cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x
+ 4*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2
*c)^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d
*x + c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos
(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c)
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sq
rt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*
cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x +
 3*c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*
d*x + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c
))*sin(3*d*x + 3*c))*sqrt(a)) + (4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 40*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x +
```

3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 24*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 24*(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
c))) + 16(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3
*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*si
n(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x
+ 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4
*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 +
 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*
c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*lo
g(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)
*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3
*cos(3/2*d*x + 3/2*c) + 5*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 3*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3
*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*co

```
s(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c)
+ 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c)
- 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3
/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x
 + 3*c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 12*sin(3/2*d*x + 3/2*c))*B/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a
^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt
(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16
*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*
c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2
)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sq
rt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c
) + 6*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + sqrt(2)*a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 12*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x
 + 3*c) + sqrt(2)*a^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sq
rt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)))/d
```

Fricas [A] time = 0.519913, size = 1303, normalized size = 8.35

$$\left[\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx\cdots}{\cdots}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((13*A - 5*B)*cos(d*x + c)^2 + (9*A - B)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32
*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*
(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sq
```

```
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c)))
+ 2*((13*A - 5*B)*cos(d*x + c)^2 + (9*A - B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.266 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -((75*A - 19*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 5*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((49*A - 9
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.571168, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4020, 4013, 3808, 206}

$$\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] -((75*A - 19*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 5*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((49*A - 9
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B)-2a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sqrt{\sec(c+dx)}\sin(c}{4d(a+a\sec(c+dx))^{5/}}$$

Mathematica [A] time = 2.30197, size = 206, normalized size = 1.01

$$\frac{\sin(c+dx)\left((49A-9B)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+(85A-13B)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+32A\sqrt{-(\sec(c+}\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (4*Sqrt[2]*(75*A - 19*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + ((85*A - 13*
B)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + (49*A - 9*B)*Sqrt[1 - Sec[c
+ d*x]]*Sec[c + d*x]^(5/2) + 32*A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]
)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.315, size = 419, normalized size = 2.1

$$\frac{(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\left(75\,A\sin(dx+c)(\cos(dx+c))^2\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(75*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-19*B*sin(d*x+c)*
cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+
c)+1))^(1/2)+150*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-38*B*sin(d*x+c)*cos(d*x+c)*arctan(
1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+75*arct
an(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*si
n(d*x+c)-64*A*cos(d*x+c)^3-19*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-106*A*cos(d*x+c)^2+26*B*cos(d*x+
c)^2+72*A*cos(d*x+c)-8*B*cos(d*x+c)+98*A-18*B)*(a*(cos(d*x+c)+1)/cos(d*x+c)
)^(1/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.535018, size = 1384, normalized size = 6.82

$$\left[-\frac{\sqrt{2}\left((75\,A-19\,B)\cos\left(dx+c\right)^3+3\,(75\,A-19\,B)\cos\left(dx+c\right)^2+3\,(75\,A-19\,B)\cos\left(dx+c\right)+75\,A-19\,B\right)\sqrt{a}\log\left(-\cdots\right.}{64\left(a^3d\cos\left(dx+c\right)^3+3\,a^3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c
)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*log(-(a*cos(d*x +
 c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 + (85*A - 13*B)*cos(d*x + c)^2 + (49*
A - 9*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(
75*A - 19*B)*cos(d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^3 + (85*A - 13*B)*
cos(d*x + c)^2 + (49*A - 9*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c) + A}{(a \sec(dx+c) + a)^{\frac{5}{2}} \sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

3.267 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(95A-39B)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\cdots}}{\sqrt{2}\sqrt{a\sec(c\cdots}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((163*A - 75*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])
/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c
+ d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((95*A - 3
9*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
 ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec
[c + d*x]])
```

Rubi [A] time = 0.761036, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4020, 4022, 4013, 3808, 206}

$$-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(95A-39B)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\cdots}}{\sqrt{2}\sqrt{a\sec(c\cdots}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] ((163*A - 75*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])
/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c
+ d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((95*A - 3
9*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
 ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec
[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A-3B)-3a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= \frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+dx}{4d\sqrt{\sec(c+dx)}(a+a\sec(c}$$

Mathematica [A] time = 1.73511, size = 193, normalized size = 0.77

$$\frac{2\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec^2(c+dx)((255B-479A)\cos(c+dx)+(48B-80A)\cos(2(c+dx))+8A\cos(3(c+}{96d\sqrt{-(\sec(c+dx)-1)\sec(c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (-12*Sqrt[2]*(163*A - 75*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(7/2)*Sin[c + d*x] + 2*(-379*A
 + 195*B + (-479*A + 255*B)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)]
```

```
+ 8*A*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])
/(96*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(5/
2))
```

Maple [B] time = 0.323, size = 449, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*A*si
n(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)-225*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(
-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+978*A*sin(d*x+c)*cos(d*
x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
1/2)+64*A*cos(d*x+c)^4-450*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+489*arctan(1/2*sin(d*x+c
)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-384*A*c
os(d*x+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d
*x+c)+1))^(1/2)*B*sin(d*x+c)+192*B*cos(d*x+c)^3-686*A*cos(d*x+c)^2+318*B*co
s(d*x+c)^2+408*A*cos(d*x+c)-216*B*cos(d*x+c)+598*A-294*B)*cos(d*x+c)^2*(1/c
os(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.543725, size = 1503, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*
x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^
3 - (503*A - 255*B)*cos(d*x + c)^2 - (299*A - 147*B)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos
```

```
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96
*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^
2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(-a)*arctan(sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d
*x + c))) - 2*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^3 - (503*A
 - 255*B)*cos(d*x + c)^2 - (299*A - 147*B)*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

3.268 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(157A-85B)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)}{240a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx}}$$

```
[Out] -((283*A - 163*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x
])/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A - 13*B)*Sin
[c + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((157*A
 - 85*B)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]
]) - ((787*A - 475*B)*Sin[c + d*x])/(240*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]]) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240
*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.956027, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4020, 4022, 4013, 3808, 206}

$$\frac{(157A-85B)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)}{240a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] -((283*A - 163*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x
])/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A - 13*B)*Sin
[c + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((157*A
 - 85*B)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]
]) - ((787*A - 475*B)*Sin[c + d*x])/(240*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]]) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240
*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(13A-5B)-4a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= -\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sin(c+a}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec}$$

Mathematica [A] time = 2.1297, size = 196, normalized size = 0.66

$$\frac{\sec^2(c+dx)\left(\frac{30\sqrt{2}(283A-163B)\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}}+\sin(c+dx)\sqrt{\sec(c+dx)}(5(887A-479B)\cos(c+dx$$

$$240d(a(\sec(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (Sec[c + d*x]^2*((3491*A - 1895*B + 5*(887*A - 479*B)*Cos[c + d*x] + 16*(52
*A - 25*B)*Cos[2*(c + d*x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)]
 + 12*A*Cos[4*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (30*Sqrt[2]*(28
3*A - 163*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^4*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]]))/(240*d*(a*(1 + Sec[
c + d*x]))^(5/2))
```

Maple [A] time = 0.348, size = 471, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/480/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(4245*A*s
in(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2
/(cos(d*x+c)+1))^(1/2)-192*A*cos(d*x+c)^5-2445*B*sin(d*x+c)*cos(d*x+c)^2*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+84
90*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))
*(-2/(cos(d*x+c)+1))^(1/2)+512*A*cos(d*x+c)^4-4890*B*sin(d*x+c)*cos(d*x+c)*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-
320*B*cos(d*x+c)^4+4245*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-
2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*arctan(1/2*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+
1920*B*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*x+c)^2+3768*A*cos(d*x+
c)-2040*B*cos(d*x+c)+5342*A-2990*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d
*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.557339, size = 1611, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/960*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos
(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(a)*log(-
(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(96*A*cos(d*x + c)^5 - 160*(A - B)*cos(d*x + c
)^4 + 32*(49*A - 25*B)*cos(d*x + c)^3 + 5*(911*A - 503*B)*cos(d*x + c)^2 +
(2671*A - 1495*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d
*x + c)^3 + 3*(283*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x +
c) + 283*A - 163*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x +
c)^5 - 160*(A - B)*cos(d*x + c)^4 + 32*(49*A - 25*B)*cos(d*x + c)^3 + 5*(91
1*A - 503*B)*cos(d*x + c)^2 + (2671*A - 1495*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

3.269 $\int (a+a\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=406

$$-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{\ }}{\sqrt[3]{\ }}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*Sqrt[2]*A*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
 + (3*B*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) -
 (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*
(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1
- Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2)])
```

Rubi [A] time = 0.631601, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 50, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}+\frac{3B\tan(c+dx)(a\sec(c+dx}{2d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (3*Sqrt[2]*A*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
 + (3*B*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) -
 (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*
(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1
- Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2)])
```


Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = A\int (a+a\sec(c+dx))^{2/3}\,dx + B\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$$

$$= \frac{\left(A(a+a\sec(c+dx))^{2/3}\right)\int (1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}} + \frac{\left(B(a+a\sec(c+dx\right.}{}$$

$$= -\frac{\left(A(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-xx}}\,dx, x, \sec(c+dx\right.}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+}{7d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+}{7d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+}{7d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 20.0402, size = 4445, normalized size = 10.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*B*Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d
*x]))^(2/3)*(A + B*Sec[c + d*x])*Tan[(c + d*x)/2])/(2*d*(B + A*Cos[c + d*x]
)*(1 + Sec[c + d*x])^(2/3)) + (Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(A + B*Sec[c + d*x])*((A*Cos[c + d*x]
*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/2 + Sec[(c + d*x)/2]^2*((A*(1
 + Sec[c + d*x])^(2/3))/2 + (B*(1 + Sec[c + d*x])^(2/3))/4))*Tan[(c + d*x)/
2]*(2*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos
[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/
3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d
*x)/2]^2 + B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2)))/(3*2^(1
/3)*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^(2/3)*(9*AppellF1[1/2, 2/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3,
2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)*((Sec[(c
 + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(2*B*AppellF1[3/2, 2/3
, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2
```

]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2)))/(6*2^(1/3)*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(2*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2))*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))))/(3*2^(1/3)*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(4*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^3 + 2*B*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^4*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (4*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^4*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) + 9*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9)*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2) + 2*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^4*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])) + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^

```
2, -Tan[(c + d*x)/2]^2]*(-3*(4*A + B)*Cos[(c + d*x)/2]*Sin[(c + d*x)/2] + B
*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
 + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + B*(
Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2
, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*B*AppellF1[3
/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^
2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)))))/(3*2^(1/3)*(
9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(
-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
 + d*x)/2]^2)) + (2^(2/3)*Tan[(c + d*x)/2]*(2*B*AppellF1[3/2, 2/3, 1, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)
*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3/2, 2/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^(2/3)*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d
*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/
2]^2*Sec[c + d*x])^(1/3)*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int (a + a\sec(dx+c))^{\frac{2}{3}} (A + B\sec(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)

[Out] int((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c) + A)(a\sec(dx+c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}}(A+B\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*(A + B*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(2/3), x)
```

3.270 $\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=354

$$\frac{3\sqrt{2}A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{(\sec(\cdots}}{\sqrt[3]{2}d(1-\sec(c+dx\cdots}$$

```
[Out] (3*Sqrt[2]*A*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) -
(3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.366077, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{(\sec(\cdots}}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Sqrt[2]*A*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) -
(3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
```

```
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = A\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx + B\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}} + \frac{\left(B\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{(A\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}} - \frac{(B\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{}{\sqrt{1-x(}}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}} - \frac{(6B\tan(c+dx))\,\mathrm{Subs}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}} - \frac{3^{3/4}BF\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-}{\sqrt[3]{2}-(1+}\right.\right.}{}$$

Mathematica [B] time = 19.1237, size = 2709, normalized size = 7.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(2/3)*Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c +
d*x])^(1/3)*(A + B*Sec[c + d*x])*((B*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^
(2/3))/2 + (A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/2)*
Tan[(c + d*x)/2]*((-A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2
]^2 + (27*(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*d*(B + A*Cos[c + d*x])*(
a*(1 + Sec[c + d*x]))^(1/3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(2/3)*((-A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2
 + (27*(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)) + (2^(2/3)*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*((-A + B)*AppellF1[3/2, 2
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (-A + B)*(Cos[c + d
*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 2/3, 2,
 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*(-A + B)*AppellF1[3/2
, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*
(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (27*(A + B)*App
ellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d
*x)/2]*Sin[(c + d*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
```

```
-Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (27*(A + B)*Cos[(c + d*x)/2]^2*(-
(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*
AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3
*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*Ap
pellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2) - (27*(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9
*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) +
 2*Tan[(c + d*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF
1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + Appell
F1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/3 + (2*2^(2/3)*Tan[(c + d*x)/
2]*((-A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(A +
 B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos
[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c +
d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)
/2]^2*Sec[c + d*x])^(1/3))))
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int (A + B\sec(dx + c))\frac{1}{\sqrt[3]{a + a\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx + c) + A}{(a\sec(dx + c) + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \sec{(c + dx)}}{\sqrt[3]{a (\sec{(c + dx)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec{(dx + c)} + A}{(a \sec{(dx + c)} + a)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.271 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=415

$$-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)}}\right.\right.}{5\sqrt[3]{2}ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (3*B*Tan[c + d*x])/(5*a*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)) -
(3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(1/3)) - (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) +
 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)])
```

Rubi [A] time = 0.4333, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 51, 63, 225}

$$-\frac{3\sqrt{2}A\tan(c+dx)F_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}+\frac{3B\tan(c+dx)}{5ad(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*B*Tan[c + d*x])/(5*a*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)) -
(3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(1/3)) - (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) +
 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)])
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx = A\int \frac{1}{(a+a\sec(c+dx))^{4/3}}\,dx + B\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$$

$$= \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{(1+\sec(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sec(c+dx)}} + \frac{\left(B\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{(A\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{11/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}} - \frac{(B\tan(c+dx))\,\mathrm{Subst}\left(\int\right.}{ad\sqrt{1-\sec(c+dx)}\sqrt[6]{1+}}$$

$$= \frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\right.}{5ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))}$$

$$= \frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\right.}{5ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))}$$

$$= \frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\right.}{5ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))}$$

Mathematica [B] time = 19.2225, size = 2901, normalized size = 6.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(4
/3)*(A + B*Sec[c + d*x])*((3*Sec[(c + d*x)/2]*(-(A*Sin[(c + d*x)/2]) + B*Si
n[(c + d*x)/2]))/5 - (3*Sec[(c + d*x)/2]^3*(-(A*Sin[(c + d*x)/2]) + B*Sin[(
c + d*x)/2]))/10))/(d*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(4/3)) +
(2^(2/3)*Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c +
d*x])^(4/3)*(A + B*Sec[c + d*x])*((A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + S
ec[c + d*x])^(2/3))/2 + Sec[(c + d*x)/2]^2*(-(A*(1 + Sec[c + d*x])^(2/3))/1
0 + (B*(1 + Sec[c + d*x])^(2/3))/10))*Tan[(c + d*x)/2]*((-6*A + B)*AppellF1
[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*S
ec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*AppellF1[1/2, 2
/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9
*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-
3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*A
ppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
+ d*x)/2]^2)))/(15*d*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(4/3)*((Se
c[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*((-6*A + B)*Appell
F1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]
*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*AppellF1[1/2,
 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/
(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2
```

```
*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2)))/(15*2^(1/3)) + (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
(2/3)*Tan[(c + d*x)/2]*((-6*A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/
2]^2)^(2/3)*Tan[(c + d*x)/2] + (-6*A + B)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF
1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])/5) + (2*(-6*A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2
*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos
[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (27*(4*A + B)*AppellF1[1/2, 2/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x
)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])
*Tan[(c + d*x)/2]^2) + (27*(4*A + B)*Cos[(c + d*x)/2]^2*(-(AppellF1[3/2, 2/
3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, 2/3
, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/
3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3,
1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (27
*(4*A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-(AppellF1[3/
2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x
)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2,
7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1,
 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])*Tan[(c + d*x)/2]^2)^2))/15 + (2*2^(2/3)*Tan[(c + d*x)/2]*((-6*A + B)
*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c
 + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*Appell
F1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)
/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
)*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
 Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(45*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^(1/3))))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int (A + B\sec(dx + c))(a + a\sec(dx + c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec (dx + c) + A}{(a \sec (dx + c) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \sec (c + dx)}{(a (\sec (c + dx) + 1))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec (dx + c) + A}{(a \sec (dx + c) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(4/3), x)
```

3.272 $\int (a + a\sec(c + dx))^{4/3}(A + B\sec(c + dx))\,dx$

Optimal. Leaf size=787

$$\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)aB\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,\mathrm{EllipticF}\left(\right.}{4\,2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*a*A*Appe
llF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(1 + Sec[c
 + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 - Sec[c + d*
x]]) - (15*(1 + Sqrt[3])*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*
(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
) + (15*3^(1/4)*a*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt
[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/
3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*B*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(4*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(
1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.839188, antiderivative size = 787, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 50, 63, 308, 225, 1881}

$$\frac{3\sqrt{2}aA\tan(c+dx)(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{11d\sqrt{1-\sec(c+dx)}}+\frac{3aB\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (3*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*a*A*Appe
llF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(1 + Sec[c
 + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 - Sec[c + d*
x]]) - (15*(1 + Sqrt[3])*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*
(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
) + (15*3^(1/4)*a*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt
[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/
3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])
```

```
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*B*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(4*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(
1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
```

```
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = A\int (a+a\sec(c+dx))^{4/3}\,dx + B\int \sec(c+dx)(a+a\sec(c+dx))^{4/3}$$

$$= \frac{\left(aA\sqrt[3]{a+a\sec(c+dx)}\right)\int (1+\sec(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sec(c+dx)}} + \frac{\left(aB\sqrt[3]{a+a\sec(c\right.}{}$$

$$= -\frac{\left(aA\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^{5/6}}{\sqrt{1-xx}}\,dx, x, \sec(c+\right.}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{5/6}}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{s}\right.}{}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{s}\right.}{}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{s}\right.}{}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{s}\right.}{}$$

Mathematica [B] time = 19.299, size = 4110, normalized size = 5.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c + d*x])
)^(4/3)*(A + B*Sec[c + d*x])*((3*(4*A + 5*B)*Sin[c + d*x])/4 + (3*B*Tan[c +
 d*x])/4))/(d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^(4/3)) + (Cos[c + d*x
]*(a*(1 + Sec[c + d*x]))^(4/3)*(A + B*Sec[c + d*x])*(2*A*(1 + Sec[c + d*x])
^(1/3) + (5*B*(1 + Sec[c + d*x])^(1/3))/4 + Cos[c + d*x]*(-3*A*(1 + Sec[c +
 d*x])^(1/3) - (15*B*(1 + Sec[c + d*x])^(1/3))/4))*Tan[(c + d*x)/2]*(-(((4*
A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) - (9*(3*Appe
llF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B
 + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2
*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(6*2^(2/3)*d*(B + A*Cos[c + d*x]
)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(4/3)*((Sec[(c
 + d*x)/2]^2*(-(((4*A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A + 5*B)*(3*Appel
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3
/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan
[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/
```

2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(12*2^(2/3)*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (Tan[(c + d*x)/2]*(-(((4*A + 5*B
)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) -
((4*A + 5*B)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (App
ellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + (2
*(4*A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(5/3)) + (9*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*AppellF1[1/2, 1/3, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Co
s[c + d*x]) - 4*(4*A + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x
)/2]^2)^2*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2)) + (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A
 + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-(Appe
llF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c
+ d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1,
7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
x)/2])/5 + 3((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2,
 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) - (9*(-15*(4*A
+ 7*B)*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sin[c + d*x] - 4*(4*A + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 4*(
4*A + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Sin[c + d*x]*Tan[(c + d*x)/2]^2 + 3*(-4*A + 5*B + 5*(4*A + 7*B)*Cos[c +
d*x])*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9
) - 4*(4*A + 5*B)*Cos[c + d*x]*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2,
 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3,
 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)
/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]

```
^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Tan[(c + d*x)/2]^2))))/(6*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3))
- (Tan[(c + d*x)/2]*(-(((4*A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(2/3)) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A + 5*B)*(
3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - App
ellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d
*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3
, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3
, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))*(-(Cos
[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d
*x]*Tan[c + d*x]))/(9*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/3))))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^{\frac{4}{3}} (A + B\sec(dx + c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx + c) + A)(a\sec(dx + c) + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3), x)
```

3.273 $\int \sqrt[3]{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=739

$$\frac{3^{3/4}\left(1-\sqrt{3}\right)B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,\text{EllipticF}\bigl(}{2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*Sqrt[2]*A*AppellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]])
 - (3*(1 + Sqrt[3])*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[
c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^
(1/3)*3^(1/4)*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])
/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2
^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[
c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
 + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec
[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan
[c + d*x])/(2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((
1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.702391, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 63, 308, 225, 1881}

$$\frac{3\sqrt{2}A\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5d\sqrt{1-\sec(c+dx)}}-\frac{3\left(1+\sqrt{3}\right)B\tan(c+}{d(\sec(c+dx)+1)^{2/3}\bigl(\sqrt[3]{2}-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (3*Sqrt[2]*A*AppellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]])
 - (3*(1 + Sqrt[3])*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[
c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^
(1/3)*3^(1/4)*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])
/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2
^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[
c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqr
```

```
t[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
 + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec
[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan
[c + d*x])/(2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((
1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt[3]{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = A\int \sqrt[3]{a+a\sec(c+dx)}\,dx + B\int \sec(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$$

$$= \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\right)\int \sqrt[3]{1+\sec(c+dx)}\,dx}{\sqrt[3]{1+\sec(c+dx)}} + \frac{\left(B\sqrt[3]{a+a\sec(c+dx)}\right.}{\sqrt[3]{}}$$

$$= -\frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x\sqrt[6]{1+x}}\,dx, x, \sec(c+\right.}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{5/6}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a\sec(c}}{5d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a\sec(c}}{5d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a\sec(c}}{5d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 21.1781, size = 5094, normalized size = 6.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + a\sec(dx + c)}\,(A + B\sec(dx + c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx + c) + A)(a\sec(dx + c) + a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a(\sec(c + dx) + 1)}\,(A + B\sec(c + dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(1/3)*(A + B*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3), x)
```

3.274 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=764

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)B\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\operatorname{EllipticF}\left(\cos\right.}{2^{2/3}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}}$$

```
[Out] (3*B*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*AppellF1[-
1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d*
Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) + (3*(1 + Sqrt[3])*B*(1
+ Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3*2^(1/3)*3^(1/4)*B*EllipticE
[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*
x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1
/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (3^(3/4)*(1
 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)])
```

Rubi [A] time = 0.733451, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3924, 3779, 3778, 136, 3828, 3827, 51, 63, 308, 225, 1881}

$$-\frac{3\sqrt{2}A\tan(c+dx)F_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}+\frac{3B\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}+\frac{3\left(1+\sqrt{3}\right)B\tan}{d\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(2/3),x]
```


```
[Out] (3*B*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*AppellF1[-
1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d*
Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) + (3*(1 + Sqrt[3])*B*(1
+ Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3*2^(1/3)*3^(1/4)*B*EllipticE
[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*
```

```
x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1
/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (3^(3/4)*(1
 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)])
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx = A\int \frac{1}{(a+a\sec(c+dx))^{2/3}}\,dx + B\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$$

$$= \frac{\left(A(1+\sec(c+dx))^{2/3}\right)\int \frac{1}{(1+\sec(c+dx))^{2/3}}\,dx}{(a+a\sec(c+dx))^{2/3}} + \frac{\left(B(1+\sec(c+dx))^{2/3}\right)\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{2/3}}\,dx}{(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(A\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{7/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}} - \frac{\left(B\sqrt[6]{1+\right.}{}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(}{d\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(}{d\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(}{d\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(}{d\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 19.1699, size = 4066, normalized size = 5.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(2
/3)*(A + B*Sec[c + d*x])*(3*Sec[(c + d*x)/2]*(-(A*Sin[(c + d*x)/2]) + B*Sin
[(c + d*x)/2]) - 3*(-A + B)*Sin[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a*(1 +
Sec[c + d*x]))^(2/3)) - (2^(1/3)*Cos[c + d*x]*(1 + Sec[c + d*x])^(2/3)*(A +
 B*Sec[c + d*x])*(2*A*(1 + Sec[c + d*x])^(1/3) - B*(1 + Sec[c + d*x])^(1/3)
 + Cos[c + d*x]*(-3*A*(1 + Sec[c + d*x])^(1/3) + 3*B*(1 + Sec[c + d*x])^(1/
3)))*Tan[(c + d*x)/2]*(((A - B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*
x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*d*(B + A*Cos[c +
d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*
(-(Sec[(c + d*x)/2]^2*(((A - B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*
x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(
```

```
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*2^(2/3)*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(2/3)) - (2^(1/3)*Tan[(c + d*x)/2]*(((A - B)*Appel
lF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + ((A - B)
*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4
/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) - (2*(A - B)*App
ellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d
*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3)) + (9*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B
)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c
+ d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)^2*(-9*AppellF1[1/2
, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2
, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3
, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) +
(9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2, 1/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(2*
(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (Appel
lF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3
, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x
)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])
*Tan[(c + d*x)/2]^2)^2) - (9*(-3*(-5*A + 7*B)*AppellF1[1/2, 1/3, 1, 3/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[c + d*x] + 4*(A - B)*(3*AppellF1
[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2,
 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2] - 4*(A - B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[c + d*x]*Tan[(c + d*x)/2]^2 + 3*(A +
B + (-5*A + 7*B)*Cos[c + d*x])*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (Appell
F1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/9) + 4*(A - B)*Cos[c + d*x]*Tan[(c + d*x)/2]^2*((3*
AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((
-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(
2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]
```

```
^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*(Cos[(c + d*x)/2]^2*Sec[c +
d*x])^(2/3)) + (2*2^(1/3)*Tan[(c + d*x)/2]*(((A - B)*AppellF1[3/2, 1/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A
- B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos
[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/
2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))*
(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec
[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/3))))
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int (A + B\sec (dx + c)) (a + a\sec (dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec (dx + c) + A}{(a\sec (dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec (c + dx)}{(a (\sec (c + dx) + 1))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3), x)
```

3.275 $\int (c\sec(e+fx))^n(a+a\sec(e+fx))^m(A+B\sec(e+fx))\,dx$

Optimal. Leaf size=197

$$-\frac{(A-B)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\mathrm{se}\right.}{fn\sqrt{1-\sec(e+fx)}}$$

```
[Out] -((B*AppellF1[n, 1/2, -1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[
e + f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*n*Sqrt[1 - Sec[e + f*x]])) - ((A - B)*AppellF1[n, 1/2, 1/2 - m, 1 +
n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]])
```

Rubi [A] time = 0.360822, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4023, 3828, 3827, 133}

$$-\frac{(A-B)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\mathrm{se}\right.}{fn\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] -((B*AppellF1[n, 1/2, -1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[
e + f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*n*Sqrt[1 - Sec[e + f*x]])) - ((A - B)*AppellF1[n, 1/2, 1/2 - m, 1 +
n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (c\sec(e+fx))^n(a+a\sec(e+fx))^m(A+B\sec(e+fx))\,dx = (A-B)\int (c\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx + \frac{B\int(}{}$$

$$= \left((A-B)(1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (c\,\text{se}$$

$$= -\frac{\left((A-B)c(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\,\text{ta}\right.}{f\sqrt{1-\text{se}}}$$

$$= -\frac{BF_1\left(n;\frac{1}{2},-\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(c}{fn}$$

Mathematica [B] time = 22.4053, size = 4897, normalized size = 24.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] (2^(1 + m)*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1 - n)*(c*Sec[e + f*x])^n*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*(A + B*S
ec[e + f*x])*(A*Sec[e + f*x]^n*(1 + Sec[e + f*x])^m + B*Sec[e + f*x]^(1 + n
)*(1 + Sec[e + f*x])^m)*Tan[(e + f*x)/2]*((-3*A*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/(3*AppellF1[1/
2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n
)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m + n, -n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + n, -n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2, 1 + m + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1
[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)/3)))/(f*(B + A*Cos[e + f*x])*(1 + Sec[e + f*x])^m*(-1 + Tan[(e
 + f*x)/2]^2)*(-((2^(1 + m)*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2*((-3*A*AppellF1[1/2, m + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/(3*AppellF1[
1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 +
 n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m + n, -n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + n, -n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2, 1 + m + n
, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*Appell
F1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2)/3)))/(-1 + Tan[(e + f*x)/2]^2)^2) + (2^m*(Sec[(e + f*x)/2]^2
)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*((-3*A*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/(3*
```

```
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m + n,
 -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m +
n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2,
1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m +
n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2)/3)))/(-1 + Tan[(e + f*x)/2]^2) + (2^(1 + m)*n*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2
]^2*((-3*A*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Cos[e + f*x])/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) -
(B*AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])/(AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(n*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)))/(-1 + Tan[(e + f*x)
/2]^2) + (2^(1 + m)*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^(m + n)*Tan[(e + f*x)/2]*((3*A*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x])/(3*AppellF1[1/2, m + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
 m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) - (3*A*Cos[e + f*x]*(-((1 - n)*AppellF1[3/2, m + n, 2 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*App
ellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*
((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*((n*AppellF1[3/2, 1 + m + n
, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/3 + ((1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/
(AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(n*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) + (3*A*AppellF1[1/2, m +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(2*((-
1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - n)*Appe
llF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2 - n)*AppellF1[5/2, m
+ n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m + n, 2 - n, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1
 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
```

```
x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (B*AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*((n*AppellF1[3/2, 1 + m + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2])/3 + ((1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*(n*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*Tan[(e + f*x)/2]^2
*(n*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m + n)
*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (1 + m + n)*((3*n*AppellF1[5/
2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(2 + m + n)*AppellF1[5/2, 3 + m + n, -n
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/5)))/3))/(AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)^2))/(-
1 + Tan[(e + f*x)/2]^2) + (2^(1 + m)*(m + n)*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n)*Tan[(e + f*x)/2]*((-3*A*AppellF1[1/
2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
)/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m
 + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 +
 m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 +
 m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Tan[(e + f*x)/2]^2)/3))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e +
f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(-1 + Tan[(e + f*
x)/2]^2)))
```

Maple [F] time = 1.236, size = 0, normalized size = 0.

$$\int \left(c\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{m}\left(A+B\sec\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

```
[Out] int((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(fx+e\right)+A\right)\left(a\sec\left(fx+e\right)+a\right)^{m}\left(c\sec\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(fx+e\right)+A\right)\left(a\sec\left(fx+e\right)+a\right)^{m}\left(c\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))**n*(a+a*sec(f*x+e))**m*(A+B*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(fx+e\right)+A\right)\left(a\sec\left(fx+e\right)+a\right)^{m}\left(c\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

3.276 $\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=164

$$\frac{(An+Bn+B)\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(\frac{1}{2}-n,-n,1-n,-\frac{}{}\right.}{dn(n+1)(\sec(c+dx)+1)}$$

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) + ((B +
A*n + B*n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c + d*x])/(1 - Sec
[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec[c + d*x]))^(1
/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1 + Sec[c + d*x]
))
```

Rubi [A] time = 0.255129, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4013, 3828, 3825, 132}

$$\frac{(An+Bn+B)\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2}-n,-n;1-n;-\frac{2\sec(c+dx)}{1-\sec(c+dx)}\right)}{dn(n+1)(\sec(c+dx)+1)}+\frac{A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) + ((B +
A*n + B*n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c + d*x])/(1 - Sec
[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec[c + d*x]))^(1
/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1 + Sec[c + d*x]
))
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
```

```
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{(B \cdots}{\cdots}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{((B \cdots}{\cdots}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{\left((B \cdots\right.}{\cdots}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{(B \cdots}{\cdots}$$

Mathematica [A] time = 1.07748, size = 111, normalized size = 0.68

$$\frac{\sin(c+dx)\sec^{-n}(c+dx)(a(\sec(c+dx)+1))^n\left(\frac{(An+Bn+B)\left(-\cot^2\left(\frac{1}{2}(c+dx)\right)\right)^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(\frac{1}{2}-n,-n,1-n,\csc^2\left(\frac{1}{2}(c+dx)\right)\right)}{n(\cos(c+dx)+1)} + \cdots\right.}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x])
,x]
```

```
[Out] ((A + ((B + A*n + B*n)*(-Cot[(c + d*x)/2]^2)^(1/2 - n)*Hypergeometric2F1[1/
2 - n, -n, 1 - n, Csc[(c + d*x)/2]^2])/(n*(1 + Cos[c + d*x])))*(a*(1 + Sec[
c + d*x]))^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Maple [F] time = 1.14, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{-1-n}(a+a\sec(dx+c))^n(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^n \sec(dx+c)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorith
m="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)(a\sec(dx+c)+a)^n \sec(dx+c)^{-n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^n \sec(dx+c)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1)
, x)
```

3.277 $\int \sec^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=114

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(4aA+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aA+3bB)\tan(c+dx)\sec}{8d}$$

```
[Out] ((4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/
d + ((4*a*A + 3*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*B*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.145108, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3997, 3787, 3768, 3770, 3767}

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(4aA+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aA+3bB)\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/
d + ((4*a*A + 3*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*B*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{bB\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^3(c+dx)(4aA+3bB+\\ &= \frac{bB\sec^3(c+dx)\tan(c+dx)}{4d}+(Ab+aB)\int \sec^4(c+dx)\,dx+\\ &= \frac{(4aA+3bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{bB\sec^3(c+dx)\tan(c}{4d}\\ &= \frac{(4aA+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB)\tan(c+dx)}{d}+\end{aligned}$$

Mathematica [A] time = 0.617527, size = 85, normalized size = 0.75

$$\frac{3(4aA+3bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(aB+Ab)(\cos(2(c+dx))+2)\sec(c+dx)+12aA+6bB\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*a*A + 9*b*B + 8
*(A*b + a*B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*b*B*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

Maple [A] time = 0.033, size = 171, normalized size = 1.5

$$\frac{Aa\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2Ba\tan(dx+c)}{3d}+\frac{Ba(\sec(dx+c))^2\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*A*a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*B
*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d+2/3/d*A*b*tan(d*x+c)+1/3/d*
A*b*tan(d*x+c)*sec(d*x+c)^2+1/4*b*B*sec(d*x+c)^3*tan(d*x+c)/d+3/8*b*B*sec(d
*x+c)*tan(d*x+c)/d+3/8/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.981238, size = 220, normalized size = 1.93

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab-3Bb\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\mathrm{si}\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*b - 3*B*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.825603, size = 352, normalized size = 3.09

$$\frac{3\,(4\,Aa+3\,Bb)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,Aa+3\,Bb)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(Ba+\cdots\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(4*A*a + 3*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A*a + 3
*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*cos(d*x + c
)^3 + 3*(4*A*a + 3*B*b)*cos(d*x + c)^2 + 6*B*b + 8*(B*a + A*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))\sec^{3}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.26879, size = 410, normalized size = 3.6

$$3\,(4\,Aa+3\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,Aa+3\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ba\,\mathrm{ta}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(4*A*a + 3*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*B
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*A*a*tan(1/2*d*x + 1/2*c)^7 -
 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 15*B*b*tan
(1/2*d*x + 1/2*c)^7 - 12*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*x +
1/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 9*B*b*tan(1/2*d*x + 1/2*c)^5 - 1
2*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1
/2*d*x + 1/2*c)^3 + 9*B*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*a*tan(1/2*d*x + 1/2
*c) + 24*B*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 15*B*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.278 $\int \sec^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=93

$$\frac{(3aA+2bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bB\tan(c+dx)\sec^2(c$$

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*a*A + 2*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*B*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.132709, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3997, 3787, 3767, 8, 3768, 3770}

$$\frac{(3aA+2bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bB\tan(c+dx)\sec^2(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*a*A + 2*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*B*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{bB\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec^2(c+dx)(3aA+2b\\ &= \frac{bB\sec^2(c+dx)\tan(c+dx)}{3d}+(Ab+aB)\int \sec^3(c+dx)\,d\\ &= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{bB\sec^2(c+dx)\tan(\\ &= \frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3aA+2bB)\tan(c+dx}{3d}\end{aligned}$$

Mathematica [A] time = 0.268569, size = 67, normalized size = 0.72

$$\frac{3(aB+Ab)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aB+Ab)\sec(c+dx)+6aA+2bB\tan^2(c+dx)+6bB\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*b
 + a*B)*Sec[c + d*x] + 2*b*B*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.027, size = 128, normalized size = 1.4

$$\frac{Aa\tan(dx+c)}{d}+\frac{B\sec(dx+c)\,a\tan(dx+c)}{2d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{A\sec(dx+c)\,b\tan(dx+c)}{2d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a*tan(d*x+c)+1/2*a*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*a*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*A*b*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x
+c))+2/3*b*B*tan(d*x+c)/d+1/3*b*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.974597, size = 171, normalized size = 1.84

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb-3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Ab\left(\frac{2\sin(}{\sin(dx}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*A*a*tan(d*x + c))/d
```

Fricas [A] time = 0.979597, size = 298, normalized size = 3.2

$$\frac{3\,(Ba+Ab)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(Ba+Ab)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(3\,Aa+2\,Bb)\cos\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A*a + 2*B*b)*cos(d*x + c)^2 +
 2*B*b + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.24908, size = 284, normalized size = 3.05

$$\frac{3\,(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/
2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A
*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x +
1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.279 $\int \sec(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bB\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((2*a*A + b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (b*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0775628, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A + b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (b*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{bB\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \sec(c+dx)(2aA+bB+2(A$$
$$= \frac{bB\sec(c+dx)\tan(c+dx)}{2d} + (Ab+aB)\int \sec^2(c+dx)\,dx + \frac{1}{2}$$
$$= \frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{bB\sec(c+dx)\tan(c+dx)}{2d}$$
$$= \frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(Ab+aB)\tan(c+dx)}{d} + \frac{b}{}$$

Mathematica [A] time = 0.0220697, size = 75, normalized size = 1.23

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\tan(c+dx)}{d} + \frac{Ab\tan(c+dx)}{d} + \frac{bB\tanh^{-1}(\sin(c+dx))}{2d} + \frac{bB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*ArcTanh[Sin[c + d*x]])/(2*d) + (A*b*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (b*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.029, size = 86, normalized size = 1.4

$$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Ba\tan(dx+c)}{d} + \frac{Ab\tan(dx+c)}{d} + \frac{Bb\sec(dx+c)\tan(dx+c)}{2d} + \frac{Bb\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+a*B*tan(d*x+c)/d+1/d*A*b*tan(d*x+c)+1/2*b
*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.962849, size = 119, normalized size = 1.95

$$-\frac{Bb\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4\,Aa\log(\sec(dx+c)+\tan(dx+c)) - 4\,Ba\tan(dx+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/4*(B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*A*a*log(sec(d*x + c) + tan(d*x + c)) - 4*B*a*tan(d
*x + c) - 4*A*b*tan(d*x + c))/d
```

Fricas [A] time = 0.80849, size = 247, normalized size = 4.05

$$\frac{(2\,Aa+Bb)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(2\,Aa+Bb)\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)}+2\,(Bb+2\,(Ba+A}{4\,d\cos{(dx+c)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*A*a + B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A*a + B*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b + 2*(B*a + A*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec{(c+dx)})\,(a+b\sec{(c+dx)})\sec{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 1.24083, size = 207, normalized size = 3.39

$$\frac{(2\,Aa+Bb)\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)}-(2\,Aa+Bb)\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)}-\frac{2\left(2\,Ba\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^3+2\,Ab\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((2*A*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A*a + B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan
(1/2*d*x + 1/2*c)^3 - B*b*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*
c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

3.280 $\int (a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{bB\tan(c+dx)}{d}$$

```
[Out] a*A*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*B*Tan[c + d*x])/d
```

Rubi [A] time = 0.0346277, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3914, 3767, 8, 3770}

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{bB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*B*Tan[c + d*x])/d
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= aAx+(bB)\int \sec^2(c+dx)\,dx+(Ab+aB)\int \sec(c+dx)\,dx\\ &= aAx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(bB)\operatorname{Subst}(\int 1\,dx,x,-\tan(c+d\end{aligned}$$
$$= aAx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0098725, size = 43, normalized size = 1.23

$$aAx + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} + \frac{Ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{bB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*
B*Tan[c + d*x])/d
```

Maple [A] time = 0.029, size = 65, normalized size = 1.9

$$aAx + \frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Aac}{d} + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bb\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*A*x+1/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*c+1/d*B*a*ln(sec(d*x+c)+tan
(d*x+c))+b*B*tan(d*x+c)/d
```

Maxima [A] time = 0.98273, size = 76, normalized size = 2.17

$$\frac{(dx+c)Aa + Ba\log(\sec(dx+c)+\tan(dx+c)) + Ab\log(\sec(dx+c)+\tan(dx+c)) + Bb\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*A*a + B*a*log(sec(d*x + c) + tan(d*x + c)) + A*b*log(sec(d*x + c
) + tan(d*x + c)) + B*b*tan(d*x + c))/d
```

Fricas [B] time = 0.503204, size = 225, normalized size = 6.43

$$\frac{2\,Aadx\cos(dx+c) + (Ba+Ab)\cos(dx+c)\log(\sin(dx+c)+1) - (Ba+Ab)\cos(dx+c)\log(-\sin(dx+c)+1) + \cdots}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*b*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [A] time = 10.1132, size = 71, normalized size = 2.03

$$\begin{cases} \frac{Aa(c+dx)+Ab\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+Ba\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+Bb\tan\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(A+B\sec\left(c\right)\right)\left(a+b\sec\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*(c + d*x) + A*b*log(tan(c + d*x) + sec(c + d*x)) + B*a*log(
tan(c + d*x) + sec(c + d*x)) + B*b*tan(c + d*x))/d, Ne(d, 0)), (x*(A + B*se
c(c))*(a + b*sec(c)), True))
```

Giac [B] time = 1.21419, size = 113, normalized size = 3.23

$$\frac{(dx+c)Aa+(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A*a + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*B*b*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.281 $\int \cos(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$x(aB+Ab)+\frac{aA\sin(c+dx)}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (A*b + a*B)*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d
```

Rubi [A] time = 0.0546407, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {3996, 3770}

$$x(aB+Ab)+\frac{aA\sin(c+dx)}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*b + a*B)*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx&=\frac{aA\sin(c+dx)}{d}-\int(-Ab-aB-bB\sec(c+dx))\,dx\\&=(Ab+aB)x+\frac{aA\sin(c+dx)}{d}+(bB)\int\sec(c+dx)\,dx\\&=(Ab+aB)x+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0281508, size = 46, normalized size = 1.31

$$\frac{aA\sin(c)\cos(dx)}{d}+\frac{aA\cos(c)\sin(dx)}{d}+aBx+Abx+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] A*b*x + a*B*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (
a*A*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.047, size = 56, normalized size = 1.6

$$Abx + Bax + \frac{A\sin(dx+c)a}{d} + \frac{Abc}{d} + \frac{Bb\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] A*b*x+B*a*x+a*A*sin(d*x+c)/d+1/d*A*b*c+1/d*B*b*ln(sec(d*x+c)+tan(d*x+c))+1/
d*B*a*c
```

Maxima [A] time = 0.963863, size = 78, normalized size = 2.23

$$\frac{2(dx+c)Ba + 2(dx+c)Ab + Bb(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + 2*(d*x + c)*A*b + B*b*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*A*a*sin(d*x + c))/d
```

Fricas [A] time = 0.493503, size = 142, normalized size = 4.06

$$\frac{2(Ba+Ab)dx + Bb\log(\sin(dx+c)+1) - Bb\log(-\sin(dx+c)+1) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a + A*b)*d*x + B*b*log(sin(d*x + c) + 1) - B*b*log(-sin(d*x + c)
+ 1) + 2*A*a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec(c+dx))(a + b\sec(c+dx))\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [B] time = 1.21985, size = 107, normalized size = 3.06

$$\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Bb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ba+Ab)(dx+c)+\frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + A*b)*(d*x + c) + 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.282 $\int \cos^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(aA+2bB)+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] ((a*A + 2*b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rubi [A] time = 0.0959616, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3996, 3787, 2637, 8}

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(aA+2bB)+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a*A + 2*b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{aA\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}\int \cos(c+dx)(-2(Ab+aB)-\\ &= \frac{aA\cos(c+dx)\sin(c+dx)}{2d}-(-Ab-aB)\int \cos(c+dx)\,dx-\\ &= \frac{1}{2}(aA+2bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{aA\cos(c+dx)\sin(}{2d}\end{aligned}$$

Mathematica [A] time = 0.0822269, size = 51, normalized size = 0.98

$$\frac{4(aB+Ab)\sin(c+dx)+aA\sin(2(c+dx))+2aAc+2aAdx+4bBdx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*A*c + 2*a*A*d*x + 4*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + a*A*Sin[2*(
c + d*x)])/(4*d)
```

Maple [A] time = 0.051, size = 57, normalized size = 1.1

$$\frac{1}{d}\left(Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+Ba\sin(dx+c)+Bb(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b*sin(d*x+c)+B*a*sin(d
*x+c)+B*b*(d*x+c))
```

Maxima [A] time = 0.959907, size = 74, normalized size = 1.42

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa+4\,(dx+c)Bb+4\,Ba\sin(dx+c)+4\,Ab\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*B*b + 4*B*a*sin(d*x
 + c) + 4*A*b*sin(d*x + c))/d
```

Fricas [A] time = 0.468127, size = 104, normalized size = 2.

$$\frac{(Aa+2\,Bb)dx+(Aa\cos(dx+c)+2\,Ba+2\,Ab)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((A*a + 2*B*b)*d*x + (A*a*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec{(c + dx)})(a + b\sec{(c + dx)})\cos^2{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] time = 1.16937, size = 163, normalized size = 3.13

$$\frac{(Aa + 2\,Bb)(dx + c) - \frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*b)*(d*x + c) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/
2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c)
- 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^2)/d
```

3.283 $\int \cos^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{(2aA+3bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] ((A*b + a*B)*x)/2 + ((2*a*A + 3*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.125352, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3996, 3787, 2635, 8, 2637}

$$\frac{(2aA+3bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*x)/2 + ((2*a*A + 3*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)(-3(Ab+aB$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - (-Ab-aB)\int \cos^2(c+dx)\,dx$$
$$= \frac{(2aA+3bB)\sin(c+dx)}{3d} + \frac{(Ab+aB)\cos(c+dx)\sin(c+dx)}{2d}$$
$$= \frac{1}{2}(Ab+aB)x + \frac{(2aA+3bB)\sin(c+dx)}{3d} + \frac{(Ab+aB)\cos(c+}{2a}$$

Mathematica [A] time = 0.156339, size = 75, normalized size = 0.89

$$\frac{3(3aA+4bB)\sin(c+dx)+3(aB+Ab)\sin(2(c+dx))+aA\sin(3(c+dx))+6aBc+6aBdx+6Abc+6Abdx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(3*a*A + 4*b*B)*Sin[c + d*x]
 + 3*(A*b + a*B)*Sin[2*(c + d*x)] + a*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.059, size = 85, normalized size = 1.

$$\frac{1}{d}\left(\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*sin(d*x+c)*b)
```

Maxima [A] time = 0.964429, size = 107, normalized size = 1.27

$$-\frac{4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Aa - 3(2dx+2c+\sin(2dx+2c))Ba - 3(2dx+2c+\sin(2dx+2c))Ab - 12Bb\sin}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b - 12*B*b*sin(d*x + c)
)/d
```

Fricas [A] time = 0.474375, size = 149, normalized size = 1.77

$$\frac{3\,(Ba+Ab)dx+\left(2\,Aa\cos\left(dx+c\right)^{2}+4\,Aa+6\,Bb+3\,(Ba+Ab)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(B*a + A*b)*d*x + (2*A*a*cos(d*x + c)^2 + 4*A*a + 6*B*b + 3*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16053, size = 243, normalized size = 2.89

$$\frac{3\,(Ba+Ab)(dx+c)+\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Bb\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*(d*x + c) + 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan(
1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/2
*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*
a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1
/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.284 $\int \cos^4(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=105

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(3aA+4bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3aA+4bB)+\frac{aA\sin(c+$$

```
[Out] ((3*a*A + 4*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((3*a*A + 4*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.138741, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3996, 3787, 2633, 2635, 8}

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(3aA+4bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3aA+4bB)+\frac{aA\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a*A + 4*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((3*a*A + 4*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)(-4(Ab+$$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - (-Ab-aB)\int \cos^3(c+dx)$$
$$= \frac{(3aA+4bB)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA\cos^3(c+dx)\sin}{4d}$$
$$= \frac{1}{8}(3aA+4bB)x + \frac{(Ab+aB)\sin(c+dx)}{d} + \frac{(3aA+4bB)c}{}$$

Mathematica [A] time = 0.235245, size = 91, normalized size = 0.87

$$\frac{-32(aB+Ab)\sin^3(c+dx)+96(aB+Ab)\sin(c+dx)+24(aA+bB)\sin(2(c+dx))+3aA\sin(4(c+dx))+36aAc+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (36*a*A*c + 48*b*B*c + 36*a*A*d*x + 48*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*x
] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*a*A
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.063, size = 107, normalized size = 1.

$$\frac{1}{d}\left(Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{Ba\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.960661, size = 136, normalized size = 1.3

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa-32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba-32\left(\sin(dx+c)^3-3\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(s
in(d*x + c)^3 - 3*sin(d*x + c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*
```

```
A*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b)/d
```

Fricas [A] time = 0.476038, size = 205, normalized size = 1.95

$$\frac{3\,(3\,Aa+4\,Bb)dx+\left(6\,Aa\cos\left(dx+c\right)^{3}+8\,(Ba+Ab)\cos\left(dx+c\right)^{2}+16\,Ba+16\,Ab+3\,(3\,Aa+4\,Bb)\cos\left(dx+c\right)\right)\sin\left(\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*d*x + (6*A*a*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*x
 + c)^2 + 16*B*a + 16*A*b + 3*(3*A*a + 4*B*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23204, size = 367, normalized size = 3.5

$$3\,(3\,Aa+4\,Bb)(dx+c)-\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*(d*x + c) - 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 - 24*B
*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan(1/2*
d*x + 1/2*c)^7 - 9*A*a*tan(1/2*d*x + 1/2*c)^5 - 40*B*a*tan(1/2*d*x + 1/2*c)
^5 - 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a*
tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 - 15*A*a*tan(1/2*d*x + 1/2*c) -
 24*B*a*tan(1/2*d*x + 1/2*c) - 24*A*b*tan(1/2*d*x + 1/2*c) - 12*B*b*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.285 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=198

$$\frac{(4a^2A+6abB+3Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2A+6abB+3Ab^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{(5a(aB+2Ab)+4b}{15a}$$

```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*b^2*B + 5
*a*(2*A*b + a*B))*Tan[c + d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*(5*A*b + 6*a*B)*Sec[c + d*x]^3*Tan[c + d*
x])/(20*d) + (b*B*Sec[c + d*x]^3*(a + b*Sec[c + d*x])*Tan[c + d*x])/(5*d) +
 ((4*b^2*B + 5*a*(2*A*b + a*B))*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.290915, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4026, 4047, 3767, 4046, 3768, 3770}

$$\frac{(4a^2A+6abB+3Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2A+6abB+3Ab^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{(5a(aB+2Ab)+4b}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*b^2*B + 5
*a*(2*A*b + a*B))*Tan[c + d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*(5*A*b + 6*a*B)*Sec[c + d*x]^3*Tan[c + d*
x])/(20*d) + (b*B*Sec[c + d*x]^3*(a + b*Sec[c + d*x])*Tan[c + d*x])/(5*d) +
 ((4*b^2*B + 5*a*(2*A*b + a*B))*Tan[c + d*x]^3)/(15*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{bB\sec^3(c+dx)(a+b\sec(c+dx))\tan(c+dx)}{5d} + \frac{1}{5}\int \sec^3(c$$

$$= \frac{bB\sec^3(c+dx)(a+b\sec(c+dx))\tan(c+dx)}{5d} + \frac{1}{5}\int \sec^3(c$$

$$= \frac{b(5Ab+6aB)\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{bB\sec^3(c+dx)(a$$

$$= \frac{\left(4b^2B+5a(2Ab+aB)\right)\tan(c+dx)}{5d} + \frac{\left(4a^2A+3Ab^2+6abB\right.}{}$$

$$= \frac{\left(4a^2A+3Ab^2+6abB\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\left(4b^2B+5a(2\right.}{}$$

Mathematica [A] time = 1.51414, size = 150, normalized size = 0.76

$$\frac{15\left(4a^2A+6abB+3Ab^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5\left(a^2B+2aAb+2b^2B\right)\tan^2(c+dx)+15\left(a^2B+2aAb\right.\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*
(4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[c + d*x] + 30*b*(A*b + 2*a*B)*Sec[c + d*x
]^3 + 8*(15*(2*a*A*b + a^2*B + b^2*B) + 5*(2*a*A*b + a^2*B + 2*b^2*B)*Tan[c
 + d*x]^2 + 3*b^2*B*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.042, size = 312, normalized size = 1.6

$$\frac{a^2A\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{2Ba^2\tan(dx+c)}{3d} + \frac{Ba^2\tan(dx+c)(\sec(dx+c}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/3
/d*B*a^2*tan(d*x+c)+1/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+4/3/d*A*a*b*tan(d*x
+c)+2/3/d*A*a*b*tan(d*x+c)*sec(d*x+c)^2+1/2/d*B*a*b*tan(d*x+c)*sec(d*x+c)^3
+3/4/d*B*a*b*sec(d*x+c)*tan(d*x+c)+3/4/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/
4/d*A*b^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*b^2*sec(d*x+c)*tan(d*x+c)+3/8/d*A
*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/15*b^2*B*tan(d*x+c)/d+1/5/d*B*b^2*tan(d*x+
c)*sec(d*x+c)^4+4/15/d*B*b^2*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.979049, size = 373, normalized size = 1.88

$$80\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{2}+160\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aab+16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 160*(tan(d*x + c)^3 + 3
*tan(d*x + c))*A*a*b + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*B*b^2 - 30*B*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 15*A*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.526381, size = 521, normalized size = 2.63

$$15\left(4\,Aa^{2}+6\,Bab+3\,Ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,Aa^{2}+6\,Bab+3\,Ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1
) - 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1)
+ 2*(16*(5*B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^4 + 15*(4*A*a^2 + 6*B*a
*b + 3*A*b^2)*cos(d*x + c)^3 + 24*B*b^2 + 8*(5*B*a^2 + 10*A*a*b + 4*B*b^2)*
cos(d*x + c)^2 + 30*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))(a+b\sec(c+dx))^{2}\sec^{3}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] time = 1.25681, size = 713, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(
60*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 120*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*
a*b*tan(1/2*d*x + 1/2*c)^9 + 150*B*a*b*tan(1/2*d*x + 1/2*c)^9 + 75*A*b^2*ta
n(1/2*d*x + 1/2*c)^9 - 120*B*b^2*tan(1/2*d*x + 1/2*c)^9 - 120*A*a^2*tan(1/2
*d*x + 1/2*c)^7 + 320*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 640*A*a*b*tan(1/2*d*x
+ 1/2*c)^7 - 60*B*a*b*tan(1/2*d*x + 1/2*c)^7 - 30*A*b^2*tan(1/2*d*x + 1/2*c
)^7 + 160*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 400*B*a^2*tan(1/2*d*x + 1/2*c)^5 -
 800*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 464*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*
A*a^2*tan(1/2*d*x + 1/2*c)^3 + 320*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 640*A*a*b
*tan(1/2*d*x + 1/2*c)^3 + 60*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 30*A*b^2*tan(1/
2*d*x + 1/2*c)^3 + 160*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 60*A*a^2*tan(1/2*d*x
+ 1/2*c) - 120*B*a^2*tan(1/2*d*x + 1/2*c) - 240*A*a*b*tan(1/2*d*x + 1/2*c)
- 150*B*a*b*tan(1/2*d*x + 1/2*c) - 75*A*b^2*tan(1/2*d*x + 1/2*c) - 120*B*b^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.286 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{(4a^2Ab+a^3(-B)+8ab^2B+4Ab^3)\tan(c+dx)}{6bd}+\frac{(4a^2B+8aAb+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(-2a^2B+8aAb+9}{}$$

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a^2*A*b +
 4*A*b^3 - a^3*B + 8*a*b^2*B)*Tan[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B +
 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(12*b*d) + (B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(
4*b*d)
```

Rubi [A] time = 0.322393, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(4a^2Ab+a^3(-B)+8ab^2B+4Ab^3)\tan(c+dx)}{6bd}+\frac{(4a^2B+8aAb+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(-2a^2B+8aAb+9}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```



```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a^2*A*b +
 4*A*b^3 - a^3*B + 8*a*b^2*B)*Tan[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B +
 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(12*b*d) + (B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(
4*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
```

```
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{B(a+b\sec(c+dx))^3\tan(c+dx)}{4bd}+\frac{\int \sec(c+dx)(a+b\sec(\\ &= \frac{(4Ab-aB)(a+b\sec(c+dx))^2\tan(c+dx)}{12bd}+\frac{B(a+b\sec(c\\ &= \frac{\left(8aAb-2a^2B+9b^2B\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4Ab-aB\\ &= \frac{\left(8aAb-2a^2B+9b^2B\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4Ab-aB\\ &= \frac{\left(8aAb+4a^2B+3b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(8aAb-2a^2B\\ &= \frac{\left(8aAb+4a^2B+3b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(4a^2Ab+4A\end{aligned}$$

Mathematica [A] time = 0.735026, size = 120, normalized size = 0.67

$$\frac{3\left(4a^2B+8aAb+3b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(4a^2B+8aAb+3b^2B\right)\sec(c+dx)+24\left(a^2A+2abB+A\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
a^2*A + A*b^2 + 2*a*b*B) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*Sec[c + d*x] + 6
*b^2*B*Sec[c + d*x]^3 + 8*b*(A*b + 2*a*B)*Tan[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.038, size = 241, normalized size = 1.4

$$\frac{a^2 A \tan(dx+c)}{d} + \frac{Ba^2 \sec(dx+c)\tan(dx+c)}{2d} + \frac{Ba^2 \ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{Aab \sec(dx+c)\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*a^2*A*tan(d*x+c)+1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^2*ln(sec(d
*x+c)+tan(d*x+c))+1/d*A*a*b*sec(d*x+c)*tan(d*x+c)+1/d*A*a*b*ln(sec(d*x+c)+t
an(d*x+c))+4/3/d*B*a*b*tan(d*x+c)+2/3/d*B*a*b*tan(d*x+c)*sec(d*x+c)^2+2/3/d
*A*b^2*tan(d*x+c)+1/3/d*A*b^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*b^2*tan(d*x+c
)*sec(d*x+c)^3+3/8/d*B*b^2*sec(d*x+c)*tan(d*x+c)+3/8/d*B*b^2*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 1.03059, size = 308, normalized size = 1.72

$$\frac{32\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Bab + 16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ab^2 - 3Bb^2\left(\frac{2\left(3\sin(dx+c)^3 - 5\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1} - 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*b^2 - 3*B*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*A*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^2*tan(d*x +
 c))/d
```

Fricas [A] time = 0.520458, size = 443, normalized size = 2.47

$$\frac{3\left(4Ba^2 + 8Aab + 3Bb^2\right)\cos(dx+c)^4 \log(\sin(dx+c)+1) - 3\left(4Ba^2 + 8Aab + 3Bb^2\right)\cos(dx+c)^4 \log(-\sin(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(8*(3*A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^3 + 6*B*b^2 + 3*(4*B*a^2 + 8
*A*a*b + 3*B*b^2)*cos(d*x + c)^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^2 \sec^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] time = 1.22297, size = 645, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*
A*a^2*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*A*a*b*t
an(1/2*d*x + 1/2*c)^7 + 48*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^2*tan(1/2*
d*x + 1/2*c)^7 - 15*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*tan(1/2*d*x + 1
/2*c)^5 + 12*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b*tan(1/2*d*x + 1/2*c)^5
 - 80*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 40*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*B*
b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*B*a^2*tan
(1/2*d*x + 1/2*c)^3 + 24*A*a*b*tan(1/2*d*x + 1/2*c)^3 + 80*B*a*b*tan(1/2*d*
x + 1/2*c)^3 + 40*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*B*b^2*tan(1/2*d*x + 1/2*
c)^3 - 24*A*a^2*tan(1/2*d*x + 1/2*c) - 12*B*a^2*tan(1/2*d*x + 1/2*c) - 24*A
*a*b*tan(1/2*d*x + 1/2*c) - 48*B*a*b*tan(1/2*d*x + 1/2*c) - 24*A*b^2*tan(1/
2*d*x + 1/2*c) - 15*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

3.287 $\int \sec(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{2\left(a^2B+3aAb+b^2B\right)\tan(c+dx)}{3d}+\frac{\left(2a^2A+2abB+Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2aB+3Ab)\tan(c+dx)\sec(c+dx)}{6d}$$

```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a*A*b + a
^2*B + b^2*B)*Tan[c + d*x])/(3*d) + (b*(3*A*b + 2*a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(6*d) + (B*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.179885, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2\left(a^2B+3aAb+b^2B\right)\tan(c+dx)}{3d}+\frac{\left(2a^2A+2abB+Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2aB+3Ab)\tan(c+dx)\sec(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a*A*b + a
^2*B + b^2*B)*Tan[c + d*x])/(3*d) + (b*(3*A*b + 2*a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(6*d) + (B*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{B(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)(a+b\operatorname{se}\\ &= \frac{b(3Ab+2aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{B(a+b\sec(c+dx))^2}{3d}\\ &= \frac{b(3Ab+2aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{B(a+b\sec(c+dx))^2}{3d}\\ &= \frac{\left(2a^2A+Ab^2+2abB\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(3Ab+2aB)\operatorname{s}}{}\\ &= \frac{\left(2a^2A+Ab^2+2abB\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2\left(3aAb+a^2B\right.}{}\end{aligned}$$

Mathematica [A] time = 0.465689, size = 92, normalized size = 0.79

$$\frac{3\left(2a^2A+2abB+Ab^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2\left(3a^2B+6aAb+b^2B\tan^2(c+dx)+3b^2B\right)+3b(2aB+Ab\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(A
*b + 2*a*B)*Sec[c + d*x] + 2*(6*a*A*b + 3*a^2*B + 3*b^2*B + b^2*B*Tan[c + d
*x]^2)))/(6*d)
```

Maple [A] time = 0.035, size = 174, normalized size = 1.5

$$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba^2\tan(dx+c)}{d}+2\frac{Aab\tan(dx+c)}{d}+\frac{Bab\sec(dx+c)\tan(dx+c)}{d}+\frac{Bab\ln(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^2*tan(d*x+c)+2/d*A*a*b*tan(d*x+
c)+1/d*B*a*b*sec(d*x+c)*tan(d*x+c)+1/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/
d*A*b^2*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/3*b^2
*B*tan(d*x+c)/d+1/3/d*B*b^2*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.00315, size = 223, normalized size = 1.92

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Bb^{2}-6\,Bab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ab^{2}\left(\frac{2}{\sin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^2 - 6*B*a*b*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*
b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 12*B*a^2*tan(d
*x + c) + 24*A*a*b*tan(d*x + c))/d
```

Fricas [A] time = 0.504269, size = 371, normalized size = 3.2

$$\frac{3\left(2\,Aa^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,Aa^{2}+2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right.\right.}{12\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(2*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
B*b^2 + 2*(3*B*a^2 + 6*A*a*b + 2*B*b^2)*cos(d*x + c)^2 + 3*(2*B*a*b + A*b^2
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))^{2}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.20154, size = 397, normalized size = 3.42

$$\frac{3\left(2\,Aa^{2}+2\,Bab+Ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^{2}+2\,Bab+Ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Ba^{2}\,\mathrm{t}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
2*A*a^2 + 2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a^2*
tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*
d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2
*c)^5 - 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*A*a*b*tan(1/2*d*x + 1/2*c)^3 -
 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) + 12*A*a*b*t
an(1/2*d*x + 1/2*c) + 6*B*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x +
1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.288 $\int (a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{(2a^2B+4aAb+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{b(3aB+2Ab)\tan(c+dx)}{2d}+\frac{bB\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] a^2*A*x + ((4*a*A*b + 2*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(2
*A*b + 3*a*B)*Tan[c + d*x])/(2*d) + (b*B*(a + b*Sec[c + d*x])*Tan[c + d*x])
/(2*d)
```

Rubi [A] time = 0.0807489, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3918, 3770, 3767, 8}

$$\frac{(2a^2B+4aAb+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{b(3aB+2Ab)\tan(c+dx)}{2d}+\frac{bB\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*A*x + ((4*a*A*b + 2*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(2
*A*b + 3*a*B)*Tan[c + d*x])/(2*d) + (b*B*(a + b*Sec[c + d*x])*Tan[c + d*x])
/(2*d)
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{bB(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2a^2A+\left(4aAb+2a^2B+b^2B\right)\right.$$
$$= a^2Ax+\frac{bB(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}(b(2Ab+3aB))\int \sec^2(c+$$
$$= a^2Ax+\frac{\left(4aAb+2a^2B+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bB(a+b\sec(c+dx}{2d}$$
$$= a^2Ax+\frac{\left(4aAb+2a^2B+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2Ab+3aB)\tan(}{2d}$$

Mathematica [A] time = 0.262533, size = 67, normalized size = 0.78

$$\frac{\left(2a^2B+4aAb+b^2B\right)\tanh^{-1}(\sin(c+dx))+2a^2Adx+b\tan(c+dx)(4aB+2Ab+bB\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*A*d*x + (4*a*A*b + 2*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]] + b*(2*A*b
 + 4*a*B + b*B*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.032, size = 133, normalized size = 1.6

$$a^2Ax+\frac{Aa^2c}{d}+\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{Aab\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{Bab\tan(dx+c)}{d}+\frac{Ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] a^2*A*x+1/d*A*a^2*c+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a*b*ln(sec(d*
x+c)+tan(d*x+c))+2/d*B*a*b*tan(d*x+c)+1/d*A*b^2*tan(d*x+c)+1/2/d*B*b^2*sec(
d*x+c)*tan(d*x+c)+1/2/d*B*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.965132, size = 170, normalized size = 1.98

$$\frac{4(dx+c)Aa^2-Bb^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Ba^2\log(\sec(dx+c)+\tan(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^2 - B*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a^2*log(sec(d*x + c) + tan(
d*x + c)) + 8*A*a*b*log(sec(d*x + c) + tan(d*x + c)) + 8*B*a*b*tan(d*x + c)
 + 4*A*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.506255, size = 335, normalized size = 3.9

$$\frac{4\,Aa^2dx\cos\left(dx+c\right)^2+\left(2\,Ba^2+4\,Aab+Bb^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ba^2+4\,Aab+Bb^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(Bb^2+2\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*A*a^2*d*x*cos(d*x + c)^2 + (2*B*a^2 + 4*A*a*b + B*b^2)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (2*B*a^2 + 4*A*a*b + B*b^2)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) + 2*(B*b^2 + 2*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.20235, size = 259, normalized size = 3.01

$$\frac{2\left(dx+c\right)Aa^2+\left(2\,Ba^2+4\,Aab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ba^2+4\,Aab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(4\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + (2*B*a^2 + 4*A*a*b + B*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (2*B*a^2 + 4*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*(4*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - B
*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b*tan(1/2*d*x + 1/2*c) - 2*A*b^2*tan(1/
2*d*x + 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

3.289 $\int \cos(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a^2A\sin(c+dx)}{d}+\frac{b(2aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+ax(aB+2Ab)+\frac{b^2B\tan(c+dx)}{d}$$

```
[Out] a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*A*Sin[
c + d*x])/d + (b^2*B*Tan[c + d*x])/d
```

Rubi [A] time = 0.102278, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4024, 3770, 3767, 8}

$$\frac{a^2A\sin(c+dx)}{d}+\frac{b(2aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+ax(aB+2Ab)+\frac{b^2B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*A*Sin[
c + d*x])/d + (b^2*B*Tan[c + d*x])/d
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\sin(c+dx)}{d} - \int \left(-a(2Ab+aB)+\left(-Ab^2-2abB\right)\text{sec}\right.$$
$$= a(2Ab+aB)x + \frac{a^2A\sin(c+dx)}{d} + \left(b^2B\right)\int \sec^2(c+dx)\,d$$
$$= a(2Ab+aB)x + \frac{b(Ab+2aB)\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2A\,\text{s}}{}$$
$$= a(2Ab+aB)x + \frac{b(Ab+2aB)\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2A\,\text{s}}{}$$

Mathematica [A] time = 0.478934, size = 109, normalized size = 1.82

$$\frac{a^2A\sin(c+dx)+a(c+dx)(aB+2Ab)-b(2aB+Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b(2aB+Ab)\log\left(\text{sin}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(2*A*b + a*B)*(c + d*x) - b*(A*b + 2*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + b*(A*b + 2*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2
*A*Sin[c + d*x] + b^2*B*Tan[c + d*x])/d
```

Maple [A] time = 0.045, size = 104, normalized size = 1.7

$$2\,Aabx+Ba^2x+\frac{Ab^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2A\sin(dx+c)}{d}+2\,\frac{Aabc}{d}+2\,\frac{Bab\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 2*A*a*b*x+B*a^2*x+1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*A*sin(d*x+c)/d+2/
d*A*a*b*c+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+b^2*B*tan(d*x+c)/d+1/d*B*a^2*
c
```

Maxima [A] time = 0.989084, size = 139, normalized size = 2.32

$$\frac{2\,(dx+c)Ba^2+4\,(dx+c)Aab+2\,Bab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Ab^2(\log(\sin(dx+c)+1)-\text{lo}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + 4*(d*x + c)*A*a*b + 2*B*a*b*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*A*a^2*sin(d*x + c) + 2*B*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.503146, size = 294, normalized size = 4.9

$$\frac{2\left(Ba^2+2\,Aab\right)dx\cos\left(dx+c\right)+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\log\left(-\sin\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*(B*a^2 + 2*A*a*b)*d*x*cos(d*x + c) + (2*B*a*b + A*b^2)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (2*B*a*b + A*b^2)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(A*a^2*cos(d*x + c) + B*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

Giac [B] time = 1.23978, size = 208, normalized size = 3.47

$$\frac{\left(Ba^2+2\,Aab\right)\left(dx+c\right)+\left(2\,Bab+Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Bab+Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^2\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((B*a^2 + 2*A*a*b)*(d*x + c) + (2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x +
 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.290 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}x\left(a^2A+4abB+2Ab^2\right)+\frac{a^2A\sin(c+dx)\cos(c+dx)}{2d}+\frac{a(aB+2Ab)\sin(c+dx)}{d}+\frac{b^2B\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] ((a^2*A + 2*A*b^2 + 4*a*b*B)*x)/2 + (b^2*B*ArcTanh[Sin[c + d*x]])/d + (a*(2
*A*b + a*B)*Sin[c + d*x])/d + (a^2*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.173926, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4024, 4047, 8, 4045, 3770}

$$\frac{1}{2}x\left(a^2A+4abB+2Ab^2\right)+\frac{a^2A\sin(c+dx)\cos(c+dx)}{2d}+\frac{a(aB+2Ab)\sin(c+dx)}{d}+\frac{b^2B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a^2*A + 2*A*b^2 + 4*a*b*B)*x)/2 + (b^2*B*ArcTanh[Sin[c + d*x]])/d + (a*(2
*A*b + a*B)*Sin[c + d*x])/d + (a^2*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)\left(-2a(2Ab+\right.$$
$$= \frac{a^2A\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)\left(-2a(2Ab+\right.$$
$$= \frac{1}{2}\left(a^2A+2Ab^2+4abB\right)x + \frac{a(2Ab+aB)\sin(c+dx)}{d} + \frac{a^2A}{}$$
$$= \frac{1}{2}\left(a^2A+2Ab^2+4abB\right)x + \frac{b^2B\tanh^{-1}(\sin(c+dx))}{d} + \frac{a(2A}{}$$

Mathematica [A] time = 0.21232, size = 120, normalized size = 1.5

$$\frac{2(c+dx)\left(a^2A+4abB+2Ab^2\right)+a^2A\sin(2(c+dx))+4a(aB+2Ab)\sin(c+dx)-4b^2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 + 4*a*b*B)*(c + d*x) - 4*b^2*B*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*b^2*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*a*(
2*A*b + a*B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.054, size = 120, normalized size = 1.5

$$\frac{a^2A\cos(dx+c)\sin(dx+c)}{2d} + \frac{a^2Ax}{2} + \frac{a^2Ac}{2d} + \frac{Ba^2\sin(dx+c)}{d} + 2\frac{Aab\sin(dx+c)}{d} + 2Babx + 2\frac{Babc}{d} + Ab^2x + \frac{Ab^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2*a^2*A*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*A*x+1/2/d*A*a^2*c+1/d*B*a^2*sin(d
*x+c)+2/d*A*a*b*sin(d*x+c)+2*B*a*b*x+2/d*B*a*b*c+A*b^2*x+1/d*A*b^2*c+1/d*B*
b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.959678, size = 134, normalized size = 1.68

$$\frac{(2dx+2c+\sin(2dx+2c))Aa^2+8(dx+c)Bab+4(dx+c)Ab^2+2Bb^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 8*(d*x + c)*B*a*b + 4*(d*x +
c)*A*b^2 + 2*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^
2*sin(d*x + c) + 8*A*a*b*sin(d*x + c))/d
```

Fricas [A] time = 0.510376, size = 213, normalized size = 2.66

$$\frac{Bb^2\log(\sin(dx+c)+1) - Bb^2\log(-\sin(dx+c)+1) + \left(Aa^2+4\,Bab+2\,Ab^2\right)dx + \left(Aa^2\cos(dx+c)+2\,Ba^2+4\,A\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*b^2*log(sin(d*x + c) + 1) - B*b^2*log(-sin(d*x + c) + 1) + (A*a^2 +
4*B*a*b + 2*A*b^2)*d*x + (A*a^2*cos(d*x + c) + 2*B*a^2 + 4*A*a*b)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19371, size = 240, normalized size = 3.

$$\frac{2\,Bb^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,Bb^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \left(Aa^2+4\,Bab+2\,Ab^2\right)(dx+c) - \frac{2\left(Aa^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*B*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*b^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (A*a^2 + 4*B*a*b + 2*A*b^2)*(d*x + c) - 2*(A*a^2*tan(1/
2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b*tan(1/2*d*x + 1
/2*c)^3 - A*a^2*tan(1/2*d*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c) - 4*A*a
*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.291 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{(2a^2A+6abB+3Ab^2)\sin(c+dx)}{3d}+\frac{1}{2}x\left(a^2B+2aAb+2b^2B\right)+\frac{a^2A\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{a(aB+2Ab)\sin(c+dx)}{2d}$$

```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*x)/2 + ((2*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c +
d*x])/(3*d) + (a*(2*A*b + a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*A*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.215749, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4024, 4047, 2637, 4045, 8}

$$\frac{(2a^2A+6abB+3Ab^2)\sin(c+dx)}{3d}+\frac{1}{2}x\left(a^2B+2aAb+2b^2B\right)+\frac{a^2A\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{a(aB+2Ab)\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*x)/2 + ((2*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c +
d*x])/(3*d) + (a*(2*A*b + a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*A*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)\left(-3a(2A\right.$$
$$= \frac{a^2A\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)\left(-3a(2A\right.$$
$$= \frac{\left(2a^2A+3Ab^2+6abB\right)\sin(c+dx)}{3d} + \frac{a(2Ab+aB)\cos(c}{2a}$$
$$= \frac{1}{2}\left(2aAb+a^2B+2b^2B\right)x + \frac{\left(2a^2A+3Ab^2+6abB\right)\sin(c}{3d}$$

Mathematica [A] time = 0.223909, size = 90, normalized size = 0.84

$$\frac{6(c+dx)\left(a^2B+2aAb+2b^2B\right)+3\left(3a^2A+8abB+4Ab^2\right)\sin(c+dx)+a^2A\sin(3(c+dx))+3a(aB+2Ab)\sin(2(c-}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*(2*a*A*b + a^2*B + 2*b^2*B)*(c + d*x) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*
Sin[c + d*x] + 3*a*(2*A*b + a*B)*Sin[2*(c + d*x)] + a^2*A*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A] time = 0.059, size = 114, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2A\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + 2\,Aab\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2) + Ba^2\left(\frac{\cos(dx+c)\sin}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*A*a*b*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*sin
(d*x+c)+2*B*a*b*sin(d*x+c)+B*b^2*(d*x+c))
```

Maxima [A] time = 0.968484, size = 146, normalized size = 1.36

$$-\frac{4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^2-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2-6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab-12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^2 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b - 12*(d*x + c)*
```

B*b^2 - 24*B*a*b*sin(d*x + c) - 12*A*b^2*sin(d*x + c))/d

Fricas [A] time = 0.479716, size = 201, normalized size = 1.88

$$\frac{3\left(Ba^2 + 2\,Aab + 2\,Bb^2\right)dx + \left(2\,Aa^2\cos\left(dx+c\right)^2 + 4\,Aa^2 + 12\,Bab + 6\,Ab^2 + 3\left(Ba^2 + 2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*d*x + (2*A*a^2*cos(d*x + c)^2 + 4*A*a^2
+ 12*B*a*b + 6*A*b^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22963, size = 343, normalized size = 3.21

$$3\left(Ba^2 + 2\,Aab + 2\,Bb^2\right)(dx+c) + \frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 6\,Ab^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*(d*x + c) + 2*(6*A*a^2*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 1
2*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*t
an(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*
d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c
) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.292 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{(a^2B+2aAb+b^2B)\sin(c+dx)}{d}+\frac{(3a^2A+8abB+4Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a^2A+8abB+4Ab^2)+\frac{a^2}{}$$

```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*x)/8 + ((2*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/d + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a^2*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(2*A*b + a*B)*Sin[c + d*x]^3)
/(3*d)
```

Rubi [A] time = 0.259949, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4024, 4047, 2635, 8, 4044, 3013}

$$\frac{(a^2B+2aAb+b^2B)\sin(c+dx)}{d}+\frac{(3a^2A+8abB+4Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a^2A+8abB+4Ab^2)+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*x)/8 + ((2*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/d + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a^2*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(2*A*b + a*B)*Sin[c + d*x]^3)
/(3*d)
```


Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)\left(-4a(2Ab\right.$$
$$= \frac{a^2A\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)\left(-4a(2Ab\right.$$
$$= \frac{\left(3a^2A+4Ab^2+8abB\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2A\cos^3(}{}$$
$$= \frac{1}{8}\left(3a^2A+4Ab^2+8abB\right)x + \frac{\left(3a^2A+4Ab^2+8abB\right)\cos(c}{8d}$$
$$= \frac{1}{8}\left(3a^2A+4Ab^2+8abB\right)x + \frac{\left(2aAb+a^2B+b^2B\right)\sin(c+d}{d}$$

Mathematica [A] time = 0.447607, size = 118, normalized size = 0.87

$$\frac{12(c+dx)\left(3a^2A+8abB+4Ab^2\right)+24\left(3a^2B+6aAb+4b^2B\right)\sin(c+dx)+24\left(a^2A+2abB+Ab^2\right)\sin(2(c+dx))+3a}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*(c + d*x) + 24*(6*a*A*b + 3*a^2*B + 4*b^2
*B)*Sin[c + d*x] + 24*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 8*a*(2*A
*b + a*B)*Sin[3*(c + d*x)] + 3*a^2*A*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.068, size = 152, normalized size = 1.1

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ba^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2Aab(2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*A*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2*
B*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+B*b^2*sin(d*x+c))
```

Maxima [A] time = 0.992416, size = 192, normalized size = 1.41

$$\frac{3 (12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\,\sin(2\,dx + 2\,c))Aa^2 - 32\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Ba^2 - 64\left(\sin(dx + c)^3 - \right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x +
c))*A*a*b + 48*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 24*(2*d*x + 2*c + s
in(2*d*x + 2*c))*A*b^2 + 96*B*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.492599, size = 274, normalized size = 2.01

$$\frac{3\left(3\,Aa^2 + 8\,Bab + 4\,Ab^2\right)dx + \left(6\,Aa^2\cos(dx + c)^3 + 16\,Ba^2 + 32\,Aab + 24\,Bb^2 + 8\left(Ba^2 + 2\,Aab\right)\cos(dx + c)^2 + 3\left(\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*d*x + (6*A*a^2*cos(d*x + c)^3 + 16*B*
a^2 + 32*A*a*b + 24*B*b^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c)^2 + 3*(3*A*a^2
 + 8*B*a*b + 4*A*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1952, size = 590, normalized size = 4.34

$$\frac{3\left(3\,Aa^2 + 8\,Bab + 4\,Ab^2\right)(dx + c) - \frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 48\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 12\,A\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*(d*x + c) - 2*(15*A*a^2*tan(1/2*d*x +
 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*tan(1/2*d*x + 1/2*c)
^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 24
*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^2*t
an(1/2*d*x + 1/2*c)^5 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b*tan(1/2*
d*x + 1/2*c)^5 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*b^2*tan(1/2*d*x + 1
/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^3
- 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*
b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^2*tan
(1/2*d*x + 1/2*c) - 24*B*a^2*tan(1/2*d*x + 1/2*c) - 48*A*a*b*tan(1/2*d*x +
1/2*c) - 24*B*a*b*tan(1/2*d*x + 1/2*c) - 12*A*b^2*tan(1/2*d*x + 1/2*c) - 24
*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.293 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$-\frac{(4a^2A+10abB+5Ab^2)\sin^3(c+dx)}{15d}+\frac{(4a^2A+10abB+5Ab^2)\sin(c+dx)}{5d}+\frac{(3a^2B+6aAb+4b^2B)\sin(c+dx)\,\mathrm{c}}{8d}$$

```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B)*x)/8 + ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c
 + d*x])/(5*d) + ((6*a*A*b + 3*a^2*B + 4*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a*(2*A*b + a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a^2*A*Cos[c
+ d*x]^4*Sin[c + d*x])/(5*d) - ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x]
^3)/(15*d)
```

Rubi [A] time = 0.268378, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4024, 4047, 2633, 4045, 2635, 8}

$$-\frac{(4a^2A+10abB+5Ab^2)\sin^3(c+dx)}{15d}+\frac{(4a^2A+10abB+5Ab^2)\sin(c+dx)}{5d}+\frac{(3a^2B+6aAb+4b^2B)\sin(c+dx)\,\mathrm{c}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B)*x)/8 + ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c
 + d*x])/(5*d) + ((6*a*A*b + 3*a^2*B + 4*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a*(2*A*b + a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a^2*A*Cos[c
+ d*x]^4*Sin[c + d*x])/(5*d) - ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x]
^3)/(15*d)
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)\left(-5a(2Ab\right.$$

$$= \frac{a^2A\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)\left(-5a(2Ab\right.$$

$$= \frac{a(2Ab+aB)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^2A\cos^4(c+dx)\sin}{5d}$$

$$= \frac{\left(4a^2A+5Ab^2+10abB\right)\sin(c+dx)}{5d} + \frac{\left(6aAb+3a^2B+4b^2B\right.}{}$$

$$= \frac{1}{8}\left(6aAb+3a^2B+4b^2B\right)x + \frac{\left(4a^2A+5Ab^2+10abB\right)\sin(c}{5d}$$

Mathematica [A] time = 0.545049, size = 146, normalized size = 0.81

$$\frac{60(c+dx)\left(3a^2B+6aAb+4b^2B\right)+60\left(5a^2A+12abB+6Ab^2\right)\sin(c+dx)+120\left(a^2B+2aAb+b^2B\right)\sin(2(c+dx))+1}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (60*(6*a*A*b + 3*a^2*B + 4*b^2*B)*(c + d*x) + 60*(5*a^2*A + 6*A*b^2 + 12*a*
b*B)*Sin[c + d*x] + 120*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 10*(5*
a^2*A + 4*A*b^2 + 8*a*b*B)*Sin[3*(c + d*x)] + 15*a*(2*A*b + a*B)*Sin[4*(c +
 d*x)] + 6*a^2*A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.068, size = 184, normalized size = 1.

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*a*b*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*B*a*b*(2+cos(d*x+c)^2)*
sin(d*x+c)+1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b^2*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.963242, size = 238, normalized size = 1.32

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{2}+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 + 30*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b - 320*(sin(d*x + c
)^3 - 3*sin(d*x + c))*B*a*b - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^2)/d
```

Fricas [A] time = 0.50198, size = 350, normalized size = 1.94

$$\frac{15\left(3\,Ba^{2}+6\,Aab+4\,Bb^{2}\right)dx+\left(24\,Aa^{2}\cos\left(dx+c\right)^{4}+30\left(Ba^{2}+2\,Aab\right)\cos\left(dx+c\right)^{3}+64\,Aa^{2}+160\,Bab+80\,Ab^{2}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*d*x + (24*A*a^2*cos(d*x + c)^4 + 30
*(B*a^2 + 2*A*a*b)*cos(d*x + c)^3 + 64*A*a^2 + 160*B*a*b + 80*A*b^2 + 8*(4*
A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x + c)^2 + 15*(3*B*a^2 + 6*A*a*b + 4*B*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19317, size = 657, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*(d*x + c) + 2*(120*A*a^2*tan(1/2*d*
x + 1/2*c)^9 - 75*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 150*A*a*b*tan(1/2*d*x + 1/
2*c)^9 + 240*B*a*b*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^2*tan(1/2*d*x + 1/2*c)^
9 - 60*B*b^2*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 30
*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 60*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 640*B*a*b
*tan(1/2*d*x + 1/2*c)^7 + 320*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*B*b^2*tan(
1/2*d*x + 1/2*c)^7 + 464*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*a*b*tan(1/2*d
*x + 1/2*c)^5 + 400*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^2*tan(1/2*d*x +
1/2*c)^3 + 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 60*A*a*b*tan(1/2*d*x + 1/2*c)^
3 + 640*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 320*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 1
20*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^2*tan(1/2*d*x + 1/2*c) + 75*B*a^2
*tan(1/2*d*x + 1/2*c) + 150*A*a*b*tan(1/2*d*x + 1/2*c) + 240*B*a*b*tan(1/2*
d*x + 1/2*c) + 120*A*b^2*tan(1/2*d*x + 1/2*c) + 60*B*b^2*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.294 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=252

$$\frac{(15a^3Ab+52a^2b^2B-3a^4B+60aAb^3+16b^4B)\tan(c+dx)}{30bd}+\frac{(12a^2Ab+4a^3B+9ab^2B+3Ab^3)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((15*a^3*A*b + 60*a*A*b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Tan[c + d*
x])/(30*b*d) + ((30*a^2*A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Sec[c + d*x]
*Tan[c + d*x])/(120*d) + ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Sec[c + d*
x])^2*Tan[c + d*x])/(60*b*d) + ((5*A*b - a*B)*(a + b*Sec[c + d*x])^3*Tan[c
+ d*x])/(20*b*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.47939, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(15a^3Ab+52a^2b^2B-3a^4B+60aAb^3+16b^4B)\tan(c+dx)}{30bd}+\frac{(12a^2Ab+4a^3B+9ab^2B+3Ab^3)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((15*a^3*A*b + 60*a*A*b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Tan[c + d*
x])/(30*b*d) + ((30*a^2*A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Sec[c + d*x]
*Tan[c + d*x])/(120*d) + ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Sec[c + d*
x])^2*Tan[c + d*x])/(60*b*d) + ((5*A*b - a*B)*(a + b*Sec[c + d*x])^3*Tan[c
+ d*x])/(20*b*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
```

```
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c+dx)(a+b\sec(}{}$$
$$= \frac{(5Ab-aB)(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} + \frac{B(a+b\sec(c}{}$$
$$= \frac{\left(15aAb-3a^2B+16b^2B\right)(a+b\sec(c+dx))^2\tan(c+dx)}{60bd} +$$
$$= \frac{\left(30a^2Ab+45Ab^3-6a^3B+71ab^2B\right)\sec(c+dx)\tan(c+dx)}{120d}$$
$$= \frac{\left(30a^2Ab+45Ab^3-6a^3B+71ab^2B\right)\sec(c+dx)\tan(c+dx)}{120d}$$
$$= \frac{\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(3}{}$$
$$= \frac{\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(1}{}$$

Mathematica [A] time = 3.34059, size = 181, normalized size = 0.72

$$\frac{15\left(12a^2Ab+4a^3B+9ab^2B+3Ab^3\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5b\left(3a^2B+3aAb+2b^2B\right)\tan^2(c+dx)+15\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sec[c + d*x] +
30*b^2*(A*b + 3*a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b*B
```

```
+ b^3*B) + 5*b*(3*a*A*b + 3*a^2*B + 2*b^2*B)*Tan[c + d*x]^2 + 3*b^3*B*Tan[c
 + d*x]^4)))/(120*d)
```

Maple [A] time = 0.042, size = 382, normalized size = 1.5

$$\frac{Aa^3\tan(dx+c)}{d}+\frac{Ba^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{3Aa^2b\sec(dx+c)\tan(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a^3*tan(d*x+c)+1/2/d*B*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^3*ln(sec(d
*x+c)+tan(d*x+c))+3/2/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+2/d*B*a^2*b*tan(d*x+c)+1/d*B*a^2*b*tan(d*x+c)*sec(d*x+c)
^2+2/d*A*a*b^2*tan(d*x+c)+1/d*A*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/4/d*B*a*b^2
*tan(d*x+c)*sec(d*x+c)^3+9/8/d*B*a*b^2*sec(d*x+c)*tan(d*x+c)+9/8/d*B*a*b^2*
ln(sec(d*x+c)+tan(d*x+c))+1/4/d*A*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*b^3*s
ec(d*x+c)*tan(d*x+c)+3/8/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*B*b^3*tan
(d*x+c)+1/5/d*B*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*B*b^3*tan(d*x+c)*sec(d*x
+c)^2
```

Maxima [A] time = 1.00424, size = 460, normalized size = 1.83

$$\frac{240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2b+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab^2+16\left(3\tan(dx+c)^5+10\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3
+ 3*tan(d*x + c))*A*a*b^2 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*B*b^3 - 45*B*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 15*A*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) - 180*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*tan(d*x +
 c))/d
```

Fricas [A] time = 0.545972, size = 612, normalized size = 2.43

$$\frac{15\left(4Ba^3+12Aa^2b+9Bab^2+3Ab^3\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(4Ba^3+12Aa^2b+9Bab^2+3Ab^3\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(15*A*a^3 + 30*B*a^2*b + 30*A*a*b^2 +
8*B*b^3)*cos(d*x + c)^4 + 24*B*b^3 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 +
 3*A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d*x +
c)^2 + 30*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^3 \sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] time = 1.26441, size = 975, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 60*B*a^3*t
an(1/2*d*x + 1/2*c)^9 - 180*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*B*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 75*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d
*x + 1/2*c)^9 - 480*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^3*tan(1/2*d*x +
1/2*c)^7 + 360*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1
/2*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 90*B*a*b^2*tan(1/2*d*x + 1/2
*c)^7 + 30*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 160*B*b^3*tan(1/2*d*x + 1/2*c)^7
+ 720*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 464*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 48
0*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*a
^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*A*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 90*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 30*A*b^3*t
an(1/2*d*x + 1/2*c)^3 - 160*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/
2*d*x + 1/2*c) + 60*B*a^3*tan(1/2*d*x + 1/2*c) + 180*A*a^2*b*tan(1/2*d*x +
1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c
) + 225*B*a*b^2*tan(1/2*d*x + 1/2*c) + 75*A*b^3*tan(1/2*d*x + 1/2*c) + 120*
B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.295 $\int \sec(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{(16a^2Ab+3a^3B+12ab^2B+4Ab^3)\tan(c+dx)}{6d}+\frac{(8a^3A+12a^2bB+12aAb^2+3b^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\,(6a^2B$$

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((16*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Tan[c + d*x])/(6*d) + (b*
(20*a*A*b + 6*a^2*B + 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b
+ 3*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.333184, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{(16a^2Ab+3a^3B+12ab^2B+4Ab^3)\tan(c+dx)}{6d}+\frac{(8a^3A+12a^2bB+12aAb^2+3b^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\,(6a^2B$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((16*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Tan[c + d*x])/(6*d) + (b*
(20*a*A*b + 6*a^2*B + 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b
+ 3*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/(4*d)
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int \sec(c+dx)(a+b\,\mathrm{se}$$
$$= \frac{(4Ab+3aB)(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{B(a+b\sec(c}{}$$
$$= \frac{b\left(20aAb+6a^2B+9b^2B\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4Ab+3}{}$$
$$= \frac{b\left(20aAb+6a^2B+9b^2B\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4Ab+3}{}$$
$$= \frac{\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(}{}$$
$$= \frac{\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(1}{}$$

Mathematica [A] time = 0.949985, size = 140, normalized size = 0.78

$$\frac{3\left(8a^3A+12a^2bB+12aAb^2+3b^3B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(9b\left(4a^2B+4aAb+b^2B\right)\sec(c+dx)+24\left(3a^2A\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) + 9*b*(4*a*A*b + 4*a
^2*B + b^2*B)*Sec[c + d*x] + 6*b^3*B*Sec[c + d*x]^3 + 8*b^2*(A*b + 3*a*B)*T
an[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.041, size = 290, normalized size = 1.6

$$\frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba^3\tan(dx+c)}{d}+3\frac{Aa^2b\tan(dx+c)}{d}+\frac{3\,Ba^2b\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{3\,Ba^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^3*tan(d*x+c)+3/d*A*a^2*b*tan(d*
x+c)+3/2/d*B*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a^2*b*ln(sec(d*x+c)+tan(d*
```

```
x+c))+3/2/d*A*a*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a*b^2*ln(sec(d*x+c)+tan(d
*x+c))+2/d*B*a*b^2*tan(d*x+c)+1/d*B*a*b^2*tan(d*x+c)*sec(d*x+c)^2+2/3/d*A*b
^3*tan(d*x+c)+1/3/d*A*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*b^3*tan(d*x+c)*se
c(d*x+c)^3+3/8/d*B*b^3*sec(d*x+c)*tan(d*x+c)+3/8/d*B*b^3*ln(sec(d*x+c)+tan(
d*x+c))
```

Maxima [A] time = 0.989862, size = 359, normalized size = 1.99

$$\frac{48\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Bab^{2}+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ab^{3}-3\,Bb^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b^2 + 16*(tan(d*x + c)^3 + 3
*tan(d*x + c))*A*b^3 - 3*B*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^3*log
(sec(d*x + c) + tan(d*x + c)) + 48*B*a^3*tan(d*x + c) + 144*A*a^2*b*tan(d*x
 + c))/d
```

Fricas [A] time = 0.52755, size = 510, normalized size = 2.83

$$\frac{3\left(8\,Aa^{3}+12\,Ba^{2}b+12\,Aab^{2}+3\,Bb^{3}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,Aa^{3}+12\,Ba^{2}b+12\,Aab^{2}+3\,Bb^{3}\right)\cos \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*B*b^3 + 8*(3*B*a^3 + 9*A*a^2*b + 6*B*a*
b^2 + 2*A*b^3)*cos(d*x + c)^3 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x +
 c)^2 + 8*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + B\sec\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^{3}\sec\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] time = 1.25369, size = 791, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b*t
an(1/2*d*x + 1/2*c)^7 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*A*a*b^2*tan(
1/2*d*x + 1/2*c)^7 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan(1/2*d
*x + 1/2*c)^7 - 15*B*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^3*tan(1/2*d*x + 1/
2*c)^5 - 216*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*
c)^5 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 120*B*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 40*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*
B*a^3*tan(1/2*d*x + 1/2*c)^3 + 216*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^
2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*b^
2*tan(1/2*d*x + 1/2*c)^3 + 40*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*B*b^3*tan(1/
2*d*x + 1/2*c)^3 - 24*B*a^3*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b*tan(1/2*d*x +
 1/2*c) - 36*B*a^2*b*tan(1/2*d*x + 1/2*c) - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)
 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c) - 24*A*b^3*tan(1/2*d*x + 1/2*c) - 15*B*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.296 $\int (a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=137

$$\frac{b(8a^2B+9aAb+2b^2B)\tan(c+dx)}{3d}+\frac{(6a^2Ab+2a^3B+3ab^2B+Ab^3)\tanh^{-1}(\sin(c+dx))}{2d}+a^3Ax+\frac{b^2(5aB+3Ab}{}$$

```
[Out] a^3*A*x + ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Tan[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*B*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(3*d)
```

Rubi [A] time = 0.189928, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3918, 4048, 3770, 3767, 8}

$$\frac{b(8a^2B+9aAb+2b^2B)\tan(c+dx)}{3d}+\frac{(6a^2Ab+2a^3B+3ab^2B+Ab^3)\tanh^{-1}(\sin(c+dx))}{2d}+a^3Ax+\frac{b^2(5aB+3Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] a^3*A*x + ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Tan[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*B*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(3*d)
```


Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{bB(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\sec(c+dx))\left(3a^2A+(6a\right.$$
$$=\frac{b^2(3Ab+5aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{bB(a+b\sec(c+dx))^2\tan(c+}{3d}$$
$$=a^3Ax+\frac{b^2(3Ab+5aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{bB(a+b\sec(c+dx))^2}{3d}$$
$$=a^3Ax+\frac{\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2(3Ab+}{}$$
$$=a^3Ax+\frac{\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\left(9aAb\right.}{}$$

Mathematica [A] time = 0.571772, size = 108, normalized size = 0.79

$$\frac{3\left(6a^2Ab+2a^3B+3ab^2B+Ab^3\right)\tanh^{-1}(\sin(c+dx))+3b\tan(c+dx)\left(6a^2B+b(3aB+Ab)\sec(c+dx)+6aAb+2b^2B\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*a^3*A*d*x + 3*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*ArcTanh[Sin[c +
d*x]] + 3*b*(6*a*A*b + 6*a^2*B + 2*b^2*B + b*(A*b + 3*a*B)*Sec[c + d*x])*Ta
n[c + d*x] + 2*b^3*B*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.04, size = 223, normalized size = 1.6

$$a^3Ax+\frac{Aa^3c}{d}+\frac{Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\frac{Aa^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\frac{Ba^2b\tan(dx+c)}{d}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] a^3*A*x+1/d*A*a^3*c+1/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+3/d*B*a^2*b*tan(d*x+c)+3/d*A*a*b^2*tan(d*x+c)+3/2/d*B*a*
b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b
^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*b^3*
tan(d*x+c)+1/3/d*B*b^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.975375, size = 273, normalized size = 1.99

$$\frac{12(dx+c)Aa^3+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb^3-9Bab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^3 - 9*B*
a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(si
n(d*x + c) - 1)) - 3*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*B*a^3*log(sec(d*x + c) + tan(d*x
 + c)) + 36*A*a^2*b*log(sec(d*x + c) + tan(d*x + c)) + 36*B*a^2*b*tan(d*x +
 c) + 36*A*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.549973, size = 458, normalized size = 3.34

$$\frac{12\,Aa^3dx\cos\left(dx+c\right)^3+3\left(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,Ba^3+6\,Aa^2b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*A*a^3*d*x*cos(d*x + c)^3 + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*
b^2 + A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*B*b^3 + 2*(9*B*a^
2*b + 9*A*a*b^2 + 2*B*b^3)*cos(d*x + c)^2 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.26516, size = 454, normalized size = 3.31

$$\frac{6\left(dx+c\right)Aa^3+3\left(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^3 + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A
*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^2*b*t
```

```
an(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^3*tan(1/2
*d*x + 1/2*c)^3 + 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x
+ 1/2*c) + 9*B*a*b^2*tan(1/2*d*x + 1/2*c) + 3*A*b^3*tan(1/2*d*x + 1/2*c) +
6*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.297 $\int \cos(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{b\left(6a^2B+6aAb+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(2aA-bB)\sin(c+dx)}{2d}+a^2x(aB+3Ab)+\frac{b^2(2aB+Ab)\tan(c+dx)}{d}$$

```
[Out] a^2*(3*A*b + a*B)*x + (b*(6*a*A*b + 6*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a^2*(2*a*A - b*B)*Sin[c + d*x])/(2*d) + (b*B*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(2*d) + (b^2*(A*b + 2*a*B)*Tan[c + d*x])/d
```

Rubi [A] time = 0.223205, antiderivative size = 131, normalized size of antiderivative = 1.1, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4025, 4048, 3770, 3767, 8}

$$-\frac{b\left(2a^2A-3abB-Ab^2\right)\tan(c+dx)}{d}+\frac{b\left(6a^2B+6aAb+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(aB+3Ab)-\frac{b^2(2aA-bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*(3*A*b + a*B)*x + (b*(6*a*A*b + 6*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (b*(2*a^2*A - A*b^2
- 3*a*b*B)*Tan[c + d*x])/d - (b^2*(2*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x])/
(2*d)
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \int (a+b\sec(c+dx))\,\Big(-$$
$$= \frac{aA(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \frac{b^2(2aA-bB)\sec(c+dx}{2d}$$
$$= a^2(3Ab+aB)x + \frac{aA(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \frac{b^2(2aA}{}$$
$$= a^2(3Ab+aB)x + \frac{b\left(6aAb+6a^2B+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}$$
$$= a^2(3Ab+aB)x + \frac{b\left(6aAb+6a^2B+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}$$

Mathematica [B] time = 0.962975, size = 399, normalized size = 3.35

$$\frac{\sec^2(c+dx)\left(\left(a^3A+2b^3B\right)\sin(c+dx)+\cos(2(c+dx))\left(-b\left(6a^2B+6aAb+b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^2*(6*a^2*A*b*c + 2*a^3*B*c + 6*a^2*A*b*d*x + 2*a^3*B*d*x - 6*
a*A*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*a^2*b*B*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - b^3*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 6*a*A*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 6*a^2*b*B*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] + b^3*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + Cos[2*(c + d*x)]*(2*a^2*(3*A*b + a*B)*(c + d*x) - b*(6*a*A*b + 6*a^
2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(6*a*A*b + 6*a^2*
B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^3*A + 2*b^3*B)*Si
n[c + d*x] + 2*A*b^3*Sin[2*(c + d*x)] + 6*a*b^2*B*Sin[2*(c + d*x)] + a^3*A*
Sin[3*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.054, size = 172, normalized size = 1.5

$$\frac{Aa^3\sin(dx+c)}{d} + Ba^3x + \frac{Ba^3c}{d} + 3\,Aa^2bx + 3\,\frac{Aa^2bc}{d} + 3\,\frac{Ba^2b\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\,\frac{Aab^2\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] a^3*A*sin(d*x+c)/d+B*a^3*x+1/d*B*a^3*c+3*A*a^2*b*x+3/d*A*a^2*b*c+3/d*B*a^2*
b*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a*b
^2*tan(d*x+c)+1/d*A*b^3*tan(d*x+c)+1/2/d*B*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*
B*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.982723, size = 228, normalized size = 1.92

$$\frac{4\,(dx+c)Ba^3+12\,(dx+c)Aa^2b-Bb^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,Ba^2b(\log\left(\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^3 + 12*(d*x + c)*A*a^2*b - B*b^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c) + 12*B*a*b^2*tan(
d*x + c) + 4*A*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.562001, size = 401, normalized size = 3.37

$$\frac{4\left(Ba^3+3\,Aa^2b\right)dx\cos\left(dx+c\right)^2+\left(6\,Ba^2b+6\,Aab^2+Bb^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(6\,Ba^2b+6\,Aab^2+\right.}{4\,d\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(4*(B*a^3 + 3*A*a^2*b)*d*x*cos(d*x + c)^2 + (6*B*a^2*b + 6*A*a*b^2 + B*
b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (6*B*a^2*b + 6*A*a*b^2 + B*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*a^3*cos(d*x + c)^2 + B*b^3
+ 2*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22502, size = 325, normalized size = 2.73

$$\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(Ba^3+3\,Aa^2b\right)(dx+c)+\left(6\,Ba^2b+6\,Aab^2+Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,Ba^2b+6\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(B*a^3 +
 3*A*a^2*b)*(d*x + c) + (6*B*a^2*b + 6*A*a*b^2 + B*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (6*B*a^2*b + 6*A*a*b^2 + B*b^3)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2
*c)^3 - B*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A
*b^3*tan(1/2*d*x + 1/2*c) - B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

3.298 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{1}{2}ax\left(a^2A+6abB+6Ab^2\right)+\frac{a^2(aB+2Ab)\sin(c+dx)}{d}-\frac{b^2(aA-2bB)\tan(c+dx)}{2d}+\frac{b^2(3aB+Ab)\tanh^{-1}(\sin(c+d$$

```
[Out] (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*x)/2 + (b^2*(A*b + 3*a*B)*ArcTanh[Sin[c + d*
x]])/d + (a^2*(2*A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*(a + b*Sec[
c + d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(a*A - 2*b*B)*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.333451, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4025, 4076, 4047, 8, 4045, 3770}

$$\frac{1}{2}ax\left(a^2A+6abB+6Ab^2\right)+\frac{a^2(aB+2Ab)\sin(c+dx)}{d}-\frac{b^2(aA-2bB)\tan(c+dx)}{2d}+\frac{b^2(3aB+Ab)\tanh^{-1}(\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*x)/2 + (b^2*(A*b + 3*a*B)*ArcTanh[Sin[c + d*
x]])/d + (a^2*(2*A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*(a + b*Sec[
c + d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(a*A - 2*b*B)*Tan[c + d*x])/(2*d)
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c$$
$$= \frac{aA\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2(aA-2b$$
$$= \frac{aA\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2(aA-2b$$
$$= \frac{1}{2}a\left(a^2A+6Ab^2+6abB\right)x + \frac{a^2(2Ab+aB)\sin(c+dx)}{d} + \frac{aA$$
$$= \frac{1}{2}a\left(a^2A+6Ab^2+6abB\right)x + \frac{b^2(Ab+3aB)\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A] time = 0.681226, size = 217, normalized size = 1.75

$$\frac{2a(c+dx)\left(a^2A+6abB+6Ab^2\right)+4a^2(aB+3Ab)\sin(c+dx)+a^3A\sin(2(c+dx))-4b^2(3aB+Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*(c + d*x) - 4*b^2*(A*b + 3*a*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*b^2*(A*b + 3*a*B)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*a^2*(3*A*b + a*B)*Sin[c + d*x] + a^3*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.056, size = 168, normalized size = 1.4

$$\frac{Aa^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{a^3Ax}{2} + \frac{Aa^3c}{2d} + \frac{Ba^3\sin(dx+c)}{d} + 3\frac{Aa^2b\sin(dx+c)}{d} + 3Ba^2bx + 3\frac{Ba^2bc}{d} + 3Aab^2x +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*A*a^3*cos(d*x+c)*sin(d*x+c)+1/2*a^3*A*x+1/2/d*A*a^3*c+a^3*B*sin(d*x+c
)/d+3/d*A*a^2*b*sin(d*x+c)+3*B*a^2*b*x+3/d*B*a^2*b*c+3*A*a*b^2*x+3/d*A*a*b^
```

```
2*c+3/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c
))+1/d*B*b^3*tan(d*x+c)
```

Maxima [A] time = 0.980297, size = 194, normalized size = 1.56

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^3 + 12\,(dx + c)Ba^2b + 12\,(dx + c)Aab^2 + 6\,Bab^2(\log(\sin(dx + c) + 1) - \log(\sin(dx +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*B*a^2*b + 12*(d*
x + c)*A*a*b^2 + 6*B*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x
 + c) + 12*A*a^2*b*sin(d*x + c) + 4*B*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.529613, size = 369, normalized size = 2.98

$$\frac{\left(Aa^3 + 6\,Ba^2b + 6\,Aab^2\right)dx\cos(dx + c) + \left(3\,Bab^2 + Ab^3\right)\cos(dx + c)\log(\sin(dx + c) + 1) - \left(3\,Bab^2 + Ab^3\right)\cos(dx}{2\,d\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*d*x*cos(d*x + c) + (3*B*a*b^2 + A*b^3)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (3*B*a*b^2 + A*b^3)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (A*a^3*cos(d*x + c)^2 + 2*B*b^3 + 2*(B*a^3 + 3*A*a^2*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29103, size = 316, normalized size = 2.55

$$-\frac{\frac{4\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \left(Aa^3 + 6\,Ba^2b + 6\,Aab^2\right)(dx + c) - 2\left(3\,Bab^2 + Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 2\left(3\,Bab^2 + A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (A*a^3 +
6*B*a^2*b + 6*A*a*b^2)*(d*x + c) - 2*(3*B*a*b^2 + A*b^3)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(3*B*a*b^2 + A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a
^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c) - 2*B*a^3*tan(1/2*
d*x + 1/2*c) - 6*A*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

3.299 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=145

$$\frac{a\left(2a^2A+9abB+8Ab^2\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)+\frac{a^2(3aB+5Ab)\sin(c+dx)\cos(c+dx)}{6d}$$

```
[Out] ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*x)/2 + (b^3*B*ArcTanh[Sin[c + d*
x]])/d + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Sin[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec
[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.347466, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4025, 4074, 4047, 8, 4045, 3770}

$$\frac{a\left(2a^2A+9abB+8Ab^2\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)+\frac{a^2(3aB+5Ab)\sin(c+dx)\cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*x)/2 + (b^3*B*ArcTanh[Sin[c + d*
x]])/d + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Sin[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec
[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2($$
$$= \frac{a^2(5Ab+3aB)\cos(c+dx)\sin(c+dx)}{6d} + \frac{aA\cos^2(c+dx)(a}{}$$
$$= \frac{a^2(5Ab+3aB)\cos(c+dx)\sin(c+dx)}{6d} + \frac{aA\cos^2(c+dx)(a}{}$$
$$= \frac{1}{2}\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)x + \frac{a\left(2a^2A+8Ab^2+9al\right.}{3d}$$
$$= \frac{1}{2}\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)x + \frac{b^3B\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A] time = 0.355049, size = 159, normalized size = 1.1

$$\frac{6(c+dx)\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)+9a\left(a^2A+4abB+4Ab^2\right)\sin(c+dx)+3a^2(aB+3Ab)\sin(2(c+dx))+a^3A\,\mathrm{s}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*(3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*(c + d*x) - 12*b^3*B*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*B*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*a*(a^2*A + 4*A*b^2 + 4*a*b*B)*Sin[c + d*x] + 3*a^2*(3*A*b + a*
B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.06, size = 207, normalized size = 1.4

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^3}{3d} + \frac{2Aa^3\sin(dx+c)}{3d} + \frac{Ba^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{Ba^3x}{2} + \frac{Ba^3c}{2d} + \frac{3Aa^2b\cos(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^3+2/3*a^3*A*sin(d*x+c)/d+1/2/d*B*a^3*cos(
d*x+c)*sin(d*x+c)+1/2*B*a^3*x+1/2/d*B*a^3*c+3/2/d*A*a^2*b*cos(d*x+c)*sin(d*
```

```
x+c)+3/2*A*a^2*b*x+3/2/d*A*a^2*b*c+3/d*B*a^2*b*sin(d*x+c)+3/d*A*a*b^2*sin(d
*x+c)+3*B*a*b^2*x+3/d*B*a*b^2*c+A*b^3*x+1/d*A*b^3*c+1/d*B*b^3*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 0.963334, size = 205, normalized size = 1.41

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{2}b-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b - 36*(d*x + c
)*B*a*b^2 - 12*(d*x + c)*A*b^3 - 6*B*b^3*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) - 36*B*a^2*b*sin(d*x + c) - 36*A*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.541763, size = 317, normalized size = 2.19

$$\frac{3\,Bb^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,Bb^{3}\log\left(-\sin\left(dx+c\right)+1\right)+3\left(Ba^{3}+3\,Aa^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)dx+\left(2\,Aa^{3}\cos\left(dx+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*B*b^3*log(sin(d*x + c) + 1) - 3*B*b^3*log(-sin(d*x + c) + 1) + 3*(B*
a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*d*x + (2*A*a^3*cos(d*x + c)^2 + 4*A*
a^3 + 18*B*a^2*b + 18*A*a*b^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2674, size = 424, normalized size = 2.92

$$\frac{6\,Bb^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Bb^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(Ba^{3}+3\,Aa^{2}b+6\,Bab^{2}+2\,Ab^{3}\right)\left(dx+c\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*B*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*(d*x + c) +
 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a
^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^
2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^2*b*tan(
1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*
x + 1/2*c) + 3*B*a^3*tan(1/2*d*x + 1/2*c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c)
+ 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.300 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{(6a^2Ab+2a^3B+9ab^2B+3Ab^3)\sin(c+dx)}{3d}+\frac{a(3a^2A+12abB+10Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^3A+12a\right.$$

```
[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*x)/8 + ((6*a^2*A*b + 3*A*b^3
 + 2*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*d) + (a*(3*a^2*A + 10*A*b^2 + 12*a
*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(3*A*b + 2*a*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(6*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(4*d)
```

Rubi [A] time = 0.423431, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4025, 4074, 4047, 2637, 4045, 8}

$$\frac{(6a^2Ab+2a^3B+9ab^2B+3Ab^3)\sin(c+dx)}{3d}+\frac{a(3a^2A+12abB+10Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^3A+12a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*x)/8 + ((6*a^2*A*b + 3*A*b^3
 + 2*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*d) + (a*(3*a^2*A + 10*A*b^2 + 12*a
*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(3*A*b + 2*a*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(6*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(4*d)
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
```

```
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3($$

$$= \frac{a^2(3Ab+2aB)\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{aA\cos^3(c+dx)(a}{}$$

$$= \frac{a^2(3Ab+2aB)\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{aA\cos^3(c+dx)(a}{}$$

$$= \frac{\left(6a^2Ab+3Ab^3+2a^3B+9ab^2B\right)\sin(c+dx)}{3d} + \frac{a\left(3a^2A+1\right.}{}$$

$$= \frac{1}{8}\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)x + \frac{\left(6a^2Ab+3Ab^3+\right.}{}$$

Mathematica [A] time = 0.407768, size = 140, normalized size = 0.78

$$\frac{12(c+dx)\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)+24a\left(a^2A+3abB+3Ab^2\right)\sin(2(c+dx))+24\left(9a^2Ab+3a^3B+12ab^2B\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*(c + d*x) + 24*(9*a^2*A*b
 + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x] + 24*a*(a^2*A + 3*A*b^2 + 3
*a*b*B)*Sin[2*(c + d*x)] + 8*a^2*(3*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^3*A*S
in[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.063, size = 180, normalized size = 1.

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+Aa^2b\left(2+(\cos(dx+c))^2\right)\sin(dx+c)+\frac{Ba^3\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a
^2*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*
a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*B*a^2*b*(1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+A*b^3*sin(d*x+c)+3*B*a*b^2*sin(d*x+c)+B*b^3*(d*x+c
))
```

Maxima [A] time = 0.966683, size = 231, normalized size = 1.29

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^3-96\left(\sin(dx+c)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*A*a^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b + 72*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*A*a*b^2 + 96*(d*x + c)*B*b^3 + 288*B*a*b^2*sin(d*x + c
) + 96*A*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.52533, size = 321, normalized size = 1.79

$$\frac{3\left(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3\right)dx+\left(6\,Aa^3\cos(dx+c)^3+16\,Ba^3+48\,Aa^2b+72\,Bab^2+24\,Ab^3+8\left(Ba^3+3\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*d*x + (6*A*a^3*cos(d*
x + c)^3 + 16*B*a^3 + 48*A*a^2*b + 72*B*a*b^2 + 24*A*b^3 + 8*(B*a^3 + 3*A*a
^2*b)*cos(d*x + c)^2 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22419, size = 724, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*(d*x + c) - 2*(15*A*a
^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*ta
n(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1
/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*
x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*
c)^5 - 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)
^5 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^5
 - 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*
a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*
b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 216*B*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^3*tan(1/2
*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b*tan(1/2*d*x + 1/
2*c) - 36*B*a^2*b*tan(1/2*d*x + 1/2*c) - 36*A*a*b^2*tan(1/2*d*x + 1/2*c) -
72*B*a*b^2*tan(1/2*d*x + 1/2*c) - 24*A*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

3.301 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=221

$$-\frac{a\left(4a^2A+15abB+12Ab^2\right)\sin^3(c+dx)}{15d}+\frac{\left(4a^3A+15a^2bB+14aAb^2+5b^3B\right)\sin(c+dx)}{5d}+\frac{\left(9a^2Ab+3a^3B+12ab\right.}{}$$

```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*x)/8 + ((4*a^3*A + 14*a*A*b^2
 + 15*a^2*b*B + 5*b^3*B)*Sin[c + d*x])/(5*d) + ((9*a^2*A*b + 4*A*b^3 + 3*a^
3*B + 12*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(7*A*b + 5*a*B)*C
os[c + d*x]^3*Sin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(5*d) - (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sin[c + d*x]^
3)/(15*d)
```

Rubi [A] time = 0.494193, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4025, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{a\left(4a^2A+15abB+12Ab^2\right)\sin^3(c+dx)}{15d}+\frac{\left(4a^3A+15a^2bB+14aAb^2+5b^3B\right)\sin(c+dx)}{5d}+\frac{\left(9a^2Ab+3a^3B+12ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*x)/8 + ((4*a^3*A + 14*a*A*b^2
 + 15*a^2*b*B + 5*b^3*B)*Sin[c + d*x])/(5*d) + ((9*a^2*A*b + 4*A*b^3 + 3*a^
3*B + 12*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(7*A*b + 5*a*B)*C
os[c + d*x]^3*Sin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(5*d) - (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sin[c + d*x]^
3)/(15*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
```

```
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4($$
$$= \frac{a^2(7Ab+5aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{aA\cos^4(c+dx)(a}{}$$
$$= \frac{a^2(7Ab+5aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{aA\cos^4(c+dx)(a}{}$$
$$= \frac{\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)\cos(c+dx)\sin(c+dx)}{8d} +$$
$$= \frac{1}{8}\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)x + \frac{\left(9a^2Ab+4Ab^3+3\right.}{}$$
$$= \frac{1}{8}\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)x + \frac{\left(4a^3A+14aAb^2+\right.}{}$$

Mathematica [A] time = 0.697969, size = 176, normalized size = 0.8

$$\frac{60(c+dx)\left(9a^2Ab+3a^3B+12ab^2B+4Ab^3\right)+10a\left(5a^2A+12abB+12Ab^2\right)\sin(3(c+dx))+60\left(5a^3A+18a^2bB+18a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (60*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*(c + d*x) + 60*(5*a^3*A +
18*a*A*b^2 + 18*a^2*b*B + 8*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3 +
a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*a*(5*a^2*A + 12*A*b^2 + 12*a*b*B)*
Sin[3*(c + d*x)] + 15*a^2*(3*A*b + a*B)*Sin[4*(c + d*x)] + 6*a^3*A*Sin[5*(c
 + d*x)])/(480*d)
```

Maple [A] time = 0.071, size = 227, normalized size = 1.

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^3*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^2*b*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^2*b*(2+cos(d*x+c)^2)*
sin(d*x+c)+A*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a*b^2*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+A*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b
^3*sin(d*x+c))
```

Maxima [A] time = 0.971718, size = 293, normalized size = 1.33

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 45*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*B*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a
*b^2 + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^2 + 120*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*b^3 + 480*B*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.548895, size = 423, normalized size = 1.91

$$\frac{15\left(3Ba^3+9Aa^2b+12Bab^2+4Ab^3\right)dx+\left(24Aa^3\cos(dx+c)^4+64Aa^3+240Ba^2b+240Aab^2+120Bb^3+30\left(B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*d*x + (24*A*a^3*cos(
d*x + c)^4 + 64*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 120*B*b^3 + 30*(B*a^3 +
 3*A*a^2*b)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x
+ c)^2 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25234, size = 907, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*(d*x + c) + 2*(120*A
*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b
*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*
tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(
1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^3*tan(1/2*d
*x + 1/2*c)^7 - 30*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*A*a^2*b*tan(1/2*d*x +
1/2*c)^7 + 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 960*A*a*b^2*tan(1/2*d*x + 1
/2*c)^7 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*A*b^3*tan(1/2*d*x + 1/2*
c)^7 + 480*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1/2*c)^5
+ 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5
 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 30
*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*B*a
^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 360*B*a*
b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*B*b^3*t
an(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d
*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x +
1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c
) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.302 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=334

$$\frac{\left(224a^2Ab^3+24a^4Ab+121a^3b^2B-4a^5B+128ab^4B+32Ab^5\right)\tan(c+dx)}{60bd}+\frac{\left(32a^3Ab+36a^2b^2B+8a^4B+24aAb^3\right.}{16d}$$

```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((24*a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*
a^3*b^2*B + 128*a*b^4*B)*Tan[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^
3 - 8*a^4*B + 178*a^2*b^2*B + 75*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(240*d)
+ ((24*a^2*A*b + 32*A*b^3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(120*b*d) + ((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Sec[c + d*x
])^3*Tan[c + d*x])/(120*b*d) + ((6*A*b - a*B)*(a + b*Sec[c + d*x])^4*Tan[c
+ d*x])/(30*b*d) + (B*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.711026, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(224a^2Ab^3+24a^4Ab+121a^3b^2B-4a^5B+128ab^4B+32Ab^5\right)\tan(c+dx)}{60bd}+\frac{\left(32a^3Ab+36a^2b^2B+8a^4B+24aAb^3\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((24*a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*
a^3*b^2*B + 128*a*b^4*B)*Tan[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^
3 - 8*a^4*B + 178*a^2*b^2*B + 75*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(240*d)
+ ((24*a^2*A*b + 32*A*b^3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(120*b*d) + ((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Sec[c + d*x
])^3*Tan[c + d*x])/(120*b*d) + ((6*A*b - a*B)*(a + b*Sec[c + d*x])^4*Tan[c
+ d*x])/(30*b*d) + (B*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c+dx)(a+b\sec(}{}$$

$$= \frac{(6Ab-aB)(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} + \frac{B(a+b\sec(c}{}$$

$$= \frac{\left(24aAb-4a^2B+25b^2B\right)(a+b\sec(c+dx))^3\tan(c+dx)}{120bd} +$$

$$= \frac{\left(24a^2Ab+32Ab^3-4a^3B+53ab^2B\right)(a+b\sec(c+dx))^2\tan}{120bd}$$

$$= \frac{\left(48a^3Ab+232aAb^3-8a^4B+178a^2b^2B+75b^4B\right)\sec(c+d}{240d}$$

$$= \frac{\left(48a^3Ab+232aAb^3-8a^4B+178a^2b^2B+75b^4B\right)\sec(c+d}{240d}$$

$$= \frac{\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B\right)\tanh^{-1}(\sin(c}{16d}$$

$$= \frac{\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B\right)\tanh^{-1}(\sin(c}{16d}$$

Mathematica [A] time = 2.86417, size = 244, normalized size = 0.73

$$\frac{15\left(32a^3Ab+36a^2b^2B+8a^4B+24aAb^3+5b^4B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(160b\left(3a^2Ab+2a^3B+4ab^2B+Ab\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(240*(a^4*A + 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B +
4*a*b^3*B) + 15*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B
)*Sec[c + d*x] + 10*b^2*(24*a*A*b + 36*a^2*B + 5*b^2*B)*Sec[c + d*x]^3 + 40
*b^4*B*Sec[c + d*x]^5 + 160*b*(3*a^2*A*b + A*b^3 + 2*a^3*B + 4*a*b^2*B)*Tan
[c + d*x]^2 + 48*b^3*(A*b + 4*a*B)*Tan[c + d*x]^4))/(240*d)
```

Maple [A] time = 0.047, size = 550, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/6/d*B*b^4*tan(d*x+c)*sec(d*x+c)^5+5/24/d*B*b^4*tan(d*x+c)*sec(d*x+c)^3+5/
16/d*B*b^4*sec(d*x+c)*tan(d*x+c)+4/15/d*A*b^4*tan(d*x+c)*sec(d*x+c)^2+32/15
/d*B*a*b^3*tan(d*x+c)+8/3/d*B*a^3*b*tan(d*x+c)+4/d*A*a^2*b^2*tan(d*x+c)+1/2
/d*B*a^4*sec(d*x+c)*tan(d*x+c)+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+9/4/d*
B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))
+5/16/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^4*tan(d*x+c)+1/2/d*B*a^4*ln
(sec(d*x+c)+tan(d*x+c))+8/15/d*A*b^4*tan(d*x+c)+2/d*A*a^3*b*sec(d*x+c)*tan(
d*x+c)+3/2/d*B*a^2*b^2*tan(d*x+c)*sec(d*x+c)^3+9/4/d*B*a^2*b^2*sec(d*x+c)*t
an(d*x+c)+1/d*A*a*b^3*tan(d*x+c)*sec(d*x+c)^3+3/2/d*A*a*b^3*sec(d*x+c)*tan(
d*x+c)+4/5/d*B*a*b^3*tan(d*x+c)*sec(d*x+c)^4+16/15/d*B*a*b^3*tan(d*x+c)*sec
(d*x+c)^2+4/3/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)^2+1/5/d*A*b^4*tan(d*x+c)*sec(
d*x+c)^4+2/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.996663, size = 640, normalized size = 1.92

$$\frac{640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3b+960\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2b^2+128\left(3\tan(dx+c)^5+10\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 960*(tan(d*x + c)^3
+ 3*tan(d*x + c))*A*a^2*b^2 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 1
5*tan(d*x + c))*B*a*b^3 + 32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan
(d*x + c))*A*b^4 - 5*B*b^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*B*a^2*b^2*(2*(3*s
in(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3
*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a*b^3*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^4*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*
A*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 480*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.702187, size = 797, normalized size = 2.39

$$\frac{15\left(8Ba^4+32Aa^3b+36Ba^2b^2+24Aab^3+5Bb^4\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left(8Ba^4+32Aa^3b+36Ba^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2
+ 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(15*A
*a^4 + 40*B*a^3*b + 60*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^5 + 4
0*B*b^4 + 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*c
os(d*x + c)^4 + 32*(10*B*a^3*b + 15*A*a^2*b^2 + 8*B*a*b^3 + 2*A*b^4)*cos(d*
x + c)^3 + 10*(36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^2 + 48*(4*
B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(A+B\sec(c+dx))\,(a+b\sec(c+dx))^4\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

Giac [B] time = 1.28367, size = 1601, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 +
24*A*a*b^3 + 5*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(240*A*a^4*tan
(1/2*d*x + 1/2*c)^11 - 120*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 480*A*a^3*b*tan(
1/2*d*x + 1/2*c)^11 + 960*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 1440*A*a^2*b^2*
tan(1/2*d*x + 1/2*c)^11 - 900*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 600*A*a*b
^3*tan(1/2*d*x + 1/2*c)^11 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 240*A*b^
4*tan(1/2*d*x + 1/2*c)^11 - 165*B*b^4*tan(1/2*d*x + 1/2*c)^11 - 1200*A*a^4*
tan(1/2*d*x + 1/2*c)^9 + 360*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 1440*A*a^3*b*ta
n(1/2*d*x + 1/2*c)^9 - 3520*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 5280*A*a^2*b^2
*tan(1/2*d*x + 1/2*c)^9 + 1260*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 840*A*a*b
^3*tan(1/2*d*x + 1/2*c)^9 - 2240*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 560*A*b^4
*tan(1/2*d*x + 1/2*c)^9 - 25*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 2400*A*a^4*tan(
1/2*d*x + 1/2*c)^7 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 960*A*a^3*b*tan(1/2
```

```
*d*x + 1/2*c)^7 + 5760*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8640*A*a^2*b^2*tan(
1/2*d*x + 1/2*c)^7 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 240*A*a*b^3*tan
(1/2*d*x + 1/2*c)^7 + 4992*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1248*A*b^4*tan(
1/2*d*x + 1/2*c)^7 - 450*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*A*a^4*tan(1/2*
d*x + 1/2*c)^5 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 960*A*a^3*b*tan(1/2*d*x
 + 1/2*c)^5 - 5760*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 8640*A*a^2*b^2*tan(1/2*
d*x + 1/2*c)^5 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 240*A*a*b^3*tan(1/2
*d*x + 1/2*c)^5 - 4992*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 1248*A*b^4*tan(1/2*
d*x + 1/2*c)^5 - 450*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 360*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1440*A*a^3*b*tan(1/2*d*x +
1/2*c)^3 + 3520*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 5280*A*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 1260*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 840*A*a*b^3*tan(1/2*d
*x + 1/2*c)^3 + 2240*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 560*A*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 25*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 240*A*a^4*tan(1/2*d*x + 1/2
*c) - 120*B*a^4*tan(1/2*d*x + 1/2*c) - 480*A*a^3*b*tan(1/2*d*x + 1/2*c) - 9
60*B*a^3*b*tan(1/2*d*x + 1/2*c) - 1440*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 900
*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 600*A*a*b^3*tan(1/2*d*x + 1/2*c) - 960*B*
a*b^3*tan(1/2*d*x + 1/2*c) - 240*A*b^4*tan(1/2*d*x + 1/2*c) - 165*B*b^4*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.303 $\int \sec(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=250

$$\frac{\left(95a^3Ab+112a^2b^2B+12a^4B+80aAb^3+16b^4B\right)\tan(c+dx)}{30d}+\frac{\left(24a^2Ab^2+8a^4A+16a^3bB+12ab^3B+3Ab^4\right)\tanh^{-1}(}{8d}$$

```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((95*a^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16
*b^4*B)*Tan[c + d*x])/(30*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*
a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^
2*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*
Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*
x])/(5*d)
```

Rubi [A] time = 0.519875, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(95a^3Ab+112a^2b^2B+12a^4B+80aAb^3+16b^4B\right)\tan(c+dx)}{30d}+\frac{\left(24a^2Ab^2+8a^4A+16a^3bB+12ab^3B+3Ab^4\right)\tanh^{-1}(}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((95*a^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16
*b^4*B)*Tan[c + d*x])/(30*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*
a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^
2*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*
Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*
x])/(5*d)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{1}{5}\int \sec(c+dx)(a+b$$

$$= \frac{(5Ab+4aB)(a+b\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{B(a+b\sec$$

$$= \frac{\left(35aAb+12a^2B+16b^2B\right)(a+b\sec(c+dx))^2\tan(c+dx)}{60d}$$

$$= \frac{b\left(130a^2Ab+45Ab^3+24a^3B+116ab^2B\right)\sec(c+dx)\tan(}{120d}$$

$$= \frac{b\left(130a^2Ab+45Ab^3+24a^3B+116ab^2B\right)\sec(c+dx)\tan(}{120d}$$

$$= \frac{\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)\tanh^{-1}(\sin}{8d}$$

$$= \frac{\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)\tanh^{-1}(\sin}{8d}$$

Mathematica [A] time = 3.91443, size = 198, normalized size = 0.79

$$15\left(24a^2Ab^2+8a^4A+16a^3bB+12ab^3B+3Ab^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(80b^2\left(3a^2B+2aAb+b^2B\right)\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(120*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^2*
B + b^4*B) + 15*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 12*a*b^2*B)*Sec[c + d*
x] + 30*b^3*(A*b + 4*a*B)*Sec[c + d*x]^3 + 80*b^2*(2*a*A*b + 3*a^2*B + b^2*
B)*Tan[c + d*x]^2 + 24*b^4*B*Tan[c + d*x]^4))/(120*d)
```

Maple [A] time = 0.049, size = 431, normalized size = 1.7

$$\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba^4\tan(dx+c)}{d}+4\frac{Aa^3b\tan(dx+c)}{d}+2\frac{Ba^3b\sec(dx+c)\tan(dx+c)}{d}+2\frac{Ba^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^4*tan(d*x+c)+4/d*A*a^3*b*tan(d*
x+c)+2/d*B*a^3*b*sec(d*x+c)*tan(d*x+c)+2/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c)
)+3/d*A*a^2*b^2*sec(d*x+c)*tan(d*x+c)+3/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c
))+4/d*B*a^2*b^2*tan(d*x+c)+2/d*B*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+8/3/d*A*a
*b^3*tan(d*x+c)+4/3/d*A*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/d*B*a*b^3*tan(d*x+c
)*sec(d*x+c)^3+3/2/d*B*a*b^3*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a*b^3*ln(sec(d*x
+c)+tan(d*x+c))+1/4/d*A*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*b^4*sec(d*x+c)*
tan(d*x+c)+3/8/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*B*b^4*tan(d*x+c)+1/
5/d*B*b^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*B*b^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.983032, size = 512, normalized size = 2.05

$$\frac{480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2b^2+320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab^3+16\left(3\tan(dx+c)^5+10\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b^2 + 320*(tan(d*x + c)^
3 + 3*tan(d*x + c))*A*a*b^3 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15
*tan(d*x + c))*B*b^4 - 60*B*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(s
in(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin
(d*x + c) - 1)) - 15*A*b^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 240*B*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
 c) + 1) + log(sin(d*x + c) - 1)) - 360*A*a^2*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^4*lo
g(sec(d*x + c) + tan(d*x + c)) + 240*B*a^4*tan(d*x + c) + 960*A*a^3*b*tan(d
*x + c))/d
```

Fricas [A] time = 0.593967, size = 687, normalized size = 2.75

$$\frac{15\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*cos(
d*x + c)^5*log(sin(d*x + c) + 1) - 15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
```

```
+ 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*B*b^4
 + 8*(15*B*a^4 + 60*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 8*B*b^4)*cos(d*x
+ c)^4 + 15*(16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c)
^3 + 16*(15*B*a^2*b^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 30*(4*B*a*b^
3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^4 \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

Giac [B] time = 1.27717, size = 1148, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
12*B*a*b^3 + 3*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*B*a^4*tan
(1/2*d*x + 1/2*c)^9 + 480*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 240*B*a^3*b*tan(
1/2*d*x + 1/2*c)^9 - 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 720*B*a^2*b^2*t
an(1/2*d*x + 1/2*c)^9 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 300*B*a*b^3*ta
n(1/2*d*x + 1/2*c)^9 - 75*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^4*tan(1/2*
d*x + 1/2*c)^9 - 480*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 1920*A*a^3*b*tan(1/2*d*
x + 1/2*c)^7 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 720*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 1920*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 1280*A*a*b^3*tan(1/
2*d*x + 1/2*c)^7 + 120*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 30*A*b^4*tan(1/2*d*
x + 1/2*c)^7 - 160*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^4*tan(1/2*d*x + 1
/2*c)^5 + 2880*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 1600*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 464*B*b^4*tan(1/2*d*x +
1/2*c)^5 - 480*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 1920*A*a^3*b*tan(1/2*d*x + 1/
2*c)^3 - 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 720*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^3 - 1920*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 1280*A*a*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 120*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*A*b^4*tan(1/2*d*x + 1/
2*c)^3 - 160*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^4*tan(1/2*d*x + 1/2*c)
+ 480*A*a^3*b*tan(1/2*d*x + 1/2*c) + 240*B*a^3*b*tan(1/2*d*x + 1/2*c) + 360
*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*
A*a*b^3*tan(1/2*d*x + 1/2*c) + 300*B*a*b^3*tan(1/2*d*x + 1/2*c) + 75*A*b^4*
tan(1/2*d*x + 1/2*c) + 120*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

3.304 $\int (a+b\sec(c+dx))^4 (A+B\sec(c+dx))\,dx$

Optimal. Leaf size=200

$$\frac{b\left(34a^2Ab+19a^3B+16ab^2B+4Ab^3\right)\tan(c+dx)}{6d}+\frac{\left(32a^3Ab+24a^2b^2B+8a^4B+16aAb^3+3b^4B\right)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] a^4*A*x + ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Tan[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*
x])/(12*d) + (b*B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.327352, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3918, 4056, 4048, 3770, 3767, 8}

$$\frac{b\left(34a^2Ab+19a^3B+16ab^2B+4Ab^3\right)\tan(c+dx)}{6d}+\frac{\left(32a^3Ab+24a^2b^2B+8a^4B+16aAb^3+3b^4B\right)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] a^4*A*x + ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Tan[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*
x])/(12*d) + (b*B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```


Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{bB(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\sec(c+dx))^2\left(4a^2A+\right.$$
$$= \frac{b(4Ab+7aB)(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{bB(a+b\sec(c+dx))}{4d}$$
$$= \frac{b^2\left(32aAb+26a^2B+9b^2B\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{b(4Ab+7aB)(}{}$$
$$= a^4Ax+\frac{b^2\left(32aAb+26a^2B+9b^2B\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{b(4Ab}{}$$
$$= a^4Ax+\frac{\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)\tanh^{-1}(\sin(c+}{8d}$$
$$= a^4Ax+\frac{\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)\tanh^{-1}(\sin(c+}{8d}$$

Mathematica [A] time = 1.02719, size = 160, normalized size = 0.8

$$\frac{3\left(32a^3Ab+24a^2b^2B+8a^4B+16aAb^3+3b^4B\right)\tanh^{-1}(\sin(c+dx))+3b\tan(c+dx)\left(b\left(24a^2B+16aAb+3b^2B\right)\text{se}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (24*a^4*A*d*x + 3*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4
*B)*ArcTanh[Sin[c + d*x]] + 3*b*(8*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B
) + b*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sec[c + d*x] + 2*b^3*B*Sec[c + d*x]^3
)*Tan[c + d*x] + 8*b^3*(A*b + 4*a*B)*Tan[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.049, size = 338, normalized size = 1.7

$$a^4Ax+\frac{Aa^4c}{d}+\frac{Ba^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\frac{Aa^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\frac{Ba^3b\tan(dx+c)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] a^4*A*x+1/d*A*a^4*c+1/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^3*b*ln(sec(
d*x+c)+tan(d*x+c))+4/d*B*a^3*b*tan(d*x+c)+6/d*A*a^2*b^2*tan(d*x+c)+3/d*B*a^
2*b^2*sec(d*x+c)*tan(d*x+c)+3/d*B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a
*b^3*sec(d*x+c)*tan(d*x+c)+2/d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*B*a*
b^3*tan(d*x+c)+4/3/d*B*a*b^3*tan(d*x+c)*sec(d*x+c)^2+2/3/d*A*b^4*tan(d*x+c)
+1/3/d*A*b^4*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*b^4*tan(d*x+c)*sec(d*x+c)^3+3/
8/d*B*b^4*sec(d*x+c)*tan(d*x+c)+3/8/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.980455, size = 409, normalized size = 2.04

$$48\,(dx+c)Aa^4 + 64\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Bab^3 + 16\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Ab^4 - 3\,Bb^4\left(\frac{2\left(3\,\sin(dx+c)^3 \cdots\right.}{\sin(dx+c)^4-2\,\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(48*(d*x + c)*A*a^4 + 64*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b^3 + 1
6*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^4 - 3*B*b^4*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 72*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 48*A*a*b^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 48*B*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*A*a^3*b*log(sec(
d*x + c) + tan(d*x + c)) + 192*B*a^3*b*tan(d*x + c) + 288*A*a^2*b^2*tan(d*x
 + c))/d
```

Fricas [A] time = 0.604623, size = 603, normalized size = 3.02

$$48\,Aa^4dx\cos\left(dx+c\right)^4 + 3\left(8\,Ba^4 + 32\,Aa^3b + 24\,Ba^2b^2 + 16\,Aab^3 + 3\,Bb^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right) - 3\left(8\,Ba^4 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*A*a^4*d*x*cos(d*x + c)^4 + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2
+ 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*B*a^4 +
 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4*log(-sin(
d*x + c) + 1) + 2*(6*B*b^4 + 16*(6*B*a^3*b + 9*A*a^2*b^2 + 4*B*a*b^3 + A*b^
4)*cos(d*x + c)^3 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^2
+ 8*(4*B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + B\sec\left(c+dx\right)\right)\left(a + b\sec\left(c+dx\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.31484, size = 857, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*A*a^4 + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*
b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^4 + 32*A*a^3*b
 + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
- 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*b^2*tan(1/2*d*x + 1/2*c)
^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^
7 + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 1
5*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 432*A
*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*
A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 160*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*
b^4*tan(1/2*d*x + 1/2*c)^5 - 9*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*B*a^3*b*t
an(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*B*a^2*b^2
*tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 160*B*a*b^3*t
an(1/2*d*x + 1/2*c)^3 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*B*b^4*tan(1/2*d
*x + 1/2*c)^3 - 96*B*a^3*b*tan(1/2*d*x + 1/2*c) - 144*A*a^2*b^2*tan(1/2*d*x
 + 1/2*c) - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 48*A*a*b^3*tan(1/2*d*x + 1/
2*c) - 96*B*a*b^3*tan(1/2*d*x + 1/2*c) - 24*A*b^4*tan(1/2*d*x + 1/2*c) - 15
*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.305 $\int \cos(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=195

$$-\frac{b\left(6a^3A-17a^2bB-12aAb^2-2b^3B\right)\tan(c+dx)}{3d}+\frac{b\left(12a^2Ab+8a^3B+4ab^2B+Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(6a^2A\right.}{}$$

```
[Out] a^3*(4*A*b + a*B)*x + (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) + (a*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d - (b*(6
*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Tan[c + d*x])/(3*d) - (b^2*(6*a
^2*A - 3*A*b^2 - 8*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(3*a*A - b*
B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.368302, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4025, 4056, 4048, 3770, 3767, 8}

$$-\frac{b\left(6a^3A-17a^2bB-12aAb^2-2b^3B\right)\tan(c+dx)}{3d}+\frac{b\left(12a^2Ab+8a^3B+4ab^2B+Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(6a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] a^3*(4*A*b + a*B)*x + (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) + (a*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d - (b*(6
*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Tan[c + d*x])/(3*d) - (b^2*(6*a
^2*A - 3*A*b^2 - 8*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(3*a*A - b*
B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
```

```
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \int (a+b\sec(c+dx))$$

$$= \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b(3aA-bB)(a+b\,\mathrm{se}}{}$$

$$= \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b^2\left(6a^2A-3Ab^2-8\right.}{}$$

$$= a^3(4Ab+aB)x + \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b^2\left(\right.}{}$$

$$= a^3(4Ab+aB)x + \frac{b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)\tanh}{2d}$$

$$= a^3(4Ab+aB)x + \frac{b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)\tanh}{2d}$$

Mathematica [B] time = 6.28756, size = 1051, normalized size = 5.39

$$\frac{\left(-Ab^4-4aBb^3-12a^2Ab^2-8a^3Bb\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,\mathrm{cc}}{2d(b+a\cos(c+dx))^4(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(4*A*b + a*B)*(c + d*x)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*S
ec[c + d*x]))/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((-12*a^2*A
*b^2 - A*b^4 - 8*a^3*b*B - 4*a*b^3*B)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(2*d*(b + a
*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((12*a^2*A*b^2 + A*b^4 + 8*a^3*b*B
 + 4*a*b^3*B)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a +
b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(2*d*(b + a*Cos[c + d*x])^4*(B + A*
Cos[c + d*x])) + ((3*A*b^4 + 12*a*b^3*B + b^4*B)*Cos[c + d*x]^5*(a + b*Sec[
c + d*x])^4*(A + B*Sec[c + d*x]))/(12*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c
 + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (b^4*B*Cos[c + d*x]^5*(
a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sin[(c + d*x)/2])/(6*d*(b + a*Co
```

```
s[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
 + (b^4*B*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sin[(c
 + d*x)/2])/(6*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^3) + ((-3*A*b^4 - 12*a*b^3*B - b^4*B)*Cos[c + d*x]^
5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(12*d*(b + a*Cos[c + d*x])^4
*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*Cos[c +
 d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(6*a*A*b^3*Sin[(c + d*x
)/2] + 9*a^2*b^2*B*Sin[(c + d*x)/2] + b^4*B*Sin[(c + d*x)/2]))/(3*d*(b + a*
Cos[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + (2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(6*a*A*b^3
*Sin[(c + d*x)/2] + 9*a^2*b^2*B*Sin[(c + d*x)/2] + b^4*B*Sin[(c + d*x)/2]))
/(3*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) + (a^4*A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c +
 d*x])*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))
```

Maple [A] time = 0.063, size = 262, normalized size = 1.3

$$\frac{Aa^4\sin(dx+c)}{d}+Ba^4x+\frac{Ba^4c}{d}+4\,Aa^3bx+4\,\frac{Aa^3bc}{d}+4\,\frac{Ba^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+6\,\frac{Aa^2b^2\ln(\sec(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+B*a^4*x+1/d*B*a^4*c+4*A*a^3*b*x+4/d*A*a^3*b*c+4/d*B*a^
3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+6/d*B
*a^2*b^2*tan(d*x+c)+4/d*A*a*b^3*tan(d*x+c)+2/d*B*a*b^3*sec(d*x+c)*tan(d*x+c
)+2/d*B*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b^4*sec(d*x+c)*tan(d*x+c)+1
/2/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*b^4*tan(d*x+c)+1/3/d*B*b^4*tan
(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.989914, size = 331, normalized size = 1.7

$$12\,(dx+c)Ba^4+48\,(dx+c)Aa^3b+4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Bb^4-12\,Bab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^4 + 48*(d*x + c)*A*a^3*b + 4*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*b^4 - 12*B*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b^4*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*a^3*b*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*A*a^2*b^2*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^4*sin(d*x + c) + 72*B*a^2*b^2*
tan(d*x + c) + 48*A*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.573885, size = 524, normalized size = 2.69

$$12\left(Ba^4+4\,Aa^3b\right)dx\cos(dx+c)^3+3\left(8\,Ba^3b+12\,Aa^2b^2+4\,Bab^3+Ab^4\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(8\,Ba^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(12*(B*a^4 + 4*A*a^3*b)*d*x*cos(d*x + c)^3 + 3*(8*B*a^3*b + 12*A*a^2*b
^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(8*B*a^3*b
 + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
+ 2*(6*A*a^4*cos(d*x + c)^3 + 2*B*b^4 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 + B*b^4)
*cos(d*x + c)^2 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2449, size = 522, normalized size = 2.68

$$\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 6\left(Ba^4 + 4\,Aa^3b\right)(dx+c) + 3\left(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(B*a^4
+ 4*A*a^3*b)*(d*x + c) + 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*l
og(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3
 + A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 24*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1
/2*c)^5 - 3*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 -
 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 -
4*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*A*a
*b^3*tan(1/2*d*x + 1/2*c) + 12*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3*A*b^4*tan(1
/2*d*x + 1/2*c) + 6*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^3)/d
```

3.306 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=209

$$-\frac{b\left(13a^2Ab+4a^3B-8ab^2B-2Ab^3\right)\tan(c+dx)}{2d}+\frac{b^2\left(12a^2B+8aAb+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(2a^2B+6aAb-\right.}{}$$

```
[Out] (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*x)/2 + (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*
ArcTanh[Sin[c + d*x]])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2*B)*Tan[c + d*x])/(2*d)
 - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.462845, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4025, 4094, 4048, 3770, 3767, 8}

$$-\frac{b\left(13a^2Ab+4a^3B-8ab^2B-2Ab^3\right)\tan(c+dx)}{2d}+\frac{b^2\left(12a^2B+8aAb+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(2a^2B+6aAb-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*x)/2 + (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*
ArcTanh[Sin[c + d*x]])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2*B)*Tan[c + d*x])/(2*d)
 - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
```

```
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{2d} - \frac{1}{2}\int \cos$$
$$= \frac{a(5Ab+2aB)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{aA\cos(c}{}$$
$$= \frac{a(5Ab+2aB)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{aA\cos(c}{}$$
$$= \frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x + \frac{a(5Ab+2aB)(a+b\sec(c}{2d}$$
$$= \frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x + \frac{b^2\left(8aAb+12a^2B+b^2B\right)}{2d}$$
$$= \frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x + \frac{b^2\left(8aAb+12a^2B+b^2B\right)}{2d}$$

Mathematica [A] time = 1.89767, size = 310, normalized size = 1.48

$$2a^2(c+dx)\left(a^2A+8abB+12Ab^2\right) - 2b^2\left(12a^2B+8aAb+b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b^2\left(12a^2B\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*(c + d*x) - 2*b^2*(8*a*A*b + 12*a^2*B +
 b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*b^2*(8*a*A*b + 12*a^2*
B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^4*B)/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2 + (4*b^3*(A*b + 4*a*B)*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) - (b^4*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (4*b^3*(A*b + 4*a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]) + 4*a^3*(4*A*b + a*B)*Sin[c + d*x] + a^4*A*Sin[2*(c + d*x)])/(4*d
)
```

Maple [A] time = 0.064, size = 236, normalized size = 1.1

$$\frac{Aa^4\sin(dx+c)\cos(dx+c)}{2d}+\frac{a^4Ax}{2}+\frac{Aa^4c}{2d}+\frac{Ba^4\sin(dx+c)}{d}+4\,\frac{Aa^3b\sin(dx+c)}{d}+4\,Ba^3bx+4\,\frac{Ba^3bc}{d}+6\,Aa^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/2/d*A*a^4*sin(d*x+c)*cos(d*x+c)+1/2*a^4*A*x+1/2/d*A*a^4*c+1/d*B*a^4*sin(d
*x+c)+4/d*A*a^3*b*sin(d*x+c)+4*B*a^3*b*x+4/d*B*a^3*b*c+6*A*a^2*b^2*x+6/d*A*
a^2*b^2*c+6/d*B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a*b^3*ln(sec(d*x+c)
+tan(d*x+c))+4/d*B*a*b^3*tan(d*x+c)+1/d*A*b^4*tan(d*x+c)+1/2/d*B*b^4*sec(d*
x+c)*tan(d*x+c)+1/2/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.981407, size = 282, normalized size = 1.35

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+16\,(dx+c)Ba^3b+24\,(dx+c)Aa^2b^2-Bb^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 16*(d*x + c)*B*a^3*b + 24*(d*
x + c)*A*a^2*b^2 - B*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 12*B*a^2*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 8*A*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 4*B*a^4*sin(d*x + c) + 16*A*a^3*b*sin(d*x + c) + 16*B*a*b^3*tan(
d*x + c) + 4*A*b^4*tan(d*x + c))/d
```

Fricas [A] time = 0.60597, size = 479, normalized size = 2.29

$$\frac{2\left(Aa^4+8\,Ba^3b+12\,Aa^2b^2\right)dx\cos(dx+c)^2+\left(12\,Ba^2b^2+8\,Aab^3+Bb^4\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-\left(12\,Ba^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(A*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*d*x*cos(d*x + c)^2 + (12*B*a^2*b^
2 + 8*A*a*b^3 + B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (12*B*a^2*b^2
 + 8*A*a*b^3 + B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4*cos(
d*x + c)^3 + B*b^4 + 2*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^2 + 2*(4*B*a*b^3 +
A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30352, size = 713, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((A*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*(d*x + c) + (12*B*a^2*b^2 + 8*A*a*b
^3 + B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*B*a^2*b^2 + 8*A*a*b^3
+ B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^4*tan(1/2*d*x + 1/2*c)
^7 - 2*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8*
B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^7 - B*b^4*tan
(1/2*d*x + 1/2*c)^7 - 3*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 2*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 + 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 8*B*a*b^3*tan(1/2*d*x + 1/2
*c)^5 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 3
*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^4*tan(1/2
*d*x + 1/2*c)^3 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^3 - A*a^4*tan(1/2*d*x + 1/2*
c) - 2*B*a^4*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b*tan(1/2*d*x + 1/2*c) - 8*B*a*
b^3*tan(1/2*d*x + 1/2*c) - 2*A*b^4*tan(1/2*d*x + 1/2*c) - B*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.307 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=198

$$\frac{a^2(2a^2A+9abB+9Ab^2)\sin(c+dx)}{3d}-\frac{b^2(3a^2B+8aAb-6b^2B)\tan(c+dx)}{6d}+\frac{1}{2}ax(4a^2Ab+a^3B+12ab^2B+8Ab^3)+$$

```
[Out] (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*x)/2 + (b^3*(A*b + 4*a*B)*Arc
Tanh[Sin[c + d*x]])/d + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Sin[c + d*x])/(3
*d) + (a*(2*A*b + a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2
*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) - (b^2
*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.590815, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4025, 4094, 4076, 4047, 8, 4045, 3770}

$$\frac{a^2(2a^2A+9abB+9Ab^2)\sin(c+dx)}{3d}-\frac{b^2(3a^2B+8aAb-6b^2B)\tan(c+dx)}{6d}+\frac{1}{2}ax(4a^2Ab+a^3B+12ab^2B+8Ab^3)+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*x)/2 + (b^3*(A*b + 4*a*B)*Arc
Tanh[Sin[c + d*x]])/d + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Sin[c + d*x])/(3
*d) + (a*(2*A*b + a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2
*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) - (b^2
*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Tan[c + d*x])/(6*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
```

```
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{3d} - \frac{1}{3}\int \mathrm{co}$$

$$= \frac{a(2Ab+aB)\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$

$$= \frac{a(2Ab+aB)\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$

$$= \frac{a(2Ab+aB)\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$

$$= \frac{1}{2}a\left(4a^2Ab+8Ab^3+a^3B+12ab^2B\right)x + \frac{a^2\left(2a^2A+9Ab\right.}{}$$

$$= \frac{1}{2}a\left(4a^2Ab+8Ab^3+a^3B+12ab^2B\right)x + \frac{b^3(Ab+4aB)\mathrm{ta}}{}$$

Mathematica [A] time = 1.07503, size = 257, normalized size = 1.3

$$\frac{6a(c+dx)\left(4a^2Ab+a^3B+12ab^2B+8Ab^3\right)+3a^2\left(3a^2A+16abB+24Ab^2\right)\sin(c+dx)+3a^3(aB+4Ab)\sin(2(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*(c + d*x) - 12*b^3*(A*b + 4
*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*(A*b + 4*a*B)*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*b^4*B*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) + (12*b^4*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 3*a^2*(3*a^2*A + 24*A*b^2 + 16*a*b*B)*Sin[c + d*x]
+ 3*a^3*(4*A*b + a*B)*Sin[2*(c + d*x)] + a^4*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.063, size = 255, normalized size = 1.3

$$\frac{A\sin(dx+c)(\cos(dx+c))^2 a^4}{3d} + \frac{2Aa^4\sin(dx+c)}{3d} + \frac{Ba^4\sin(dx+c)\cos(dx+c)}{2d} + \frac{Ba^4x}{2} + \frac{Ba^4c}{2d} + 2\frac{Aa^3b\sin(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3/d*A*a^4*sin(d*x+c)+1/2/d*B*a^4*sin(
d*x+c)*cos(d*x+c)+1/2*B*a^4*x+1/2/d*B*a^4*c+2/d*A*a^3*b*sin(d*x+c)*cos(d*x+
c)+2*A*a^3*b*x+2/d*A*a^3*b*c+4/d*B*a^3*b*sin(d*x+c)+6/d*A*a^2*b^2*sin(d*x+c
)+6*B*a^2*b^2*x+6/d*B*a^2*b^2*c+4*A*a*b^3*x+4/d*A*a*b^3*c+4/d*B*a*b^3*ln(se
c(d*x+c)+tan(d*x+c))+1/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b^4*tan(d*x+
c)
```

Maxima [A] time = 0.989478, size = 266, normalized size = 1.34

$$-\frac{4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Aa^4 - 3(2dx+2c+\sin(2dx+2c))Ba^4 - 12(2dx+2c+\sin(2dx+2c))Aa^3b - 72(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^4 - 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3*b - 72*(d*x +
c)*B*a^2*b^2 - 48*(d*x + c)*A*a*b^3 - 24*B*a*b^3*(log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) - 6*A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
 1)) - 48*B*a^3*b*sin(d*x + c) - 72*A*a^2*b^2*sin(d*x + c) - 12*B*b^4*tan(d
*x + c))/d
```

Fricas [A] time = 0.593142, size = 471, normalized size = 2.38

$$\frac{3\left(Ba^4 + 4Aa^3b + 12Ba^2b^2 + 8Aab^3\right)dx\cos(dx+c) + 3\left(4Bab^3 + Ab^4\right)\cos(dx+c)\log(\sin(dx+c)+1) - 3\left(4Bab^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*d*x*cos(d*x + c) + 3*
(4*B*a*b^3 + A*b^4)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(4*B*a*b^3 + A*b
```

```
^4)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*A*a^4*cos(d*x + c)^3 + 6*B*b^4
 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^2 + 4*(A*a^4 + 6*B*a^3*b + 9*A*a^2*b^
2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28818, size = 501, normalized size = 2.53

$$-\frac{\frac{12\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\left(Ba^4 + 4\,Aa^3b + 12\,Ba^2b^2 + 8\,Aab^3\right)(dx+c) - 6\left(4\,Bab^3 + Ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*B*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(B*a^4
 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*(d*x + c) - 6*(4*B*a*b^3 + A*b^4)*
log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(4*B*a*b^3 + A*b^4)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(6*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*tan(1/2*d*
x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^3*b*tan(1/2*d*x +
 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^4*tan(1/2*d*x + 1/2
*c)^3 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan(1/2*d*x + 1/2*c) + 12*A*a
^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*tan(1/2*d*x + 1/2*c) + 36*A*a^2*b^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.308 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=216

$$\frac{a\left(16a^2Ab+4a^3B+34ab^2B+19Ab^3\right)\sin(c+dx)}{6d}+\frac{a^2\left(9a^2A+32abB+26Ab^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(24a^2Ab^2\right.$$

```
[Out] ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*x)/8 + (b^4*B
*ArcTanh[Sin[c + d*x]])/d + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Sin[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[
c + d*x])/(4*d)
```

Rubi [A] time = 0.609493, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4025, 4094, 4074, 4047, 8, 4045, 3770}

$$\frac{a\left(16a^2Ab+4a^3B+34ab^2B+19Ab^3\right)\sin(c+dx)}{6d}+\frac{a^2\left(9a^2A+32abB+26Ab^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(24a^2Ab^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*x)/8 + (b^4*B
*ArcTanh[Sin[c + d*x]])/d + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Sin[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[
c + d*x])/(4*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{4d} - \frac{1}{4}\int \mathrm{co}$$

$$= \frac{a(7Ab+4aB)\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx}{12d}$$

$$= \frac{a^2\left(9a^2A+26Ab^2+32abB\right)\cos(c+dx)\sin(c+dx)}{24d} + \frac{a(}{}$$

$$= \frac{a^2\left(9a^2A+26Ab^2+32abB\right)\cos(c+dx)\sin(c+dx)}{24d} + \frac{a(}{}$$

$$= \frac{1}{8}\left(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B\right)x + \frac{a\,(1}{}$$

$$= \frac{1}{8}\left(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B\right)x + \frac{b^4B}{}$$

Mathematica [A] time = 0.601376, size = 210, normalized size = 0.97

$$\frac{12(c+dx)\left(24a^2Ab^2+3a^4A+16a^3bB+32ab^3B+8Ab^4\right)+24a^2\left(a^2A+4abB+6Ab^2\right)\sin(2(c+dx))+24a\left(12a^2Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*(c + d*x)
- 96*b^4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*b^4*B*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 24*a*(12*a^2*A*b + 16*A*b^3 + 3*a^3*B + 24*a
*b^2*B)*Sin[c + d*x] + 24*a^2*(a^2*A + 6*A*b^2 + 4*a*b*B)*Sin[2*(c + d*x)]
+ 8*a^3*(4*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^4*A*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.071, size = 319, normalized size = 1.5

$$\frac{Aa^4 \sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{3Aa^4 \sin(dx+c)\cos(dx+c)}{8d} + \frac{3a^4Ax}{8} + \frac{3Aa^4c}{8d} + \frac{B\sin(dx+c)(\cos(dx+c))^2 a^4}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+3/8/d*A*a^4*sin(d*x+c)*cos(d*x+c)+3/8*a
^4*A*x+3/8/d*A*a^4*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3/d*B*a^4*sin(d*
x+c)+4/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^3*b+8/3/d*A*a^3*b*sin(d*x+c)+2/d*B*a
^3*b*sin(d*x+c)*cos(d*x+c)+2*B*a^3*b*x+2/d*B*a^3*b*c+3/d*A*a^2*b^2*sin(d*x+
c)*cos(d*x+c)+3*A*a^2*b^2*x+3/d*A*a^2*b^2*c+6/d*B*a^2*b^2*sin(d*x+c)+4/d*A*
a*b^3*sin(d*x+c)+4*B*a*b^3*x+4/d*B*a*b^3*c+A*b^4*x+1/d*A*b^4*c+1/d*B*b^4*ln
(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.9832, size = 290, normalized size = 1.34

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa^4 - 32\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Ba^4 - 128\left(\sin(dx+c)^3 - 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 - 128*(sin(d*x + c)^3 - 3*sin(d*x +
 c))*A*a^3*b + 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b + 144*(2*d*x + 2
*c + sin(2*d*x + 2*c))*A*a^2*b^2 + 384*(d*x + c)*B*a*b^3 + 96*(d*x + c)*A*b
^4 + 48*B*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 576*B*a^2*b
^2*sin(d*x + c) + 384*A*a*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.594929, size = 447, normalized size = 2.07

$$\frac{12Bb^4\log(\sin(dx+c)+1) - 12Bb^4\log(-\sin(dx+c)+1) + 3\left(3Aa^4 + 16Ba^3b + 24Aa^2b^2 + 32Bab^3 + 8Ab^4\right)dx + \left(6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(12*B*b^4*log(sin(d*x + c) + 1) - 12*B*b^4*log(-sin(d*x + c) + 1) + 3*
(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*d*x + (6*A*a^4
```

```
*cos(d*x + c)^3 + 16*B*a^4 + 64*A*a^3*b + 144*B*a^2*b^2 + 96*A*a*b^3 + 8*(B
*a^4 + 4*A*a^3*b)*cos(d*x + c)^2 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*
cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)

[Out] Timed out
```

Giac [B] time = 1.30325, size = 814, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")

[Out] 1/24*(24*B*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*B*b^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3
+ 8*A*b^4)*(d*x + c) - 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/
2*d*x + 1/2*c)^7 - 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d
*x + 1/2*c)^7 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2
*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^4*tan(1/2*d*x +
 1/2*c)^5 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*A*a^3*b*tan(1/2*d*x + 1/2
*c)^5 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^5 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*A*a*b^3*tan(1/2*d*x + 1/2
*c)^5 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 -
160*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72
*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) - 24*B*a
^4*tan(1/2*d*x + 1/2*c) - 96*A*a^3*b*tan(1/2*d*x + 1/2*c) - 48*B*a^3*b*tan(
1/2*d*x + 1/2*c) - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 144*B*a^2*b^2*tan(1/
2*d*x + 1/2*c) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

3.309 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=258

$$\frac{\left(60a^2Ab^2+8a^4A+40a^3bB+60ab^3B+15Ab^4\right)\sin(c+dx)}{15d}+\frac{a^2\left(8a^2A+25abB+18Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{a}{}$$

```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*x)/8 + ((8*a^
4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b^3*B)*Sin[c + d*x])/(15*
d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b^2*B)*Cos[c + d*x]*Sin[c
 + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b*B)*Cos[c + d*x]^2*Sin[c
 + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(5*d)
```

Rubi [A] time = 0.69051, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4025, 4094, 4074, 4047, 2637, 4045, 8}

$$\frac{\left(60a^2Ab^2+8a^4A+40a^3bB+60ab^3B+15Ab^4\right)\sin(c+dx)}{15d}+\frac{a^2\left(8a^2A+25abB+18Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```



```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*x)/8 + ((8*a^
4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b^3*B)*Sin[c + d*x])/(15*
d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b^2*B)*Cos[c + d*x]*Sin[c
 + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b*B)*Cos[c + d*x]^2*Sin[c
 + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(5*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{5d} - \frac{1}{5}\int \mathrm{co}$$
$$= \frac{a(8Ab+5aB)\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx}{20d}$$
$$= \frac{a^2\left(8a^2A+18Ab^2+25abB\right)\cos^2(c+dx)\sin(c+dx)}{30d} + \frac{a}{}$$
$$= \frac{a^2\left(8a^2A+18Ab^2+25abB\right)\cos^2(c+dx)\sin(c+dx)}{30d} + \frac{a}{}$$
$$= \frac{\left(8a^4A+60a^2Ab^2+15Ab^4+40a^3bB+60ab^3B\right)\sin(c+}{15d}$$
$$= \frac{1}{8}\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\right)x + \frac{(8a}{}$$

Mathematica [A] time = 0.632869, size = 263, normalized size = 1.02

$$\frac{120a\left(4a^2Ab+a^3B+6ab^2B+4Ab^3\right)\sin(2(c+dx))+60\left(36a^2Ab^2+5a^4A+24a^3bB+32ab^3B+8Ab^4\right)\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (720*a^3*A*b*c + 960*a*A*b^3*c + 180*a^4*B*c + 1440*a^2*b^2*B*c + 480*b^4*B
*c + 720*a^3*A*b*d*x + 960*a*A*b^3*d*x + 180*a^4*B*d*x + 1440*a^2*b^2*B*d*x
 + 480*b^4*B*d*x + 60*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a
*b^3*B)*Sin[c + d*x] + 120*a*(4*a^2*A*b + 4*A*b^3 + a^3*B + 6*a*b^2*B)*Sin[
2*(c + d*x)] + 50*a^4*A*Sin[3*(c + d*x)] + 240*a^2*A*b^2*Sin[3*(c + d*x)] +
 160*a^3*b*B*Sin[3*(c + d*x)] + 60*a^3*A*b*Sin[4*(c + d*x)] + 15*a^4*B*Sin[
4*(c + d*x)] + 6*a^4*A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.073, size = 258, normalized size = 1.

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+4Aa^3b\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^3*b*(1/
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^4*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*a^2*b^2*(2+cos(d*x+c)
^2)*sin(d*x+c)+4/3*B*a^3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+4*A*a*b^3*(1/2*cos(d
*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*B*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*
d*x+1/2*c)+A*b^4*sin(d*x+c)+4*B*a*b^3*sin(d*x+c)+B*b^4*(d*x+c))
```

Maxima [A] time = 0.984833, size = 332, normalized size = 1.29

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Ba^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 60*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3*b - 640*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*B*a^3*b - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a
^2*b^2 + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b^2 + 480*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*A*a*b^3 + 480*(d*x + c)*B*b^4 + 1920*B*a*b^3*sin(d*x +
c) + 480*A*b^4*sin(d*x + c))/d
```

Fricas [A] time = 0.593607, size = 478, normalized size = 1.85

$$\frac{15\left(3Ba^4+12Aa^3b+24Ba^2b^2+16Aab^3+8Bb^4\right)dx+\left(24Aa^4\cos(dx+c)^4+64Aa^4+320Ba^3b+480Aa^2b^2+480B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*d*x
+ (24*A*a^4*cos(d*x + c)^4 + 64*A*a^4 + 320*B*a^3*b + 480*A*a^2*b^2 + 480*B
*a*b^3 + 120*A*b^4 + 30*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^3 + 16*(2*A*a^4 +
10*B*a^3*b + 15*A*a^2*b^2)*cos(d*x + c)^2 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B
*a^2*b^2 + 16*A*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3127, size = 1068, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*(d*x
 + c) + 2*(120*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)
^9 - 300*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^
9 + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 240*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c
)^9 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^4*tan(1/2*d*x + 1/2*c)^7 -
 30*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 128
0*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 -
720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 +
 1920*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 4
64*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 240
0*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 +
720*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 30*B*
a^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 1280*B*a^
3*b*tan(1/2*d*x + 1/2*c)^3 + 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*B*
a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 1920*
B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a
^4*tan(1/2*d*x + 1/2*c) + 75*B*a^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1
/2*d*x + 1/2*c) + 480*B*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x
 + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.310 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=309

$$-\frac{a\left(16a^2Ab+4a^3B+27ab^2B+13Ab^3\right)\sin^3(c+dx)}{15d}+\frac{\left(48a^3Ab+87a^2b^2B+12a^4B+53aAb^3+15b^4B\right)\sin(c+dx)}{15d}+\frac{a}{}$$

```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*x)/16 + ((48*
a^3*A*b + 53*a*A*b^3 + 12*a^4*B + 87*a^2*b^2*B + 15*b^4*B)*Sin[c + d*x])/(1
5*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) + (a^2*(25*a^2*A + 48*A*b^2 + 72*a*b*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(120*d) + (a*(3*A*b + 2*a*B)*Cos[c + d*x]^4*(a + b*Se
c[c + d*x])^2*Sin[c + d*x])/(10*d) + (a*A*Cos[c + d*x]^5*(a + b*Sec[c + d*x
])^3*Sin[c + d*x])/(6*d) - (a*(16*a^2*A*b + 13*A*b^3 + 4*a^3*B + 27*a*b^2*B
)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.819945, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4025, 4094, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{a\left(16a^2Ab+4a^3B+27ab^2B+13Ab^3\right)\sin^3(c+dx)}{15d}+\frac{\left(48a^3Ab+87a^2b^2B+12a^4B+53aAb^3+15b^4B\right)\sin(c+dx)}{15d}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```



```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*x)/16 + ((48*
a^3*A*b + 53*a*A*b^3 + 12*a^4*B + 87*a^2*b^2*B + 15*b^4*B)*Sin[c + d*x])/(1
5*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) + (a^2*(25*a^2*A + 48*A*b^2 + 72*a*b*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(120*d) + (a*(3*A*b + 2*a*B)*Cos[c + d*x]^4*(a + b*Se
c[c + d*x])^2*Sin[c + d*x])/(10*d) + (a*A*Cos[c + d*x]^5*(a + b*Sec[c + d*x
])^3*Sin[c + d*x])/(6*d) - (a*(16*a^2*A*b + 13*A*b^3 + 4*a^3*B + 27*a*b^2*B
)*Sin[c + d*x]^3)/(15*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^5(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{6d} - \frac{1}{6}\int \cos^5($$
$$= \frac{a(3Ab+2aB)\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{10d} +$$
$$= \frac{a^2\left(25a^2A+48Ab^2+72abB\right)\cos^3(c+dx)\sin(c+dx)}{120d} + \frac{a(3}{}$$
$$= \frac{a^2\left(25a^2A+48Ab^2+72abB\right)\cos^3(c+dx)\sin(c+dx)}{120d} + \frac{a(3}{}$$
$$= \frac{\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)\cos(c+dx)\,\mathrm{s}}{16d}$$
$$= \frac{1}{16}\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)x + \frac{(5a^4}{}$$
$$= \frac{1}{16}\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)x + \frac{(48a^3}{}$$

Mathematica [A] time = 1.22771, size = 333, normalized size = 1.08

$$\frac{120\left(20a^3Ab+36a^2b^2B+5a^4B+24aAb^3+8b^4B\right)\sin(c+dx)+15\left(96a^2Ab^2+15a^4A+64a^3bB+64ab^3B+16Ab^4\right)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (300*a^4*A*c + 2160*a^2*A*b^2*c + 480*A*b^4*c + 1440*a^3*b*B*c + 1920*a*b^3
*B*c + 300*a^4*A*d*x + 2160*a^2*A*b^2*d*x + 480*A*b^4*d*x + 1440*a^3*b*B*d*
x + 1920*a*b^3*B*d*x + 120*(20*a^3*A*b + 24*a*A*b^3 + 5*a^4*B + 36*a^2*b^2*
B + 8*b^4*B)*Sin[c + d*x] + 15*(15*a^4*A + 96*a^2*A*b^2 + 16*A*b^4 + 64*a^3
*b*B + 64*a*b^3*B)*Sin[2*(c + d*x)] + 400*a^3*A*b*Sin[3*(c + d*x)] + 320*a*
A*b^3*Sin[3*(c + d*x)] + 100*a^4*B*Sin[3*(c + d*x)] + 480*a^2*b^2*B*Sin[3*(
c + d*x)] + 45*a^4*A*Sin[4*(c + d*x)] + 180*a^2*A*b^2*Sin[4*(c + d*x)] + 12
0*a^3*b*B*Sin[4*(c + d*x)] + 48*a^3*A*b*Sin[5*(c + d*x)] + 12*a^4*B*Sin[5*(
c + d*x)] + 5*a^4*A*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.079, size = 316, normalized size = 1.

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{Ba^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
/5*A*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*B*a^3*b*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^2*b^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*B*a^2*b^2*(2+cos(d*x+
c)^2)*sin(d*x+c)+4/3*A*a*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+4*B*a*b^3*(1/2*cos
(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+
1/2*c)+B*b^4*sin(d*x+c))
```

Maxima [A] time = 0.986661, size = 414, normalized size = 1.34

$$-\frac{5\left(4\sin\left(2dx+2c\right)^{3}-60dx-60c-9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)Aa^{4}-64\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+\dots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^4 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*B*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
c))*A*a^3*b - 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B
*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*
b^2 + 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b^2 + 1280*(sin(d*x + c)
^3 - 3*sin(d*x + c))*A*a*b^3 - 960*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^3
 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 960*B*b^4*sin(d*x + c))/d
```

Fricas [A] time = 0.609289, size = 587, normalized size = 1.9

$$\frac{15\left(5Aa^{4}+24Ba^{3}b+36Aa^{2}b^{2}+32Bab^{3}+8Ab^{4}\right)dx+\left(40Aa^{4}\cos\left(dx+c\right)^{5}+128Ba^{4}+512Aa^{3}b+960Ba^{2}b^{2}+\dots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*d*x
+ (40*A*a^4*cos(d*x + c)^5 + 128*B*a^4 + 512*A*a^3*b + 960*B*a^2*b^2 + 640*
A*a*b^3 + 240*B*b^4 + 48*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^4 + 10*(5*A*a^4 +
 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^3 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B
*a^2*b^2 + 10*A*a*b^3)*cos(d*x + c)^2 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2
*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2839, size = 1521, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*(d*x
 + c) - 2*(165*A*a^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*
c)^11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2
*c)^11 + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*
x + 1/2*c)^11 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x +
 1/2*c)^11 - 25*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 560*B*a^4*tan(1/2*d*x + 1/2*
c)^9 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 840*B*a^3*b*tan(1/2*d*x + 1/2*
c)^9 + 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 5280*B*a^2*b^2*tan(1/2*d*x +
 1/2*c)^9 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1440*B*a*b^3*tan(1/2*d*x
+ 1/2*c)^9 + 360*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 1200*B*b^4*tan(1/2*d*x + 1/
2*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)
^7 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)
^7 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/
2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/
2*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)
^7 - 450*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 -
 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 -
 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)
^5 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 960*B*a*b^3*tan(1/2*d*x + 1/2*c)
^5 - 240*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 +
 25*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*
A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*
A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 -
 360*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 165
*A*a^4*tan(1/2*d*x + 1/2*c) - 240*B*a^4*tan(1/2*d*x + 1/2*c) - 960*A*a^3*b*
tan(1/2*d*x + 1/2*c) - 600*B*a^3*b*tan(1/2*d*x + 1/2*c) - 900*A*a^2*b^2*tan
(1/2*d*x + 1/2*c) - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*A*a*b^3*tan(1
/2*d*x + 1/2*c) - 480*B*a*b^3*tan(1/2*d*x + 1/2*c) - 120*A*b^4*tan(1/2*d*x
+ 1/2*c) - 240*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/
d
```

3.311 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(-3a^2B+3aAb-2b^2B\right)\tan(c+dx)}{3b^3d}+\frac{\left(2a^2+b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a+}}\right.}{b^4d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ((2*a^2 + b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(A*b -
 a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4
*Sqrt[a + b]*d) - ((3*a*A*b - 3*a^2*B - 2*b^2*B)*Tan[c + d*x])/(3*b^3*d) +
((A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*b*d)
```

Rubi [A] time = 0.675946, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4033, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-3a^2B+3aAb-2b^2B\right)\tan(c+dx)}{3b^3d}+\frac{\left(2a^2+b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a+}}\right.}{b^4d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(A*b -
 a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4
*Sqrt[a + b]*d) - ((3*a*A*b - 3*a^2*B - 2*b^2*B)*Tan[c + d*x])/(3*b^3*d) +
((A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*b*d)
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}dx = \frac{B\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)\left(2aB+2bB\sec(c+dx)+3(Ab-aB)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}d}{3b}$$

$$= \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{B\sec^2(c+dx)\tan(c+dx)}{3bd} + \int \frac{\sec(c+dx}{}$$

$$= -\frac{\left(3aAb-3a^2B-2b^2B\right)\tan(c+dx)}{3b^3d} + \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} +$$

$$= -\frac{\left(3aAb-3a^2B-2b^2B\right)\tan(c+dx)}{3b^3d} + \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} +$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{\left(3aAb-3a^2B-2b^2B\right)\tan(c+}{3b^3d}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{\left(3aAb-3a^2B-2b^2B\right)\tan(c+}{3b^3d}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(}{\sqrt{a+}}\right.}{\sqrt{a-b}b^4\sqrt{a+b}d}$$

Mathematica [B] time = 2.36635, size = 422, normalized size = 2.26

$$\frac{\frac{4b\left(3a^2B-3aAb+2b^2B\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4b\left(3a^2B-3aAb+2b^2B\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} + \frac{24a^3(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 6\left(2a^2+b^2\right)(aB-Ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((24*a^3*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + 6*(2*a^2 + b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 6*(2*a^2 + b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + (b^2*(3*A*b + (-3*a + b)*B))/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2 + (2*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^3 + (4*b*(-3*a*A*b + 3*a^2*B + 2*b^2*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]) + (2*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3 - (b^2*(3*A*b + (-3*a + b)*B))/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (4*b*(-3*a*A*b + 3*a^2*B + 2*b^2*B)*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]))/(12*b^4*d)
```

Maple [B] time = 0.083, size = 688, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2*A+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*B+1/2/d
/b*ln(tan(1/2*d*x+1/2*c)+1)*A+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)*A-1/d/b/(tan(1
```

```
/2*d*x+1/2*c)+1)*B-1/3/d*B/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b*ln(tan(1/2*d*
x+1/2*c)-1)*A+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)*A-1/d/b/(tan(1/2*d*x+1/2*c)-1)
*B-1/3/d*B/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*A-1/
2/d/b/(tan(1/2*d*x+1/2*c)-1)^2*B-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*a^4/b^4/((a+b)*(a-b))^(1/
2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d/b^2/(tan(1/2
*d*x+1/2*c)+1)*A*a-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*B*a^2-1/2/d/b^2/(tan(1/2*
d*x+1/2*c)+1)*B*a-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*B*a-1/d/b^3/(tan(1/2*d
*x+1/2*c)-1)*B*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)*B*a+1/2/d/b^2/(tan(1/2*
d*x+1/2*c)+1)^2*B*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*A*a^2-1/d/b^4*ln(tan(1
/2*d*x+1/2*c)+1)*B*a^3-1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*B*a-1/d/b^3*ln(ta
n(1/2*d*x+1/2*c)-1)*A*a^2+1/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*B*a^3+1/2/d/b^2*
ln(tan(1/2*d*x+1/2*c)-1)*B*a+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*A*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35656, size = 1650, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")

[Out] [-1/12*(6*(B*a^4 - A*a^3*b)*sqrt(a^2 - b^2)*cos(d*x + c)^3*log((2*a*b*cos(d
*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c)
+ a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 b^2)) + 3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*
cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 +
 A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*
B*a^2*b^3 - 2*B*b^5 + 2*(3*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4 -
2*B*b^5)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d
*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*(B*a^4
- A*a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*B*a^5 - 2*A*a^4*b - B*a^3*
b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3
*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x +
c)^3*log(-sin(d*x + c) + 1) + 2*(2*B*a^2*b^3 - 2*B*b^5 + 2*(3*B*a^4*b - 3*A
*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*cos(d*x + c)^2 - 3*(B*a^3*b^2 -
 A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*
d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.26332, size = 556, normalized size = 2.97

$$-\frac{\frac{3\left(2Ba^3-2Aa^2b+Bab^2-Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2Ba^3-2Aa^2b+Bab^2-Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}-\frac{12\left(Ba^4-Aa^3b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/b^4 - 3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^4 - 12*(B*a^4 - A*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) + 2*(6*B*a^2*tan(1/2*d*x + 1/2
*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3
*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*t
an(1/2*d*x + 1/2*c)^3 + 12*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^2*tan(1/2*d
*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c)
 - 3*B*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*ta
n(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

3.312 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{\left(-2a^2B+2aAb-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{2a^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(Ab-aB)\tan(c+dx)}{b^2d}+\frac{B\,\text{ta}}{}$$

```
[Out] -((2*a*A*b - 2*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + (2*a^2*(A*
b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
b^3*Sqrt[a + b]*d) + ((A*b - a*B)*Tan[c + d*x])/(b^2*d) + (B*Sec[c + d*x]*T
an[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.395496, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4033, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-2a^2B+2aAb-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{2a^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(Ab-aB)\tan(c+dx)}{b^2d}+\frac{B\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((2*a*A*b - 2*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + (2*a^2*(A*
b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
b^3*Sqrt[a + b]*d) + ((A*b - a*B)*Tan[c + d*x])/(b^2*d) + (B*Sec[c + d*x]*T
an[c + d*x])/(2*b*d)
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(aB+bB\sec(c+dx)+2(Ab-aB)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2b}$$

$$= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(abB-\left(2aAb-2a\ldots\right.\right.}{a+b\sec(c\ldots}}{2b^2\ldots}$$

$$= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\left(a^2(Ab-aB)\right)\int \frac{\sec(\ldots}{a+b\,\mathrm{se}\ldots}}{b^3}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\,\mathrm{se}\ldots}{}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\,\mathrm{se}\ldots}{}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{2a^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{ta}\ldots}{\sqrt{\ldots}}\right.}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}$$

Mathematica [B] time = 1.77521, size = 300, normalized size = 2.1

$$\frac{8a^2(aB-Ab)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2\left(2a^2B-2aAb+b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\left(2a^2B-2aAb+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*a^2*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] - 2*(-2*a*A*b + 2*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*(-2*a*A*b + 2*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (b^2*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b*(A
*b - a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^2*B)
/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b*(A*b - a*B)*Sin[(c + d*x)/2
])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*b^3*d)
```

Maple [B] time = 0.074, size = 410, normalized size = 2.9

$$2\frac{a^2A}{db^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-2\frac{Ba^3}{db^3\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*B-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*B-1/d/b/(tan(1/2
*d*x+1/2*c)+1)*A+1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*B*a+1/2/d/b/(tan(1/2*d*x+1/
2*c)+1)*B-1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*A*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c
)+1)*B*a^2+1/2/d/b*ln(tan(1/2*d*x+1/2*c)+1)*B+1/2/d/b/(tan(1/2*d*x+1/2*c)-1
)^2*B-1/d/b/(tan(1/2*d*x+1/2*c)-1)*A+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*B*a+1/2
/d/b/(tan(1/2*d*x+1/2*c)-1)*B+1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*A*a-1/d/b^3*
ln(tan(1/2*d*x+1/2*c)-1)*B*a^2-1/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.3259, size = 1353, normalized size = 9.46

$$\left[-\frac{2\left(Ba^3-Aa^2b\right)\sqrt{a^2-b^2}\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(2\,Ba^4-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/4*(2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
```

```
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2)) - (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^2*b^2 - B*b^4 - 2*(B*a^
3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d*cos(d*x + c)^2), -1/4*(4*(B*a^3 - A*a^2*b)*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x +
 c)^2 - (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^2*b^2 - B*b^4 - 2*(B*a^
3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^3(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.30943, size = 363, normalized size = 2.54

$$\frac{\left(2\,Ba^2-2\,Aab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}-\frac{\left(2\,Ba^2-2\,Aab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^3}-\frac{4\left(Ba^3-Aa^2b\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan}{}\right.\right.}{\sqrt{-a^2+b^2}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 - (
2*B*a^2 - 2*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 4*(B*a^
3 - A*a^2*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a
^2 + b^2)*b^3) + 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*
c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(
1/2*d*x + 1/2*c) + B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^
2*b^2))/d
```

3.313 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{B\tan(c+dx)}{bd}$$

```
[Out] ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*a*(A*b - a*B)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) +
(B*Tan[c + d*x])/(b*d)
```

Rubi [A] time = 0.228801, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4010, 12, 3789, 3770, 3831, 2659, 208}

$$\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{B\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*a*(A*b - a*B)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) +
(B*Tan[c + d*x])/(b*d)
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{B\tan(c+dx)}{bd}+\frac{\int \frac{(Ab-aB)\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{B\tan(c+dx)}{bd}+\frac{(Ab-aB)\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{B\tan(c+dx)}{bd}+\frac{(Ab-aB)\int \sec(c+dx)\,dx}{b^2}-\frac{(a(Ab-aB))\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^2}$$

$$= \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{B\tan(c+dx)}{bd}-\frac{(a(Ab-aB))\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}}{b^3}$$

$$= \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{B\tan(c+dx)}{bd}-\frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+$$

Mathematica [A] time = 0.573219, size = 130, normalized size = 1.33

$$\frac{-\frac{2a(aB-Ab)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-(Ab-aB)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-2*a*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] - (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b*B*Tan[c + d*x])/(b^2*d)
```

Maple [B] time = 0.061, size = 228, normalized size = 2.3

$$-2\,\frac{Aa}{db\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{Ba^2}{db^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*A+2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*B-1/d/b/(tan(1/2*d*x+1/2*c)+1)*B+1/d/b*ln(tan(1/2*d*x+
1/2*c)+1)*A-1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*B*a-1/d/b/(tan(1/2*d*x+1/2*c)-
1)*B-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)*A+1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.875603, size = 1065, normalized size = 10.87

$$\left[-\frac{\left(Ba^2 - Aab\right)\sqrt{a^2-b^2}\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right) + \left(Ba^3 - Aa^2 b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/2*((B*a^2 - A*a*b)*sqrt(a^2 - b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
 - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*si
n(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
+ (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) -
(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2
*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c)), 1/2*(2*(
B*a^2 - A*a*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 +
A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*
b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^2*b - B*b^3)*sin(d*x + c)
)/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right)\right)\sec^{2}\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.22739, size = 238, normalized size = 2.43

$$-\frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b}-\frac{2\left(Ba^2-Aab\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\right.\right.}{\sqrt{-a^2+b^2}b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - (B*a - A*b)*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*b) - 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/d
```

3.314 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{B\tanh^{-1}(\sin(c+dx))}{bd}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b*d) + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.126499, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{B\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b*d) + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{B\int \sec(c+dx)\,dx}{b} + \frac{(Ab-aB)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{bd} + \frac{(Ab-aB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^2}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{bd} + \frac{(2(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+\right.\right.}{b^2d}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{bd} + \frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

Mathematica [A] time = 0.179255, size = 112, normalized size = 1.47

$$\frac{\frac{2(aB-Ab)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + B\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + B*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]))/(b*d)
```

Maple [A] time = 0.063, size = 135, normalized size = 1.8

$$2\,\frac{A}{d\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right) - 2\,\frac{Ba}{db\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right) + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*A-2/d/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*B*a+1/d/b*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/b*ln(tan(1/2*d*x+1/2*c)-
1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima
")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.91895, size = 707, normalized size = 9.3

$$\left[-\frac{(Ba-Ab)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(Ba^2-Bb^2\right)\log\left(\sin\left(dx+\right.\right.}{2\left(a^2b-b^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((B*a - A*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (B*a^2 - B*b^2)
*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^2*b -
b^3)*d), -1/2*(2*(B*a - A*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*log(sin(d*x
+ c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.23159, size = 171, normalized size = 2.25

$$\frac{\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Ba-Ab)}{\sqrt{-a^2+b^2}\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b - B*log(abs(tan(1/2*d*x + 1/2*c) -
1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(B*a - A*b)/(
sqrt(-a^2 + b^2)*b))/d
```

3.315 $\int \frac{A+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{Ax}{a}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (A*x)/a - (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.099078, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3919, 3831, 2659, 208}

$$\frac{Ax}{a}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a - (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{Ax}{a} - \frac{(Ab-aB)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$
$$= \frac{Ax}{a} - \frac{(Ab-aB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{ab}$$
$$= \frac{Ax}{a} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$
$$= \frac{Ax}{a} - \frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.122792, size = 68, normalized size = 1.01

$$\frac{\frac{2(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + A(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (A*(c + d*x) + (2*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/Sqrt[a^2 - b^2])/(a*d)
```

Maple [A] time = 0.072, size = 113, normalized size = 1.7

$$2\,\frac{A\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad} - 2\,\frac{Ab}{ad\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right) + 2\,\frac{B}{d\sqrt{(a+b)(a-b)}}\text{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*A/a*arctan(tan(1/2*d*x+1/2*c))-2/d/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b+2/d/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.520771, size = 540, normalized size = 8.06

$$\left[\frac{2\left(Aa^2 - Ab^2\right)dx - (Ba - Ab)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 - 2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c) + 2\,a^2 - b^2}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right)}{2\left(a^3 - ab^2\right)d}, \frac{\left(Aa^2 - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(A*a^2 - A*b^2)*d*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2)))/((a^3 - a*b^2)*d), ((A*a^2 - A*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.20014, size = 136, normalized size = 2.03

$$\frac{\frac{(dx+c)A}{a} + \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Ba-Ab)}{\sqrt{-a^2+b^2}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A/a + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arct
an(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(B
*a - A*b)/(sqrt(-a^2 + b^2)*a))/d
```

3.316 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(Ab-aB)}{a^2}+\frac{A\sin(c+dx)}{ad}$$

```
[Out] -(((A*b - a*B)*x)/a^2) + (2*b*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c + d*x])/(a*d
)
```

Rubi [A] time = 0.149073, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4034, 12, 3783, 2659, 208}

$$\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(Ab-aB)}{a^2}+\frac{A\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] -(((A*b - a*B)*x)/a^2) + (2*b*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c + d*x])/(a*d
)
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx &= \frac{A\sin(c+dx)}{ad}-\frac{\int \frac{Ab-aB}{a+b\sec(c+dx)}\,dx}{a}\\ &= \frac{A\sin(c+dx)}{ad}-\frac{(Ab-aB)\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}\\ &= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\frac{(Ab-aB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}\\ &= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\frac{(2(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\right.}{a^2d}\\ &= -\frac{(Ab-aB)x}{a^2}+\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d}+\frac{A\sin(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.208126, size = 85, normalized size = 0.94

$$\frac{-\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+(c+dx)(aB-Ab)+aA\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-(A*b) + a*B)*(c + d*x) - (2*b*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*A*Sin[c + d*x])/(a^2*d)
```

Maple [B] time = 0.097, size = 172, normalized size = 1.9

$$2\,\frac{A\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{A\arctan(\tan(1/2\,dx+c/2))\,b}{da^2}+2\,\frac{B\arctan(\tan(1/2\,dx+c/2))}{ad}+2\,\frac{Ab^2}{da^2\sqrt{(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/a^2*A*arctan(tan(1/
2*d*x+1/2*c))*b+2/d/a*B*arctan(tan(1/2*d*x+1/2*c))+2/d*b^2/a^2/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*b/a/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.542725, size = 702, normalized size = 7.8

$$\left[\frac{2\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)dx-\left(Bab-Ab^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{2\left(a^4-a^2b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a*b - A*b^2)*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a
^2*b^2)*d), ((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a*b - A*b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec(c+dx))\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.17341, size = 190, normalized size = 2.11

$$\frac{\frac{(Ba-Ab)(dx+c)}{a^2}+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}-\frac{2\left(Bab-Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((B*a - A*b)*(d*x + c)/a^2 + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*a) - 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^
2 + b^2)))/(sqrt(-a^2 + b^2)*a^2))/d
```

3.317 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=134

$$-\frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2A-2abB+2Ab^2\right)}{2a^3}-\frac{(Ab-aB)\sin(c+dx)}{a^2d}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] ((a^2*A + 2*A*b^2 - 2*a*b*B)*x)/(2*a^3) - (2*b^2*(A*b - a*B)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - ((
A*b - a*B)*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.403134, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4034, 4104, 3919, 3831, 2659, 208}

$$-\frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2A-2abB+2Ab^2\right)}{2a^3}-\frac{(Ab-aB)\sin(c+dx)}{a^2d}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2*A + 2*A*b^2 - 2*a*b*B)*x)/(2*a^3) - (2*b^2*(A*b - a*B)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - ((
A*b - a*B)*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(2(Ab-aB)-aA\sec(c+dx)-Ab\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int \frac{a^2A+2Ab^2-2abB+aAb\,s}{a+b\sec(c+dx)}}{2a^2}$$

$$= \frac{\left(a^2A+2Ab^2-2abB\right)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx}{2ad}$$

$$= \frac{\left(a^2A+2Ab^2-2abB\right)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx}{2ad}$$

$$= \frac{\left(a^2A+2Ab^2-2abB\right)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx}{2ad}$$

$$= \frac{\left(a^2A+2Ab^2-2abB\right)x}{2a^3} - \frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} - \frac{(Ab-}{}$$

Mathematica [A] time = 0.329075, size = 121, normalized size = 0.9

$$\frac{2(c+dx)\left(a^2A-2abB+2Ab^2\right)+\frac{8b^2(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a^2A\sin(2(c+dx))+4a(aB-Ab)\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 - 2*a*b*B)*(c + d*x) + (8*b^2*(A*b - a*B)*ArcTanh[((-a
 + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*(-(A*b) + a*
B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] time = 0.099, size = 367, normalized size = 2.7

$$-\frac{A}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-2\,\frac{(\tan(1/2\,dx+c/2))^{3}\,Ab}{da^{2}\left(1+(\tan(1/2\,dx+c/2))^{2}\right)^{2}}+2\,\frac{(\tan(1/2\,dx+c/2))^{3}\,B}{ad\left(1+(\tan(1/2\,dx+c/2))^{2}\right)^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)^3*B+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*
A-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b+2/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B+1/d*A/a*arctan(tan(1/2*d*x+1/2*c))+2
/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A*b^2-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*
B*b-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*A+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.565854, size = 934, normalized size = 6.97

$$\left[\frac{\left(Aa^{4}-2\,Ba^{3}b+Aa^{2}b^{2}+2\,Bab^{3}-2\,Ab^{4}\right)dx-\left(Bab^{2}-Ab^{3}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+c)}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right.}{2\left(a^{5}-a^{3}b^{2}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*d*x - (B*a*b^2
- A*b^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*B*a^4 - 2*A*a^3*b - 2*
B*a^2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a
^5 - a^3*b^2)*d), 1/2*((A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4
)*d*x + 2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
s(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^4 - 2*A*a^3*b - 2*B*a^
2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 -
 a^3*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.21447, size = 306, normalized size = 2.28

$$\frac{\frac{(Aa^2-2\,Bab+2\,Ab^2)(dx+c)}{a^3}+\frac{4\,(Bab^2-Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^3}-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*(d*x + c)/a^3 + 4*(B*a*b^2 - A*b^3)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) - 2*
(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan(1/2*
d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 2*
A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.318 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{\left(2a^2A-3abB+3Ab^2\right)\sin(c+dx)}{3a^3d}+\frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(a^2+2b^2\right)(Ab-aB)}{2a^4}-\frac{(Ab-aB)\sin(}{}$$

```
[Out] -((a^2 + 2*b^2)*(A*b - a*B)*x)/(2*a^4) + (2*b^3*(A*b - a*B)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2
*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[c + d*x])/(3*a^3*d) - ((A*b - a*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.641879, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4034, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(2a^2A-3abB+3Ab^2\right)\sin(c+dx)}{3a^3d}+\frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(a^2+2b^2\right)(Ab-aB)}{2a^4}-\frac{(Ab-aB)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```



```
[Out] -((a^2 + 2*b^2)*(A*b - a*B)*x)/(2*a^4) + (2*b^3*(A*b - a*B)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2
*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[c + d*x])/(3*a^3*d) - ((A*b - a*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)\left(3(Ab-aB)-2aA\sec(c+dx)-2Ab\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,d}{3a}$$

$$= -\frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int \frac{\cos(c+d}{}}{}$$

$$= \frac{\left(2a^2A+3Ab^2-3abB\right)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A}{}$$

$$= -\frac{\left(a^2+2b^2\right)(Ab-aB)x}{2a^4} + \frac{\left(2a^2A+3Ab^2-3abB\right)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\,c}{}$$

$$= -\frac{\left(a^2+2b^2\right)(Ab-aB)x}{2a^4} + \frac{\left(2a^2A+3Ab^2-3abB\right)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\,c}{}$$

$$= -\frac{\left(a^2+2b^2\right)(Ab-aB)x}{2a^4} + \frac{\left(2a^2A+3Ab^2-3abB\right)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\,c}{}$$

$$= -\frac{\left(a^2+2b^2\right)(Ab-aB)x}{2a^4} + \frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(2a^2A\right.}{}$$

Mathematica [A] time = 0.487204, size = 152, normalized size = 0.85

$$\frac{6\left(a^2+2b^2\right)(c+dx)(aB-Ab)+3a\left(3a^2A-4abB+4Ab^2\right)\sin(c+dx)-\frac{24b^3(Ab-aB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+3a^2(aB-}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*(a^2 + 2*b^2)*(-(A*b) + a*B)*(c + d*x) - (24*b^3*(A*b - a*B)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2*A +
```

```
4*A*b^2 - 4*a*b*B)*Sin[c + d*x] + 3*a^2*(-(A*b) + a*B)*Sin[2*(c + d*x)] +
a^3*A*Sin[3*(c + d*x)])/(12*a^4*d)
```

Maple [B] time = 0.102, size = 641, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+1/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^5*A*b^2-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)^5*B-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B*b+4/3/d/
a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+4/d/a^3/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A*b^2-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^3*B*b+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)
*A+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b^2-2/d/a^2/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*B*b-1/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)*A*b+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)*B-1/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))*b-2/d/a^4*arctan(tan(1/2*d*x+1
/2*c))*A*b^3+1/d/a*B*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arctan(tan(1/2*d*x+
1/2*c))*B*b^2+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*A-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.583899, size = 1177, normalized size = 6.61

$$\left[\frac{3\left(Ba^5 - Aa^4b + Ba^3b^2 - Aa^2b^3 - 2\,Bab^4 + 2\,Ab^5\right)dx - 3\left(Bab^3 - Ab^4\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{a^2 -}}{a^2\cos(dx+c)^2 + 2\,ab\,\mathrm{c}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*d*x
 - 3*(B*a*b^3 - A*b^4)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
```

```
^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (4*A*a^5 -
6*B*a^4*b + 2*A*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*a^3*b^2)*c
os(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^6 - a^4*b^2)*d), 1/6*(3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a
^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*d*x - 6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (
4*A*a^5 - 6*B*a^4*b + 2*A*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*
a^3*b^2)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d
*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos^3(c + dx)}{a + b\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.22299, size = 486, normalized size = 2.73

$$\frac{3\left(Ba^3 - Aa^2b + 2\,Bab^2 - 2\,Ab^3\right)(dx+c)}{a^4} - \frac{12\left(Bab^3 - Ab^4\right)\left(\pi\left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,a+2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^4} + \frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*(d*x + c)/a^4 - 12*(B*a*b^3
- A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 +
 b^2)*a^4) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*
c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*
A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*
x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) -
6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^3*a^3))/d
```

3.319 $\int \frac{\cos^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=240

$$-\frac{\left(2a^2+3b^2\right)(Ab-aB)\sin(c+dx)}{3a^4d}+\frac{\left(3a^2A-4abB+4Ab^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^4(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+b}}\right.}{a^5d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ((3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*x)/(8*a^5) - (2*
b^4*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*S
qrt[a - b]*Sqrt[a + b]*d) - ((2*a^2 + 3*b^2)*(A*b - a*B)*Sin[c + d*x])/(3*a
^4*d) + ((3*a^2*A + 4*A*b^2 - 4*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d)
 - ((A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*
Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.982321, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4034, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(2a^2+3b^2\right)(Ab-aB)\sin(c+dx)}{3a^4d}+\frac{\left(3a^2A-4abB+4Ab^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^4(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+b}}\right.}{a^5d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*x)/(8*a^5) - (2*
b^4*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*S
qrt[a - b]*Sqrt[a + b]*d) - ((2*a^2 + 3*b^2)*(A*b - a*B)*Sin[c + d*x])/(3*a
^4*d) + ((3*a^2*A + 4*A*b^2 - 4*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d)
 - ((A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*
Sin[c + d*x])/(4*a*d)
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^3(c+dx)\left(4(Ab-aB)-3aA\sec(c+dx)-3Ab\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,d}{4a}$$

$$= -\frac{(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} + \int \frac{\cos^2(c}{}$$

$$= \frac{\left(3a^2A+4Ab^2-4abB\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{(Ab-aB)\cos^2(c+dx)\sin}{3a^2d}$$

$$= -\frac{\left(2a^2+3b^2\right)(Ab-aB)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2A+4Ab^2-4abB\right)\cos(c+dx)\sin}{8a^3d}$$

$$= \frac{\left(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B\right)x}{8a^5} - \frac{\left(2a^2+3b^2\right)(Ab-aB)\sin(c}{3a^4d}$$

$$= \frac{\left(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B\right)x}{8a^5} - \frac{\left(2a^2+3b^2\right)(Ab-aB)\sin(c}{3a^4d}$$

$$= \frac{\left(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B\right)x}{8a^5} - \frac{\left(2a^2+3b^2\right)(Ab-aB)\sin(c}{3a^4d}$$

$$= \frac{\left(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B\right)x}{8a^5} - \frac{2b^4(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,t}{}\right.}{a^5\sqrt{a-b}\sqrt{a+ba}}$$

Mathematica [A] time = 0.639956, size = 202, normalized size = 0.84

$$\frac{12(c+dx)\left(4a^2Ab^2+3a^4A-4a^3bB-8ab^3B+8Ab^4\right)+24a^2\left(a^2A-abB+Ab^2\right)\sin(2(c+dx))+24a\left(3a^2+4b^2\right)(aB}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*(c + d*x) + (
192*b^4*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + 24*a*(3*a^2 + 4*b^2)*(-(A*b) + a*B)*Sin[c + d*x] + 24*a^2*
(a^2*A + A*b^2 - a*b*B)*Sin[2*(c + d*x)] + 8*a^3*(-(A*b) + a*B)*Sin[3*(c +
d*x)] + 3*a^4*A*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [B] time = 0.108, size = 1212, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -10/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*b-6/d/a^4/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*b^3-1/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*b^2+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*ta
n(1/2*d*x+1/2*c)^5*B*b-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c
)*B*b-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*A*b-1/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*B*b+3/4/d*A/a*arctan(tan(1/2*d
*x+1/2*c))+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*B*b^2+10
/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*B+1/d/a^3*arctan(tan
(1/2*d*x+1/2*c))*A*b^2-10/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^3*A*b-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A*b^3+1/
d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*A*b^2+1/d/a^3/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A*b^2+6/d/a^3/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^3*B*b^2+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)*B*b^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*
b-2/d*b^5/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*A+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))*B-1/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B*b-3/4/d/a/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A+10/3/d/a/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*B+5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)*A+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*B-2/d/a^
4*arctan(tan(1/2*d*x+1/2*c))*B*b^3+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*A*b^4
-5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A+2/d/a/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*B+3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*
tan(1/2*d*x+1/2*c)^5*A-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c
)*A*b^3-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b^2-2/d/a
^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b^3+1/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*B*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^7*B*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.669333, size = 1504, normalized size = 6.27

$$\left[\frac{3\left(3\,Aa^6-4\,Ba^5b+Aa^4b^2-4\,Ba^3b^3+4\,Aa^2b^4+8\,Bab^5-8\,Ab^6\right)dx-12\left(Bab^4-Ab^5\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/24*(3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 - 4*B*a^3*b^3 + 4*A*a^2*b^4 + 8*B
*a*b^5 - 8*A*b^6)*d*x - 12*(B*a*b^4 - A*b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos
(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c
) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + b^2)) + (16*B*a^6 - 16*A*a^5*b + 8*B*a^4*b^2 - 8*A*a^3*b^3 - 24*B*a^2*b^
4 + 24*A*a*b^5 + 6*(A*a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b
- B*a^4*b^2 + A*a^3*b^3)*cos(d*x + c)^2 + 3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^
2 + 4*B*a^3*b^3 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)
*d), 1/24*(3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 - 4*B*a^3*b^3 + 4*A*a^2*b^4 +
 8*B*a*b^5 - 8*A*b^6)*d*x + 24*(B*a*b^4 - A*b^5)*sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (16*B*a^
6 - 16*A*a^5*b + 8*B*a^4*b^2 - 8*A*a^3*b^3 - 24*B*a^2*b^4 + 24*A*a*b^5 + 6*
(A*a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b - B*a^4*b^2 + A*a^3
*b^3)*cos(d*x + c)^2 + 3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 + 4*B*a^3*b^3 - 4
*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20034, size = 867, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a^4 - 4*B*a^3*b + 4*A*a^2*b^2 - 8*B*a*b^3 + 8*A*b^4)*(d*x + c)
/a^5 + 48*(B*a*b^4 - A*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*
b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
```

```
b^2)))/(sqrt(-a^2 + b^2)*a^5) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a
^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*b*
tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a*b^2*tan
(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x
 + 1/2*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^2*b*tan(1/2*d*x + 1/
2*c)^5 - 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c
)^5 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 +
 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^
2*b*tan(1/2*d*x + 1/2*c)^3 + 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^3*tan(
1/2*d*x + 1/2*c)^3 - 15*A*a^3*tan(1/2*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x +
 1/2*c) + 24*A*a^2*b*tan(1/2*d*x + 1/2*c) + 12*B*a^2*b*tan(1/2*d*x + 1/2*c)
 - 12*A*a*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*A
*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

3.320 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=272

$$\frac{\left(2a^2Ab-3a^3B+2ab^2B-Ab^3\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-6a^2B+4aAb-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(2a^2Ab-3a^3B+4a\right.}{b^4a}$$

```
[Out] -((4*a*A*b - 6*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^2*(2*
a^2*A*b - 3*A*b^3 - 3*a^3*B + 4*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*b^
3 - 3*a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*
a^2*B + b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b -
 a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.86559, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4029, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(2a^2Ab-3a^3B+2ab^2B-Ab^3\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-6a^2B+4aAb-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(2a^2Ab-3a^3B+4a\right.}{b^4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```



```
[Out] -((4*a*A*b - 6*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^2*(2*
a^2*A*b - 3*A*b^3 - 3*a^3*B + 4*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*b^
3 - 3*a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*
a^2*B + b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b -
 a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
```

```
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec^2(c+dx)\left(2a(Ab-aB)-b(Ab-aB)\sec(c+dx)-\right.}{a+b\sec(c+dx}}{b\left(a^2-b^2\right.}$$

$$= -\frac{\left(2aAb-3a^2B+b^2B\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^2(c+dx)\,\mathrm{ta}}{b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2aAb-3a^2B+b^2B\right)\sec(c+}{2b^2\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2aAb-3a^2B+b^2B\right)\sec(c+}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2\right.}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2\right.}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^2\left(2a^2Ab-3Ab^3-3a^3B+\right.}{(a-b)^{3/2}}$$

Mathematica [A] time = 6.27154, size = 438, normalized size = 1.61

$$\frac{a^4B\sin(c+dx)-a^3Ab\sin(c+dx)}{b^3d(b-a)(a+b)(a\cos(c+dx)+b)} - \frac{2a^2\left(-2a^2Ab+3a^3B-4ab^2B+3Ab^3\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}\left(b^2-a^2\right)} + \frac{\left(-6a^2B+4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*(-2*a^2*A*b + 3*A*b^3 + 3*a^3*B - 4*a*b^2*B)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)*d) + ((4*
a*A*b - 6*a^2*B - b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*b^4*d
) + ((-4*a*A*b + 6*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/(2*b^4*d) + B/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - B/(4*b^2
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (A*b*Sin[(c + d*x)/2] - 2*a*B
*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (A*b*Sin
[(c + d*x)/2] - 2*a*B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (-(a^3*A*b*Sin[c + d*x]) + a^4*B*Sin[c + d*x])/(b^3*(-a + b)*(
a + b)*d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.098, size = 698, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)*A+2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*a^2/b
/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*A-6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+8/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-1/2/d*B/b^2
/(tan(1/2*d*x+1/2*c)+1)^2-1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*A+2/d/b^3/(tan(1/2
*d*x+1/2*c)+1)*B*a+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*B-2/d/b^3*ln(tan(1/2*d*
x+1/2*c)+1)*A*a+3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*B*a^2+1/2/d/b^2*ln(tan(1/2
*d*x+1/2*c)+1)*B+1/2/d*B/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^2/(tan(1/2*d*x+
1/2*c)-1)*A+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*B*a+1/2/d/b^2/(tan(1/2*d*x+1/2*c
)-1)*B+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*A*a-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1
)*B*a^2-1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 49.3933, size = 2969, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*((3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3)*cos(d*x + c)^3 +
 (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*cos(d*x + c)^2)*sqrt
(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
 c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 +
 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^3 + (6*B*a
^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 +
 B*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*B*a^7 - 4*A*a^6*b - 11*
B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)
^3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 -
4*A*a*b^6 + B*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(B*a^4*b^3 -
2*B*a^2*b^5 + B*b^7 - 2*(3*B*a^6*b - 2*A*a^5*b^2 - 5*B*a^4*b^3 + 3*A*a^3*b^
4 + 2*B*a^2*b^5 - A*a*b^6)*cos(d*x + c)^2 - (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*
B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/
((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^
9)*d*cos(d*x + c)^2), -1/4*(4*((3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3
*b^3)*cos(d*x + c)^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4
)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
 + a)/((a^2 - b^2)*sin(d*x + c))) - ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 +
```

```
8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^3 + (6*B*a^
6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 +
B*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*B*a^7 - 4*A*a^6*b - 11*B
*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^
3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4
*A*a*b^6 + B*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^4*b^3 - 2
*B*a^2*b^5 + B*b^7 - 2*(3*B*a^6*b - 2*A*a^5*b^2 - 5*B*a^4*b^3 + 3*A*a^3*b^4
 + 2*B*a^2*b^5 - A*a*b^6)*cos(d*x + c)^2 - (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*B
*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/(
(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9
)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.25255, size = 518, normalized size = 1.9

$$-\frac{\frac{4\left(3\,Ba^5-2\,Aa^4b-4\,Ba^3b^2+3\,Aa^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{4\left(Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aa^3b\tan\left(\frac{1}{2}\,d\right.\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(3*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 3*A*a^2*b^3)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - 4
*(B*a^4*tan(1/2*d*x + 1/2*c) - A*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^
5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (6*B*a^
2 - 4*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (6*B*a^2 - 4*
A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(4*B*a*tan(1/2*d*
x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 -
4*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c) + B*b*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

3.321 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(Ab-aB)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(Ab-2aB)\tanh^{-1}(s}{b^3d}$$

```
[Out] ((A*b - 2*a*B)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(a^2*A*b - 2*A*b^3 - 2
*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (B*Tan[c + d*x])/(b^2*d) - (a^2*(A*b -
a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.57818, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4028, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(Ab-aB)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(Ab-2aB)\tanh^{-1}(s}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(a^2*A*b - 2*A*b^3 - 2
*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (B*Tan[c + d*x])/(b^2*d) - (a^2*(A*b -
a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab(Ab-aB)-\left(a^2-b^2\right)(Ab-aB)\sec(c+d}{a+b\sec(c+dx)}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab^2(Ab-aB)-b(a}{a+b\sec(}}{b^3\left(a^2\right.}$$

$$= \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(Ab-2aB)\int \sec(c+dx}{b^3}$$

$$= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+}{b^2\left(a^2-b^2\right)d(a+b\sec(}$$

$$= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+}{b^2\left(a^2-b^2\right)d(a+b\sec(}$$

$$= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a\left(a^2Ab-2Ab^3-2a^3B+3ab^2B\right)\tanh^{-}}{(a-b)^{3/2}b^3(a+b)^{3/2}d}$$

Mathematica [A] time = 2.08354, size = 240, normalized size = 1.46

$$-\frac{2a\left(-a^2Ab+2a^3B-3ab^2B+2Ab^3\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a^2b(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} + 2aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - A*b*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 2*a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*b
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*a*B*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b +
 a*Cos[c + d*x])) + b*B*Tan[c + d*x])/(b^3*d)
```

Maple [B] time = 0.085, size = 510, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)*A-2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d*a/(a+b)/
(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*A+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-1/d/b^2/(tan(1/2*d*x
+1/2*c)+1)*B+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*A-2/d/b^3*ln(tan(1/2*d*x+1/2*
c)+1)*B*a-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*B-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)
*A+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 31.3625, size = 2433, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3)*cos(d*x + c)^2 + (2*
B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2
```

```
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) - ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3
+ 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*
b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*
cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a
*b^5 - A*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b^2 - 2*B*a^2
*b^4 + B*b^6 + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)), 1/2*(2*((2*B*a^5 - A*a^4*b
- 3*B*a^3*b^2 + 2*A*a^2*b^3)*cos(d*x + c)^2 + (2*B*a^4*b - A*a^3*b^2 - 3*B*
a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((2*B*a^6 - A*a^5*b -
4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^
5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x +
c))*log(sin(d*x + c) + 1) + ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3
 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3
*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1
) + 2*(B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6 + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b
^3 + A*a^2*b^4 + B*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5
 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c+dx))\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.26911, size = 545, normalized size = 3.32

$$\frac{2\left(2Ba^4 - Aa^3b - 3Ba^2b^2 + 2Aab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{-a^2+b^2}} - \frac{2\left(2Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - 2*(2*B*a^3
*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*
d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*b^3*tan(1/2*d*x + 1/2*c
)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^2*b
*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) + B*b^3*tan(1/2*d*x +
1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
```

```
*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) - (2*B*a - A*b)*log(abs(tan(1/2*d
*x + 1/2*c) + 1))/b^3 + (2*B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
3)/d
```

3.322 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$-\frac{2\left(a^3B-2ab^2B+Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{B\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) + (a*(A*b - a*B)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.300546, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4009, 3998, 3770, 3831, 2659, 208}

$$-\frac{2\left(a^3B-2ab^2B+Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{B\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) + (a*(A*b - a*B)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(-b(Ab-aB)+\left(a^2-b^2\right)B\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{B\int\sec(c+dx)\,dx}{b^2} - \frac{\left(Ab^3+a\left(a^2-2b^2\right)B\right)\int}{b^2\left(a^2-b^2\right.}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(Ab^3+a\left(a^2-2b^2\right)\right.}{b^3\left(a^2\right.}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(Ab^3+a\left(a^2-2b\right.\right.\right.}{}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2\left(Ab^3+a^3B-2ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} + \frac{}{b}$$

Mathematica [A] time = 0.697127, size = 191, normalized size = 1.46

$$\frac{\cos(c+dx)(A+B\sec(c+dx))\left(\frac{2\left(aB\left(a^2-2b^2\right)+Ab^3\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab(aB-Ab)\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)} - B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{b^2d(A\cos(c+dx)+B)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[
((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - B*Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + (a*b*(-(A*b) + a*B)*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*
x]))))/(b^2*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 0.079, size = 350, normalized size = 2.7

$$-2\frac{a\tan(1/2\,dx+c/2)\,A}{d\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}+2\frac{a^2\tan(1/2\,dx+c/2}{db\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)*A+2/d/b*a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^3/b^2/(a+b)/(a-b)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
B+4/d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*B*a+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/b^2*ln(tan(1/2*
d*x+1/2*c)-1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.57901, size = 1551, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((B*a^3*b - 2*B*a*b^3 + A*b^4 + (B*a^4 - 2*B*a^2*b^2 + A*a*b^3)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (B*a^4*b - 2*B*a^2*b^3 + B
*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1)
- (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4
)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 -
2*a^2*b^5 + b^7)*d), -1/2*(2*(B*a^3*b - 2*B*a*b^3 + A*b^4 + (B*a^4 - 2*B*a^
2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (B*a^4*b - 2*B*a^2*b^3 + B
*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4
)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 -
```

```
2*a^2*b^5 + b^7)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec{(c + dx)})\sec^2{(c + dx)}}{(a + b\sec{(c + dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.26215, size = 312, normalized size = 2.38

$$-\frac{\frac{2\left(Ba^3-2\,Bab^2+Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{-a^2+b^2}}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^
2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) - B*log(abs(tan(1/2*d*x + 1/2
*c) + 1))/b^2 + B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 - 2*(B*a^2*tan(1/2
*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d*x +
 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

3.323 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b
*Sec[c + d*x]))
```

Rubi [A] time = 0.13444, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b
*Sec[c + d*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = -\frac{(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{(-aA+bB)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(aA-bB)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(aA-bB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\right.\right.}{b\left(a^2-b^2\right)d}$$

$$= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.34591, size = 97, normalized size = 0.97

$$\frac{\frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - \frac{2(aA-bB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*(a*A - b*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c
 + d*x])))/d
```

Maple [A] time = 0.078, size = 132, normalized size = 1.3

$$\frac{1}{d}\left(2\frac{(Ab-Ba)\tan(1/2\,dx+c/2)}{\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)} + 2\frac{Aa-Bb}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(2*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh(
```

```
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.550038, size = 861, normalized size = 8.61

$$\left[\frac{\left(Aab - Bb^2 + \left(Aa^2 - Bab\right)\cos\left(dx + c\right)\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c) + 2\,a^2 - b}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right.}{2\left(\left(a^5 - 2\,a^3b^2 + ab^4\right)d\cos\left(dx + c\right) + \left(a^4b - 2\,a^2b^3 + b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((A*a*b - B*b^2 + (A*a^2 - B*a*b)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2
*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + b^2)) + 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*sin(d*x + c))/((a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d), ((A*a
*b - B*b^2 + (A*a^2 - B*a*b)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^3 - A*a^2
*b - B*a*b^2 + A*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c) + (a^4*b - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.22012, size = 232, normalized size = 2.32

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Aa-Bb)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}+\frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(A*a - B*b)/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) + (B*a*tan(1/2*d*x + 1/2*c) - A*b*tan(1/2*d*x + 1/2
*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 -
b^2)))/d
```

3.324 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{2\left(2a^2Ab+a^3(-B)-Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Ax}{a^2}$$

```
[Out] (A*x)/a^2 - (2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(A*b - a*B)*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.207455, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3923, 3919, 3831, 2659, 208}

$$-\frac{2\left(2a^2Ab+a^3(-B)-Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Ax}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (A*x)/a^2 - (2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(A*b - a*B)*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-A\left(a^2-b^2\right)+a(Ab-aB)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{Ax}{a^2} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2Ab-Ab^3-a^3B\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{Ax}{a^2} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2Ab-Ab^3-a^3B\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2b\left(a^2-b^2\right)}$$

$$= \frac{Ax}{a^2} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(2a^2Ab-Ab^3-a^3B\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,\right.}{a^2b\left(a^2-b^2\right)d}$$

$$= \frac{Ax}{a^2} - \frac{2\left(2a^2Ab-Ab^3-a^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.655569, size = 155, normalized size = 1.25

$$\frac{\frac{Ab\left(a^2-b^2\right)(c+dx)+aA\left(a^2-b^2\right)(c+dx)\cos(c+dx)-ab(aB-Ab)\sin(c+dx)}{a\cos(c+dx)+b} - \frac{2\left(-2a^2Ab+a^3B+Ab^3\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{a^2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/Sqrt[a^2 - b^2] + (A*b*(a^2 - b^2)*(c + d*x) + a*A*(a^2 - b^2)
*(c + d*x)*Cos[c + d*x] - a*b*(-(A*b) + a*B)*Sin[c + d*x])/(b + a*Cos[c + d
*x]))/(a^2*(a - b)*(a + b)*d)
```

Maple [B] time = 0.091, size = 328, normalized size = 2.7

$$2\,\frac{A\arctan(\tan(1/2\,dx+c/2))}{da^2} - 2\,\frac{b^2\tan(1/2\,dx+c/2)\,A}{ad\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)} + 2\,\frac{}{d\left(a^2-b^2\right)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*A/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*b/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-4/d*b/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A+2/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^3+2/d/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.585183, size = 1226, normalized size = 9.89

$$\left[\frac{2\left(Aa^5-2\,Aa^3b^2+Aab^4\right)dx\cos\left(dx+c\right)+2\left(Aa^4b-2\,Aa^2b^3+Ab^5\right)dx-\left(Ba^3b-2\,Aa^2b^2+Ab^4+\left(Ba^4-2\,Aa^3b\right.\right.}{2\left(\left(a^7-2\,a^5b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + 2*(A*a^4*b - 2*A
*a^2*b^3 + A*b^5)*d*x - (B*a^3*b - 2*A*a^2*b^2 + A*b^4 + (B*a^4 - 2*A*a^3*b
 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
 + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(B*a^4
*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((A*a^5 - 2*A*a^3*
b^2 + A*a*b^4)*d*x*cos(d*x + c) + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*d*x + (B*
a^3*b - 2*A*a^2*b^2 + A*b^4 + (B*a^4 - 2*A*a^3*b + A*a*b^3)*cos(d*x + c))*s
qrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*
sin(d*x + c))) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/
((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.21517, size = 271, normalized size = 2.19

$$\frac{\frac{2\left(Ba^3-2\,Aa^2b+Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{-a^2+b^2}}+\frac{(dx+c)A}{a^2}+\frac{2\left(Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) + (d*x + c)*A/a^2 + 2*(B*a*b*t
an(1/2*d*x + 1/2*c) - A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

3.325 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=180

$$\frac{\left(a^2A+abB-2Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2b\left(3a^2Ab-2a^3B+ab^2B-2Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\sin}{ad\left(a^2-b^2\right)(a+b}$$

```
[Out] -(((2*A*b - a*B)*x)/a^3) + (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a +
 b)^(3/2)*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.568946, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4030, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(a^2A+abB-2Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2b\left(3a^2Ab-2a^3B+ab^2B-2Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\sin}{ad\left(a^2-b^2\right)(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((2*A*b - a*B)*x)/a^3) + (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a +
 b)^(3/2)*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-a^2A+2Ab^2-abB+a(Ab-aB)\sec(c+dx)-b(Ab-\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A-2Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \int \frac{-\left(a^2-b^2\right)(2}{}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{\left(a^2A-2Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+a}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{\left(a^2A-2Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+a}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{\left(a^2A-2Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+a}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{2b\left(3a^2Ab-2Ab^3-2a^3B+ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} +$$

Mathematica [A] time = 1.09333, size = 221, normalized size = 1.23

$$\frac{(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(\frac{2b\left(-3a^2Ab+2a^3B-ab^2B+2Ab^3\right)\sec(c+dx)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab^2(aB-Ab)\tan}{(a-b)(a+b)}\right.}{a^3d(a+b\sec(c+dx))^2(A\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*((-2*A*b + a*B)*(c + d*x)*(b + a
*Cos[c + d*x])*Sec[c + d*x] + (2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*
B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]
)*Sec[c + d*x])/(a^2 - b^2)^(3/2) + (a*b^2*(-(A*b) + a*B)*Tan[c + d*x])/((a
 - b)*(a + b)) + a*A*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(a^3*d*(B + A*Cos[
c + d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.116, size = 453, normalized size = 2.5

$$2\,\frac{A\tan(1/2\,dx+c/2)}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\,\frac{A\arctan(\tan(1/2\,dx+c/2))\,b}{da^3}+2\,\frac{B\arctan(\tan(1/2\,dx+c/2))}{da^2}+2\,\frac{}{da^2\left(a^2-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^3*A*arctan(tan(
1/2*d*x+1/2*c))*b+2/d/a^2*B*arctan(tan(1/2*d*x+1/2*c))+2/d/a^2*b^3/(a^2-b^2
)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-
2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)*B+6/d/a*b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-4/d/a^3*b^4/(a+b)/(a-b)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-4/d*b/(a+
b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*B+2/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.660387, size = 1715, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*
b^5)*d*x*cos(d*x + c) + 2*(B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^
```

```
4 + B*a*b^5 - 2*A*b^6)*d*x + (2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5
 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a
^2*b^4 + 2*A*a*b^5 + (A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 +
a^3*b^5)*d), ((B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 -
2*A*a*b^5)*d*x*cos(d*x + c) + (B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^
2*b^4 + B*a*b^5 - 2*A*b^6)*d*x - (2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A
*b^5 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5 +
(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6
*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a + b\sec(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.39815, size = 505, normalized size = 2.81

$$-\frac{\frac{2\left(2Ba^3b-3Aa^2b^2-Bab^3+2Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{-a^2+b^2}}-\frac{2\left(Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-A\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(2*(2*B*a^3*b - 3*A*a^2*b^2 - B*a*b^3 + 2*A*b^4)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/((a^5 - a^3*b^2)*sqrt(-a^2 + b^2)) - 2*(A*a^3*
tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d
*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*
c)^3 - A*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*
tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x +
 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/
2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) - (B*a - 2*A*b)*(d*x + c)/a^3)/d
```

3.326 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(2a^2Ab+a^3(-B)+2ab^2B-3Ab^3\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2A+2abB-3Ab^2\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b^2\left(4a^2Ab-3a^3\right.}{}$$

```
[Out] ((a^2*A + 6*A*b^2 - 4*a*b*B)*x)/(2*a^4) - (2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a
^3*B + 2*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*a^2*A*b - 3*A*b^3 - a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.890839, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4030, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(2a^2Ab+a^3(-B)+2ab^2B-3Ab^3\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2A+2abB-3Ab^2\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b^2\left(4a^2Ab-3a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((a^2*A + 6*A*b^2 - 4*a*b*B)*x)/(2*a^4) - (2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a
^3*B + 2*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*a^2*A*b - 3*A*b^3 - a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
```

```
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(-a^2A+3Ab^2-2abB+a(Ab-aB)\sec(c+dx\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\cos(c+dx)\sin(c+}{a\left(a^2-b^2\right)d(a+b\sec(c+dx}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2abB\right)\cos(c+}{2a^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A\right.}{}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A\right.}{}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A\right.}{}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{2b^2\left(4a^2Ab-3Ab^3-3a^3B+2ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan}{\sqrt{}}\right.}{a^4(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 1.07366, size = 184, normalized size = 0.7

$$\frac{2(c+dx)\left(a^2A-4abB+6Ab^2\right)-\frac{8b^2\left(-4a^2Ab+3a^3B-2ab^2B+3Ab^3\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+a^2A\sin(2(c+dx))-\frac{4ab^3(aB-Ab)\cdots}{(a-b)(a+b)(a\cdots}}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2*A + 6*A*b^2 - 4*a*b*B)*(c + d*x) - (8*b^2*(-4*a^2*A*b + 3*A*b^3 + 3
*a^3*B - 2*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(3/2) + 4*a*(-2*A*b + a*B)*Sin[c + d*x] - (4*a*b^3*(-(A*b) + a*B
)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*A*Sin[2*(c + d
*x)])/(4*a^4*d)
```

Maple [B] time = 0.114, size = 651, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-4/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)^3*B+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)*A-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b+2/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B+1/d*A/a^2*arctan(tan(1/2*d*x
+1/2*c))+6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*A*b^2-4/d/a^3*arctan(tan(1/2*d*
x+1/2*c))*B*b-2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-8/d/a^2/(a+b)/(a-b)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*
b^3+6/d*b^5/a^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*b^2/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d*b^4/a^3/(a+b)/(a-b)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.740963, size = 2136, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a
^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + (A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3
 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x + (3*B*a^3*b^3 - 4
*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4
 + 3*A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*B*a
^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 10*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 +
 (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b -
4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*
x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 +
a^4*b^5)*d), 1/2*((A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3
*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + (A*a^6*b - 4*B*a^5*b^2 +
 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x + 2*(3
*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6 + (3*B*a^4*b^2 - 4*A*a^3*b^3
 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^6*b - 4*
A*a^5*b^2 - 6*B*a^4*b^3 + 10*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 + (A*a^7 -
 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b
^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
(a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.50577, size = 459, normalized size = 1.76

$$\frac{\frac{4\left(3Ba^3b^2-4Aa^2b^3-2Bab^4+3Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}+\frac{4\left(Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2)) + 4*
(B*a*b^3*tan(1/2*d*x + 1/2*c) - A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2
)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + (A*a^2 -
 4*B*a*b + 6*A*b^2)*(d*x + c)/a^4 - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*t
an(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/
2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 4*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^2*a^3))/d
```

3.327 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=346

$$\frac{\left(7a^2Ab^2+2a^4A-6a^3bB+9ab^3B-12Ab^4\right)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}+\frac{\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{\left(2a^2Ab+a^3\right.}{}$$

```
[Out] -((2*a^2*A*b + 8*A*b^3 - a^3*B - 6*a*b^2*B)*x)/(2*a^5) + (2*b^3*(5*a^2*A*b
- 4*A*b^3 - 4*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4*A + 7*a^2*A*b^2 -
12*A*b^4 - 6*a^3*b*B + 9*a*b^3*B)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - ((2
*a^2*A*b - 4*A*b^3 - a^3*B + 3*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(
a^2 - b^2)*d) + ((a^2*A - 4*A*b^2 + 3*a*b*B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2
- b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.27456, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4030, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(7a^2Ab^2+2a^4A-6a^3bB+9ab^3B-12Ab^4\right)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}+\frac{\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{\left(2a^2Ab+a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -((2*a^2*A*b + 8*A*b^3 - a^3*B - 6*a*b^2*B)*x)/(2*a^5) + (2*b^3*(5*a^2*A*b
- 4*A*b^3 - 4*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4*A + 7*a^2*A*b^2 -
12*A*b^4 - 6*a^3*b*B + 9*a*b^3*B)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - ((2
*a^2*A*b - 4*A*b^3 - a^3*B + 3*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(
a^2 - b^2)*d) + ((a^2*A - 4*A*b^2 + 3*a*b*B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2
- b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^3(c+dx)\left(-a^2A+4Ab^2-3abB+a(Ab-aB)\sec(c+a\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A-4Ab^2+3abB\right)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\cos^2(c+dx)\sin(c}{a\left(a^2-b^2\right)d(a+b\sec(c+a}$$

$$= -\frac{\left(2a^2Ab-4Ab^3-a^3B+3ab^2B\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-4Ab^2+3ab\right.}{3a^2}$$

$$= \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab^3B\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{\left(2a^2Ab-4Ab^3-a^3\right.}{}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab\right.}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab\right.}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab\right.}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{2b^3\left(5a^2Ab-4Ab^3-4a^3B+3ab^2B\right)\tanh}{a^5(a-b)^{3/2}(a+b)^{3/2}}$$

Mathematica [A] time = 1.37415, size = 224, normalized size = 0.65

$$\frac{6(c+dx)\left(-2a^2Ab+a^3B+6ab^2B-8Ab^3\right)+3a\left(3a^2A-8abB+12Ab^2\right)\sin(c+dx)+\frac{24b^3\left(-5a^2Ab+4a^3B-3ab^2B+4Ab^3\right)\tanh^{-1}\left(\right.}{\left(a^2-b^2\right)^{3/2}}}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (6*(-2*a^2*A*b - 8*A*b^3 + a^3*B + 6*a*b^2*B)*(c + d*x) + (24*b^3*(-5*a^2*A
*b + 4*A*b^3 + 4*a^3*B - 3*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 3*a*(3*a^2*A + 12*A*b^2 - 8*a*b*B)*Sin[c
 + d*x] + (12*a*b^4*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Co
s[c + d*x])) + 3*a^2*(-2*A*b + a*B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x
)])/(12*a^5*d)
```

Maple [B] time = 0.12, size = 926, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/a^3/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)^5*A*b^2-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*
x+1/2*c)^5*B-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B*b+4/
3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+12/d/a^4/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A*b^2-8/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^3*B*b+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d
*x+1/2*c)*A+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b^2-4/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*B*b-2/d/a^3/(1+tan(1/2*d
*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)*B-2/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))*b-8/d/a^5*arctan(ta
n(1/2*d*x+1/2*c))*A*b^3+1/d/a^2*B*arctan(tan(1/2*d*x+1/2*c))+6/d/a^4*arctan
(tan(1/2*d*x+1/2*c))*B*b^2+2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-2/d*b^4/a^3/(a^2-b^2)*tan(1/
2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+10/d/a^3
*b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*A-8/d*b^6/a^5/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-8/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d*b^5
/a^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.815434, size = 2592, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/6*(3*(B*a^8 - 2*A*a^7*b + 4*B*a^6*b^2 - 4*A*a^5*b^3 - 11*B*a^4*b^4 + 14*
A*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*cos(d*x + c) + 3*(B*a^7*b - 2*A*a^
6*b^2 + 4*B*a^5*b^3 - 4*A*a^4*b^4 - 11*B*a^3*b^5 + 14*A*a^2*b^6 + 6*B*a*b^7
 - 8*A*b^8)*d*x + 3*(4*B*a^3*b^4 - 5*A*a^2*b^5 - 3*B*a*b^6 + 4*A*b^7 + (4*B
*a^4*b^3 - 5*A*a^3*b^4 - 3*B*a^2*b^5 + 4*A*a*b^6)*cos(d*x + c))*sqrt(a^2 -
b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2)) + (4*A*a^7*b - 12*B*a^6*b^2 + 10*A*a^5*b^3 + 30
*B*a^4*b^4 - 38*A*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 - 2*A*a^6*
b^2 + A*a^4*b^4)*cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b^2 + 8*A*
a^5*b^3 + 3*B*a^4*b^4 - 4*A*a^3*b^5)*cos(d*x + c)^2 + (4*A*a^8 - 9*B*a^7*b
+ 4*A*a^6*b^2 + 18*B*a^5*b^3 - 20*A*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a^2*b^6)*c
os(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (
```

```
a^9*b - 2*a^7*b^3 + a^5*b^5)*d), 1/6*(3*(B*a^8 - 2*A*a^7*b + 4*B*a^6*b^2 -
4*A*a^5*b^3 - 11*B*a^4*b^4 + 14*A*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*co
s(d*x + c) + 3*(B*a^7*b - 2*A*a^6*b^2 + 4*B*a^5*b^3 - 4*A*a^4*b^4 - 11*B*a^
3*b^5 + 14*A*a^2*b^6 + 6*B*a*b^7 - 8*A*b^8)*d*x - 6*(4*B*a^3*b^4 - 5*A*a^2*
b^5 - 3*B*a*b^6 + 4*A*b^7 + (4*B*a^4*b^3 - 5*A*a^3*b^4 - 3*B*a^2*b^5 + 4*A*
a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
 c) + a)/((a^2 - b^2)*sin(d*x + c))) + (4*A*a^7*b - 12*B*a^6*b^2 + 10*A*a^5
*b^3 + 30*B*a^4*b^4 - 38*A*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 -
 2*A*a^6*b^2 + A*a^4*b^4)*cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b
^2 + 8*A*a^5*b^3 + 3*B*a^4*b^4 - 4*A*a^3*b^5)*cos(d*x + c)^2 + (4*A*a^8 - 9
*B*a^7*b + 4*A*a^6*b^2 + 18*B*a^5*b^3 - 20*A*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a
^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x
 + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49033, size = 639, normalized size = 1.85

$$-\frac{\frac{12\left(4\,Ba^3b^3-5\,Aa^2b^4-3\,Bab^5+4\,Ab^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-a^5b^2\right)\sqrt{-a^2+b^2}}+\frac{12\left(Bab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^5\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(4*B*a^3*b^3 - 5*A*a^2*b^4 - 3*B*a*b^5 + 4*A*b^6)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2)) +
12*(B*a*b^4*tan(1/2*d*x + 1/2*c) - A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*
b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 3*(B*
a^3 - 2*A*a^2*b + 6*B*a*b^2 - 8*A*b^3)*(d*x + c)/a^5 - 2*(6*A*a^2*tan(1/2*d
*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a*b*tan(1/2*d*x + 1/2*
c)^5 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^2*tan(1/2*d*x + 1/2*c)^5 +
4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^2
*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*
x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 12*B*a*b*tan(1/2*d*x + 1/2*c) +
 18*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4))/d
```

3.328 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=407

$$\frac{\left(-11a^2Ab^3+6a^4Ab+21a^3b^2B-12a^5B-6ab^4B+2Ab^5\right)\tan(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{\left(-12a^2B+6aAb-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^5d}$$

```
[Out] -((6*a*A*b - 12*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(2*b^5*d) + (a^2*(6*a
^4*A*b - 15*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a +
b)^(5/2)*d) + ((6*a^4*A*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*
B - 6*a*b^4*B)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*
b^3 - 6*a^4*B + 10*a^2*b^2*B - b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^3*(a^
2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(2*b*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B
)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.95939, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {4029, 4098, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(-11a^2Ab^3+6a^4Ab+21a^3b^2B-12a^5B-6ab^4B+2Ab^5\right)\tan(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{\left(-12a^2B+6aAb-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -((6*a*A*b - 12*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(2*b^5*d) + (a^2*(6*a
^4*A*b - 15*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a +
b)^(5/2)*d) + ((6*a^4*A*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*
B - 6*a*b^4*B)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*
b^3 - 6*a^4*B + 10*a^2*b^2*B - b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^3*(a^
2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(2*b*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B
)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)\left(3a(Ab-aB)-2b(Ab-aB)\sec(c+dx\right.}{(a+b\sec(c+dx}}{2b\left(a^2-b^2\right.}$$

$$= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2Ab-5Ab^3-4a^3B+7ab^2B\right)\text{se}}{2b^2\left(a^2-b^2\right)^2d(a+b\,\text{se}}$$

$$= -\frac{\left(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B\right)\sec(c+dx)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2d} + \frac{a(Ab}{2b\,(}$$

$$= \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B\right)\tan(c+dx)}{2b^4\left(a^2-b^2\right)^2d} - \frac{(3a^3}{}$$

$$= \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B\right)\tan(c+dx)}{2b^4\left(a^2-b^2\right)^2d} - \frac{(3a^3}{}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-\right.}{2b}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-\right.}{2b}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{a^2\left(6a^4Ab-15a^2Ab^3+12A\right.}{}$$

Mathematica [A] time = 2.9472, size = 507, normalized size = 1.25

$$\frac{\frac{16a^2\left(15a^2Ab^3-6a^4Ab-29a^3b^2B+12a^5B+20ab^4B-12Ab^5\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - 8\left(12a^2B-6aAb+b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
20*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) - 8*(-6*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 8*(-6*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*b*(18*a^5*A*b^2 - 32*a^3*A*b^4 + 8*a*A*b^6 - 36*a^6*b*B + 68*
a^4*b^3*B - 30*a^2*b^5*B + 4*b^7*B + (18*a^6*A*b - 25*a^4*A*b^3 - 10*a^2*A*
b^5 + 8*A*b^7 - 36*a^7*B + 47*a^5*b^2*B + 14*a^3*b^4*B - 16*a*b^6*B)*Cos[c
+ d*x] - 2*a*b*(-9*a^4*A*b + 16*a^2*A*b^3 - 4*A*b^5 + 18*a^5*B - 32*a^3*b^2
*B + 11*a*b^4*B)*Cos[2*(c + d*x)] + 6*a^6*A*b*Cos[3*(c + d*x)] - 11*a^4*A*b
^3*Cos[3*(c + d*x)] + 2*a^2*A*b^5*Cos[3*(c + d*x)] - 12*a^7*B*Cos[3*(c + d*
x)] + 21*a^5*b^2*B*Cos[3*(c + d*x)] - 6*a^3*b^4*B*Cos[3*(c + d*x)])*Sec[c +
 d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(16*b^5*d)
```

Maple [B] time = 0.106, size = 1599, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-12/d*a^7
/b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*B+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-20/d*a^3/b/(a^4
-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*B+10/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a
^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*
tan(1/2*d*x+1/2*c)*A-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-4/d*a^5/b^3/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*A+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-10/d*a^
4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+8/d*a^3/b
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*A+6/d*a^6/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d*a^6/b^4/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*B+1/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d*B/b^3/(tan(1/2*d*x+1/2
*c)+1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*A+1/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*
B-1/2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*B
+1/2/d*B/b^3/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*A+3/d/
b^4*ln(tan(1/2*d*x+1/2*c)-1)*A*a-6/d/b^5*ln(tan(1/2*d*x+1/2*c)-1)*B*a^2+3/d
/b^4/(tan(1/2*d*x+1/2*c)+1)*B*a-3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*A*a+6/d/b^
5*ln(tan(1/2*d*x+1/2*c)+1)*B*a^2+3/d/b^4/(tan(1/2*d*x+1/2*c)-1)*B*a+12/d*a^
2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 171.058, size = 5414, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(((12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4
- 12*A*a^4*b^5)*cos(d*x + c)^4 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3
 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c)^3 + (12*B*a^7*b
^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b^
7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - ((12*B*a^10 - 6
*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*
a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*A*a^8
*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*
b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*b^3 -
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((12*B*a^10 - 6
*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*
a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*A*a^8
*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*
b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*b^3 -
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^6*b^4 -
 3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10 - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*
b^3 + 17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^
8)*cos(d*x + c)^3 - (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^5*b
^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c)^2
 - 2*(2*B*a^7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3
*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^5 - 3*
a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d*cos(d*x + c)^4 + 2*(a^7*b^6 - 3*a^5*b^8 +
 3*a^3*b^10 - a*b^12)*d*cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11
- b^13)*d*cos(d*x + c)^2), -1/4*(2*((12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 +
15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5)*cos(d*x + c)^4 + 2*(12*B*a^8*b
- 6*A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*
cos(d*x + c)^3 + (12*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5
+ 20*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((12*B*a^1
0 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 -
 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*
A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B
*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*
b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b
^8 + 6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((12*B*a^1
0 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 -
 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*
A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B
*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*
b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b
^8 + 6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^6*
b^4 - 3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10 - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B
*a^7*b^3 + 17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a
^2*b^8)*cos(d*x + c)^3 - (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*
a^5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x +
 c)^2 - 2*(2*B*a^7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^
7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^5
 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d*cos(d*x + c)^4 + 2*(a^7*b^6 - 3*a^5*
b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 + 3*a^2*
b^11 - b^13)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^5(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.70943, size = 1878, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(12*B*a^7 - 6*A*a^6*b - 29*B*a^5*b^2 + 15*A*a^4*b^3 + 20*B*a^3*b^4
- 12*A*a^2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(
-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4
*b^5 - 2*a^2*b^7 + b^9)*sqrt(-a^2 + b^2)) - 2*(12*B*a^7*tan(1/2*d*x + 1/2*c
)^7 - 6*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^7
+ 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7
+ 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7
- 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7
+ 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7
+ 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*A
*b^7*tan(1/2*d*x + 1/2*c)^7 + B*b^7*tan(1/2*d*x + 1/2*c)^7 - 36*B*a^7*tan(1
/2*d*x + 1/2*c)^5 + 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^6*b*tan(1/2*
d*x + 1/2*c)^5 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 67*B*a^5*b^2*tan(1/2*
d*x + 1/2*c)^5 - 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*B*a^4*b^3*tan(1/2
*d*x + 1/2*c)^5 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 26*B*a^3*b^4*tan(1/
2*d*x + 1/2*c)^5 + 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 5*B*a^2*b^5*tan(1/
2*d*x + 1/2*c)^5 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 4*B*a*b^6*tan(1/2*d*x
 + 1/2*c)^5 - 2*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^7*tan(1/2*d*x + 1/2*c)
^5 + 36*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 +
18*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67
*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 2
9*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 +
26*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 +
 5*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*
B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^3 + 3*B*b^7*t
an(1/2*d*x + 1/2*c)^3 - 12*B*a^7*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b*tan(1/2*d
*x + 1/2*c) - 18*B*a^6*b*tan(1/2*d*x + 1/2*c) + 9*A*a^5*b^2*tan(1/2*d*x + 1
/2*c) + 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c) - 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c
) + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c) - 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c) +
 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c) - 13*B
*a^2*b^5*tan(1/2*d*x + 1/2*c) + 4*A*a*b^6*tan(1/2*d*x + 1/2*c) - 4*B*a*b^6*
tan(1/2*d*x + 1/2*c) + 2*A*b^7*tan(1/2*d*x + 1/2*c) + B*b^7*tan(1/2*d*x + 1
/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d
```

```
*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) - (12*B*a^2 - 6*A*a*
b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 + (12*B*a^2 - 6*A*a*b + B
*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5)/d
```

3.329 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$-\frac{\left(-3a^2B+aAb+2b^2B\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2Ab^3+2a^4Ab+15a^3b^2B-6a^5B-12ab^4B+6Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((A*b - 3*a*B)*ArcTanh[Sin[c + d*x]])/(b^4*d) - (a*(2*a^4*A*b - 5*a^2*A*b^3
 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b -
 3*a^2*B + 2*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Se
c[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^
2*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.42402, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4029, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-3a^2B+aAb+2b^2B\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2Ab^3+2a^4Ab+15a^3b^2B-6a^5B-12ab^4B+6Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((A*b - 3*a*B)*ArcTanh[Sin[c + d*x]])/(b^4*d) - (a*(2*a^4*A*b - 5*a^2*A*b^3
 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b -
 3*a^2*B + 2*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Se
c[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^
2*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
```

```
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec^2(c+dx)\left(2a(Ab-aB)-2b(Ab-aB)\sec(c+dx)-(a\ldots\right.}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\tan(c\ldots}{2b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))\ldots}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\ldots}{\ldots}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\ldots}{\ldots}$$

$$= \frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-\ldots}{2b\left(a^2\ldots\right.}$$

$$= \frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-\ldots}{2b\left(a^2\ldots\right.}$$

$$= \frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a\left(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2\ldots\right.}{(a-b)^{5/2}b^4(a\ldots}$$

Mathematica [A] time = 6.47046, size = 418, normalized size = 1.45

$$\frac{a^2Ab\sin(c+dx)-a^3B\sin(c+dx)}{2b^2d(b-a)(a+b)(a\cos(c+dx)+b)^2} + \frac{5a^2Ab^3\sin(c+dx)-2a^4Ab\sin(c+dx)-7a^3b^2B\sin(c+dx)+4a^5B\sin(c+dx\ldots}{2b^3d(b-a)^2(a+b)^2(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B
)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2
]*(-a^2 + b^2)^2*d) + ((-(A*b) + 3*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]])/(b^4*d) + ((A*b - 3*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(
b^4*d) + (B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + (B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^
2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(2*b^2*(-a + b)*(a + b)*d*(b + a*C
os[c + d*x])^2) + (-2*a^4*A*b*Sin[c + d*x] + 5*a^2*A*b^3*Sin[c + d*x] + 4*a
^5*B*Sin[c + d*x] - 7*a^3*b^2*B*Sin[c + d*x])/(2*b^3*(-a + b)^2*(a + b)^2*d
*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.097, size = 1406, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a
^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^4/b^2/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*B+8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^4/b^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+
1/2*c)*A-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+4/d*a^5/b^3/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*B+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*a^5/b^3/(a^4-2*
a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*A+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6
/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+12/d*a^2/
(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*B-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*B+1/d/b^3*ln(tan(1/2*d*
x+1/2*c)+1)*A-3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*B*a-1/d/b^3/(tan(1/2*d*x+1/2
*c)-1)*B-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*A+3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1
)*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 114.913, size = 4591, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(((6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 -
6*A*a^3*b^5)*cos(d*x + c)^3 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5
```

```
*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + (6*B*a^6*b^2 - 2*
A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*((3*B*a^9 - A*a^8*b - 9*B*
a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7
)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9
*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7
*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 -
3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((3*B*a^9 - A*a^
8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 +
 A*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a
^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2
+ (3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*
a^2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*B*
a^6*b^3 - 6*B*a^4*b^5 + 6*B*a^2*b^7 - 2*B*b^9 + (6*B*a^8*b - 2*A*a^7*b^2 -
17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7)*cos(
d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B
*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 -
 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^
7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^1
0 - b^12)*d*cos(d*x + c)), 1/2*(((6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*
a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5)*cos(d*x + c)^3 + 2*(6*B*a^7*b - 2*A*a
^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x +
 c)^2 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*
b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((3*B*a^9 - A*a^8*b - 9*B*
a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7
)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9
*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7
*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 -
3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((3*B*a^9 - A*a^8*
b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A
*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5
*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 +
(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^
2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*B*a^6*
b^3 - 6*B*a^4*b^5 + 6*B*a^2*b^7 - 2*B*b^9 + (6*B*a^8*b - 2*A*a^7*b^2 - 17*B
*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7)*cos(d*x
+ c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B*a^3
*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a
^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 +
3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 -
b^12)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.58403, size = 784, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a*b
^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(-a^2 + b^2)) - (4*B*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^5*b
*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*b^3*tan
(1/2*d*x + 1/2*c)^3 + 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^4*tan(
1/2*d*x + 1/2*c)^3 - 4*B*a^6*tan(1/2*d*x + 1/2*c) + 2*A*a^5*b*tan(1/2*d*x +
 1/2*c) - 5*B*a^5*b*tan(1/2*d*x + 1/2*c) + 3*A*a^4*b^2*tan(1/2*d*x + 1/2*c)
 + 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 5*A*a^3*b^3*tan(1/2*d*x + 1/2*c) + 8*
B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^
3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 -
 a - b)^2) - (3*B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (3*B*a
- A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*B*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

3.330 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=220

$$\frac{\left(a^2Ab^3+5a^3b^2B-2a^5B-6ab^4B+2Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2Ab-3\ldots\right.}{2b^2d\left(\ldots\right.}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3
*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*(A*b - a*B)*Tan[c + d*x])/(2*b^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*
a*b^2*B)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.686463, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4028, 4080, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(a^2Ab^3+5a^3b^2B-2a^5B-6ab^4B+2Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2Ab-3\ldots\right.}{2b^2d\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3
*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*(A*b - a*B)*Tan[c + d*x])/(2*b^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*
a*b^2*B)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(-2ab(Ab-aB)-\left(a^2-2b^2\right)(Ab-aB)\sec\ldots}{(a+b\sec(c+dx)\ldots}}{2b^2\left(a^2-b\ldots\right.}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\tan\ldots}{2b^2\left(a^2-b^2\right)^2d(a+b\sec(c+d\ldots}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right)\tan\ldots}{2b^2\left(a^2-b^2\right)^2d(a+b\sec(c+d\ldots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2Ab-4A\ldots\right.}{2b^2\left(a^2\ldots\right.}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2Ab-4A\ldots\right.}{2b^2\left(a^2\ldots\right.}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{\left(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B\right)\tanh^{-1}\ldots}{(a-b)^{5/2}b^3(a+b)^{5/2}d}$$

Mathematica [A] time = 1.82697, size = 270, normalized size = 1.23

$$\frac{\cos(c+dx)(A+B\sec(c+dx))\left(\frac{ab\left(-2a^3B+5ab^2B-3Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)}+\frac{2\left(-a^2Ab^3-5a^3b^2B+2a^5B+6ab^4B-2Ab^5\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{ab}{(b-a}\cdots\right.}{2b^3d(A\cos(c+d\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B -
5*a^3*b^2*B + 6*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(5/2) - 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*B*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*
x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])^2) + (a*b*(-3*A*b^3 - 2*a^3*B +
5*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/(2*b^
3*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 0.095, size = 1085, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d*a^4/b^2
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*B+1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-2/d*a^4/b^2/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*
x+1/2*c)*B-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/
(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2*a^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^2/(a^4-2*a^2
*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*A+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+5/
d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a+1/d/b^3*ln(tan(1/
2*d*x+1/2*c)+1)*B-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 43.5527, size = 3051, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*((2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 + (2*B*
a^7 - 5*B*a^5*b^2 - A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5)*cos(d*x + c)^2 +
 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(B*a^6*b^2 - 3*B*a^4*b^4 +
 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*cos(
d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c)
)*log(sin(d*x + c) + 1) + 2*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8
+ (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*cos(d*x + c)^2 + 2*(B*a^7
*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c))*log(-sin(d*x + c) +
 1) + 2*(3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6
- 4*A*a*b^7 + (2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^
2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*
b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(
d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*B*a^5*b^2
- 5*B*a^3*b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 + (2*B*a^7 - 5*B*a^5*b^2 -
A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*
a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) - (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 +
 3*B*a^4*b^4 - B*a^2*b^6)*cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a
^3*b^5 - B*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + (B*a^6*b^2 - 3*B*a^
4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^
6)*cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d
*x + c))*log(-sin(d*x + c) + 1) + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 +
5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7 + (2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*
b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*
a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 +
3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^1
1)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec (c + dx))\sec^3 (c + dx)}{(a + b\sec (c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.51929, size = 656, normalized size = 2.98

$$-\frac{\frac{\left(2\,Ba^5-5\,Ba^3b^2-Aa^2b^3+6\,Bab^4-2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{-a^2+b^2}}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}+\frac{B\log\left(\left|\tan\left(\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((2*B*a^5 - 5*B*a^3*b^2 - A*a^2*b^3 + 6*B*a*b^4 - 2*A*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a
^2 + b^2)) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + B*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/b^3 - (2*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^4*b*tan(1/2
*d*x + 1/2*c)^3 + A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^2*tan(1/2*d*
x + 1/2*c)^3 + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^3 - 4*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^5*tan(1/2*d*x + 1/2*
c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*B*
a^3*b^2*tan(1/2*d*x + 1/2*c) - 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*B*a^2*b
^3*tan(1/2*d*x + 1/2*c) - 4*A*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2
*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2
))/d
```

3.331 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a(Ab-aB)}{2bd\left(a^2-b^2\right)(a}$$

```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Tan[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*
a*b^2*B)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.335514, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4009, 4003, 12, 3831, 2659, 208}

$$-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a(Ab-aB)}{2bd\left(a^2-b^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Tan[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*
a*b^2*B)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(-2b(Ab-aB)+\left(aAb+a^2B-2b^2B\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= -\frac{\left(3aAb-a^2B-2b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Ab-aB)\tan(c+dx}{2b\left(a^2-b^2\right)d(a+b\sec(c+}$$

Mathematica [A] time = 0.677794, size = 157, normalized size = 0.87

$$\frac{\frac{\left(2a^2A-3abB+Ab^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{2\left(a^2B-3aAb+2b^2B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a
+ b)*(b + a*Cos[c + d*x])^2) + ((2*a^2*A + A*b^2 - 3*a*b*B)*Sin[c + d*x])/(
```

```
(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*d)
```

Maple [A] time = 0.084, size = 238, normalized size = 1.3

$$\frac{1}{d}\left(2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^2}\left(-1/2\,\frac{\left(2\,a^2A+Aab+2\,Ab^2-Ba^2-4\,Bab\right)(\tan(1/2\,d}{(a-b)\left(a^2+2\,ab+b^2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(2*(-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-4*B*a*b)/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3+1/2*(2*A*a^2-A*a*b+2*A*b^2+B*a^2-4*B*a*b)/(a+b)/(a^2-2*a
*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^2-(3*A*a*b-B*a^2-2*B*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.656479, size = 1631, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*((B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4 + (B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2)*
cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2
*b^3 + 4*B*a*b^4 - 2*A*b^5 + (2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3
 - A*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((B*a^2*b^2 - 3*A*
a*b^3 + 2*B*b^4 + (B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2)*cos(d*x + c)^2 + 2*(B*a
^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^5 + A*
a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*b^4 - 2*A*b^5 + (2*A*a^5 - 3*B*a^4*
b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 -
```

```
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + dx))*sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.44321, size = 540, normalized size = 3.

$$\frac{\frac{(Ba^2-3\,Aab+2\,Bb^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}-\frac{2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*A*a^3*tan(1/2*d*x +
 1/2*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
 3*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1
/2*d*x + 1/2*c) - B*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c)
 + 3*B*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*tan(1/2*d*x + 1/2*c) + 4*B*a*b^
2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 +
b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.332 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(Ab-aB)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Tan[c + d*x])/(2*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Tan[c + d*
x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.264168, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(Ab-aB)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Tan[c + d*x])/(2*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Tan[c + d*
x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2(aA-bB)+(Ab-aB)\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{(2a^2A}{}}{}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(2a^2A}{}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(2a^2A}{}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(2a^2A}{}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx}$$

Mathematica [A] time = 0.855897, size = 172, normalized size = 1.05

$$\frac{\frac{\left(-4a^2Ab+2a^3B+ab^2B+Ab^3\right)\sin(c+dx)}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{2\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b(Ab-aB)\sin(c+dx)}{a(a-b)(a+b)(a\cos(c+dx)+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a - b)*(a
+ b)*(b + a*Cos[c + d*x])^2) + ((-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*Si
n[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*d)
```

Maple [A] time = 0.084, size = 236, normalized size = 1.4

$$\frac{1}{d}\left(-2\frac{1}{\left(\left(\tan(1/2\,dx+c/2)\right)^2a-\left(\tan(1/2\,dx+c/2)\right)^2b-a-b\right)^2}\left(-1/2\,\frac{\left(4\,Aab+Ab^2-2\,Ba^2-Bab-2\,Bb^2\right)\left(\tan(1/2\,dx\right.}{(a-b)\left(a^2+2\,ab+b^2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.665143, size = 1631, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4 + (2*A*a^4 - 3*B*a^3*b + A*a^2*b^2)*
cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3
*A*a*b^4 - 2*B*b^5 + (2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b
^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((2*A*a^2*b^2 - 3*
B*a*b^3 + A*b^4 + (2*A*a^4 - 3*B*a^3*b + A*a^2*b^2)*cos(d*x + c)^2 + 2*(2*A
*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^4*b -
3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5 + (2*B*a^5 - 4*A*a^4*b - B*a^
3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.47987, size = 539, normalized size = 3.29

$$\frac{\left(2\,Aa^2-3\,Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*B*a^3*tan(1/2*d*x +
 1/2*c)^3 - 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*
b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1
/2*d*x + 1/2*c) + 4*A*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^2*b*tan(1/2*d*x + 1/
2*c) + 3*A*a*b^2*tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) - A*b^
3*tan(1/2*d*x + 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 +
b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.333 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=205

$$-\frac{\left(-5a^2Ab^3+6a^4Ab-a^3b^2B-2a^5B+2Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2Ab-3a^3B-2Ab^3\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{}{2}$$

```
[Out] (A*x)/a^3 - ((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)
^(5/2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Tan[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.536315, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3923, 4060, 3919, 3831, 2659, 208}

$$-\frac{\left(-5a^2Ab^3+6a^4Ab-a^3b^2B-2a^5B+2Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2Ab-3a^3B-2Ab^3\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] (A*x)/a^3 - ((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)
^(5/2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Tan[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2A\left(a^2-b^2\right)+2a(Ab-aB)\sec(c+dx)-b(Ab-aB)\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\int \frac{2A\left(a^2-b^2\right)^2-a(\cdots}{\cdots}}{\cdots}$$

$$= \frac{Ax}{a^3} + \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} - \frac{\left(6a^4Ab\cdots\right.}{\cdots}$$

$$= \frac{Ax}{a^3} + \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} - \frac{\left(6a^4Ab\cdots\right.}{\cdots}$$

$$= \frac{Ax}{a^3} + \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} - \frac{\left(6a^4Ab\cdots\right.}{\cdots}$$

$$= \frac{Ax}{a^3} - \frac{\left(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b(Ab-a\cdots}{2a\left(a^2-b^2\right)d\cdots}$$

Mathematica [A] time = 1.41674, size = 267, normalized size = 1.3

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(-\frac{ab\left(-6a^2Ab+4a^3B-ab^2B+3Ab^3\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} - \frac{2\left(5a^2Ab^3-6a^4Ab+a^3b^2B+\cdots\right.}{\cdots}\right.}{2a^3d(a+b\sec(c+dx))^3(A\cos(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*(2*A*(c + d*x)*(b
 + a*Cos[c + d*x])^2 - (2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a
^3*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c
 + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b
)*(a + b)) - (a*b*(-6*a^2*A*b + 3*A*b^3 + 4*a^3*B - a*b^2*B)*(b + a*Cos[c +
 d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(B + A*Cos[c + d*x])*
(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.099, size = 1063, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/a/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*a/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*B+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-
1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a-b)
^2*tan(1/2*d*x+1/2*c)*A-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-4/d*a/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*ta
n(1/2*d*x+1/2*c)*B-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+5/d/a/(a^4-2*a^2*b^2+b^4)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*
b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^5+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*B*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.750819, size = 2479, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2
+ 8*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 4*(A
*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x - (2*B*a^5*b^2 - 6*A*a^4*
b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 6*A*a^6*b + B*a^5*b^2
+ 5*A*a^4*b^3 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*a^5*b^2 +
B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d
*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) - 2*(3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 -
 2*A*a*b^7 + (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b
^5 - 3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^
4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7
)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(
A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2 + 4*(A*a^
7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 2*(A*a^6*b^2
- 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x + (2*B*a^5*b^2 - 6*A*a^4*b^3 + B*a
^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4
*b^3 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3
 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*B*a^6*b^2 -
5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A*a*b^7 + (4*B*a^7*b - 6*A*a^6*
b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 - 3*A*a^2*b^6)*cos(d*x + c))*si
n(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*
(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^
7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.43989, size = 617, normalized size = 3.01

$$\frac{\left(2\,Ba^5-6\,Aa^4b+Ba^3b^2+5\,Aa^2b^3-2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}+\frac{(dx+c)A}{a^3}+\frac{4\,Ba^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,Aa^3b}{}$$

$$\frac{\qquad}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^
2 + b^2)) + (d*x + c)*A/a^3 + (4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^
3*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*tan
(1/2*d*x + 1/2*c)^3 - 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^4*b*tan(1/2*d*
x + 1/2*c) + 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 3*B*a^3*b^2*tan(1/2*d*x + 1
/2*c) + 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + B*a^2*b^3*tan(1/2*d*x + 1/2*c) -
 3*A*a*b^4*tan(1/2*d*x + 1/2*c) - 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a
^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a
- b)^2))/d
```

3.334 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=290

$$\frac{\left(-11a^2Ab^2+2a^4A+5a^3bB-2ab^3B+6Ab^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-15a^2Ab^3+12a^4Ab+5a^3b^2B-6a^5B-2ab^4B+6Ab^5\right)\text{t}}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] -(((3*A*b - a*B)*x)/a^4) + (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*
B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4*A - 11*a^2*A*b^2 + 6*A
*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A
*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b*(6*
a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.53495, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-11a^2Ab^2+2a^4A+5a^3bB-2ab^3B+6Ab^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-15a^2Ab^3+12a^4Ab+5a^3b^2B-6a^5B-2ab^4B+6Ab^5\right)\text{t}}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -(((3*A*b - a*B)*x)/a^4) + (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*
B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4*A - 11*a^2*A*b^2 + 6*A
*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A
*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b*(6*
a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
```

```
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a^2A+3Ab^2-abB+2a(Ab-aB)\sec(c+dx)-2\right.}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(6a^2Ab-3Ab^3-4a^3B+ab^2B\right)\sin(c+dx}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b(Ab-aB)\sin(}{2a\left(a^2-b^2\right)d(a+b\,\mathrm{se}}$$

$$= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{}{2a(}$$

$$= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{}{2a(}$$

$$= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{}{2a(}$$

$$= -\frac{(3Ab-aB)x}{a^4} + \frac{b\left(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B\right)\tanh^{}}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 1.96861, size = 306, normalized size = 1.06

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(\frac{ab^2\left(-8a^2Ab+6a^3B-3ab^2B+5Ab^3\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} - \frac{2b\left(-15a^2Ab^3+12a^4Ab+5a\right.}{}\right.}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*(2*(-3*A*b + a*B)
*(c + d*x)*(b + a*Cos[c + d*x])^2 - (2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b
^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*(A*b -
 a*B)*Sin[c + d*x])/((a - b)*(a + b)) + (a*b^2*(-8*a^2*A*b + 5*A*b^3 + 6*a^
3*B - 3*a*b^2*B)*(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) +
 2*a*A*(b + a*Cos[c + d*x])^2*Sin[c + d*x]))/(2*a^4*d*(B + A*Cos[c + d*x])*
(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.123, size = 1349, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^4*A*arctan(tan(
1/2*d*x+1/2*c))*b+2/d/a^3*B*arctan(tan(1/2*d*x+1/2*c))+8/d/a/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*A+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*b^5/a^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*A-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a
-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*
B+2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d/a^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*
d*x+1/2*c)*A+4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-1/d*b^3/a/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x
+1/2*c)*B-2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-15/d*b^4/
a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*A+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-6/d*b/(a^4-2*a^2*
b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*B*a+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.916193, size = 3394, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*(4*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*
a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*cos(d*x + c)^2 + 8*(B*a^8*b - 3*A*a^
7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 +
 3*A*a*b^8)*d*x*cos(d*x + c) + 4*(B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9
*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*d*x - (6*B*a^5*
b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^7 - 6*A*b^8 + (6*
B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2
```

```
*b^6)*cos(d*x + c)^2 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4 + 15*A*a
^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*
cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
+ c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + b^2)) + 2*(2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 1
7*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a^5*b^
4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9
*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*
x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^
11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*
b^4 + 3*a^6*b^6 - a^4*b^8)*d), 1/2*(2*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*
A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*cos(d*
x + c)^2 + 4*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b
^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*d*x*cos(d*x + c) + 2*(B*a^7*b^2 -
 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*
b^8 + 3*A*b^9)*d*x - (6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b
^6 + 2*B*a*b^7 - 6*A*b^8 + (6*B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a
^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + 2*(6*B*a^6*b^2 - 12*A*
a^5*b^3 - 5*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + (2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^
5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a
^5*b^4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^
3 - 9*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*s
in(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 +
2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3
*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46853, size = 737, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*
b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 2*a^6*b^
2 + a^4*b^4)*sqrt(-a^2 + b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A*
a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a
^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A*a*
b^5*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6*tan
```

```
(1/2*d*x + 1/2*c)^3 - 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b^3*tan(1/
2*d*x + 1/2*c) - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*A*a^2*b^4*tan(1/2*d*x
 + 1/2*c) + 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*A*a*b^5*tan(1/2*d*x + 1/2*
c) + 2*B*a*b^5*tan(1/2*d*x + 1/2*c) - 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 -
 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2) - (B*a - 3*A*b)*(d*x + c)/a^4 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(
1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

3.335 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=393

$$-\frac{\left(-21a^2Ab^3+6a^4Ab+11a^3b^2B-2a^5B-6ab^4B+12Ab^5\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2Ab^2+a^4A+6a^3bB-3ab^3B+6Ab^4\right)\text{si}}{2a^3d\left(a^2-b^2\right)^2}$$

```
[Out] ((a^2*A + 12*A*b^2 - 6*a*b*B)*x)/(2*a^5) - (b^2*(20*a^4*A*b - 29*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d) - ((6*a^4*A
*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*b^2 + 6*A*b^4 + 6*a^3*b*
B - 3*a*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A*b
 - a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)*Cos[c + d*x]*Sin[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.99893, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(-21a^2Ab^3+6a^4Ab+11a^3b^2B-2a^5B-6ab^4B+12Ab^5\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2Ab^2+a^4A+6a^3bB-3ab^3B+6Ab^4\right)\text{si}}{2a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((a^2*A + 12*A*b^2 - 6*a*b*B)*x)/(2*a^5) - (b^2*(20*a^4*A*b - 29*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d) - ((6*a^4*A
*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*b^2 + 6*A*b^4 + 6*a^3*b*
B - 3*a*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A*b
 - a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)*Cos[c + d*x]*Sin[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-2\left(a^2A-2Ab^2+abB\right)+2a(Ab-aB)\sec(c\cdots}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(7a^2Ab-4Ab^3-5a^3B+2ab^2B\right)\cos(c\cdots}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+\cdots}$$

$$= \frac{\left(a^4A-10a^2Ab^2+6Ab^4+6a^3bB-3ab^3B\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b(Ab-a\cdots}{2a\left(a^2-\cdots\right.}$$

$$= -\frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4A-\cdots\right.}{}$$

$$= \frac{\left(a^2A+12Ab^2-6abB\right)x}{2a^5} - \frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6a\cdots\right.}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2A+12Ab^2-6abB\right)x}{2a^5} - \frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6a\cdots\right.}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2A+12Ab^2-6abB\right)x}{2a^5} - \frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6a\cdots\right.}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2A+12Ab^2-6abB\right)x}{2a^5} - \frac{b^2\left(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B\cdots\right.}{a^5(a-b)^{5/2}(a+b)^{5\cdots}}$$

Mathematica [A] time = 4.25608, size = 734, normalized size = 1.87

$$\frac{16ab\left(a^2-b^2\right)^2(c+dx)\left(a^2A-6abB+12Ab^2\right)\cos(c+dx)+4\left(a^3-ab^2\right)^2(c+dx)\left(a^2A-6abB+12Ab^2\right)\cos(2(c+dx))-48a^6Ab^2\sin(2(c+dx))-2a^6Ab^2\sin(4(c+dx))-32a^5Ab^3\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B -
6*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2
)^(5/2) + (4*a^8*A*c + 48*a^6*A*b^2*c - 12*a^4*A*b^4*c - 136*a^2*A*b^6*c +
96*A*b^8*c - 24*a^7*b*B*c + 72*a^3*b^5*B*c - 48*a*b^7*B*c + 4*a^8*A*d*x + 4
8*a^6*A*b^2*d*x - 12*a^4*A*b^4*d*x - 136*a^2*A*b^6*d*x + 96*A*b^8*d*x - 24*
a^7*b*B*d*x + 72*a^3*b^5*B*d*x - 48*a*b^7*B*d*x + 16*a*b*(a^2 - b^2)^2*(a^2
*A + 12*A*b^2 - 6*a*b*B)*(c + d*x)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*(a^2*A
+ 12*A*b^2 - 6*a*b*B)*(c + d*x)*Cos[2*(c + d*x)] - 8*a^7*A*b*Sin[c + d*x] -
 32*a^5*A*b^3*Sin[c + d*x] + 160*a^3*A*b^5*Sin[c + d*x] - 96*a*A*b^7*Sin[c
+ d*x] + 4*a^8*B*Sin[c + d*x] + 8*a^6*b^2*B*Sin[c + d*x] - 84*a^4*b^4*B*Sin
[c + d*x] + 48*a^2*b^6*B*Sin[c + d*x] + 2*a^8*A*Sin[2*(c + d*x)] - 48*a^6*A
*b^2*Sin[2*(c + d*x)] + 130*a^4*A*b^4*Sin[2*(c + d*x)] - 72*a^2*A*b^6*Sin[2
*(c + d*x)] + 16*a^7*b*B*Sin[2*(c + d*x)] - 64*a^5*b^3*B*Sin[2*(c + d*x)] +
 36*a^3*b^5*B*Sin[2*(c + d*x)] - 8*a^7*A*b*Sin[3*(c + d*x)] + 16*a^5*A*b^3*
Sin[3*(c + d*x)] - 8*a^3*A*b^5*Sin[3*(c + d*x)] + 4*a^8*B*Sin[3*(c + d*x)]
```

```
- 8*a^6*b^2*B*Sin[3*(c + d*x)] + 4*a^4*b^4*B*Sin[3*(c + d*x)] + a^8*A*Sin[4
*(c + d*x)] - 2*a^6*A*b^2*Sin[4*(c + d*x)] + a^4*A*b^4*Sin[4*(c + d*x)])/((
a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(16*a^5*d)
```

Maple [B] time = 0.134, size = 1552, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b-6/d/a^4/(1+tan
(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*B+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d
*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^
2*tan(1/2*d*x+1/2*c)*B-4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-20/d/a/(a^4-2*a^
2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A*b^3+29/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^5-10/d/a^2/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*A+10/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*
b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+1/
d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)
^2*tan(1/2*d*x+1/2*c)*B-1/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+8/d*b^3/a/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*B+1/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*b^5/a^3/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*A-8/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)
/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*A/a^3*arctan(ta
n(1/2*d*x+1/2*c))+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B
+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A+2/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B+12/d/a^5*arctan(tan(1/2*d*x+1/2*c))*A
*b^2-6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B*b+12/d/(a^4-2*a^2*b^2+b^4)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b^2-1
/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.11101, size = 4018, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*(A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 1
8*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2
 + 4*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 1
8*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^2*b^8 - 12*A*a*b^9)*d*x*cos(d*x + c) + 2
*(A*a^8*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*
B*a^3*b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*d*x - (12*B*a^5*b^4 - 20*
A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 + 6*B*a*b^8 - 12*A*b^9 + (12*B*a^7*
b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2*b
^7)*cos(d*x + c)^2 + 2*(12*B*a^6*b^3 - 20*A*a^5*b^4 - 15*B*a^4*b^5 + 29*A*a
^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) + 2*(2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^6*b^4 + 27*A*a^5*b^5 +
 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9 + (A*a^10 - 3*A*a^8
*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 2*(B*a^10 - 2*A*a^9*b - 3*
B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B*a^4*b^6 + 2*A*a^3*b
^7)*cos(d*x + c)^2 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43*A*a^6*b^
4 + 25*B*a^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c))
*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2
+ 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2
- 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*((A*a^10 - 6*B*a^9*b + 9*A*a^8*b
^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^
7 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2 + 2*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a^7*b
^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^2*b^
8 - 12*A*a*b^9)*d*x*cos(d*x + c) + (A*a^8*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 +
 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12
*A*b^10)*d*x + (12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 +
 6*B*a*b^8 - 12*A*b^9 + (12*B*a^7*b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*
a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(12*B*a^6*b^3 - 20
*A*a^5*b^4 - 15*B*a^4*b^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*
x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c))) + (2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^6*b^4 + 27*A*
a^5*b^5 + 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9 + (A*a^10
- 3*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 2*(B*a^10 - 2*A*a
^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B*a^4*b^6 +
2*A*a^3*b^7)*cos(d*x + c)^2 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43
*A*a^6*b^4 + 25*B*a^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(
d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*
x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (
a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.53663, size = 1827, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(12*B*a^5*b^2 - 20*A*a^4*b^3 - 15*B*a^3*b^4 + 29*A*a^2*b^5 + 6*B*a*b
^6 - 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-
(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^9
- 2*a^7*b^2 + a^5*b^4)*sqrt(-a^2 + b^2)) - 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7
- 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a
^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5
*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4
*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3
*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2
*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6
*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^7*tan(1/2
*d*x + 1/2*c)^5 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^6*b*tan(1/2*d*x +
1/2*c)^5 + 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 5*A*a^5*b^2*tan(1/2*d*x + 1/2
*c)^5 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 26*A*a^4*b^3*tan(1/2*d*x + 1/
2*c)^5 - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*A*a^3*b^4*tan(1/2*d*x + 1
/2*c)^5 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 67*A*a^2*b^5*tan(1/2*d*x +
1/2*c)^5 + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a*b^6*tan(1/2*d*x + 1/
2*c)^5 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^
5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a
^6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^
2*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b
^3*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*
b^4*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2
*b^5*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b
^6*tan(1/2*d*x + 1/2*c)^3 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*ta
n(1/2*d*x + 1/2*c)^3 - A*a^7*tan(1/2*d*x + 1/2*c) - 2*B*a^7*tan(1/2*d*x + 1
/2*c) + 4*A*a^6*b*tan(1/2*d*x + 1/2*c) - 4*B*a^6*b*tan(1/2*d*x + 1/2*c) + 1
3*A*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*A*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*A*a^3*
b^4*tan(1/2*d*x + 1/2*c) + 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c) - 17*A*a^2*b^5*
tan(1/2*d*x + 1/2*c) - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*A*a*b^6*tan(1/
2*d*x + 1/2*c) - 6*B*a*b^6*tan(1/2*d*x + 1/2*c) + 12*A*b^7*tan(1/2*d*x + 1/
2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*
x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (A*a^2 - 6*B*a*b +
12*A*b^2)*(d*x + c)/a^5)/d
```

3.336 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=418

$$-\frac{\left(3a^3Ab+23a^2b^2B-12a^4B-8aAb^3-6b^4B\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{a\left(-7a^4Ab^3+8a^2Ab^5+2a^6Ab+28a^5b^2B-35a^3b^4B-8a^7\cdots\right.}{b^5d(a-b)^{7/2}(a+\cdots}$$

```
[Out] ((A*b - 4*a*B)*ArcTanh[Sin[c + d*x]])/(b^5*d) - (a*(2*a^6*A*b - 7*a^4*A*b^3
 + 8*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6
*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5
*(a + b)^(7/2)*d) - ((3*a^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b
^4*B)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^3
*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6
*A*b^3 - 4*a^3*B + 9*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*b^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*
a^5*B + 11*a^3*b^2*B - 12*a*b^4*B)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a
+ b*Sec[c + d*x]))
```

Rubi [A] time = 5.27344, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {4029, 4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(3a^3Ab+23a^2b^2B-12a^4B-8aAb^3-6b^4B\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{a\left(-7a^4Ab^3+8a^2Ab^5+2a^6Ab+28a^5b^2B-35a^3b^4B-8a^7\cdots\right.}{b^5d(a-b)^{7/2}(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((A*b - 4*a*B)*ArcTanh[Sin[c + d*x]])/(b^5*d) - (a*(2*a^6*A*b - 7*a^4*A*b^3
 + 8*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6
*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5
*(a + b)^(7/2)*d) - ((3*a^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b
^4*B)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^3
*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6
*A*b^3 - 4*a^3*B + 9*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*b^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*
a^5*B + 11*a^3*b^2*B - 12*a*b^4*B)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a
+ b*Sec[c + d*x]))
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)\left(3a(Ab-aB)-3b(Ab-aB)\sec(c+dx)-(a\ldots\right.}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\sec^2(c\ldots}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+\ldots}$$

$$= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\sec^2(c\ldots}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+\ldots}$$

$$= -\frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} + \frac{a(Ab-aB)\sec^3(c\ldots}{3b\left(a^2-b^2\right)d(a\ldots}$$

$$= -\frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} + \frac{a(Ab-aB)\sec^3(c\ldots}{3b\left(a^2-b^2\right)d(a\ldots}$$

$$= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\ldots}{6b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\ldots}{6b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{a\left(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B\ldots\right.}{(a\ldots}$$

Mathematica [A] time = 2.99968, size = 548, normalized size = 1.31

$$\frac{2b\tan(c+dx)\left(-6a^2b\left(15a^3Ab^3-5a^5Ab-57a^4b^2B+53a^2b^4B+20a^6B-20aAb^5-6b^6B\right)\cos(2(c+dx))+a\left(-7a^5Ab^3-50a^3Ab^5+18a^7Ab+28a^6b^2B+305a^4b^4B-438a^2b^6B\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-48*a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^
5*b^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 48*(A*b - 4*a*B)*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 48*(A*b - 4*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + (2*b*(30*a^7*A*b^2 - 90*a^5*A*b^4 + 120*a^3*A*b^6 - 120*a^8*b*B + 318
*a^6*b^3*B - 246*a^4*b^5*B - 36*a^2*b^7*B + 24*b^9*B + a*(18*a^7*A*b - 7*a^
5*A*b^3 - 50*a^3*A*b^5 + 144*a*A*b^7 - 72*a^8*B + 28*a^6*b^2*B + 305*a^4*b^
4*B - 438*a^2*b^6*B + 72*b^8*B)*Cos[c + d*x] - 6*a^2*b*(-5*a^5*A*b + 15*a^3
*A*b^3 - 20*a*A*b^5 + 20*a^6*B - 57*a^4*b^2*B + 53*a^2*b^4*B - 6*b^6*B)*Cos
[2*(c + d*x)] + 6*a^8*A*b*Cos[3*(c + d*x)] - 17*a^6*A*b^3*Cos[3*(c + d*x)]
```

```
+ 26*a^4*A*b^5*Cos[3*(c + d*x)] - 24*a^9*B*Cos[3*(c + d*x)] + 68*a^7*b^2*B*
Cos[3*(c + d*x)] - 65*a^5*b^4*B*Cos[3*(c + d*x)] + 6*a^3*b^6*B*Cos[3*(c + d
*x)])*Tan[c + d*x])/((-a^2 + b^2)^3*(b + a*Cos[c + d*x])^3))/(48*b^5*d)
```

Maple [B] time = 0.108, size = 2948, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -28/d*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+35/d*a^4/b/(a^6-3*a^4*b^2+3*a^
2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*B-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-20/d*a^3/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)*B-20/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+4/d*a^3/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^
2+b^3)*tan(1/2*d*x+1/2*c)^5*A+40/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/
d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-
b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))*A-4/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^
2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a^4/b/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)*A+1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d*a^6/b^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-24/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*
A+12/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2
*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^6/b^3/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)*B-1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*a^4/b/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a
*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^
5*B+18/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-116/3/d*a^5/b^2/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B+44/3/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/
d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+
3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5*A-5/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+12/d*a^2*b/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5*A+18/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+5
/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+12/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a
```

```
-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*A-6/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/
(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+8/d*a*b^2/(a^6-3*a^4*b
^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*A+8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^
(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+7/d*a^5/b^2/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*A+1/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*A-1/d*B/b^4/
(tan(1/2*d*x+1/2*c)-1)-1/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*A-1/d*B/b^4/(tan(1/
2*d*x+1/2*c)+1)-4/d/b^5*ln(tan(1/2*d*x+1/2*c)+1)*B*a-8/d*a^3/(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*A+4/d/b^5*ln(tan(1/2*d*x+1/2*c)-1)*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^5(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.61392, size = 1357, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(8*B*a^8 - 2*A*a^7*b - 28*B*a^6*b^2 + 7*A*a^5*b^3 + 35*B*a^4*b^4 - 8
*A*a^3*b^5 - 20*B*a^2*b^6 + 8*A*a*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(-a^2 + b^2
)) - (18*B*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^8*b*tan(1/2*d*x + 1/2*c)^5 -
42*B*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
4*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 1
17*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 -
 24*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 -
 105*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5
 + 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^
5 - 36*B*a^9*tan(1/2*d*x + 1/2*c)^3 + 12*A*a^8*b*tan(1/2*d*x + 1/2*c)^3 + 1
52*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 -
 236*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^
3 + 120*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^7*tan(1/2*d*x + 1/2*c
)^3 + 18*B*a^9*tan(1/2*d*x + 1/2*c) - 6*A*a^8*b*tan(1/2*d*x + 1/2*c) + 42*B
*a^8*b*tan(1/2*d*x + 1/2*c) - 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a^7*
b^2*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c) - 117*B*a^6*b^3
*tan(1/2*d*x + 1/2*c) + 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^4*ta
n(1/2*d*x + 1/2*c) + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*B*a^4*b^5*tan(1
/2*d*x + 1/2*c) - 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*B*a^3*b^6*tan(1/2*
d*x + 1/2*c) - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3
*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a -
 b)^3) - 3*(4*B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 + 3*(4*B*a
- A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 - 6*B*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 - 1)*b^4))/d
```

3.337 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=310

$$-\frac{\left(3a^2Ab^5-7a^5b^2B+8a^3b^4B+2a^7B-8ab^6B+2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^4*d) - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a
^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Sec[c +
d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a^2*(5*A
*b^3 + 3*a^3*B - 8*a*b^2*B)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) - (a*(a^2*A*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^
4*B)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.36639, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4029, 4090, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(3a^2Ab^5-7a^5b^2B+8a^3b^4B+2a^7B-8ab^6B+2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^4*d) - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a
^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Sec[c +
d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a^2*(5*A
*b^3 + 3*a^3*B - 8*a*b^2*B)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) - (a*(a^2*A*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^
4*B)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
```

```
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec^2(c+dx)\left(2a(Ab-aB)-3b(Ab-aB)\sec(c+dx)+3(\right.}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a\right.}{6b^3\left(a^2-b^2\right)}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a\right.}{6b^3\left(a^2-b^2\right)}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

Mathematica [A] time = 1.8103, size = 369, normalized size = 1.19

$$\frac{\cos(c+dx)(A+B\sec(c+dx))\left(-\frac{2ab\sin(c+dx)\left(a^2\left(4a^2Ab^3+17a^3b^2B-6a^5B-26ab^4B+11Ab^5\right)\cos(2(c+dx))-6ab\left(-a^2Ab^3-15a^3b^2B+5a^5B+20ab^4B-9\right.\right.}{\left(b^2-a^2\right)^3(a\cos(c+dx)+b)^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B -
7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 24*B*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 24*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(8*a^4*A*
b^3 + a^2*A*b^5 + 36*A*b^7 - 6*a^7*B - 5*a^5*b^2*B + 38*a^3*b^4*B - 72*a*b^
6*B - 6*a*b*(-(a^2*A*b^3) - 9*A*b^5 + 5*a^5*B - 15*a^3*b^2*B + 20*a*b^4*B)*
Cos[c + d*x] + a^2*(4*a^2*A*b^3 + 11*A*b^5 - 6*a^5*B + 17*a^3*b^2*B - 26*a*
b^4*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/((-a^2 + b^2)^3*(b + a*Cos[c + d*x])
^3)))/(24*b^4*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 0.122, size = 2264, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 4/3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*a*b+
b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+8/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*B*a-2/d/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^7-3/d*b/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*A*a^2+7/d/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^5-4/d*a^3/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)*B+4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d*a^
3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A
-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a^2-2*a
*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+44/3/d/b/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*B-24/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3*a^2/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+12/d*b/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A
+12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/d*b^2/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*A+12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d*a^6/b^3/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)*B+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-1/
d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+
3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5*B-3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d*a^5/b^2/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)*B-6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+3/d*a^2
*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+1/d*B/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/d*B
/b^4*ln(tan(1/2*d*x+1/2*c)-1)-8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^3-2/d*b^
3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 116.544, size = 5023, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*(2*B*a^7*b^3 - 7*B*a^5*b^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^
9 + 2*A*b^10 + (2*B*a^10 - 7*B*a^8*b^2 + 8*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^
4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*
b^5 + 3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*
b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a*b^9)*co
s(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*(B*a^8*b^3 - 4*B*a^6
*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7
*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3
+ 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*
B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B*a*b^10)*cos(d*x + c))*log(sin(d*x
 + c) + 1) + 6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^1
1 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x
+ c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)
*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 +
B*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(11*B*a^8*b^3 - 2*A*a^7*
b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*B*a^2*b
^9 + 18*A*a*b^10 + (6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*a^6*b^5
- 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(5*B*a^9*b^
2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 +
9*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8
 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^
6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 +
6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6
*a^4*b^11 - 4*a^2*b^13 + b^15)*d), -1/6*(3*(2*B*a^7*b^3 - 7*B*a^5*b^5 + 8*B
*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10 + (2*B*a^10 - 7*B*a^8*b^2 + 8
*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2
*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*
b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*
b^7 - 8*B*a^2*b^8 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(
-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(B*a^8*b^3
 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2
 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*
B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^
9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B*a*b^10)*cos(d*x + c))*l
og(sin(d*x + c) + 1) + 3*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b
^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8
)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 +
B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*
a^3*b^8 + B*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*B*a^8*b^3 -
2*A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36
*B*a^2*b^9 + 18*A*a*b^10 + (6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*
a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(5*
B*a^9*b^2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^
3*b^8 + 9*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6
```

```
*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^
7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^
7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6
*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.65944, size = 1139, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*
A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*
a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a^2 + b^2)) - 3*B*log(abs(tan(1/2*d*x + 1
/2*c) + 1))/b^4 + 3*B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - (6*B*a^8*tan
(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^2*tan(1
/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b^3*tan(1
/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b^4*tan(1/
2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^5*tan(1/
2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b^6*tan(1
/2*d*x + 1/2*c)^5 - 18*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^8*tan(1/2*d*
x + 1/2*c)^3 + 56*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*A*a^5*b^3*tan(1/2*d*
x + 1/2*c)^3 - 116*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 32*A*a^3*b^5*tan(1/2*
d*x + 1/2*c)^3 + 72*B*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^7*tan(1/2*d
*x + 1/2*c)^3 + 6*B*a^8*tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*tan(1/2*d*x + 1/2
*c) - 6*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 45*B*a^5*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*B
*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 60*B*a^3
*b^5*tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b^
6*tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4
*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^3))/d
```

3.338 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=274

$$\frac{\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(a^2Ab-4a^3B+9ab^2\right.}{6b^2d\left(a^2-b^2\right)^2(a}$$

```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Tan
[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6*A
*b^3 - 4*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B
+ 18*a*b^4*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.699892, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4028, 4080, 4003, 12, 3831, 2659, 208}

$$\frac{\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(a^2Ab-4a^3B+9ab^2\right.}{6b^2d\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Tan
[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6*A
*b^3 - 4*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B
+ 18*a*b^4*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
```

```
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(-3ab(Ab-aB)-\left(a^2-3b^2\right)(Ab-aB)\sec(c+}{(a+b\sec(c+dx))^3}}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\tan(c-}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\tan(c-}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\tan(c-}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\tan(c-}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2B\right)\tan(c-}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(a^3A+4aAb^2-3a^2bB-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2(Ab-aB)\,\mathrm{t}}{3b^2\left(a^2-b^2\right)d(a+}$$

Mathematica [A] time = 2.23161, size = 226, normalized size = 0.82

$$\frac{\frac{\left(-13a^2Ab+4a^3B+11ab^2B-2Ab^3\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)} + \frac{\left(3a^2A-5abB+2Ab^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} - \frac{6\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{2(aB-Ab)\,\mathrm{si}}{(a-b)(a+b)(a\,\mathrm{co}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-6*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d*x
])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^3) + ((3*a^2*A + 2*A*b^2 - 5*a*b*B
)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + ((-13*a^2*A*
b - 2*A*b^3 + 4*a^3*B + 11*a*b^2*B)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(b +
 a*Cos[c + d*x]))/(6*d)
```

Maple [A] time = 0.094, size = 375, normalized size = 1.4

$$\frac{1}{d}\left(-2\frac{1}{\left(\left(\tan(1/2\,dx+c/2)\right)^2a-\left(\tan(1/2\,dx+c/2)\right)^2b-a-b\right)^3}\left(-1/2\frac{\left(Aa^3+6\,Aa^2b+2\,Aab^2+2\,Ab^3-2\,Ba^3-3\,Ba^2b\right.}{(a-b)\left(a^3+3\,a^2b+3\,ab^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(A*a^3+6*A*a^2*b+2*A*a*b^2+2*A*b^3-2*B*a^3-3*B*a^2*b-6*B*a*b^
2)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(7*A*a^2*b+3*A*
b^3-B*a^3-9*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1
/2*(A*a^3-6*A*a^2*b+2*A*a*b^2-2*A*b^3+2*B*a^3-3*B*a^2*b+6*B*a*b^2)/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3+(A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.799262, size = 2707, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(A*a^3*b^3 - 3*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6 + (A*a^6 - 3*B*a^5*
b + 4*A*a^4*b^2 - 2*B*a^3*b^3)*cos(d*x + c)^3 + 3*(A*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c)^2 + 3*(A*a^4*b^2 - 3*B*a^3*b^3 + 4*
A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c
) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*s
in(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
 + 2*(2*B*a^7 + A*a^6*b - 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a
^2*b^5 - 18*B*a*b^6 + 6*A*b^7 + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*
a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b -
 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*
x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9
)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^1
0)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)
, 1/6*(3*(A*a^3*b^3 - 3*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6 + (A*a^6 - 3*B*a^5*
b + 4*A*a^4*b^2 - 2*B*a^3*b^3)*cos(d*x + c)^3 + 3*(A*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c)^2 + 3*(A*a^4*b^2 - 3*B*a^3*b^3 + 4*
A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^7 + A*a^6*b -
 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A
*b^7 + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 +
2*A*a^2*b^5)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b - 10*A*a^5*b^2 + 8*B*a^4*b
^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^
10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*
```

```
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^3(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.55525, size = 936, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(A*a^3 - 3*B*a^2*b + 4*A*a*b^2 - 2*B*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2
+ b^2)) + (3*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*tan(1/2*d*x + 1/2*c)^5
+ 12*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27
*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 12
*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6
*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b
^5*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 28*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 + 32*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 36*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^5*tan(1/2
*d*x + 1/2*c)^3 - 3*A*a^5*tan(1/2*d*x + 1/2*c) - 6*B*a^5*tan(1/2*d*x + 1/2*
c) + 12*A*a^4*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + 27*
A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*A*a^
2*b^3*tan(1/2*d*x + 1/2*c) - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a*b^4*
tan(1/2*d*x + 1/2*c) - 18*B*a*b^4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*
x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.339 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx$

Optimal. Leaf size=263

$$-\frac{\left(4a^2Ab + a^3(-B) - 4ab^2B + Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{\left(2a^3Ab - 10a^2b^2B + a^4B + 13aAb^3 - 6b^4B\right)\tan(c+ \ldots}{6bd\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Tan[
c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^
3 + a^3*B - 6*a*b^2*B)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*
x])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Tan[c +
 d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.615023, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4009, 4003, 12, 3831, 2659, 208}

$$-\frac{\left(4a^2Ab + a^3(-B) - 4ab^2B + Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{\left(2a^3Ab - 10a^2b^2B + a^4B + 13aAb^3 - 6b^4B\right)\tan(c+ \ldots}{6bd\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Tan[
c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^
3 + a^3*B - 6*a*b^2*B)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*
x])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Tan[c +
 d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)\left(-3b(Ab-aB)+\left(2aAb+a^2B-3b^2B\right)\sec(c+dx)\right.}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\tan(c+dx}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\tan(c+dx}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\tan(c+dx}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\tan(c+dx}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\tan(c+dx}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= -\frac{\left(4a^2Ab+Ab^3-a^3B-4ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a(Ab-aB)\,\mathrm{ta}}{3b\left(a^2-b^2\right)d(a+}$$

Mathematica [A] time = 1.15707, size = 252, normalized size = 0.96

$$\frac{\frac{2\sin(c+dx)\left(a\left(-10a^2Ab^2-6a^4A+13a^3bB+2ab^3B+Ab^4\right)\cos(2(c+dx))-6\left(9a^2Ab^3+2a^4Ab-9a^3b^2B+a^5B-2ab^4B-Ab^5\right)\cos(c+dx)-14a^3Ab^2-6a^5A+22a^2b^3B+11a^4b}{(a\cos(c+dx)+b)^3}}{24d\left(b^2-a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(-6*a^5*A - 14*a^3*A*b^2 - 25
*a*A*b^4 + 11*a^4*b*B + 22*a^2*b^3*B + 12*b^5*B - 6*(2*a^4*A*b + 9*a^2*A*b^
3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*Cos[c + d*x] + a*(-6*a^4*A - 1
0*a^2*A*b^2 + A*b^4 + 13*a^3*b*B + 2*a*b^3*B)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(b + a*Cos[c + d*x])^3)/(24*(-a^2 + b^2)^3*d)
```

Maple [A] time = 0.089, size = 388, normalized size = 1.5

$$\frac{1}{d}\left(2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)^{3}}\left(-1/2\,\frac{\left(2\,Aa^{3}+2\,Aa^{2}b+6\,Aab^{2}+Ab^{3}-Ba^{3}-6\,Ba^{2}b\right.}{\left(a-b\right)\left(a^{3}+3\,a^{2}b+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3-B*a^3-6*B*a^2*b-2*B*a*b^2-2
*B*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(3*A*a^3+7
*A*a*b^2-7*B*a^2*b-3*B*b^3)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3-1/2*(2*A*a^3-2*A*a^2*b+6*A*a*b^2-A*b^3+B*a^3-6*B*a^2*b+2*B*a*b^2-2*B*
b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.844989, size = 2715, normalized size = 10.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6 + (B*a^6 - 4*A*a^5*b
+ 4*B*a^4*b^2 - A*a^3*b^3)*cos(d*x + c)^3 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*
```

```
a^3*b^3 - A*a^2*b^4)*cos(d*x + c)^2 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4*B*a^2*
b^4 - A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*
a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^
6 + 6*B*b^7 + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3
*b^4 + 2*B*a^2*b^5 + A*a*b^6)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*
a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos
(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b
^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*
b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*
b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)
*d), 1/6*(3*(B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6 + (B*a^6 - 4*A*a^5
*b + 4*B*a^4*b^2 - A*a^3*b^3)*cos(d*x + c)^3 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4
*B*a^3*b^3 - A*a^2*b^4)*cos(d*x + c)^2 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4*B*a
^2*b^4 - A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^6*b + 2*A*a^5*b^2 -
11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7 + (6*A*a^7
 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A
*a*b^6)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3
+ 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c
))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 +
 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2
+ 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) +
 (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.58856, size = 980, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 +
b^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5 -
6*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A
*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*
A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12
*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^
```

```
5*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^5*tan(1/
2*d*x + 1/2*c)^3 + 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 - 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 28*A*a*b^4*tan(1/2*
d*x + 1/2*c)^3 - 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/
2*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B
*a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*
b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 12*B*a^2*b^3
*tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2
*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(1/2*d*x + 1/2*c)
)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/
2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.340 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx$

Optimal. Leaf size=237

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{\left(-2a^2B+\right.}{6d\left(a^2-\right.}$$

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Tan[c +
 d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b
^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((11*a^2*A
*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
b*Sec[c + d*x]))
```

Rubi [A] time = 0.509945, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{\left(-2a^2B+\right.}{6d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Tan[c +
 d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b
^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((11*a^2*A
*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
b*Sec[c + d*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = -\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3(aA-bB)+2(Ab-aB)\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \int$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - (11$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - (11$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - (11$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - (11$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{(Ab-aB)\,\mathrm{t}}{3\left(a^2-b^2\right)d(a+}$$

Mathematica [A] time = 1.05016, size = 404, normalized size = 1.7

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(\frac{24\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)(a\cos(c+dx)+b)^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+54a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((24*(2*a^3*A + 3
*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x])^3)/Sqrt[a^2 - b^2] + 18*a^4*A*b*Sin[c + d*x] +
 39*a^2*A*b^3*Sin[c + d*x] + 18*A*b^5*Sin[c + d*x] - 6*a^5*B*Sin[c + d*x] -
```

```
18*a^3*b^2*B*Sin[c + d*x] - 51*a*b^4*B*Sin[c + d*x] + 54*a^3*A*b^2*Sin[2*(
c + d*x)] + 6*a*A*b^4*Sin[2*(c + d*x)] - 12*a^4*b*B*Sin[2*(c + d*x)] - 54*a
^2*b^3*B*Sin[2*(c + d*x)] + 6*b^5*B*Sin[2*(c + d*x)] + 18*a^4*A*b*Sin[3*(c
+ d*x)] - 5*a^2*A*b^3*Sin[3*(c + d*x)] + 2*A*b^5*Sin[3*(c + d*x)] - 6*a^5*B
*Sin[3*(c + d*x)] - 10*a^3*b^2*B*Sin[3*(c + d*x)] + a*b^4*B*Sin[3*(c + d*x)
]))/(24*(-a^2 + b^2)^3*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [A] time = 0.096, size = 376, normalized size = 1.6

$$\frac{1}{d}\left(-2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^3}\left(-1/2\,\frac{\left(6\,Aa^2b+3\,Aab^2+2\,Ab^3-2\,Ba^3-2\,Ba^2b-6\,Ba\right.}{(a-b)\left(a^3+3\,a^2b+3\,ab^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(9*A*a^2*b+A*b^
3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1
/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.839856, size = 2707, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6 + (2*A*a^6 - 4*B*a^
5*b + 3*A*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 + 3*(2*A*a^5*b - 4*B*a^4*b^2
+ 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c)^2 + 3*(2*A*a^4*b^2 - 4*B*a^3*b^3 +
3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c
) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*s
in(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
```

```
 + 2*(2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6
+ 4*A*b^7 + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b
^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a
^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*
x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9
)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^1
0)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)
, 1/6*(3*(2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6 + (2*A*a^6 - 4*B*a^
5*b + 3*A*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 + 3*(2*A*a^5*b - 4*B*a^4*b^2
+ 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c)^2 + 3*(2*A*a^4*b^2 - 4*B*a^3*b^3 +
3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^5*b^2 - 11*A*
a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7 + (6*B*a^7 - 18
*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^
6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*
A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^
10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.54549, size = 936, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 +
 b^2)) + (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 - 6*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
+ 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
- 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 +
12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^
5*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 36*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 - 16*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^5*tan(1/2*
d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c) +
```

```
12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*
B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 12*B*a*b^
4*tan(1/2*d*x + 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.341 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=292

$$-\frac{\left(-8a^4Ab^3+7a^2Ab^5+8a^6Ab-3a^5b^2B-2a^7B-2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b\left(-17a^2Ab^3+26a^4Ab-4a^3b^2\right.}{6a^3d\left(a^2-b^2\right)^3\,(}$$

```
[Out] (A*x)/a^4 - ((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3
*a^5*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a -
b)^(7/2)*(a + b)^(7/2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^3 - 5*a^3*B)*Tan[c + d*x])
/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b*(26*a^4*A*b - 17*a^2*A
*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3
*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.06764, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3923, 4060, 3919, 3831, 2659, 208}

$$-\frac{\left(-8a^4Ab^3+7a^2Ab^5+8a^6Ab-3a^5b^2B-2a^7B-2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b\left(-17a^2Ab^3+26a^4Ab-4a^3b^2\right.}{6a^3d\left(a^2-b^2\right)^3\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (A*x)/a^4 - ((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3
*a^5*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a -
b)^(7/2)*(a + b)^(7/2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^3 - 5*a^3*B)*Tan[c + d*x])
/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b*(26*a^4*A*b - 17*a^2*A
*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3
*d*(a + b*Sec[c + d*x]))
```


Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3A\left(a^2-b^2\right)+3a(Ab-aB)\sec(c+dx)-2b(Ab-aB)\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{\int \frac{6A\left(a^2-b^2\right)^2-2a}{}}{}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{b\left(26a^4Ab-1\right.}{6}$$

$$= \frac{Ax}{a^4} + \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{b\left(26a^4\right.}{}$$

$$= \frac{Ax}{a^4} + \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{b\left(26a^4\right.}{}$$

$$= \frac{Ax}{a^4} + \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{b\left(26a^4\right.}{}$$

$$= \frac{Ax}{a^4} - \frac{\left(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{}{3a}$$

Mathematica [B] time = 3.37973, size = 769, normalized size = 2.63

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(\frac{36a^7Ab^2\sin(c+dx)+36a^7Ab^2\sin(3(c+dx))+120a^6Ab^3\sin(2(c+dx))+72a^5Ab^4\sin(c+dx}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((-24*(-8*a^6*A*b
 + 8*a^4*A*b^3 - 7*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^
2)^(7/2) + (36*a^8*A*b*c - 84*a^6*A*b^3*c + 36*a^4*A*b^5*c + 36*a^2*A*b^7*c
 - 24*A*b^9*c + 36*a^8*A*b*d*x - 84*a^6*A*b^3*d*x + 36*a^4*A*b^5*d*x + 36*a
^2*A*b^7*d*x - 24*A*b^9*d*x + 18*a*A*(a^2 - b^2)^3*(a^2 + 4*b^2)*(c + d*x)*
Cos[c + d*x] + 36*a^2*A*b*(a^2 - b^2)^3*(c + d*x)*Cos[2*(c + d*x)] + 6*a^9*
A*c*Cos[3*(c + d*x)] - 18*a^7*A*b^2*c*Cos[3*(c + d*x)] + 18*a^5*A*b^4*c*Cos
[3*(c + d*x)] - 6*a^3*A*b^6*c*Cos[3*(c + d*x)] + 6*a^9*A*d*x*Cos[3*(c + d*x
)] - 18*a^7*A*b^2*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^4*d*x*Cos[3*(c + d*x)]
- 6*a^3*A*b^6*d*x*Cos[3*(c + d*x)] + 36*a^7*A*b^2*Sin[c + d*x] + 72*a^5*A*b
^4*Sin[c + d*x] - 57*a^3*A*b^6*Sin[c + d*x] + 24*a*A*b^8*Sin[c + d*x] - 18*
a^8*b*B*Sin[c + d*x] - 39*a^6*b^3*B*Sin[c + d*x] - 18*a^4*b^5*B*Sin[c + d*x
] + 120*a^6*A*b^3*Sin[2*(c + d*x)] - 90*a^4*A*b^5*Sin[2*(c + d*x)] + 30*a^2
*A*b^7*Sin[2*(c + d*x)] - 54*a^7*b^2*B*Sin[2*(c + d*x)] - 6*a^5*b^4*B*Sin[2
*(c + d*x)] + 36*a^7*A*b^2*Sin[3*(c + d*x)] - 32*a^5*A*b^4*Sin[3*(c + d*x)]
 + 11*a^3*A*b^6*Sin[3*(c + d*x)] - 18*a^8*b*B*Sin[3*(c + d*x)] + 5*a^6*b^3*
B*Sin[3*(c + d*x)] - 2*a^4*b^5*B*Sin[3*(c + d*x)])/(a^2 - b^2)^3))/(24*a^4*
d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.108, size = 2242, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 3/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*A*a^2-12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a^
2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*b/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)*B+6/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+3/d*a/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^2/(a-b)/(a^3+3*a^2*b+3*a*
b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*
A+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-12/d*b^2/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*A+24/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d/a^3/(tan(1/2
```

```
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c)*A+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2
/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d/a/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d
*x+1/2*c)*A-3/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^2
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-1/d/a^2/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3
)*tan(1/2*d*x+1/2*c)*A-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*
b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-4/3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B
+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*B-7/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^5+2/d/a^4/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A*b^7+4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3*b^6/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-44/3/d/a/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a^2-2*a*b+b^2)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d*A/a^4*arctan(tan(1/2*d*x+1/2*c))+2/d
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^3+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.04381, size = 4111, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*
x*cos(d*x + c)^3 + 36*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 +
 A*a^2*b^9)*d*x*cos(d*x + c)^2 + 36*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6
- 4*A*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 12*(A*a^8*b^3 - 4*A*a^6*b^5 +
6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x - 3*(2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*
B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10 - 8*A*a^9*b +
3*B*a^8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(
2*B*a^9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2
*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5
```

```
*b^5 - 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*co
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x +
c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + b^2)) - 2*(11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4
*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10 + (18*B*a^10*b - 36*A*a^9*b^2 - 23*B
*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11*A*a
^3*b^8)*cos(d*x + c)^2 + 3*(9*B*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A
*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 + 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x +
c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^
3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x +
c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d
*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), 1/
6*(6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*x*cos
(d*x + c)^3 + 18*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^
2*b^9)*d*x*cos(d*x + c)^2 + 18*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A
*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 6*(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^
4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x + 3*(2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*
b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10 - 8*A*a^9*b + 3*B*a^
8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^
9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*
cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5*b^5 -
 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*B*a^8*b^3 - 2
6*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A
*a*b^10 + (18*B*a^10*b - 36*A*a^9*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a
^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(9*B
*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4
*b^7 + 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^1
1*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*
a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4
 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^
5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.47087, size = 1099, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*
```

```
b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) + 3*(d*x + c)*A/a^4 + (18*B*a^
7*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6
*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5
*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*
b^4*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*
b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b^
7*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^7*b*tan(
1/2*d*x + 1/2*c)^3 + 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*B*a^5*b^3*tan
(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^3*b^5*ta
n(1/2*d*x + 1/2*c)^3 + 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^8*tan(1
/2*d*x + 1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*tan(1/2*
d*x + 1/2*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(1/2*d*x
 + 1/2*c) + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*d*x + 1/
2*c) + 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)
 + 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) - 15
*A*a*b^7*tan(1/2*d*x + 1/2*c) - 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7
*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^3))/d
```

3.342 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=411

$$\frac{\left(-65a^4Ab^2+68a^2Ab^4+6a^6A-17a^3b^3B+26a^5bB+6ab^5B-24Ab^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b\left(-35a^4Ab^3+28a^2Ab^5+20a^6\right.}{}$$

```
[Out] -(((4*A*b - a*B)*x)/a^5) + (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8
*A*b^7 - 8*a^7*B + 8*a^5*b^2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTanh[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) + (
(6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3
*B + 6*a*b^5*B)*Sin[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b*(9*a^2*A*b - 4*A*
b^3 - 6*a^3*B + a*b^2*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c
+ d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 - 6*a^5*B + 2*a^3*b^2*
B - a*b^4*B)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 5.59506, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-65a^4Ab^2+68a^2Ab^4+6a^6A-17a^3b^3B+26a^5bB+6ab^5B-24Ab^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b\left(-35a^4Ab^3+28a^2Ab^5+20a^6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] -(((4*A*b - a*B)*x)/a^5) + (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8
*A*b^7 - 8*a^7*B + 8*a^5*b^2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTanh[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) + (
(6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3
*B + 6*a*b^5*B)*Sin[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b*(9*a^2*A*b - 4*A*
b^3 - 6*a^3*B + a*b^2*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c
+ d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 - 6*a^5*B + 2*a^3*b^2*
B - a*b^4*B)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-3a^2A+4Ab^2-abB+3a(Ab-aB)\sec(c+d\right.}{(a+b\sec(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(9a^2Ab-4Ab^3-6a^3B+ab^2B\right)\sin(c+}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(9a^2Ab-4Ab^3-6a^3B+ab^2B\right)\sin(c+}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B\right)\sin(c+}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B\right.}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B\right.}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B\right.}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{b\left(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B\right.}{a^5(a-b)^{7/2}(a+b)}$$

Mathematica [B] time = 6.08404, size = 1205, normalized size = 2.93

$$\frac{(b+a\cos(c+dx))\sec^3(c+dx)(A+B\sec(c+dx))\left(\frac{24b\left(8Ba^7-20Aba^6-8b^2Ba^5+35Ab^3a^4+7b^4Ba^3-28Ab^5a^2-2b^6Ba+8Ab^7\right)\tanh^{-1}\left(\frac{(b-a)}{}\right.}{\left(a^2-b^2\right)^{7/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((24*b*(-20*a^6*A
*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8*a^5*b^2*B + 7*a^3*
b^4*B - 2*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b
+ a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (-144*a^8*A*b^2*c + 336*a^6*A*b^4*
c - 144*a^4*A*b^6*c - 144*a^2*A*b^8*c + 96*A*b^10*c + 36*a^9*b*B*c - 84*a^7
*b^3*B*c + 36*a^5*b^5*B*c + 36*a^3*b^7*B*c - 24*a*b^9*B*c - 144*a^8*A*b^2*d
*x + 336*a^6*A*b^4*d*x - 144*a^4*A*b^6*d*x - 144*a^2*A*b^8*d*x + 96*A*b^10*
d*x + 36*a^9*b*B*d*x - 84*a^7*b^3*B*d*x + 36*a^5*b^5*B*d*x + 36*a^3*b^7*B*d
*x - 24*a*b^9*B*d*x + 18*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(-4*A*b + a*B)*(c +
d*x)*Cos[c + d*x] + 36*a^2*b*(a^2 - b^2)^3*(-4*A*b + a*B)*(c + d*x)*Cos[2*(
c + d*x)] - 24*a^9*A*b*c*Cos[3*(c + d*x)] + 72*a^7*A*b^3*c*Cos[3*(c + d*x)]
 - 72*a^5*A*b^5*c*Cos[3*(c + d*x)] + 24*a^3*A*b^7*c*Cos[3*(c + d*x)] + 6*a^
10*B*c*Cos[3*(c + d*x)] - 18*a^8*b^2*B*c*Cos[3*(c + d*x)] + 18*a^6*b^4*B*c*
Cos[3*(c + d*x)] - 6*a^4*b^6*B*c*Cos[3*(c + d*x)] - 24*a^9*A*b*d*x*Cos[3*(c
```

```
 + d*x)] + 72*a^7*A*b^3*d*x*Cos[3*(c + d*x)] - 72*a^5*A*b^5*d*x*Cos[3*(c +
d*x)] + 24*a^3*A*b^7*d*x*Cos[3*(c + d*x)] + 6*a^10*B*d*x*Cos[3*(c + d*x)] -
 18*a^8*b^2*B*d*x*Cos[3*(c + d*x)] + 18*a^6*b^4*B*d*x*Cos[3*(c + d*x)] - 6*
a^4*b^6*B*d*x*Cos[3*(c + d*x)] + 18*a^9*A*b*Sin[c + d*x] - 90*a^7*A*b^3*Sin
[c + d*x] - 135*a^5*A*b^5*Sin[c + d*x] + 228*a^3*A*b^7*Sin[c + d*x] - 96*a*
A*b^9*Sin[c + d*x] + 36*a^8*b^2*B*Sin[c + d*x] + 72*a^6*b^4*B*Sin[c + d*x]
- 57*a^4*b^6*B*Sin[c + d*x] + 24*a^2*b^8*B*Sin[c + d*x] + 6*a^10*A*Sin[2*(c
 + d*x)] + 18*a^8*A*b^2*Sin[2*(c + d*x)] - 300*a^6*A*b^4*Sin[2*(c + d*x)] +
 336*a^4*A*b^6*Sin[2*(c + d*x)] - 120*a^2*A*b^8*Sin[2*(c + d*x)] + 120*a^7*
b^3*B*Sin[2*(c + d*x)] - 90*a^5*b^5*B*Sin[2*(c + d*x)] + 30*a^3*b^7*B*Sin[2
*(c + d*x)] + 18*a^9*A*b*Sin[3*(c + d*x)] - 114*a^7*A*b^3*Sin[3*(c + d*x)]
+ 125*a^5*A*b^5*Sin[3*(c + d*x)] - 44*a^3*A*b^7*Sin[3*(c + d*x)] + 36*a^8*b
^2*B*Sin[3*(c + d*x)] - 32*a^6*b^4*B*Sin[3*(c + d*x)] + 11*a^4*b^6*B*Sin[3*
(c + d*x)] + 3*a^10*A*Sin[4*(c + d*x)] - 9*a^8*A*b^2*Sin[4*(c + d*x)] + 9*a
^6*A*b^4*Sin[4*(c + d*x)] - 3*a^4*A*b^6*Sin[4*(c + d*x)])/(a^2 - b^2)^3))/(
24*a^5*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.144, size = 2891, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -40/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+
b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*A+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-8/d*b^8/a^5/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*A+2/d/a^4*B*arctan(tan(1/2*d*x+1/2*c))-8/d*a^2*b/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*B+5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-12/d*
a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^2/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*A-18/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d/a^3/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b
^3)*tan(1/2*d*x+1/2*c)^5*A-5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-12/d*a
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^2/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-18/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*A+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+24/d*b^2*a/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*B-44/3/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*b^7/a
^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b
+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+4/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*B-2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d*b^6/a^3/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
```

```
tan(1/2*d*x+1/2*c)^5*B+116/3/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-12
/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b
+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*b^4/a/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*B+1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d*b^4/a/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)*B-1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+6/d*
b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*A-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+20/d/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)*A+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+2/d*b^7/a^4/(a^6-3*
a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-7/d*b^5/a^2/
(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d/a^4*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+
1/2*c)^2)-8/d/a^5*A*arctan(tan(1/2*d*x+1/2*c))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.36104, size = 5736, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(12*(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4
- 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x*
cos(d*x + c)^3 + 36*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 +
 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*
a^2*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 +
 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B
*a^2*b^10 - 4*A*a*b^11)*d*x*cos(d*x + c) + 12*(B*a^9*b^3 - 4*A*a^8*b^4 - 4*
B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*
a^2*b^10 + B*a*b^11 - 4*A*b^12)*d*x - 3*(8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a
^5*b^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^10 + 8*A*b^11
```

```
+ (8*B*a^10*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 2
8*A*a^5*b^6 - 2*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 - 28*A*a^4*b^7 - 2*
B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^8*b^3 - 20*A*a^7*b^4 - 8
*B*a^6*b^5 + 35*A*a^5*b^6 + 7*B*a^4*b^7 - 28*A*a^3*b^8 - 2*B*a^2*b^9 + 8*A*
a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(6*A*a^9*b
^3 + 26*B*a^8*b^4 - 71*A*a^7*b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23*B*a^4*
b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2
+ 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*cos(d*x + c)^3 + (18*A*a^11*b + 36
*B*a^10*b^2 - 132*A*a^9*b^3 - 68*B*a^8*b^4 + 239*A*a^7*b^5 + 43*B*a^6*b^6 -
 169*A*a^5*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(6*A*a^10*
b^2 + 20*B*a^9*b^3 - 59*A*a^8*b^4 - 35*B*a^7*b^5 + 110*A*a^6*b^6 + 20*B*a^5
*b^7 - 77*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x +
c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)
^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x +
 c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos
(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d),
1/6*(6*(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 - 2
4*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x*cos
(d*x + c)^3 + 18*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 + 6*
B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*a^2
*b^10)*d*x*cos(d*x + c)^2 + 18*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 + 16
*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B*a^
2*b^10 - 4*A*a*b^11)*d*x*cos(d*x + c) + 6*(B*a^9*b^3 - 4*A*a^8*b^4 - 4*B*a^
7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^2*
b^10 + B*a*b^11 - 4*A*b^12)*d*x - 3*(8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a^5*b
^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^10 + 8*A*b^11 + (8
*B*a^10*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 28*A*
a^5*b^6 - 2*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - 20*A
*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 - 28*A*a^4*b^7 - 2*B*a^
3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^8*b^3 - 20*A*a^7*b^4 - 8*B*a
^6*b^5 + 35*A*a^5*b^6 + 7*B*a^4*b^7 - 28*A*a^3*b^8 - 2*B*a^2*b^9 + 8*A*a*b^
10)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
 + a)/((a^2 - b^2)*sin(d*x + c))) + (6*A*a^9*b^3 + 26*B*a^8*b^4 - 71*A*a^7*
b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23*B*a^4*b^8 - 92*A*a^3*b^9 - 6*B*a^2*
b^10 + 24*A*a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2 + 6*A*a^8*b^4 - 4*A*a^6*b^6 +
 A*a^4*b^8)*cos(d*x + c)^3 + (18*A*a^11*b + 36*B*a^10*b^2 - 132*A*a^9*b^3 -
 68*B*a^8*b^4 + 239*A*a^7*b^5 + 43*B*a^6*b^6 - 169*A*a^5*b^7 - 11*B*a^4*b^8
 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(6*A*a^10*b^2 + 20*B*a^9*b^3 - 59*A*a^8
*b^4 - 35*B*a^7*b^5 + 110*A*a^6*b^6 + 20*B*a^5*b^7 - 77*A*a^4*b^8 - 5*B*a^3
*b^9 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a
^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 +
 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*
b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^1
1*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5629, size = 1304, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(8*B*a^7*b - 20*A*a^6*b^2 - 8*B*a^5*b^3 + 35*A*a^4*b^4 + 7*B*a^3*b^
5 - 28*A*a^2*b^6 - 2*B*a*b^7 + 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(-a^2 + b^
2)) + (36*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^6*b^3*tan(1/2*d*x + 1/2
*c)^5 - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 105*A*a^5*b^4*tan(1/2*d*x + 1
/2*c)^5 - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^4*b^5*tan(1/2*d*x + 1
/2*c)^5 + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 117*A*a^3*b^6*tan(1/2*d*x +
 1/2*c)^5 - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^2*b^7*tan(1/2*d*x +
 1/2*c)^5 - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 42*A*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 6*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 18*A*b^9*tan(1/2*d*x + 1/2*c)
^5 - 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^6*b^3*tan(1/2*d*x + 1/2*
c)^3 + 116*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 236*A*a^4*b^5*tan(1/2*d*x + 1
/2*c)^3 - 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 152*A*a^2*b^7*tan(1/2*d*x +
 1/2*c)^3 + 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*A*b^9*tan(1/2*d*x + 1/2*
c)^3 + 36*B*a^7*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c
) + 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c)
- 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) - 45
*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 6*B*
a^3*b^6*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 15*B*a^2
*b^7*tan(1/2*d*x + 1/2*c) - 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 6*B*a*b^8*tan
(1/2*d*x + 1/2*c) - 18*A*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a
^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a
- b)^3) - 3*(B*a - 4*A*b)*(d*x + c)/a^5 - 6*A*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 + 1)*a^4))/d
```

3.343 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} dx$

Optimal. Leaf size=538

$$-\frac{\left(-146a^4Ab^3+167a^2Ab^5+24a^6Ab+65a^5b^2B-68a^3b^4B-6a^7B+24ab^6B-60Ab^7\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}+\frac{\left(-23a^4Ab^2+27a^2\right.}{}$$

```
[Out] ((a^2*A + 20*A*b^2 - 8*a*b*B)*x)/(2*a^6) - (b^2*(40*a^6*A*b - 84*a^4*A*b^3
+ 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^
6*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2
)*(a + b)^(7/2)*d) - ((24*a^6*A*b - 146*a^4*A*b^3 + 167*a^2*A*b^5 - 60*A*b^
7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^6*B)*Sin[c + d*x])/(6*a^
5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^2*A*b^4 - 10*A*b^6 + 12*
a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2
- b^2)^3*d) + (b*(A*b - a*B)*Cos[c + d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*
(a + b*Sec[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3 - 7*a^3*B + 2*a*b^2*B)*C
os[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) +
(b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B + 20*a^3*b^2*B - 8*a*b^
4*B)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x])
)
```

Rubi [A] time = 6.84428, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4030, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(-146a^4Ab^3+167a^2Ab^5+24a^6Ab+65a^5b^2B-68a^3b^4B-6a^7B+24ab^6B-60Ab^7\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}+\frac{\left(-23a^4Ab^2+27a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ((a^2*A + 20*A*b^2 - 8*a*b*B)*x)/(2*a^6) - (b^2*(40*a^6*A*b - 84*a^4*A*b^3
+ 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^
6*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2
)*(a + b)^(7/2)*d) - ((24*a^6*A*b - 146*a^4*A*b^3 + 167*a^2*A*b^5 - 60*A*b^
7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^6*B)*Sin[c + d*x])/(6*a^
5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^2*A*b^4 - 10*A*b^6 + 12*
a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2
- b^2)^3*d) + (b*(A*b - a*B)*Cos[c + d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*
(a + b*Sec[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3 - 7*a^3*B + 2*a*b^2*B)*C
os[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) +
(b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B + 20*a^3*b^2*B - 8*a*b^
4*B)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x])
)
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
```

```
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-3a^2A+5Ab^2-2abB+3a(Ab-aB)\sec(c+\right.}{(a+b\sec(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B+2ab^2B\right)\cos(c}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B+2ab^2B\right)\cos(c}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= \frac{\left(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab^5B\right)\cos(c+dx)}{2a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3b^4B+24\right.}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B\right.}{6a^5\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B\right.}{6a^5\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B\right.}{6a^5\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{b^2\left(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7\right.}{a^6(a-}$$

<u>Mathematica</u> [B] time = 5.8241, size = 1452, normalized size = 2.7

$$\frac{12Ac\cos(3(c+dx))a^{11}+12Adx\cos(3(c+dx))a^{11}+6A\sin(c+dx)a^{11}+24B\sin(2(c+dx))a^{11}+9A\sin(3(c+dx))a^{11}+12B\sin(4(c+dx))a^{11}+3A\sin(5(c+dx))a^{11}+72Abca}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B
- 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + (72*a^10*A*b*c + 1272*a^8*A*b^3*c
 - 3288*a^6*A*b^5*c + 1512*a^4*A*b^7*c + 1392*a^2*A*b^9*c - 960*A*b^11*c -
576*a^9*b^2*B*c + 1344*a^7*b^4*B*c - 576*a^5*b^6*B*c - 576*a^3*b^8*B*c + 38
4*a*b^10*B*c + 72*a^10*A*b*d*x + 1272*a^8*A*b^3*d*x - 3288*a^6*A*b^5*d*x +
1512*a^4*A*b^7*d*x + 1392*a^2*A*b^9*d*x - 960*A*b^11*d*x - 576*a^9*b^2*B*d*
x + 1344*a^7*b^4*B*d*x - 576*a^5*b^6*B*d*x - 576*a^3*b^8*B*d*x + 384*a*b^10
*B*d*x + 36*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(a^2*A + 20*A*b^2 - 8*a*b*B)*(c +
 d*x)*Cos[c + d*x] + 72*a^2*b*(a^2 - b^2)^3*(a^2*A + 20*A*b^2 - 8*a*b*B)*(c
 + d*x)*Cos[2*(c + d*x)] + 12*a^11*A*c*Cos[3*(c + d*x)] + 204*a^9*A*b^2*c*C
os[3*(c + d*x)] - 684*a^7*A*b^4*c*Cos[3*(c + d*x)] + 708*a^5*A*b^6*c*Cos[3*
```

```
(c + d*x)] - 240*a^3*A*b^8*c*Cos[3*(c + d*x)] - 96*a^10*b*B*c*Cos[3*(c + d*
x)] + 288*a^8*b^3*B*c*Cos[3*(c + d*x)] - 288*a^6*b^5*B*c*Cos[3*(c + d*x)] +
 96*a^4*b^7*B*c*Cos[3*(c + d*x)] + 12*a^11*A*d*x*Cos[3*(c + d*x)] + 204*a^9
*A*b^2*d*x*Cos[3*(c + d*x)] - 684*a^7*A*b^4*d*x*Cos[3*(c + d*x)] + 708*a^5*
A*b^6*d*x*Cos[3*(c + d*x)] - 240*a^3*A*b^8*d*x*Cos[3*(c + d*x)] - 96*a^10*b
*B*d*x*Cos[3*(c + d*x)] + 288*a^8*b^3*B*d*x*Cos[3*(c + d*x)] - 288*a^6*b^5*
B*d*x*Cos[3*(c + d*x)] + 96*a^4*b^7*B*d*x*Cos[3*(c + d*x)] + 6*a^11*A*Sin[c
 + d*x] - 270*a^9*A*b^2*Sin[c + d*x] + 750*a^7*A*b^4*Sin[c + d*x] + 1086*a^
5*A*b^6*Sin[c + d*x] - 2232*a^3*A*b^8*Sin[c + d*x] + 960*a*A*b^10*Sin[c + d
*x] + 72*a^10*b*B*Sin[c + d*x] - 360*a^8*b^3*B*Sin[c + d*x] - 540*a^6*b^5*B
*Sin[c + d*x] + 912*a^4*b^7*B*Sin[c + d*x] - 384*a^2*b^9*B*Sin[c + d*x] - 6
0*a^10*A*b*Sin[2*(c + d*x)] - 372*a^8*A*b^3*Sin[2*(c + d*x)] + 2772*a^6*A*b
^5*Sin[2*(c + d*x)] - 3300*a^4*A*b^7*Sin[2*(c + d*x)] + 1200*a^2*A*b^9*Sin[
2*(c + d*x)] + 24*a^11*B*Sin[2*(c + d*x)] + 72*a^9*b^2*B*Sin[2*(c + d*x)] -
 1200*a^7*b^4*B*Sin[2*(c + d*x)] + 1344*a^5*b^6*B*Sin[2*(c + d*x)] - 480*a^
3*b^8*B*Sin[2*(c + d*x)] + 9*a^11*A*Sin[3*(c + d*x)] - 279*a^9*A*b^2*Sin[3*
(c + d*x)] + 1143*a^7*A*b^4*Sin[3*(c + d*x)] - 1253*a^5*A*b^6*Sin[3*(c + d*
x)] + 440*a^3*A*b^8*Sin[3*(c + d*x)] + 72*a^10*b*B*Sin[3*(c + d*x)] - 456*a
^8*b^3*B*Sin[3*(c + d*x)] + 500*a^6*b^5*B*Sin[3*(c + d*x)] - 176*a^4*b^7*B*
Sin[3*(c + d*x)] - 30*a^10*A*b*Sin[4*(c + d*x)] + 90*a^8*A*b^3*Sin[4*(c + d
*x)] - 90*a^6*A*b^5*Sin[4*(c + d*x)] + 30*a^4*A*b^7*Sin[4*(c + d*x)] + 12*a
^11*B*Sin[4*(c + d*x)] - 36*a^9*b^2*B*Sin[4*(c + d*x)] + 36*a^7*b^4*B*Sin[4
*(c + d*x)] - 12*a^5*b^6*B*Sin[4*(c + d*x)] + 3*a^11*A*Sin[5*(c + d*x)] - 9
*a^9*A*b^2*Sin[5*(c + d*x)] + 9*a^7*A*b^4*Sin[5*(c + d*x)] - 3*a^5*A*b^6*Si
n[5*(c + d*x)])/((a^2 - b^2)^3*(b + a*Cos[c + d*x])^3))/(96*a^6*d)
```

Maple [B] time = 0.148, size = 3099, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+20/d*b^2/(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*B*a-12/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-35/d*b^4/a/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*B+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-8/d*
b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*A-30/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+34/d/a^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3
)*tan(1/2*d*x+1/2*c)*A-6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+34/d/a
^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^3+3*a
^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-30/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*A+6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/
(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+3/d*b^7/a^4/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*A+2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
```

```
*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d*b^6
/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+5/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*B-18/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-5/d*b^4/a/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*B-18/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-3/d*b^7/
a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+24/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*A+20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-40/d/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*B+20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3*b^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+84/d/a^2/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*A*b^5-69/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^7-21
2/3/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+60/d/a/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*A+1/d*A/a^4*arctan(tan(1/2*d*x+1/2*c))-12/d*b^8/a^5/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*A-12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d
*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+116/3/d*b^5/a^2/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*B-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-8/d/a^5/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)*A*b-1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*
c)^3*A+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+1/d/a^4/(1
+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A+2/d/a^4/(1+tan(1/2*d*x+1/2*c)
^2)^2*tan(1/2*d*x+1/2*c)*B+20/d/a^6*arctan(tan(1/2*d*x+1/2*c))*A*b^2-8/d/a^
5*arctan(tan(1/2*d*x+1/2*c))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.56026, size = 6657, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(6*(A*a^13 - 8*B*a^12*b + 16*A*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^
4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^
9 + 20*A*a^3*b^10)*d*x*cos(d*x + c)^3 + 18*(A*a^12*b - 8*B*a^11*b^2 + 16*A*
a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*
B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^11)*d*x*cos(d*x + c)^2
 + 18*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b
^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*
b^11 + 20*A*a*b^12)*d*x*cos(d*x + c) + 6*(A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a
^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*
a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*d*x - 3*(20*B*a^7*b^5 -
40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^10 -
 8*B*a*b^11 + 20*A*b^12 + (20*B*a^10*b^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84
*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*cos(
d*x + c)^3 + 3*(20*B*a^9*b^3 - 40*A*a^8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 +
 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^2
+ 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b
^8 - 69*A*a^3*b^9 - 8*B*a^2*b^10 + 20*A*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^
2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^
2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2
*a*b*cos(d*x + c) + b^2)) + 2*(6*B*a^10*b^3 - 24*A*a^9*b^4 - 71*B*a^8*b^5 +
 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 + 227*A*a^3*b
^10 + 24*B*a^2*b^11 - 60*A*a*b^12 + 3*(A*a^13 - 4*A*a^11*b^2 + 6*A*a^9*b^4
- 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x + c)^4 + 3*(2*B*a^13 - 5*A*a^12*b - 8*B*
a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^7*b^6 + 20*A
*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*cos(d*x + c)^3 + (18*B*a^12*b - 63*A*
a^11*b^2 - 132*B*a^10*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590*A*a^7*b^6 -
 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)*cos(d*x + c
)^2 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^3 - 59*B*a^9*b^4 + 146*A*a^8*b^5 + 110*
B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B*a^3*b^10 -
50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a^13*b^4
 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 + 6*a^12
*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13*b^4 +
6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^12*b^5
+ 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d), 1/6*(3*(A*a^13 - 8*B*a^12*b + 16*A
*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 3
2*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^10)*d*x*cos(d*x + c)^
3 + 9*(A*a^12*b - 8*B*a^11*b^2 + 16*A*a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^
5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^
10 + 20*A*a^2*b^11)*d*x*cos(d*x + c)^2 + 9*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*
A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32
*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*b^11 + 20*A*a*b^12)*d*x*cos(d*x + c) +
 3*(A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 -
 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12
+ 20*A*b^13)*d*x + 3*(20*B*a^7*b^5 - 40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4
*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^10 - 8*B*a*b^11 + 20*A*b^12 + (20*B*a^10*b
^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b
^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*cos(d*x + c)^3 + 3*(20*B*a^9*b^3 - 40*A*a^
8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 + 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3
*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*
B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b^8 - 69*A*a^3*b^9 - 8*B*a^2*b^10 + 20*
A*a*b^11)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*
x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*B*a^10*b^3 - 24*A*a^9*b^4 - 71
*B*a^8*b^5 + 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 +
 227*A*a^3*b^10 + 24*B*a^2*b^11 - 60*A*a*b^12 + 3*(A*a^13 - 4*A*a^11*b^2 +
6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x + c)^4 + 3*(2*B*a^13 - 5*A*a
^12*b - 8*B*a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^
```

```
7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*cos(d*x + c)^3 + (18*B*a^
12*b - 63*A*a^11*b^2 - 132*B*a^10*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590
*A*a^7*b^6 - 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)
*cos(d*x + c)^2 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^3 - 59*B*a^9*b^4 + 146*A*a^
8*b^5 + 110*B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B
*a^3*b^10 - 50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2
+ 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*
b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4
*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 -
 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63385, size = 1420, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*a^
3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2
*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(-a^
2 + b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1/2*d
*x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*tan(1/
2*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6*tan(1
/2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3*b^7*ta
n(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^2*b^8*t
an(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A*a*b^9*ta
n(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^10*tan(1/
2*d*x + 1/2*c)^5 - 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 + 180*A*a^6*b^4*tan
(1/2*d*x + 1/2*c)^3 + 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 - 392*A*a^4*b^6*
tan(1/2*d*x + 1/2*c)^3 - 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 + 284*A*a^2*b
^8*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^10*t
an(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) - 90*A*a^6*b^4*ta
n(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 162*A*a^5*b^5*tan
(1/2*d*x + 1/2*c) - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) + 48*A*a^4*b^6*tan(1/
2*d*x + 1/2*c) - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 213*A*a^3*b^7*tan(1/2
*d*x + 1/2*c) - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 48*A*a^2*b^8*tan(1/2*d*
x + 1/2*c) + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) - 81*A*a*b^9*tan(1/2*d*x + 1
/2*c) + 18*B*a*b^9*tan(1/2*d*x + 1/2*c) - 36*A*b^10*tan(1/2*d*x + 1/2*c))/(
(a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(
1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*(d*x + c)/a
```

```
^6 - 6*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 8*A*b*t
an(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*
c) + 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d
```

3.344 $\int \frac{\frac{bB}{a}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{2B\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}+\frac{bBx}{a^2}$$

```
[Out] (b*B*x)/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*d)
```

Rubi [A] time = 0.11474, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3919, 3831, 2659, 208}

$$\frac{2B\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}+\frac{bBx}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*B*x)/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*d)
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{bBx}{a^2}-\frac{\left(-aB+\frac{b^2B}{a}\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$
$$= \frac{bBx}{a^2}-\frac{\left(-aB+\frac{b^2B}{a}\right)\int\frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{ab}$$
$$= \frac{bBx}{a^2}-\frac{\left(2\left(-aB+\frac{b^2B}{a}\right)\right)\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$
$$= \frac{bBx}{a^2}+\frac{2\sqrt{a-b}\sqrt{a+b}B\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}$$

Mathematica [A] time = 0.144292, size = 61, normalized size = 1.

$$\frac{B\left(b(c+dx)-2\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*(b*(c + d*x) - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]]))/(a^2*d)
```

Maple [B] time = 0.077, size = 116, normalized size = 1.9

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)Bb}{da^2}+2\,\frac{B}{d\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right)-2\,\frac{Bb^2}{da^2\sqrt{(a+b)(a-b)}}\text{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B*b+2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*b^2/a^2/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.536089, size = 454, normalized size = 7.44

$$\left[\frac{2\,Bbdx + \sqrt{a^2 - b^2}B\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c) + 2\,a^2 - b^2}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right)}{2\,a^2d}, \frac{Bbdx + \sqrt{-a^2 + b^2}B\arctan}{a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*b*d*x + sqrt(a^2 - b^2)*B*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a^2*d), (B*b*d
*x + sqrt(-a^2 + b^2)*B*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c))))/(a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\left(\int \frac{b}{a+b\sec(c+dx)}\,dx + \int \frac{a\sec(c+dx)}{a+b\sec(c+dx)}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] B*(Integral(b/(a + b*sec(c + d*x)), x) + Integral(a*sec(c + d*x)/(a + b*sec
(c + d*x)), x))/a
```

Giac [B] time = 1.31344, size = 143, normalized size = 2.34

$$\frac{\frac{(dx+c)Bb}{a^2} + \frac{2\left(Ba^2 - Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B*b/a^2 + 2*(B*a^2 - B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2))/d
```

3.345 $\int \frac{\frac{aB}{b}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] (B*x)/b
```

Rubi [A] time = 0.0013912, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*x)/b
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.0005538, size = 6, normalized size = 1.

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*x)/b
```

Maple [A] time = 0.007, size = 7, normalized size = 1.2

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] B*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.429939, size = 9, normalized size = 1.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [A] time = 6.06857, size = 3, normalized size = 0.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] B*x/b
```

Giac [B] time = 1.38539, size = 18, normalized size = 3.

$$\frac{(dx+c)B}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)*B/(b*d)
```

3.346 $\int \frac{a+b\sec(c+dx)}{(b+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{ax}{b^2}-\frac{a\tan(c+dx)}{bd(a\sec(c+dx)+b)}$$

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(b^2*d) - (a*Tan[c + d*x])/(b*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.180483, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3923, 3919, 3831, 2659, 205}

$$-\frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{ax}{b^2}-\frac{a\tan(c+dx)}{bd(a\sec(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/(b + a*Sec[c + d*x])^2,x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(b^2*d) - (a*Tan[c + d*x])/(b*d*(b + a*Sec[c + d*x]))
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{(b+a\sec(c+dx))^2}\,dx = -\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))} + \frac{\int \frac{a(a^2-b^2)+b(a^2-b^2)\sec(c+dx)}{b+a\sec(c+dx)}\,dx}{b(a^2-b^2)}$$

$$= \frac{ax}{b^2} - \frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))} - \frac{(a^2-b^2)\int \frac{\sec(c+dx)}{b+a\sec(c+dx)}\,dx}{b^2}$$

$$= \frac{ax}{b^2} - \frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))} - \frac{(a^2-b^2)\int \frac{1}{1+\frac{b\cos(c+dx)}{a}}\,dx}{ab^2}$$

$$= \frac{ax}{b^2} - \frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))} - \frac{\left(2(a^2-b^2)\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{b}{a}+\left(1-\frac{b}{a}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ab^2d}$$

$$= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d} - \frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}$$

Mathematica [A] time = 0.35524, size = 97, normalized size = 1.13

$$\frac{2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right) + \frac{a(ac+adx-b\sin(c+dx)+b(c+dx)\cos(c+dx))}{a+b\cos(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/(b + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] +
 (a*(a*c + a*d*x + b*(c + d*x)*Cos[c + d*x] - b*Sin[c + d*x]))/(a + b*Cos[c
 + d*x]))/(b^2*d)
```

Maple [B] time = 0.092, size = 163, normalized size = 1.9

$$2\,\frac{a\arctan(\tan(1/2\,dx+c/2))}{db^2} - 2\,\frac{\tan(1/2\,dx+c/2)\,a}{db\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b+a+b\right)} - 2\,\frac{a^2}{db^2\sqrt{(a+b)(a-b)}}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d/b*tan(1/2*d*x+1/2*c)*a/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)-2/d/b^2/((a+b)*(a-b))^(1/2)*arctan
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2+2/d/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.550893, size = 670, normalized size = 7.79

$$\left[\frac{2\,abdx\cos\left(dx+c\right)+2\,a^2dx-2\,ab\sin\left(dx+c\right)+\sqrt{-a^2+b^2}\left(b\cos\left(dx+c\right)+a\right)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{}}{b^2\cos(dx+c)^2+2}\right.}{2\left(b^3d\cos\left(dx+c\right)+ab^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x*cos(d*x + c) + 2*a^2*d*x - 2*a*b*sin(d*x + c) + sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(b^3*d*cos(d*x + c) + a
*b^2*d), (a*b*d*x*cos(d*x + c) + a^2*d*x - a*b*sin(d*x + c) - sqrt(a^2 - b^
2)*(b*cos(d*x + c) + a)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d
*x + c))))/(b^3*d*cos(d*x + c) + a*b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sec\left(c+dx\right)}{\left(a\sec\left(c+dx\right)+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))/(a*sec(c + d*x) + b)**2, x)
```

Giac [A] time = 1.37274, size = 188, normalized size = 2.19

$$\frac{\frac{(dx+c)a}{b^2}-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)b}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 + a + b)*b) - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqr
t(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)/d
```

3.347 $\int \frac{3+\sec(c+dx)}{2-\sec(c+dx)} dx$

Optimal. Leaf size=87

$$-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d}+\frac{5\log\left(\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d}+\frac{3x}{2}$$

```
[Out] (3*x)/2 - (5*Log[Cos[(c + d*x)/2] - Sqrt[3]*Sin[(c + d*x)/2]])/(2*Sqrt[3]*d
) + (5*Log[Cos[(c + d*x)/2] + Sqrt[3]*Sin[(c + d*x)/2]])/(2*Sqrt[3]*d)
```

Rubi [A] time = 0.0737429, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3919, 3831, 2659, 207}

$$-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d}+\frac{5\log\left(\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d}+\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Sec[c + d*x])/(2 - Sec[c + d*x]),x]
```

```
[Out] (3*x)/2 - (5*Log[Cos[(c + d*x)/2] - Sqrt[3]*Sin[(c + d*x)/2]])/(2*Sqrt[3]*d
) + (5*Log[Cos[(c + d*x)/2] + Sqrt[3]*Sin[(c + d*x)/2]])/(2*Sqrt[3]*d)
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{3+\sec(c+dx)}{2-\sec(c+dx)}\,dx = \frac{3x}{2} + \frac{5}{2}\int \frac{\sec(c+dx)}{2-\sec(c+dx)}\,dx$$
$$= \frac{3x}{2} - \frac{5}{2}\int \frac{1}{1-2\cos(c+dx)}\,dx$$
$$= \frac{3x}{2} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1+3x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$= \frac{3x}{2} - \frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d} + \frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}d}$$

Mathematica [A] time = 0.0686279, size = 39, normalized size = 0.45

$$\frac{9(c+dx) + 10\sqrt{3}\tanh^{-1}\left(\sqrt{3}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Sec[c + d*x])/(2 - Sec[c + d*x]),x]
```

```
[Out] (9*(c + d*x) + 10*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[(c + d*x)/2]])/(6*d)
```

Maple [A] time = 0.074, size = 39, normalized size = 0.5

$$3\,\frac{\arctan\left(\tan\left(1/2\,dx + c/2\right)\right)}{d} + \frac{5\sqrt{3}}{3\,d}\mathrm{Artanh}\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sec(d*x+c))/(2-sec(d*x+c)),x)
```

```
[Out] 3/d*arctan(tan(1/2*d*x+1/2*c))+5/3/d*3^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*3^(
1/2))
```

Maxima [A] time = 1.42947, size = 108, normalized size = 1.24

$$-\frac{5\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{3\sin(dx+c)}{\cos(dx+c)+1}}{\sqrt{3}+\frac{3\sin(dx+c)}{\cos(dx+c)+1}}\right) - 18\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(5*sqrt(3)*log(-(sqrt(3) - 3*sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(3)
 + 3*sin(d*x + c)/(cos(d*x + c) + 1))) - 18*arctan(sin(d*x + c)/(cos(d*x +
c) + 1)))/d
```

Fricas [A] time = 0.497372, size = 225, normalized size = 2.59

$$\frac{18\,dx + 5\,\sqrt{3}\log\left(-\frac{2\,\cos(dx+c)^2+2\,\left(\sqrt{3}\cos(dx+c)-2\,\sqrt{3}\right)\sin(dx+c)+4\,\cos(dx+c)-7}{4\,\cos(dx+c)^2-4\,\cos(dx+c)+1}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(18*d*x + 5*sqrt(3)*log(-(2*cos(d*x + c)^2 + 2*(sqrt(3)*cos(d*x + c) -
 2*sqrt(3))*sin(d*x + c) + 4*cos(d*x + c) - 7)/(4*cos(d*x + c)^2 - 4*cos(d*
x + c) + 1)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sec(c+dx)}{\sec(c+dx)-2}\,dx - \int \frac{3}{\sec(c+dx)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x)
```

```
[Out] -Integral(sec(c + d*x)/(sec(c + d*x) - 2), x) - Integral(3/(sec(c + d*x) -
2), x)
```

Giac [A] time = 1.39533, size = 78, normalized size = 0.9

$$\frac{9\,dx - 5\,\sqrt{3}\log\left(\frac{\left|-2\,\sqrt{3}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{3}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right) + 9\,c}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(9*d*x - 5*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(1/2*d*x + 1/2*c))/abs(2*s
qrt(3) + 6*tan(1/2*d*x + 1/2*c))) + 9*c)/d
```

3.348 $\int \sec^4(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=485

$$-\frac{2(a-b)\sqrt{a+b}\left(12a^2b(2A-B)-16a^3B+18ab^2(A-2B)+3b^3(25A-49B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{315b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B
+ 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))]])/(315*b^5*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*
A - 49*B) + 18*a*b^2*(A - 2*B) + 12*a^2*b*(2*A - B) - 16*a^3*B)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^4*d) - (2*(12*a^2*A*b - 75*A*b^3 - 8*a^3*B - 13*a*b^2*B)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^3*d) + (2*(9*a*A*b - 6*a^2*B + 49*b^2
*B)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9
*A*b + a*B)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d)
+ (2*B*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```

Rubi [A] time = 1.43671, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4031, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(-6a^2B+9aAb+49b^2B\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2\left(12a^2Ab-8a^3B-13ab^2B-75Ab^3\right)\tan(c+d}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B
+ 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))]])/(315*b^5*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*
A - 49*B) + 18*a*b^2*(A - 2*B) + 12*a^2*b*(2*A - B) - 16*a^3*B)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^4*d) - (2*(12*a^2*A*b - 75*A*b^3 - 8*a^3*B - 13*a*b^2*B)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^3*d) + (2*(9*a*A*b - 6*a^2*B + 49*b^2
*B)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9
*A*b + a*B)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d)
+ (2*B*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```


Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2B\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{9d} + \frac{2}{9}\int \frac{\sec^3(c$$

$$= \frac{2(9Ab+aB)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{63bd} + \frac{2.}{}$$

$$= \frac{2\left(9aAb-6a^2B+49b^2B\right)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(}{315b^2d}$$

$$= -\frac{2\left(12a^2Ab-75Ab^3-8a^3B-13ab^2B\right)\sqrt{a+b\sec(c+dx)}\,\text{ta}}{315b^3d}$$

$$= -\frac{2\left(12a^2Ab-75Ab^3-8a^3B-13ab^2B\right)\sqrt{a+b\sec(c+dx)}\,\text{ta}}{315b^3d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+14\right.}{}$$

Mathematica [B] time = 25.544, size = 3734, normalized size = 7.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*
b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^4) + (2*Sec[c + d*x]^3*(9*A*b*Sin[c
 + d*x] + a*B*Sin[c + d*x]))/(63*b) + (2*Sec[c + d*x]^2*(9*a*A*b*Sin[c + d*
x] - 6*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x]))/(315*b^2) + (2*Sec[c +
d*x]*(-12*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] + 8*a^3*B*Sin[c + d*
x] + 13*a*b^2*B*Sin[c + d*x]))/(315*b^3) + (2*B*Sec[c + d*x]^3*Tan[c + d*x]
)/9))/d + (2*((-19*a*A)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (8*a^3*A)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^4*
B)/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a^2*B)/(105*b
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*b*B)/(15*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*A*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[
b + a*Cos[c + d*x]]) - (17*a^2*A*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[
c + d*x]]) + (5*A*b*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (4*
a*B*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*B*Sqrt[Sec[
c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*B*Sqrt[Sec[c + d*x]]
)/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^2*A*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) - (7*a*B*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*B*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (8*a^3*B*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(-24*a
^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*
a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*
a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b
```

```
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^4*d*(b + a*
Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*((a*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3
+ 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*
A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3
 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^4*(b + a*Cos[c + d*x])^(3/2)*Sq
rt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d
*x)/2]*(2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147
*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3
*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 14
7*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(315*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B +
24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^4)/2 + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 +
 Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])] + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A
+ 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d
*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3
+ 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) +
3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x
])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-24*a^
3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*Sec[
(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-24*a^3*A*b - 57*a*A*b^3 +
16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24
*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^
2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
) + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*
B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*b^4*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*
a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B
) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c
```

```
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3 + 16
*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*
x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*b^4*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]])))
```

Maple [B] time = 1.664, size = 4394, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-35*B*b^5+24*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-16*B*cos(d*x+c)^6*a^5-57*A*c
os(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^3-57*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-16*B*cos(d*x+c)^5*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-4*B*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b^2-24*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+16*B*cos(d*x+c)^5*a^5+147*
B*cos(d*x+c)^5*b^5-98*B*cos(d*x+c)^4*b^5-14*B*cos(d*x+c)^2*b^5+75*A*cos(d*x
+c)^5*b^5-30*A*cos(d*x+c)^3*b^5-45*A*cos(d*x+c)*b^5+16*B*cos(d*x+c)^5*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5-
147*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^5+75*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*B*cos(d*x+c)^4*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+16*B
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^5-147*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+75*A*cos(d*x+c)^5*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*B*cos
(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^5-40*B*cos(d*x+c)*a*b^4+57*A*cos(d*x+c)^6*a^2*b^3+75*A*cos(d*x
+c)^6*a*b^4+8*B*cos(d*x+c)^6*a^4*b-24*B*cos(d*x+c)^6*a^3*b^2+13*B*cos(d*x+c
)^6*a^2*b^3+147*B*cos(d*x+c)^6*a*b^4-24*A*cos(d*x+c)^5*a^4*b+24*A*cos(d*x+c
)^5*a^3*b^2-60*A*cos(d*x+c)^5*a^2*b^3+57*A*cos(d*x+c)^5*a*b^4-16*B*cos(d*x+
c)^5*a^4*b+26*B*cos(d*x+c)^5*a^3*b^2-24*B*cos(d*x+c)^5*a^2*b^3-85*B*cos(d*x
```

```
+c)^5*a*b^4-12*A*cos(d*x+c)^4*a^3*b^2-78*A*cos(d*x+c)^4*a*b^4+8*B*cos(d*x+c
)^4*a^4*b+10*B*cos(d*x+c)^4*a^2*b^3+3*A*cos(d*x+c)^3*a^2*b^3-2*B*cos(d*x+c)
^3*a^3*b^2-22*B*cos(d*x+c)^3*a*b^4-54*A*cos(d*x+c)^2*a*b^4+B*cos(d*x+c)^2*a
^2*b^3+24*A*cos(d*x+c)^6*a^4*b-12*A*cos(d*x+c)^6*a^3*b^2+111*B*cos(d*x+c)^5
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^4+16*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^4*b+24*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+24*B*cos(d*x+c)
^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^3-147*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+24*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+6*A*cos(d*
x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b^3+57*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-24*A*cos(d*x+c)^4*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-24*A*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*b^2-57*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-57*A*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-16*B*
cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^4*b-4*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-24*B*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+11
1*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^4+16*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+24*B*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2
+24*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b^3-147*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+6*A*cos(d*x+c)^5*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b^3+57*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^4-24*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-24*A*cos(d*x+c)^5*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
```

```
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^3*b^2)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+B\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4, x)
```

3.349 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=397

$$\frac{2(a-b)\sqrt{a+b}\left(-8a^2B+2ab(7A-3B)+b^2(63A-25B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{105b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(105*b^4*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) + 2*a*b*(7
*A - 3*B) - 8*a^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(7*a*A*b - 4*a^2*B + 25
*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^2*d) + (2*(7*A*b + a*
B)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b*d) + (2*B*Sec[
c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.932789, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4031, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(-4a^2B+7aAb+25b^2B\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^2d}+\frac{2(a-b)\sqrt{a+b}\left(-8a^2B+2ab(7A-3B)+b^2(63A-25B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(105*b^4*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) + 2*a*b*(7
*A - 3*B) - 8*a^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(7*a*A*b - 4*a^2*B + 25
*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^2*d) + (2*(7*A*b + a*
B)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b*d) + (2*B*Sec[
c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*d)
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
```

```
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2B\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7d}+\frac{2}{7}\int \frac{\sec^2(c}{}$$

$$= \frac{2(7Ab+aB)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35bd}+\frac{2B}{}$$

$$= \frac{2\left(7aAb-4a^2B+25b^2B\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^2d}+\frac{2}{}$$

$$= \frac{2\left(7aAb-4a^2B+25b^2B\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^2d}+\frac{2}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(14a^2Ab-63Ab^3-8a^3B-19ab^2B\right)\cot(c+d}{105}$$

Mathematica [B] time = 24.4237, size = 3330, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*
B)*Sin[c + d*x])/(105*b^3) + (2*Sec[c + d*x]^2*(7*A*b*Sin[c + d*x] + a*B*Si
n[c + d*x]))/(35*b) + (2*Sec[c + d*x]*(7*a*A*b*Sin[c + d*x] - 4*a^2*B*Sin[c
 + d*x] + 25*b^2*B*Sin[c + d*x]))/(105*b^2) + (2*B*Sec[c + d*x]^2*Tan[c + d
*x])/7))/d - (2*((2*a^2*A)/(15*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (3*A*b)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (19*a*B)/(10
5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(105*b^2*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqrt[Sec[c + d*x]])/(15*Sqrt
[b + a*Cos[c + d*x]]) + (2*a^3*A*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos
[c + d*x]]) - (8*a^4*B*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (17*a^2*B*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) + (5*b*
B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*Cos[2*(c + d*x
)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*B*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^
2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(
-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*
(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^
2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
)/(105*b^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x
]]*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-14*
a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*(7*A
 + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-14*a^2*A*b + 63*A*b
^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(6
3*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*(((-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-14*a^2*A*b + 63*A*b^3
 + 8*a^3*B + 19*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*S
in[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B)
+ b^2*(63*A + 25*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c
```

```
 + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B +
 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^2
*B - 2*a*b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((
a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))] - a*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x
]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-14*a^2*A*b + 63*A*b^
3 + 8*a^3*B + 19*a*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d
*x]*Tan[(c + d*x)/2] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a
 + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)]) + ((a + b)*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*
b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^3*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-14*a^2*A*b + 63*A*b^3 + 8
*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*A +
25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
 + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/
(105*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.091, size = 3438, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-15*B*b^4+8*B*cos(d*x+c)^5*a^4+2*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+19*B*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b^3+14*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+14*A*sin(d*x+c)*cos(d*x+c)^4*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-63*A*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b^3-14*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
```

x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+49*A*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*B*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b-19*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-19*B*sin(d*x+c)*cos(d*x+c)^4
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*B
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*b+19*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+14*A*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+14*A
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*b^2-63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-14*A*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+
49*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^3-8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-19*B*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2
-19*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^3-35*A*cos(d*x+c)^4*a*b^3+8*B*cos(d*x+c)^4*a^3*b-20
*B*cos(d*x+c)^4*a^2*b^2+19*B*cos(d*x+c)^4*a*b^3+7*A*cos(d*x+c)^3*a^2*b^2-4*
B*cos(d*x+c)^3*a^3*b-26*B*cos(d*x+c)^3*a*b^3-28*A*cos(d*x+c)^2*a*b^3+B*cos(
d*x+c)^2*a^2*b^2-18*B*cos(d*x+c)*a*b^3-14*A*cos(d*x+c)^5*a^3*b+7*A*cos(d*x+
c)^5*a^2*b^2+63*A*cos(d*x+c)^5*a*b^3-4*B*cos(d*x+c)^5*a^3*b+19*B*cos(d*x+c)
^5*a^2*b^2+25*B*cos(d*x+c)^5*a*b^3+14*A*cos(d*x+c)^4*a^3*b-14*A*cos(d*x+c)^
4*a^2*b^2+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+2*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-63*A*sin(d*x+c)
*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^4+63*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^4-8*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*B*sin(d*x+c)*cos(d*
x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4
-63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^4+63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-8*B*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))

```
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*B*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^4+63*A*cos(d*x+c)^4*b^4+25*B*cos(d*x+c)^4*b^4-10*B*cos(d*x+c
)^2*b^4-8*B*cos(d*x+c)^4*a^4-42*A*cos(d*x+c)^3*b^4-21*A*cos(d*x+c)*b^4)/(b+
a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+B\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.350 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=314

$$-\frac{2(a-b)\sqrt{a+b}(-2aB+5Ab-9bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3
*d) - (2*(a - b)*Sqrt[a + b]*(5*A*b - 2*a*B - 9*b*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) + (2*(5*A*b - 2*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (
2*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.597502, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4010, 4002, 4005, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\left(-2a^2B+5aAb+9b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-2($$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3
*d) - (2*(a - b)*Sqrt[a + b]*(5*A*b - 2*a*B - 9*b*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) + (2*(5*A*b - 2*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (
2*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \sec(c+dx)\sqrt{a+b}\cdots}{\cdots}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2B(a+b\sec(\cdots}{\cdots}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2B(a+b\sec(\cdots}{\cdots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(5aAb-2a^2B+9b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\cdots\right.\right.}{15b^3d}$$

Mathematica [A] time = 18.4535, size = 434, normalized size = 1.38

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2b(a+b)(-2aB+5Ab+9bB)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{Ellipti}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b
)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(5*A*b - 2*a*B + 9*b*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-5*a*A*
```

```
b + 2*a^2*B - 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))/(15*b^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]
*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Sec[c + d*x]]*((2*(5*a*A*b - 2*a^2*B + 9
*b^2*B)*Sin[c + d*x])/(15*b^2) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] + a*B*
Sin[c + d*x]))/(15*b) + (2*B*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.718, size = 2498, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(5*A*cos(d*x+c)^3*b^3+5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b-5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^2-2*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+7*B*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+
2*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5*A
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*b-5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*B*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+7*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-3*B*b^3-5
*A*cos(d*x+c)*b^3-2*B*cos(d*x+c)^4*a^3+2*B*cos(d*x+c)^3*a^3+9*B*cos(d*x+c)^
3*b^3-6*B*cos(d*x+c)^2*b^3+5*A*cos(d*x+c)^4*a*b^2+B*cos(d*x+c)^4*a^2*b+9*B*
cos(d*x+c)^4*a*b^2-5*A*cos(d*x+c)^3*a^2*b+5*A*cos(d*x+c)^3*a*b^2-2*B*cos(d*
x+c)^3*a^2*b-5*B*cos(d*x+c)^3*a*b^2-10*A*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)^2*
a^2*b-4*B*cos(d*x+c)*a*b^2+5*A*cos(d*x+c)^4*a^2*b+5*A*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*B
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
```

```
/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*A*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^3+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3
)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a} \sec(dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B \sec(dx + c)^3 + A \sec(dx + c)^2\right)\sqrt{b \sec(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B \sec(c + dx)) \sqrt{a + b \sec(c + dx)} \sec^2(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.351 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=256

$$\frac{2(a-b)\sqrt{a+b}(3A-B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2(a-b)\sqrt{a+}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*(a - b)*
Sqrt[a + b]*(3*A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*B*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.340192, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4002, 4005, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}(aB+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2(a-b)\sqrt{a+b}(3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*(a - b)*
Sqrt[a + b]*(3*A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*B*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(3*d)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\sec(c+dx)\left(\frac{1}{2}(3a\right.}{}$$

$$= \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}((a-b)(3A-B))\int$$

$$= -\frac{2(a-b)\sqrt{a+b}(3Ab+aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{3b^2d}$$

Mathematica [A] time = 14.9369, size = 408, normalized size = 1.59

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\left(2b(a+b)(3A+B)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+}{(a+b)(\cos(c+dx}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Se
c[c + d*x])*(-2*(a + b)*(3*A*b + a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*A + B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*A*b + a*B)*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*d
*(b + a*Cos[c + d*x])*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c +
 d*x]^(3/2)) + (Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x])*
((2*(3*A*b + a*B)*Sin[c + d*x])/(3*b) + (2*B*Tan[c + d*x])/3))/(d*(B + A*Co
s[c + d*x]))
```

Maple [B] time = 0.49, size = 1752, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/b*(-1+cos(d*x+c))^2*(-B*b^2+B*cos(d*x+c)^3*a^2-B*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+3
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^2-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+
3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^2-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+B*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+3*A*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b+3*A*cos(d*x+c)^3*a*b+B*cos(d*x+c)^3*a*b-3*A*cos(d*x+c)^2*a*b+B*c
os(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b-B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^2-3*A*cos(d*x+
c)*b^2+B*cos(d*x+c)^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+
1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sec\left(dx+c\right)^2+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+B\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.352 $\int \sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=320

$$\frac{2\sqrt{a+b}(B(a-b)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(A*b + (a -
b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rubi [A] time = 0.290496, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3916, 3784, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(B(a-b)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(A*b + (a -
b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rule 3916

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
```

```
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = (aA)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(Ab+aB+bB\sec(c+$$

$$= -\frac{2A\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{d}$$

$$= -\frac{2(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{bd}$$

Mathematica [C] time = 17.8871, size = 913, normalized size = 2.85

$$\frac{2B\cos(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\sin(c+dx)}{d(B+A\cos(c+dx))} + \frac{2\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\left(a\sqrt{\frac{b-a}{a+b}}B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*B*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sin[c + d*x
])/(d*(B + A*Cos[c + d*x])) + (2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*
x])*(a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + b*Sqrt[(-a + b)/(a + b)]
*B*Tan[(c + d*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + a*S
qrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*B*Tan
[(c + d*x)/2]^5 + (2*I)*a*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
+ (2*I)*a*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
```

```
)] - I*(a - b)*B*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a
- b)*(A - B)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(Sqrt[(-a
 + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*(B + A*Cos[c + d*x])*Sec[c + d*x]
^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.403, size = 1372, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*
(A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a-A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-B*co
s(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b+B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a+B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+A*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-B*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-B*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+
B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*s
in(d*x+c)-B*cos(d*x+c)^2*a+B*cos(d*x+c)*a-B*cos(d*x+c)*b+B*b)/sin(d*x+c)^5/
(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)\sqrt{b\sec (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\sec (dx+c)+A)\sqrt{b\sec (dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec (c+dx))\sqrt{a+b\sec (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)\sqrt{b\sec (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)
```

3.353 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=344

$$\frac{\sqrt{a+b}(A+2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(2aB+Ab)\,\text{co}}{}$$

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A + 2*B)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (Sqrt[a + b]*(A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (A*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.36894, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4032, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(A+2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(2aB+Ab)\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A + 2*B)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (Sqrt[a + b]*(A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (A*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{\frac{1}{2}(Ab+2aB)+bB\,\mathrm{se}}{\sqrt{a+}}$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}(Ab)\int \frac{\sec(c+dx)(1}{\sqrt{a+b\,\mathrm{se}}}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{bd}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{bd}$$

Mathematica [C] time = 17.9205, size = 1107, normalized size = 3.22

$$\sqrt{a+b\sec(c+dx)}\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(aA\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-Ab\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-2aA\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*A*Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]^3 + a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
- A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (2*I)*A*b*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - I*A*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + (2*I)*(a - b)*B*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)]))/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b - b*Tan[(c +
d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2))
```

Maple [B] time = 0.385, size = 1389, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(2*A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*
A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*b+2*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-4*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
```

```
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*sin(d*x+c)-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b*sin(d*x+c)-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-4*B*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-A*cos(d*x+c)^3*a+A*
a*cos(d*x+c)^2-A*cos(d*x+c)^2*b+A*b*cos(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\sqrt{a+b\sec(c+dx)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

3.354 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=429

$$\frac{\sqrt{a+b}(2a(A+2B)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4ad}-\sqrt{a+b}\,(\cdots$$

```
[Out] ((a - b)*Sqrt[a + b]*(A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
A*b + 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2*A - A*b^2
+ 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((A*b + 4*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.732771, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4032, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2A+4abB-Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{(4aB+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
A*b + 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2*A - A*b^2
+ 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((A*b + 4*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```


Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(c}{}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{A\cos(c+dx}{}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{A\cos(c+dx}{}$$

$$= \frac{(a-b)\sqrt{a+b}(Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{4abd}$$

$$= \frac{(a-b)\sqrt{a+b}(Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{4abd}$$

Mathematica [B] time = 18.83, size = 1161, normalized size = 2.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d
*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*b*Tan[(c + d*x)/2] + A*b^2*Ta
n[(c + d*x)/2] + 4*a^2*B*Tan[(c + d*x)/2] + 4*a*b*B*Tan[(c + d*x)/2] - 2*a*
A*b*Tan[(c + d*x)/2]^3 - 8*a^2*B*Tan[(c + d*x)/2]^3 + a*A*b*Tan[(c + d*x)/2
]^5 - A*b^2*Tan[(c + d*x)/2]^5 + 4*a^2*B*Tan[(c + d*x)/2]^5 - 4*a*b*B*Tan[(
c + d*x)/2]^5 - 8*a^2*A*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + 2*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*b*B*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a^2*A*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 2*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*b*B*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] + (a + b)*(A*b + 4*a*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(2*
a*A - A*b + 4*b*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*d*Sqrt[b + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.379, size = 2065, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/a*(-1+cos(d*x+c))^2*(-4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*A*cos(d*x+c)^4*a^2+8*A*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+4*B
*cos(d*x+c)^3*a^2+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+3*A*cos(d*x+c)^3*a*
b-A*cos(d*x+c)^2*a*b+4*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b+2*A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b-8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*B*cos(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*a*b-8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b*sin(d*x+c)+8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*A*cos(d*x+c)^2*a^2-2*A*cos(d*x+c)*a*b-
2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^2*sin(d*x+c)+A*cos(d*x+c)^2*b^2-4*B*cos(d*x+c)^2*a^2-A*cos(
d*x+c)*b^2-4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b^2+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)*a*b+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*a*b+4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*a*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c)
)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.355 $\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=509

$$\frac{\sqrt{a+b}(2a+b)(8aA+6aB-3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{24a^2d}+\frac{(16\ldots}{\ldots}$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*b
*d) + (Sqrt[a + b]*(2*a + b)*(8*a*A - 3*A*b + 6*a*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*
d) - (Sqrt[a + b]*(4*a^2*A*b + A*b^3 + 8*a^3*B - 2*a*b^2*B)*Cot[c + d*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(8*a^3*d) + ((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(24*a^2*d) + ((A*b + 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(12*a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.12949, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4032, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2A+6abB-3Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24a^2d}+\frac{(a-b)\sqrt{a+b}\left(16a^2A+6abB-3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\ldots}{24a^2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*b
*d) + (Sqrt[a + b]*(2*a + b)*(8*a*A - 3*A*b + 6*a*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*
d) - (Sqrt[a + b]*(4*a^2*A*b + A*b^3 + 8*a^3*B - 2*a*b^2*B)*Cot[c + d*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(8*a^3*d) + ((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(24*a^2*d) + ((A*b + 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(12*a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(3*d)
```


Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\cos^2(c\cdots}{\cdots}$$

$$= \frac{(Ab+6aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12ad} + \frac{A\cos\cdots}{\cdots}$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^2d} + \frac{(A\cdots}{\cdots}$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^2d} + \frac{(A\cdots}{\cdots}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A-3Ab^2+6abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\cdots\right.\right.}{24a^2bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A-3Ab^2+6abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\cdots\right.\right.}{24a^2bd}$$

Mathematica [B] time = 20.1862, size = 1565, normalized size = 3.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((A*Sin[c + d*x])/12 + ((A*b + 6*a*B)*Sin[2*(c +
d*x)])/(24*a) + (A*Sin[3*(c + d*x)])/12))/d + (Sqrt[a + b*Sec[c + d*x]]*Sqr
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-16*a^3*A*Tan[(c + d*x)/2] - 16
*a^2*A*b*Tan[(c + d*x)/2] + 3*a*A*b^2*Tan[(c + d*x)/2] + 3*A*b^3*Tan[(c + d
*x)/2] - 6*a^2*b*B*Tan[(c + d*x)/2] - 6*a*b^2*B*Tan[(c + d*x)/2] + 32*a^3*A
*Tan[(c + d*x)/2]^3 - 6*a*A*b^2*Tan[(c + d*x)/2]^3 + 12*a^2*b*B*Tan[(c + d*
x)/2]^3 - 16*a^3*A*Tan[(c + d*x)/2]^5 + 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 3*a
*A*b^2*Tan[(c + d*x)/2]^5 - 3*A*b^3*Tan[(c + d*x)/2]^5 - 6*a^2*b*B*Tan[(c +
 d*x)/2]^5 + 6*a*b^2*B*Tan[(c + d*x)/2]^5 + 24*a^2*A*b*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A*b^3*Elli
pticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + 48*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] - 12*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b*EllipticPi[-1, -ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(
c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 1
2*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(16*a^2*A - 3*A*b^2 + 6*a*b*B)
```

```
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + 2*a*(-(A*b^2) + 2*a*b*(7*A - 3*B) + 12*a^2*B)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)]))/(24*a^2*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan
[(c + d*x)/2]^2)))
```

Maple [B] time = 0.453, size = 2954, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d/a^2*(-1+cos(d*x+c))^2*(16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+6*B*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+6*B*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+12*B*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-
12*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*a+16*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*A*cos(d*x+c)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3+48*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+16*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*A*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-
28*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*b+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*a+24*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-12*B*a^3*cos(d*x+c)^2+8*A*c
os(d*x+c)^5*a^3+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3-3*A*cos(d*x+c)^2
*b^3+6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*b^3*sin(d*x+c)-24*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
```

```
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*sin(d*x+c)+48*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+3*A*cos(d*x+c)*b^3+12*B*cos(d*x+c
)^4*a^3+6*A*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b
-2*A*cos(d*x+c)*a*b^2-12*B*cos(d*x+c)*a^2*b+12*B*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-12*B*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a*b^2+16*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-28*A*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*b+2*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+24*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+6*B*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2-A*cos(d*x+c)^3*a*b^2+18*B*cos(d*x+c)^3*a^2*b+
3*A*cos(d*x+c)^2*a*b^2-6*B*cos(d*x+c)^2*a^2*b-6*B*cos(d*x+c)*a*b^2+10*A*cos
(d*x+c)^4*a^2*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*
cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos(dx+c)^3\sec(dx+c)+A\cos(dx+c)^3\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.356 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=475

$$-\frac{2(a-b)\sqrt{a+b}\left(-6a^2b(3A-B)+8a^3B-3ab^2(57A-13B)+3b^3(25A-49B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+}{a-b}}}{315b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B -
 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))]])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*A
 - 49*B) - 3*a*b^2*(57*A - 13*B) - 6*a^2*b*(3*A - B) + 8*a^3*B)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^3*d) - (2*(18*a^2*A*b - 75*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^
2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a
*B)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*B*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 1.22462, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4033, 4082, 4002, 4005, 3832, 4004}

$$-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2Ab-8a^3B-39ab^2B-75Ab^3\right)\tan(c+dx)\sqrt{a+}}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B -
 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))]])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*A
 - 49*B) - 3*a*b^2*(57*A - 13*B) - 6*a^2*b*(3*A - B) + 8*a^3*B)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^3*d) - (2*(18*a^2*A*b - 75*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^
2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a
*B)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*B*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
```

```
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(}{}$$

$$= \frac{2(9Ab-4aB)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d} + \frac{2B\sec(c}{}$$

$$= -\frac{2\left(18aAb-8a^2B-49b^2B\right)(a+b\sec(c+dx))^{3/2}\tan(c+}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\sec(c+dx)}}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\sec(c+dx)}}{315b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B-14\right.}{}$$

Mathematica [B] time = 25.7647, size = 3766, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(-18*a^3*A*b + 246*a*A*b^3 + 8
*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x
]^3*(9*A*b*Sin[c + d*x] + 10*a*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(72*
a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x]))/(315*b)
 + (2*Sec[c + d*x]*(9*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] - 4*a^3*
B*Sin[c + d*x] + 88*a*b^2*B*Sin[c + d*x]))/(315*b^2) + (2*b*B*Sec[c + d*x]^
3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])) - (2*((2*a^3*A)/(35*b*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82*a*A*b)/(105*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) - (11*a^2*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (8*a^4*B)/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (7*b^2*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a^2*
A*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*A*Sqrt[Sec[c
+ d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) + (5*A*b^2*Sqrt[Sec[c + d*x]])/(
21*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b
 + a*Cos[c + d*x]]) - (31*a^3*B*Sqrt[Sec[c + d*x]])/(315*b*Sqrt[b + a*Cos[c
 + d*x]]) + (13*a*b*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) -
(82*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]
]) + (2*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c
 + d*x]]) - (8*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b +
 a*Cos[c + d*x]]) - (11*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*S
qrt[b + a*Cos[c + d*x]]) - (7*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(1
5*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*S
ec[c + d*x])^(3/2)*(2*(a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2
*b^2*B + 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A
 + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + (-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^
2*b^2*B + 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(315*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2
```

]*Sec[c + d*x]^(3/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x
]*(2*(a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*
B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
- 2*b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*
(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + (-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4
*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))
/(315*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-18*a^3*A*b + 246
*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*B - 6*a^2*
b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-18*a^3*A*b + 24
6*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^3*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((
(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Cos[c + d*
x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-18*a^3*A*b + 246
*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
 Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^3*
B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
] + ((a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*
B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b
+ a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^3*B - 6*a^2
b(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))] - a*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a
^2*b^2*B + 147*b^4*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
 d*x)/2] - (-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-
18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(8
*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1
- ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-18*a^3*A*b + 246*a*A*
b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)
/2]^2]))/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*
(a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b
(a + b)(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A
 + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a

```
 + b)] + (-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Co
s[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c +
d*x]*Tan[c + d*x]))/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]
^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.639, size = 4394, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-35*B*b^5-18*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+8*B*cos(d*x+c)^6*a^5-246*A*c
os(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^3-246*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+8*B*cos(d*x+c)^5*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+2*B*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b^2+33*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-8*B*cos(d*x+c)^5*a^5+147*B
*cos(d*x+c)^5*b^5-98*B*cos(d*x+c)^4*b^5-14*B*cos(d*x+c)^2*b^5+75*A*cos(d*x+
c)^5*b^5-30*A*cos(d*x+c)^3*b^5-45*A*cos(d*x+c)*b^5-8*B*cos(d*x+c)^5*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5-14
7*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^5+75*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*B*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-8*B*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^5-147*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+75*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*B*cos(d*
x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^5-85*B*cos(d*x+c)*a*b^4+246*A*cos(d*x+c)^6*a^2*b^3+75*A*cos(d*x+c
)^6*a*b^4-4*B*cos(d*x+c)^6*a^4*b+33*B*cos(d*x+c)^6*a^3*b^2+88*B*cos(d*x+c)^
6*a^2*b^3+147*B*cos(d*x+c)^6*a*b^4+18*A*cos(d*x+c)^5*a^4*b-18*A*cos(d*x+c)^
5*a^3*b^2-165*A*cos(d*x+c)^5*a^2*b^3+246*A*cos(d*x+c)^5*a*b^4+8*B*cos(d*x+c
)^5*a^4*b-34*B*cos(d*x+c)^5*a^3*b^2+33*B*cos(d*x+c)^5*a^2*b^3-10*B*cos(d*x+
c)^5*a*b^4+9*A*cos(d*x+c)^4*a^3*b^2-204*A*cos(d*x+c)^4*a*b^4-4*B*cos(d*x+c)
^4*a^4*b-68*B*cos(d*x+c)^4*a^2*b^3-81*A*cos(d*x+c)^3*a^2*b^3+B*cos(d*x+c)^3
```

```
*a^3*b^2-52*B*cos(d*x+c)^3*a*b^4-117*A*cos(d*x+c)^2*a*b^4-53*B*cos(d*x+c)^2
*a^2*b^3-18*A*cos(d*x+c)^6*a^4*b+9*A*cos(d*x+c)^6*a^3*b^2+186*B*cos(d*x+c)^
5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^4-8*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^4*b-33*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-33*B*cos(d*x+c)
^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^3-147*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-18*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+153*A*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*b^3+246*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+18*A*cos(d*x+c)^4*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+18*A*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b^2-246*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-246*A*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+
8*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^4*b+2*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+33*B*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^
3+186*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^4-8*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-33*B*cos(d*x+c)^4*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
b^2-33*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b^3-147*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+153*A*cos(d*x+c)^5
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b^3+246*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+18*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+18*A*cos(d*x+c
)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^3*b^2)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec\left(dx+c\right)^{5}+Aa\sec\left(dx+c\right)^{3}+\left(Ba+Ab\right)\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^5 + A*a*sec(d*x + c)^3 + (B*a + A*b)*sec(d*x + c
)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

3.357 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=388

$$\frac{2(a-b)\sqrt{a+b}\left(6a^2B-a(21Ab-57bB)+b^2(63A-25B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\right.}{105b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(105*b^3*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) + 6*a^2*B
 - a*(21*A*b - 57*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) + (2*(21*a*A*b - 6*a^2*B
+ 25*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) + (2*(7*A*b -
2*a*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*B*(a + b*Sec[
c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.829065, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4010, 4002, 4005, 3832, 4004}

$$\frac{2\left(-6a^2B+21aAb+25b^2B\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2(a-b)\sqrt{a+b}\left(6a^2B-a(21Ab-57bB)+b^2(63A-25\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(105*b^3*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) + 6*a^2*B
 - a*(21*A*b - 57*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) + (2*(21*a*A*b - 6*a^2*B
+ 25*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) + (2*(7*A*b -
2*a*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*B*(a + b*Sec[
c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int \sec(c+dx)(a+}{}$$

$$= \frac{2(7Ab-2aB)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd} + \frac{2B(a+b}{}$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105bd}$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(21a^2Ab+63Ab^3-6a^3B+82ab^2B\right)\cot(c}{}$$

Mathematica [B] time = 24.5464, size = 3342, normalized size = 8.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*(-21*a^2*A*b - 63*A*b^3 + 6*a
^3*B - 82*a*b^2*B)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]^2*(7*A*b*Sin[c
```

```
 + d*x] + 8*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(42*a*A*b*Sin[c + d*x]
+ 3*a^2*B*Sin[c + d*x] + 25*b^2*B*Sin[c + d*x]))/(105*b) + (2*b*B*Sec[c + d
*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])) + (2*(-(a^2*A)/(5*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*A*b^2)/(5*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]) + (2*a^3*B)/(35*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (82*a*b*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(a^3*A*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (a*A*b*Sqrt[Sec
[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (31*a^2*B*Sqrt[Sec[c + d*x]])/(1
05*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b
+ a*Cos[c + d*x]]) + (5*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*
x]]) - (a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c +
d*x]]) - (3*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c
+ d*x]]) - (82*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*C
os[c + d*x]]) + (2*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[
b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)*(2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(
a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 63
*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c +
 d*x)/2]^2]*Sec[c + d*x]^(3/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Si
n[c + d*x]*(2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a
 + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 63*
A*b^3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c
+ d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*
(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^2*
B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 63*A*b^3 + 6*a^3*
B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) +
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-21*a^2*A*b - 63*A*b^3 + 6*a^3*
B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (
(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b
)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*
x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-
21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))] + (b*(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*
B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b
 + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b +
```

```
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-21*a^2*A*b - 63*A*b^3
+ 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c
 + d*x)/2] - (-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-21*a^2*A*b - 63
*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b
^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-21
*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x
)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*
x)/2]^2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((
2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^
2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 63*A*b^3 + 6*a^
3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
 d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.07, size = 3424, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/105/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(15*B*b^4+6*B*cos(d*x+c)^5*a^4-51*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-82*B*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*b^3+21*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+21*A*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+63*A*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^3-21*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-84*A*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*B*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b+82*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+82*B*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*B*
```

```
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b-82*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+21*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+21*A*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^2+63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-21*A*sin(d*x+c)*cos(d*x+c)^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-8
4*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^3-6*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+82*B*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+
82*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^3+6*B*cos(d*x+c)^4*a^3*b+55*B*cos(d*x+c)^4*a^2*b^2-8
2*B*cos(d*x+c)^4*a*b^3+63*A*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^3*a^3*b+68*
B*cos(d*x+c)^3*a*b^3+63*A*cos(d*x+c)^2*a*b^3+27*B*cos(d*x+c)^2*a^2*b^2+39*B
*cos(d*x+c)*a*b^3-21*A*cos(d*x+c)^5*a^3*b-42*A*cos(d*x+c)^5*a^2*b^2-63*A*co
s(d*x+c)^5*a*b^3-3*B*cos(d*x+c)^5*a^3*b-82*B*cos(d*x+c)^5*a^2*b^2-25*B*cos(
d*x+c)^5*a*b^3+21*A*cos(d*x+c)^4*a^3*b-21*A*cos(d*x+c)^4*a^2*b^2+6*B*sin(d*
x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^3*b-51*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+63*A*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-63*A*sin(d*
x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^4-6*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^4-25*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+63*A*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^4-63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^4-6*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-25*B*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-63
*A*cos(d*x+c)^4*b^4-25*B*cos(d*x+c)^4*b^4+10*B*cos(d*x+c)^2*b^4-6*B*cos(d*x
+c)^4*a^4+42*A*cos(d*x+c)^3*b^4+21*A*cos(d*x+c)*b^4)/(b+a*cos(d*x+c))/cos(d
*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^4 + Aa\sec(dx+c)^2 + (Ba+Ab)\sec(dx+c)^3\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^4 + A*a*sec(d*x + c)^2 + (B*a + A*b)*sec(d*x + c
)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

3.358 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=312

$$\frac{2(a-b)\sqrt{a+b}(15aA-3aB-5Ab+9bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{15bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^
2*d) + (2*(a - b)*Sqrt[a + b]*(15*a*A - 5*A*b - 3*a*B + 9*b*B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*b*d) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*
d) + (2*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.570295, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4002, 4005, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\left(3a^2B+20aAb+9b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^2d}+2($$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^
2*d) + (2*(a - b)*Sqrt[a + b]*(15*a*A - 5*A*b - 3*a*B + 9*b*B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*b*d) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*
d) + (2*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int \sec(c+dx)\sqrt{a+$$

$$=\frac{2(5Ab+3aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2B(a+b\sec$$

$$=\frac{2(5Ab+3aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2B(a+b\sec$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(20aAb+3a^2B+9b^2B\right)\cot(c+dx)E\left(\sin^{-}\right.}{15b^2d}$$

Mathematica [A] time = 18.7551, size = 502, normalized size = 1.61

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\left(\frac{2\left(3a^2B+20aAb+9b^2B\right)\sin(c+dx)}{15b}+\frac{2}{15}\sec(c+dx)(6aB\sin(c+dx)+5Ab\right.}{d(a\cos(c+dx)+b)(A\cos(c+dx)+B)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B
*Sec[c + d*x])*(2*(a + b)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*(5
*A + B) + b*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*d*(b + a*Cos[c +
d*x])^2*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)) +
 (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*((2*(20*a*
A*b + 3*a^2*B + 9*b^2*B)*Sin[c + d*x])/(15*b) + (2*Sec[c + d*x]*(5*A*b*Sin[
c + d*x] + 6*a*B*Sin[c + d*x]))/15 + (2*b*B*Sec[c + d*x]*Tan[c + d*x])/5))/
(d*(b + a*Cos[c + d*x])*(B + A*Cos[c + d*x]))
```

Maple [B] time = 0.71, size = 2683, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/15/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b-5*A*cos(d*x+c)^3*b^3-20*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^2-15*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b+20*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+20*A*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^2-3*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b-12*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
+9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2-20*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+20*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+
20*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-12*B*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3
*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*b+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*B*b^3+5*A*cos(d*x+c)*b^3-3
*B*cos(d*x+c)^4*a^3+3*B*cos(d*x+c)^3*a^3-9*B*cos(d*x+c)^3*b^3+6*B*cos(d*x+c
)^2*b^3-5*A*cos(d*x+c)^4*a*b^2-6*B*cos(d*x+c)^4*a^2*b-9*B*cos(d*x+c)^4*a*b^
2+20*A*cos(d*x+c)^3*a^2*b-20*A*cos(d*x+c)^3*a*b^2-3*B*cos(d*x+c)^3*a^2*b+25
*A*cos(d*x+c)^2*a*b^2+9*B*cos(d*x+c)^2*a^2*b+9*B*cos(d*x+c)*a*b^2-20*A*cos(
d*x+c)^4*a^2*b-5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*B*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3+9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
```

```
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3-5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*B*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*B
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin
(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^3+Aa\sec(dx+c)+(Ba+Ab)\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^3 + A*a*sec(d*x + c) + (B*a + A*b)*sec(d*x + c)^
2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.359 $\int (a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=381

$$-\frac{2\sqrt{a+b}\left(-3a^2B-a(6Ab-4bB)+b^2(3A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{3bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*Sqrt[a +
 b]*(b^2*(3*A - B) - 3*a^2*B - a*(6*A*b - 4*b*B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (
2*a*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*B*Sqrt[a + b*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.464855, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3918, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(-3a^2B-a(6Ab-4bB)+b^2(3A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*Sqrt[a +
 b]*(b^2*(3*A - B) - 3*a^2*B - a*(6*A*b - 4*b*B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (
2*a*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*B*Sqrt[a + b*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```


Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2bB\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3a^2A}{2}+\frac{1}{2}\left(6aAb+3a^2B+\right.}{}$$

$$=\frac{2bB\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3a^2A}{2}+\left(-\frac{1}{2}b(3Ab+4aB)\right.}{\sqrt{}}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{3bd}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{3bd}$$

Mathematica [B] time = 24.1177, size = 6093, normalized size = 15.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.476, size = 2340, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(B*b^2-4*B*cos(d*x+c)^3*a^2+4*B*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-3
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^2+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*B*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2+3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-6*A*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b-3*A*cos(d*x+c)^3*a*b-B*cos(d*x+c)^3*a*b+3*A*cos(d*x+c)^2*a*b-4
*B*cos(d*x+c)^2*a*b+5*B*cos(d*x+c)*a*b+3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)
^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b+4*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b-6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-4*B*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-6*A*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^2-3*B*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^2-3*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
```

```
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-3*A*cos(d*x+c)^2*b^2+4*B*
cos(d*x+c)^2*a^2+3*A*cos(d*x+c)*b^2-B*cos(d*x+c)^2*b^2+3*A*cos(d*x+c)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
6*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)
^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2), x)
```

3.360 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{a+b}(a(A+4B)+2b(A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{(a-b}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(a*A - 2*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(2*b*
(A - B) + a*(A + 4*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (Sqrt[a + b]*(3*A*b + 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d + (a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.451444, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4025, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(a(A+4B)+2b(A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(a*A - 2*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(2*b*
(A - B) + a*(A + 4*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (Sqrt[a + b]*(3*A*b + 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d + (a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
```

```
+ Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \int \frac{-\frac{1}{2}a(3Ab+2aB)}{}$$

$$= \frac{aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}(b(aA-2bB))\int$$

$$= \frac{(a-b)\sqrt{a+b}(aA-2bB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}(aA-2bB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{bd}$$

Mathematica [B] time = 18.375, size = 979, normalized size = 2.71

$$\frac{2bB\cos(c+dx)\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{d(b+a\cos(c+dx))} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(a^2A\tan^5\left(\frac{1}{2}(c+dx)\right) - aAb\tan^5\left(\frac{1}{2}(c+dx)\right) + \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*b*B*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[
c + d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)
]*(a^2*A*Tan[(c + d*x)/2] + a*A*b*Tan[(c + d*x)/2] - 2*a*b*B*Tan[(c + d*x)/
2] - 2*b^2*B*Tan[(c + d*x)/2] - 2*a^2*A*Tan[(c + d*x)/2]^3 + 4*a*b*B*Tan[(c
 + d*x)/2]^3 + a^2*A*Tan[(c + d*x)/2]^5 - a*A*b*Tan[(c + d*x)/2]^5 - 2*a*b*
B*Tan[(c + d*x)/2]^5 + 2*b^2*B*Tan[(c + d*x)/2]^5 - 6*a*A*b*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^2*B
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - 6*a*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^2*B*EllipticPi[-1, -ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (a + b)*(a*A - 2*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a*b*(A - B) + a^2
*B - b^2*(A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(b + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.464, size = 2199, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(2*B*b^2-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-A*cos(d*x+c)^3*a^2-6*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*a*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*B*cos(d*x+c)*b^2-A
*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)^2*a*b+2*B*cos(d*x+c)*a*b+4*A*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
```

```
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-6*A*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*a*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-4*B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2-4*B*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x
+c)+A*cos(d*x+c)^2*a^2+2*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+A*cos(d*x+c)*a*b-2*B*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*B*b^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))-4*B*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*a*b+4*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c)
)/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos(dx+c)\sec(dx+c)^2+Aa\cos(dx+c)+(Ba+Ab)\cos(dx+c)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)*sec(d*x + c)^2 + A*a*cos(d*x + c) + (B*a + A*b)*
cos(d*x + c)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.361 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=428

$$\frac{\sqrt{a+b}(2aA+4aB+5Ab+8bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4d}-\cdots$$

```
[Out] ((a - b)*Sqrt[a + b]*(5*A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[a + b]*(
2*a*A + 5*A*b + 4*a*B + 8*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2*A
+ 3*A*b^2 + 12*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + ((5*A*b + 4*a*B)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.792164, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4025, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2A+12abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{(4aB\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(5*A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[a + b]*(
2*a*A + 5*A*b + 4*a*B + 8*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2*A
+ 3*A*b^2 + 12*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + ((5*A*b + 4*a*B)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} - \frac{1}{2}\int$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{aA\cos(c}{}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{aA\cos(c}{}$$

$$= \frac{(a-b)\sqrt{a+b}(5Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{4bd}$$

$$= \frac{(a-b)\sqrt{a+b}(5Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{4bd}$$

Mathematica [C] time = 19.4379, size = 1598, normalized size = 3.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*
Cos[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*A*b*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2] + 5*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 4*a^
2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a*b*Sqrt[(-a + b)/(a + b)]*
B*Tan[(c + d*x)/2] - 10*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 8
*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 5*a*A*b*Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2]^5 - 5*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 4*a*b*Sqrt[(-a + b
)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - (24*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a
^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6
*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 - (24*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - I*(a - b)*(5*A*b + 4*a*B)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (2*I)*(a - b)*(2*a*A + b*(A + 4*B))*EllipticF[I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
```

```
(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c
+ d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.379, size = 2439, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(-4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*A*cos(d*x+c)^4*a^2+8*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x
+c)+4*B*cos(d*x+c)^3*a^2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+5*A*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+8*B*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2+7*A*cos(d*x+c)^3*a*b-5*A*cos(d*x+c)^2*a*b+4*B
*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+5*A*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b-16*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+24*B*cos(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-16*B*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+
c)+24*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*a*b*sin(d*x+c)-2*A*cos(d*x+c)^2*a^2-2*A*cos(d*x+c)*a*b+6*A*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*B*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+5*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*sin(d*x+c)+5*A*cos(d*x+c)^2*b^2-4*B*cos(d*x+c)^2*a^2-5*A*cos(d*x+c)*b^2+8
*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin
(d*x+c)-4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
```

```
1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*A*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2
+5*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*a*b+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*a*b+4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*a*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Bb\cos(dx+c)^2\sec(dx+c)^2+Aa\cos(dx+c)^2+(Ba+Ab)\cos(dx+c)^2\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A
*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

3.362 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=520

$$\frac{\sqrt{a+b}\left(16a^2A+12a^2B+14aAb+30abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{}}\right.\right.}{24ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*
d) + (Sqrt[a + b]*(16*a^2*A + 14*a*A*b + 3*A*b^2 + 12*a^2*B + 30*a*b*B)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(24*a*d) - (Sqrt[a + b]*(12*a^2*A*b - A*b^3 + 8*a^3*B + 6*a*b^2*
B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((16*a^2*A + 3*A*b^2 + 30*a*b*B)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a*d) + ((7*A*b + 6*a*B)*Cos[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^2*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.27528, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4025, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2A+30abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24ad}+\frac{\sqrt{a+b}\left(16a^2A+12a^2B+14aAb+30abB+3Ab^2\right)\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*
d) + (Sqrt[a + b]*(16*a^2*A + 14*a*A*b + 3*A*b^2 + 12*a^2*B + 30*a*b*B)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(24*a*d) - (Sqrt[a + b]*(12*a^2*A*b - A*b^3 + 8*a^3*B + 6*a*b^2*
B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((16*a^2*A + 3*A*b^2 + 30*a*b*B)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a*d) + ((7*A*b + 6*a*B)*Cos[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^2*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\int$$

$$= \frac{(7Ab+6aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d}$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{24ad}$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{24ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A+3Ab^2+30abB\right)\cot(c+dx)E\Big(}{24}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A+3Ab^2+30abB\right)\cot(c+dx)E\Big(}{24}$$

Mathematica [B] time = 19.0343, size = 1551, normalized size = 2.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((a*A*Sin[c + d*x])/12 + ((7*A*b +
 6*a*B)*Sin[2*(c + d*x)])/24 + (a*A*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c
+ d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*
(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 3*a*A*b^2*Tan[(c
 + d*x)/2] + 3*A*b^3*Tan[(c + d*x)/2] + 30*a^2*b*B*Tan[(c + d*x)/2] + 30*a*
b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 6*a*A*b^2*Tan[(c + d
*x)/2]^3 - 60*a^2*b*B*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16
*a^2*A*b*Tan[(c + d*x)/2]^5 + 3*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*A*b^3*Tan[(c
 + d*x)/2]^5 + 30*a^2*b*B*Tan[(c + d*x)/2]^5 - 30*a*b^2*B*Tan[(c + d*x)/2]^
5 - 72*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)] + 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*a^3*B*EllipticPi[-1, -ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 36*a*b^2*B*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
72*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(
c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 48
*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - 36*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a
+ b)*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
```

```
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(12*a^2
*B + b^2*(-7*A + 24*B) + a*(26*A*b - 6*b*B))*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2
)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24
*a*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*(1 + Tan[(c + d*x)/2]^2)
^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[
(c + d*x)/2]^2)])
```

Maple [B] time = 0.448, size = 3142, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/24/d/a*(-1+cos(d*x+c))^2*(16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+30*B*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+30*B*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+12*B*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+
36*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*a+16*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*A*cos(d*x+c)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3+48*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+16*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+3*A*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-
52*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*b+14*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*a+72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-12*B*a^3*cos(d*x+c)^2+8*A*
cos(d*x+c)^5*a^3+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+3*A*cos(d*x+c)^
2*b^3-48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2-6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
```

```
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-24*B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+48*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*A*cos(d
*x+c)*b^3+12*B*cos(d*x+c)^4*a^3-6*A*cos(d*x+c)^2*a^2*b+30*B*cos(d*x+c)^2*a*
b^2-16*A*cos(d*x+c)*a^2*b-14*A*cos(d*x+c)*a*b^2-12*B*cos(d*x+c)*a^2*b+12*B*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2*b+36*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2+16*A*cos(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+3*A*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^2-52*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+14*A*cos(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+72*A*cos(d*x+c
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*b+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-48*B*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+17*A
*cos(d*x+c)^3*a*b^2+42*B*cos(d*x+c)^3*a^2*b-3*A*cos(d*x+c)^2*a*b^2-30*B*cos
(d*x+c)^2*a^2*b-30*B*cos(d*x+c)*a*b^2+22*A*cos(d*x+c)^4*a^2*b)*(cos(d*x+c)+
1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^3\sec(dx+c)^2+Aa\cos(dx+c)^3+(Ba+Ab)\cos(dx+c)^3\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3*sec(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A
*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x
)
```

3.363 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=566

$$-\frac{2(a-b)\sqrt{a+b}\left(-15a^2b^2(121A-19B)-a^3(110Ab-30bB)+40a^4B+6ab^3(209A-505B)-3b^4(539A-225B)\right)\cot}{3465b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*
B - 255*a^3*b^2*B - 3705*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(a - b)*S
qrt[a + b]*(6*a*b^3*(209*A - 505*B) - 3*b^4*(539*A - 225*B) - 15*a^2*b^2*(1
21*A - 19*B) + 40*a^4*B - a^3*(110*A*b - 30*b*B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d
) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*
Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*A*b - 539
*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(
3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a + b*Sec[c + d*x])^(5/2)
*Tan[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a + b*Sec[c + d*x])^(7/2)
*Tan[c + d*x])/(99*b^2*d) + (2*B*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Ta
n[c + d*x])/(11*b*d)
```

Rubi [A] time = 1.78437, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4033, 4082, 4002, 4005, 3832, 4004}

$$-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2Ab-40a^3B-335ab^2B-539Ab^3\right)\tan(c+}{3465b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*
B - 255*a^3*b^2*B - 3705*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(a - b)*S
qrt[a + b]*(6*a*b^3*(209*A - 505*B) - 3*b^4*(539*A - 225*B) - 15*a^2*b^2*(1
21*A - 19*B) + 40*a^4*B - a^3*(110*A*b - 30*b*B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d
) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*
Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*A*b - 539
*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(
3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a + b*Sec[c + d*x])^(5/2)
*Tan[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a + b*Sec[c + d*x])^(7/2)
*Tan[c + d*x])/(99*b^2*d) + (2*B*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Ta
n[c + d*x])/(11*b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
```

```
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd} + \frac{2\int \mathrm{s}}{}$$

$$= \frac{2(11Ab-4aB)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d} + \frac{2B\,\mathrm{s}}{}$$

$$= -\frac{2\left(22aAb-8a^2B-81b^2B\right)(a+b\sec(c+dx))^{5/2}\tan(}{693b^2d}$$

$$= -\frac{2\left(110a^2Ab-539Ab^3-40a^3B-335ab^2B\right)(a+b\sec(}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4\right.}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-675b^4\right.}{3465b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(110a^4Ab-3069a^2Ab^3-1617Ab^5-40\right.}{}$$

Mathematica [B] time = 26.5934, size = 4227, normalized size = 7.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-110*a^4*A*b + 3069*a^2*A*b
^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sin[c + d*x])/(3
465*b^3) + (2*Sec[c + d*x]^4*(11*A*b^2*Sin[c + d*x] + 23*a*b*B*Sin[c + d*x]
))/99 + (2*Sec[c + d*x]^3*(209*a*A*b*Sin[c + d*x] + 113*a^2*B*Sin[c + d*x]
+ 81*b^2*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(825*a^2*A*b*Sin[c + d*x]
 + 539*A*b^3*Sin[c + d*x] + 15*a^3*B*Sin[c + d*x] + 1145*a*b^2*B*Sin[c + d*
x]))/(3465*b) + (2*Sec[c + d*x]*(55*a^3*A*b*Sin[c + d*x] + 1793*a*A*b^3*Sin
[c + d*x] - 20*a^4*B*Sin[c + d*x] + 1025*a^2*b^2*B*Sin[c + d*x] + 675*b^4*B
*Sin[c + d*x]))/(3465*b^2) + (2*b^2*B*Sec[c + d*x]^4*Tan[c + d*x])/11))/(d*
(b + a*Cos[c + d*x])^2) - (2*((2*a^4*A)/(63*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (31*a^2*A*b)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - (7*A*b^3)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (17*a^3
*B)/(231*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*B)/(693*b^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (247*a*b^2*B)/(231*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (124*a^3*A*Sqrt[Sec[c + d*x]])/(315*S
qrt[b + a*Cos[c + d*x]]) + (2*a^5*A*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*
Cos[c + d*x]]) + (38*a*A*b^2*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*
x]]) - (8*a^6*B*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (7
*a^4*B*Sqrt[Sec[c + d*x]])/(99*b*Sqrt[b + a*Cos[c + d*x]]) - (26*a^2*b*B*Sq
rt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]) + (15*b^3*B*Sqrt[Sec[c + d
*x]])/(77*Sqrt[b + a*Cos[c + d*x]]) - (31*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c
 + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (2*a^5*A*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (7*a*A*b^2*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (8*a^6*B*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (17*a^4*B*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*b*Sqrt[b + a*Cos[c + d*x]]) - (247*a
^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]))
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(a + b
```

)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B +
3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(12
1*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b
 + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(346
5*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*
(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-110*a^
4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4
*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B
) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^
2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^3*(b +
 a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 +
 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*B
 - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B
) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 4
0*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-110*a
^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^
4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-1
10*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*
a*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1
+ Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])] - (b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^
2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^
2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
 + ((a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^
3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a
+ b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*
(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a
 + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))] - a*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255
*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan
[(c + d*x)/2] - (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 25
5*a^3*b^2*B + 3705*a*b^4*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c +
 d*x]*Tan[(c + d*x)/2] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a
^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[

```
(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A +
 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A
+ 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-110*a^4*A*
b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)
/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3 +
1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*B
- 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225*B)
 + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40
*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c
+ d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3465*b^3*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]])))
```

Maple [B] time = 2.49, size = 5368, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2 \sec(dx+c)^6 + Aa^2 \sec(dx+c)^3 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^5 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)^4\right)\sqrt{b\sec(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^6 + A*a^2*sec(d*x + c)^3 + (2*B*a*b + A*b^2)*s
ec(d*x + c)^5 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

3.364 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=469

$$-\frac{2(a-b)\sqrt{a+b}\left(15a^2b(3A-11B)-10a^3B-6ab^2(60A-19B)+3b^3(25A-49B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec}{}}}{315b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*
B + 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(2
5*A - 49*B) - 6*a*b^2*(60*A - 19*B) + 15*a^2*b*(3*A - 11*B) - 10*a^3*B)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(315*b^2*d) + (2*(45*a^2*A*b + 75*A*b^3 - 10*a^3*B + 114*a*b^2*B
)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b*d) + (2*(45*a*A*b - 10*a^2*
B + 49*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*A*
b - 2*a*B)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*B*(a + b*
Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 1.18285, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4010, 4002, 4005, 3832, 4004}

$$\frac{2\left(-10a^2B+45aAb+49b^2B\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2Ab-10a^3B+114ab^2B+75Ab^3\right)\tan(c+dx}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*
B + 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(2
5*A - 49*B) - 6*a*b^2*(60*A - 19*B) + 15*a^2*b*(3*A - 11*B) - 10*a^3*B)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(315*b^2*d) + (2*(45*a^2*A*b + 75*A*b^3 - 10*a^3*B + 114*a*b^2*B
)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b*d) + (2*(45*a*A*b - 10*a^2*
B + 49*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*A*
b - 2*a*B)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*B*(a + b*
Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)(a+ \ldots}{\ldots}$$

$$= \frac{2(9Ab-2aB)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd} + \frac{2B(a+b \ldots}{\ldots}$$

$$= \frac{2\left(45aAb-10a^2B+49b^2B\right)(a+b\sec(c+dx))^{3/2}\tan(c+ \ldots}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\sec(c+dx \ldots}}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\sec(c+dx \ldots}}{315bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B \ldots\right.}{\ldots}$$

Mathematica [B] time = 26.2938, size = 3781, normalized size = 8.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(45*a^3*A*b + 435*a*A*b^3 -
10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^2) + (2*Sec[c +
d*x]^3*(9*A*b^2*Sin[c + d*x] + 19*a*b*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]
^2*(135*a*A*b*Sin[c + d*x] + 75*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x])
)/315 + (2*Sec[c + d*x]*(135*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] +
 5*a^3*B*Sin[c + d*x] + 163*a*b^2*B*Sin[c + d*x]))/(315*b) + (2*b^2*B*Sec[c
 + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^2) + (2*(-(a^3*A)/(7*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*A*b^2)/(21*Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^4*B)/(63*b*Sqrt[b + a*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) - (31*a^2*b*B)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) - (7*b^3*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^
4*A*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*A*b*Sqrt[Se
c[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5*A*b^3*Sqrt[Sec[c + d*x]])/(
21*Sqrt[b + a*Cos[c + d*x]]) - (124*a^3*B*Sqrt[Sec[c + d*x]])/(315*Sqrt[b +
 a*Cos[c + d*x]]) + (2*a^5*B*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c +
 d*x]]) + (38*a*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) -
(a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]])
- (29*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d
*x]]) - (31*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c
 + d*x]]) + (2*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b +
a*Cos[c + d*x]]) - (7*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt
[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c +
 d*x])^(5/2)*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2
*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60
*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 27
9*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/
2]^2]*Sec[c + d*x]^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c +
d*x]*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147
*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*
B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^
2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(315*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2])
 - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-45*
a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-
10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49
*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^
2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x
)/2]^2*Sec[c + d*x]]*(((-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*
B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((
a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c +
d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c +
```

```
d*x])] + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19
*B) + 3*b^3*(25*A + 49*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]
*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-45*a^3*A*b - 435*a*A*b^3 +
10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))] + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*
B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c
+ d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-
45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-45*a^3*A*b - 435*a*
A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-45*a^3*A*b - 435*a*A*b^3 + 10*
a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 1
1*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[
(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)]) + ((a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a
^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*b^2*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-45*a^3*A*b
 - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-10*a^3*
B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-4
5*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x
)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c
 + d*x]))/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.639, size = 4395, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-35*B*b^5+45*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-10*B*cos(d*x+c)^6*a^5-435*A*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^3-435*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*B*cos(d*x+c)^5*sin(d*x+c
```

)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+15
5*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b^2+279*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+10*B*cos(d*x+c)^5*a^5
+147*B*cos(d*x+c)^5*b^5-98*B*cos(d*x+c)^4*b^5-14*B*cos(d*x+c)^2*b^5+75*A*co
s(d*x+c)^5*b^5-30*A*cos(d*x+c)^3*b^5-45*A*cos(d*x+c)*b^5+10*B*cos(d*x+c)^5*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^5-147*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^5+75*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*B*cos(d*x+c)^4*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5
+10*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^5-147*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+75*A*cos(d*x+c)^5*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*
B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^5-130*B*cos(d*x+c)*a*b^4+435*A*cos(d*x+c)^6*a^2*b^3+75*A*
cos(d*x+c)^6*a*b^4+5*B*cos(d*x+c)^6*a^4*b+279*B*cos(d*x+c)^6*a^3*b^2+163*B*
cos(d*x+c)^6*a^2*b^3+147*B*cos(d*x+c)^6*a*b^4-45*A*cos(d*x+c)^5*a^4*b+45*A*
cos(d*x+c)^5*a^3*b^2-165*A*cos(d*x+c)^5*a^2*b^3+435*A*cos(d*x+c)^5*a*b^4-10
*B*cos(d*x+c)^5*a^4*b-199*B*cos(d*x+c)^5*a^3*b^2+279*B*cos(d*x+c)^5*a^2*b^3
+65*B*cos(d*x+c)^5*a*b^4-180*A*cos(d*x+c)^4*a^3*b^2-330*A*cos(d*x+c)^4*a*b^
4+5*B*cos(d*x+c)^4*a^4*b-272*B*cos(d*x+c)^4*a^2*b^3-270*A*cos(d*x+c)^3*a^2*
b^3-80*B*cos(d*x+c)^3*a^3*b^2-82*B*cos(d*x+c)^3*a*b^4-180*A*cos(d*x+c)^2*a*
b^4-170*B*cos(d*x+c)^2*a^2*b^3+45*A*cos(d*x+c)^6*a^4*b+135*A*cos(d*x+c)^6*a
^3*b^2+261*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-279*B*cos(d*x+c)^
5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*b^2-279*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*B*cos(d*x+c)^5*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+45*A*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^3*b^2+405*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+435*A*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-45*A
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^4*b-45*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(

```
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-435*A*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3
-435*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^4-10*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+155*B*cos(d*x+c)^4*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3
*b^2+279*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b^3+261*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*B*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^4*b-279*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-279*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*B*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*b^4+405*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+435*A*cos(d*x+c)^5*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-45*A*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^4*b-45*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2)/(b+a*cos(d*x+c))/cos(d*x+c)
^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Bb^2\sec(dx+c)^5 + Aa^2\sec(dx+c)^2 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^4 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)^3\right)\sqrt{b\sec(dx\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^5 + A*a^2*sec(d*x + c)^2 + (2*B*a*b + A*b^2)*s
ec(d*x + c)^4 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

3.365 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=384

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2(7A-B)-8ab(7A-15B)+b^2(63A-25B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{si}\right.}{105bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^2*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) - 8*a*
b*(7*A - 15*B) + 15*a^2*(7*A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b*d) + (2*(56*a*A*b +
 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(
7*A*b + 5*a*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a +
b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.806795, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4002, 4005, 3832, 4004}

$$\frac{2\left(15a^2B+56aAb+25b^2B\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\left(15a^2(7A-B)-8ab(7A-15B)+b^2(63A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^2*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) - 8*a*
b*(7*A - 15*B) + 15*a^2*(7*A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b*d) + (2*(56*a*A*b +
 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(
7*A*b + 5*a*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a +
b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
```

```
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{2}{7}\int \sec(c+dx)(a$$

$$= \frac{2(7Ab+5aB)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35d}+\frac{2B(a+}{}$$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\sec(c+dx)}\tan(c+a}{105d}$$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\sec(c+dx)}\tan(c+a}{105d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(161a^2Ab+63Ab^3+15a^3B+145ab^2B\right)}{}$$

Mathematica [B] time = 23.2795, size = 2957, normalized size = 7.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x])*((2*(161*a^
2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sin[c + d*x])/(105*b) + (2*Sec[c
 + d*x]^2*(7*A*b^2*Sin[c + d*x] + 15*a*b*B*Sin[c + d*x]))/35 + (2*Sec[c + d
*x]*(77*a*A*b*Sin[c + d*x] + 45*a^2*B*Sin[c + d*x] + 25*b^2*B*Sin[c + d*x])
)/105 + (2*b^2*B*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2
*(B + A*Cos[c + d*x])) + (2*((-23*a^2*A*b)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (3*A*b^3)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) - (a^3*B)/(7*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*b^2*B)/
(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*A*Sqrt[Sec[c + d*
x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (8*a*A*b^2*Sqrt[Sec[c + d*x]])/(15*Sqr
t[b + a*Cos[c + d*x]]) - (a^4*B*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c +
 d*x]]) - (2*a^2*b*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5
```

```
*b^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (23*a^3*A*Cos[2*
(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*b^2*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (a^4*B*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (29*a^
2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]))*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[
c + d*x])*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A
*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A*b^3 +
15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)))/(105*b
*d*(b + a*Cos[c + d*x])^2*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec
[c + d*x]^(7/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*((-
2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3
*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*
(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 14
5*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)))/(105*b*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((-2*(Cos[c + d*x]/(1 + Cos[
c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b
*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)])*Sec[c + d*x])/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))] + (161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/
2]*(-1 + Tan[(c + d*x)/2]^2)))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[b +
 a*Cos[c + d*x]]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b
 + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25
*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A
*b^3 + 15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2))*
(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec
[c + d*x]*Tan[c + d*x]))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]) + (2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*((-3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((161*a^2*A*b + 63
*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x]*((Cos[c
+ d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])
))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((Cos[c + d*x]
/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*
B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*A +
B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)])*Sec[c + d*x]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x
])^2)))/((b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])))^(3/2) + (161*a^
2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]^2 + ((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sec[(c + d*x)/2]
^2*(-1 + Tan[(c + d*x)/2]^2))/2 - (2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2
)*Sec[c + d*x]*(-(b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25
*B))*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*
Tan[(c + d*x)/2]^2)/(a + b)]) + ((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a
*b^2*B)*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/
(2*Sqrt[1 - Tan[(c + d*x)/2]^2])))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))] - (2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b
+ 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
```

```
 - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*
B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x]*Tan[
c + d*x])/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]))/(105*b*
Sqrt[Sec[(c + d*x)/2]^2])))
```

Maple [B] time = 1.082, size = 3637, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/105/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+
c))^2*(15*B*b^4-15*B*cos(d*x+c)^5*a^4-135*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-105*A*sin
(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b-105*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-145*B*sin(d*x+c)*cos(d*x+c)^3*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+161*A*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b+161*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+63*A*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-16
1*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b^2-119*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+15*B*sin(d*x+c)*cos(d*x
+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
b+145*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^2+145*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-15*B*sin(d*x+c)*cos
(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b-145*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^3+161*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+161*A*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*b^2+63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-161*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-119*A*sin(d
```

```
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^3+15*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+145*B*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+145*B*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b^3-35*A*cos(d*x+c)^4*a*b^3-15*B*cos(d*x+c)^4*a^3*b+55*B*cos
(d*x+c)^4*a^2*b^2-145*B*cos(d*x+c)^4*a*b^3+238*A*cos(d*x+c)^3*a^2*b^2+60*B*
cos(d*x+c)^3*a^3*b+110*B*cos(d*x+c)^3*a*b^3+98*A*cos(d*x+c)^2*a*b^3+90*B*co
s(d*x+c)^2*a^2*b^2+60*B*cos(d*x+c)*a*b^3-161*A*cos(d*x+c)^5*a^3*b-77*A*cos(
d*x+c)^5*a^2*b^2-63*A*cos(d*x+c)^5*a*b^3-45*B*cos(d*x+c)^5*a^3*b-145*B*cos(
d*x+c)^5*a^2*b^2-25*B*cos(d*x+c)^5*a*b^3+161*A*cos(d*x+c)^4*a^3*b-161*A*cos
(d*x+c)^4*a^2*b^2-15*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-135*B*sin(d*x+c)*cos(d*x+c)^3*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+63*A
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^4-63*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+15*B*sin(d*x+c)*cos(d*x+c)^4*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-25*B*sin(d
*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^4+63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+15*B*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^4-25*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^4-63*A*cos(d*x+c)^4*b^4-25*B*cos(d*x+c)^4*b^4+1
0*B*cos(d*x+c)^2*b^4+15*B*cos(d*x+c)^4*a^4+42*A*cos(d*x+c)^3*b^4+21*A*cos(d
*x+c)*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\sec\left(dx+c\right)^4 + Aa^2\sec\left(dx+c\right) + \left(2\,Bab + Ab^2\right)\sec\left(dx+c\right)^3 + \left(Ba^2 + 2\,Aab\right)\sec\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^4 + A*a^2*sec(d*x + c) + (2*B*a*b + A*b^2)*sec
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.366 $\int (a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=442

$$\frac{2\sqrt{a+b}\left(a^2b(45A-23B)+15a^3B-ab^2(35A-17B)+b^3(5A-9B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}}{15bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b
*d) + (2*Sqrt[a + b]*(a^2*b*(45*A - 23*B) - a*b^2*(35*A - 17*B) + b^3*(5*A
- 9*B) + 15*a^3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c + d*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(15*d) + (2*b*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.655875, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(a^2b(45A-23B)+15a^3B-ab^2(35A-17B)+b^3(5A-9B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b
*d) + (2*Sqrt[a + b]*(a^2*b*(45*A - 23*B) - a*b^2*(35*A - 17*B) + b^3*(5*A
- 9*B) + 15*a^3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c + d*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(15*d) + (2*b*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
```

```
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2bB(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\sec(c+dx)}\left(\frac{5a^2A}{2}\right.$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2bB(a+b\sec(c+dx))}{5d}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2bB(a+b\sec(c+dx))}{5d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(35aAb+23a^2B+9b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+}}\right.\right.}{15bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(35aAb+23a^2B+9b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+}}\right.\right.}{15bd}$$

Mathematica [B] time = 25.207, size = 7168, normalized size = 16.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.749, size = 3285, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(-45*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b-5*A*cos(d*x+c)^3*b^3-35*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2
-45*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b+35*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+35*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2
-23*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b-17*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+23*B*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
+9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
```

```
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2-35*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+35*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+
35*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2-23*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-17*B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+
23*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*B*b^3+5*A*cos(d*x+c)*b^3
-23*B*cos(d*x+c)^4*a^3+23*B*cos(d*x+c)^3*a^3-9*B*cos(d*x+c)^3*b^3+6*B*cos(d
*x+c)^2*b^3+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^3-30*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-15*B*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3+15*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3-30*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-15*B*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^3-5*A*cos(d*x+c)^4*a*b^2-11*B*cos(d*x+c)^4*a^2*b-9*B*cos(d*x+c)^4*a*b
^2+35*A*cos(d*x+c)^3*a^2*b-35*A*cos(d*x+c)^3*a*b^2-23*B*cos(d*x+c)^3*a^2*b-
5*B*cos(d*x+c)^3*a*b^2+40*A*cos(d*x+c)^2*a*b^2+34*B*cos(d*x+c)^2*a^2*b+14*B
*cos(d*x+c)*a*b^2-35*A*cos(d*x+c)^4*a^2*b-5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*B*sin(d*x
+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^3+23*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^3+9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*A*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^3-9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+23*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+9*B*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3)/(b+a
*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2\sec(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2), x)
```

3.367 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=433

$$\frac{\sqrt{a+b}\left(3a^2(A+6B)+2ab(9A-7B)-2b^2(3A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+\dots}}{}\right.\right.}{3d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*A - 6*A*b^2 - 14*a*b*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) +
 (Sqrt[a + b]*(2*a*b*(9*A - 7*B) - 2*b^2*(3*A - B) + 3*a^2*(A + 6*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*d) - (a*Sqrt[a + b]*(5*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 + (a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*a*A - 2*b*B)*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.703233, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4025, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(3a^2(A+6B)+2ab(9A-7B)-2b^2(3A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*A - 6*A*b^2 - 14*a*b*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) +
 (Sqrt[a + b]*(2*a*b*(9*A - 7*B) - 2*b^2*(3*A - B) + 3*a^2*(A + 6*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*d) - (a*Sqrt[a + b]*(5*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 + (a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*a*A - 2*b*B)*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \int \sqrt{a+b\sec(c+}$$

$$= \frac{aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3aA-2bB)\sqrt{a}}{}$$

$$= \frac{aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3aA-2bB)\sqrt{a}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3a^2A-6Ab^2-14abB\right)\cot(c+dx)E\left(\sin\right.}{3bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3a^2A-6Ab^2-14abB\right)\cot(c+dx)E\left(\sin\right.}{3bd}$$

Mathematica [B] time = 19.2888, size = 1146, normalized size = 2.65

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(3*a^3*A*Ta
n[(c + d*x)/2] + 3*a^2*A*b*Tan[(c + d*x)/2] - 6*a*A*b^2*Tan[(c + d*x)/2] -
6*A*b^3*Tan[(c + d*x)/2] - 14*a^2*b*B*Tan[(c + d*x)/2] - 14*a*b^2*B*Tan[(c
+ d*x)/2] - 6*a^3*A*Tan[(c + d*x)/2]^3 + 12*a*A*b^2*Tan[(c + d*x)/2]^3 + 28
*a^2*b*B*Tan[(c + d*x)/2]^3 + 3*a^3*A*Tan[(c + d*x)/2]^5 - 3*a^2*A*b*Tan[(c
 + d*x)/2]^5 - 6*a*A*b^2*Tan[(c + d*x)/2]^5 + 6*A*b^3*Tan[(c + d*x)/2]^5 -
14*a^2*b*B*Tan[(c + d*x)/2]^5 + 14*a*b^2*B*Tan[(c + d*x)/2]^5 - 30*a^2*A*b*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] - 12*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 30*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^3*B*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (a + b)*(3*a^2*A - 6*A*b^2 - 14*a*b*B)*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 2*(9*a^2*b*(A - B) + 3*a^3*B - b^3*(3*A + B) - a*b^2*(9*A +
7*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*
x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^2*(a + b
*Sec[c + d*x])^(5/2)*((2*b*(3*A*b + 7*a*B)*Sin[c + d*x])/3 + (2*b^2*B*Tan[c
 + d*x])/3))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.639, size = 3215, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

```
[Out] -1/3/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(30*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+6*A*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+3*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*A*cos(d*x+c)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*B*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3+12*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A*cos(d*x+c)^4*a^3-18*A*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b+18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-6*A*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b^2+18*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b+14*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-14*B*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b-14*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-6*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-3*
A*cos(d*x+c)^3*a^3+6*A*cos(d*x+c)^2*b^3-2*B*b^3+14*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-6*A*
cos(d*x+c)*b^3+2*B*cos(d*x+c)^2*b^3-3*A*cos(d*x+c)^2*a^2*b+14*B*cos(d*x+c)^
2*a*b^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3+18*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+3*A*cos(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-
6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
```

```
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^2-18*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+18*A*cos(d*x+c)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+30*A*co
s(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*b-14*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-14*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*A*cos(d
*x+c)^3*a^2*b+6*A*cos(d*x+c)^3*a*b^2+14*B*cos(d*x+c)^3*a^2*b+2*B*cos(d*x+c)
^3*a*b^2-6*A*cos(d*x+c)^2*a*b^2-14*B*cos(d*x+c)^2*a^2*b-16*B*cos(d*x+c)*a*b
^2+6*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)^5/(b+a*cos(d*x+c
))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)\sec(dx+c)^3 + Aa^2\cos(dx+c) + \left(2\,Bab + Ab^2\right)\cos(dx+c)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b
 + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(
d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.368 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=450

$$\frac{\sqrt{a+b}\left(2a^2(A+2B)+3ab(3A+8B)+8b^2(A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{}\right.\right.}{4d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(9*a*A*b + 4*a^2*B - 8*b^2*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) +
(Sqrt[a + b]*(8*b^2*(A - B) + 2*a^2*(A + 2*B) + 3*a*b*(3*A + 8*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*d) - (Sqrt[a + b]*(4*a^2*A + 15*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(4*d) + (a*(7*A*b + 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(4*d) + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.834781, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4025, 4094, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(2a^2(A+2B)+3ab(3A+8B)+8b^2(A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(9*a*A*b + 4*a^2*B - 8*b^2*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) +
(Sqrt[a + b]*(8*b^2*(A - B) + 2*a^2*(A + 2*B) + 3*a*b*(3*A + 8*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*d) - (Sqrt[a + b]*(4*a^2*A + 15*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(4*d) + (a*(7*A*b + 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(4*d) + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d} - \frac{1}{2}\int$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{aA\cos(}{}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{aA\cos(}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(9aAb+4a^2B-8b^2B\right)\cot(c+dx)E\left(\sin\right.}{4ba}$$

$$= \frac{(a-b)\sqrt{a+b}\left(9aAb+4a^2B-8b^2B\right)\cot(c+dx)E\left(\sin\right.}{4ba}$$

Mathematica [B] time = 19.3444, size = 1338, normalized size = 2.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(2*b^2*B*Sin[c + d*x] + (a^2*A*S
in[2*(c + d*x)])/4))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/
2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(9*a^2*A*b*Tan[(c + d*x)/2] + 9*a*A*
b^2*Tan[(c + d*x)/2] + 4*a^3*B*Tan[(c + d*x)/2] + 4*a^2*b*B*Tan[(c + d*x)/2
] - 8*a*b^2*B*Tan[(c + d*x)/2] - 8*b^3*B*Tan[(c + d*x)/2] - 18*a^2*A*b*Tan[
(c + d*x)/2]^3 - 8*a^3*B*Tan[(c + d*x)/2]^3 + 16*a*b^2*B*Tan[(c + d*x)/2]^3
 + 9*a^2*A*b*Tan[(c + d*x)/2]^5 - 9*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*a^3*B*Ta
n[(c + d*x)/2]^5 - 4*a^2*b*B*Tan[(c + d*x)/2]^5 - 8*a*b^2*B*Tan[(c + d*x)/2
]^5 + 8*b^3*B*Tan[(c + d*x)/2]^5 - 8*a^3*A*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*A*b^2*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 40*a^
2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 8*a^3*A*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*A*b^2*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - 40*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(9*a*A*b + 4*a^2*
B - 8*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a^3*A - a^2*b*(A - 12*B) + 12*
a*b^2*(A - B) - 4*b^3*(A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*d*(b + a*Cos[c
 + d*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.648, size = 3271, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-4*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+
4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*b-8*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*a-24*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+9*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+9*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*A*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-24*A*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*a+30*A*cos(d*x+c)*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*a+8*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*A*cos(d*x+c)^4*a^3-4*B*a^3*cos(d*
x+c)^2+8*B*cos(d*x+c)*b^3-4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+8*A*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+4*B*c
os(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))-8*B*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))+30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+40*B*a^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+40*B*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)^2*a^3-8*B*b^3+24*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+4*B*
cos(d*x+c)^3*a^3-9*A*cos(d*x+c)^2*a^2*b+8*B*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+
c)*a^2*b-9*A*cos(d*x+c)*a*b^2-4*B*cos(d*x+c)*a^2*b+8*A*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*B*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
```

```
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+4*
B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(
d*x+c)-8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3*sin(d*x+c)-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*A*cos(d*x+c)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+9*A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^2+2*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*b-24*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+4*B*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-
8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^2+24*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+11*A*cos(d*x+c)^3*a^2*b+9*A*cos(d*x+c)^2*
a*b^2+4*B*cos(d*x+c)^2*a^2*b-8*B*cos(d*x+c)*a*b^2)*(cos(d*x+c)+1)^2*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^2\sec(dx+c)^3+Aa^2\cos(dx+c)^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2\sec(dx+c)^2+\left(Ba^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (2*B
*a*b + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

3.369 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=518

$$\frac{\sqrt{a+b}\left(16a^2A+12a^2B+26aAb+54abB+33Ab^2+48b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{si}\right.}{24d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d
) + (Sqrt[a + b]*(16*a^2*A + 26*a*A*b + 33*A*b^2 + 12*a^2*B + 54*a*b*B + 48
*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(24*d) - (Sqrt[a + b]*(20*a^2*A*b + 5*A*b^3 + 8*a^3*B
 + 30*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2*A + 33*A*b^2 +
54*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(3*A*b + 2*a*B
)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*Cos[c +
d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.36519, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4025, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2A+54abB+33Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(16a^2A+12a^2B+26aAb+54abB+33Ab^2+48\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d
) + (Sqrt[a + b]*(16*a^2*A + 26*a*A*b + 33*A*b^2 + 12*a^2*B + 54*a*b*B + 48
*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(24*d) - (Sqrt[a + b]*(20*a^2*A*b + 5*A*b^3 + 8*a^3*B
 + 30*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2*A + 33*A*b^2 +
54*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(3*A*b + 2*a*B
)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*Cos[c +
d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
```

```
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} - \frac{1}{3}\int$$

$$= \frac{a(3Ab+2aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+d}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+d}{24d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A+33Ab^2+54abB\right)\cot(c+dx)E\Big(}{24}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A+33Ab^2+54abB\right)\cot(c+dx)E\Big(}{24}$$

Mathematica [B] time = 19.4859, size = 1567, normalized size = 3.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*A*Sin[c + d*x])/12 + (a*(1
3*A*b + 6*a*B)*Sin[2*(c + d*x)])/24 + (a^2*A*Sin[3*(c + d*x)])/12))/(d*(b +
 a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2
]^2)^(-1)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 33*a*
A*b^2*Tan[(c + d*x)/2] + 33*A*b^3*Tan[(c + d*x)/2] + 54*a^2*b*B*Tan[(c + d*
x)/2] + 54*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 66*a*A*
b^2*Tan[(c + d*x)/2]^3 - 108*a^2*b*B*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c +
 d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 33*a*A*b^2*Tan[(c + d*x)/2]^5
- 33*A*b^3*Tan[(c + d*x)/2]^5 + 54*a^2*b*B*Tan[(c + d*x)/2]^5 - 54*a*b^2*B*
Tan[(c + d*x)/2]^5 - 120*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^3*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*a^3*B*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
180*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - 120*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^3*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - 48*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a -
```

```
 b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 180*a*b^2*B*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] - 2*(24*b^3*(A - B) + 12*a^3*B + a*b^2*(-13*A + 72*B) + a^2*(38*
A*b - 6*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*d*(b + a*Cos[c + d*x])^(5/2)*S
ec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.489, size = 3511, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-48*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-16*A*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+
c)-54*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*b-54*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*a-12*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-180*B*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-16*A*cos(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-33*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-30*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+24*B*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-48*B*c
os(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b*sin(d*x+c)-33*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+76*A*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-26*A*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^
```

```
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-120*A*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*si
n(d*x+c)-33*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^3*sin(d*x+c)+48*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^3*sin(d*x+c)+12*B*a^3*cos(d*x+c)^2-8*A*cos(d*x+c)^5*a^3-8*A
*cos(d*x+c)^3*a^3+16*A*cos(d*x+c)^2*a^3-33*A*cos(d*x+c)^2*b^3+144*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2-30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^3*sin(d*x+c)+24*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-48*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+33*A*cos(d*x+c)*b^3-12*B*c
os(d*x+c)^4*a^3+18*A*cos(d*x+c)^2*a^2*b-54*B*cos(d*x+c)^2*a*b^2+16*A*cos(d*
x+c)*a^2*b+26*A*cos(d*x+c)*a*b^2+12*B*cos(d*x+c)*a^2*b-48*B*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-12*B*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b-180*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2-16*A*cos(d*x+c)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-33*A*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2+76*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-26*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-120*A*cos(d*x+c)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*b-54*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-54*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+144*B*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-59*A*
cos(d*x+c)^3*a*b^2-66*B*cos(d*x+c)^3*a^2*b+33*A*cos(d*x+c)^2*a*b^2+54*B*cos
(d*x+c)^2*a^2*b+54*B*cos(d*x+c)*a*b^2-34*A*cos(d*x+c)^4*a^2*b)*(cos(d*x+c)+
1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3\sec(dx+c)^3 + Aa^2\cos(dx+c)^3 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^3\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (2*B
*a*b + A*b^2)*cos(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c) + A)(b\sec(dx+c) + a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x
)
```

3.370 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=617

$$\frac{\sqrt{a+b}\left(4a^2b(71A+52B)+8a^3(9A+16B)+2ab^2(59A+132B)+15Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\cdots}{192ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(192*a*b*d) + (Sqrt[a + b]*(15*A*b^3 + 8*a^3*(9*A + 16*B) + 4*a^
2*b*(71*A + 52*B) + 2*a*b^2*(59*A + 132*B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sqrt
[a + b]*(48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*B + 40*a*b^3*B)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(64*a^2*d) + ((284*a^2*A*b + 15*A*b^3 + 128*a^3*B +
264*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((36*a^2*A
+ 59*A*b^2 + 104*a*b*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(96*d) + (a*(11*A*b + 8*a*B)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(4*d)
```

Rubi [A] time = 1.83221, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4025, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(284a^2Ab+128a^3B+264ab^2B+15Ab^3\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{\left(36a^2A+104abB+59Ab^2\right)\sin(c+dx)\cos\cdots}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(192*a*b*d) + (Sqrt[a + b]*(15*A*b^3 + 8*a^3*(9*A + 16*B) + 4*a^
2*b*(71*A + 52*B) + 2*a*b^2*(59*A + 132*B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sqrt
[a + b]*(48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*B + 40*a*b^3*B)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(64*a^2*d) + ((284*a^2*A*b + 15*A*b^3 + 128*a^3*B +
264*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((36*a^2*A
+ 59*A*b^2 + 104*a*b*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(96*d) + (a*(11*A*b + 8*a*B)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(4*d)
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d} - \frac{1}{4}\int$$

$$= \frac{a(11Ab+8aB)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+a}{24d}$$

$$= \frac{\left(36a^2A+59Ab^2+104abB\right)\cos(c+dx)\sqrt{a+b\sec(c+}}{96d}$$

$$= \frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\sec(c}}{192ad}$$

$$= \frac{\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)\sqrt{a+b\sec(c}}{192ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2Ab+15Ab^3+128a^3B+264ab^2B\right)}{}$$

Mathematica [B] time = 24.5393, size = 5186, normalized size = 8.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.612, size = 4231, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/192/d/a*(-1+cos(d*x+c))^2*(48*A*a^4*cos(d*x+c)^6+288*A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
```

((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+24*A*a^4
*cos(d*x+c)^4-72*A*a^4*cos(d*x+c)^2+64*B*cos(d*x+c)^3*a^4+15*A*cos(d*x+c)^2
*b^4-128*B*cos(d*x+c)^2*a^4+64*B*cos(d*x+c)^5*a^4-30*A*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+284*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x
+c)+15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^3*sin(d*x+c)+72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b*sin(d*x+c)-644*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+118*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+960*B*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+24
0*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*
b^3*sin(d*x+c)+128*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b*sin(d*x+c)+264*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+264*B*b^3*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-608*B*a^3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+208*B*
a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))-384*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^3*sin(d*x+c)+172*A*cos(d*x+c)^3*a^3*b+133*A*cos(d*x+c)^3*a*b^3+472
*B*cos(d*x+c)^3*a^2*b^2-284*A*cos(d*x+c)^2*a^3*b+30*A*cos(d*x+c)^2*a^2*b^2-
144*B*cos(d*x+c)^2*a^3*b+264*B*cos(d*x+c)^2*a*b^3-72*A*cos(d*x+c)*a^3*b-284
*A*cos(d*x+c)*a^2*b^2-118*A*cos(d*x+c)*a*b^3-128*B*cos(d*x+c)*a^3*b-208*B*c
os(d*x+c)*a^2*b^2+272*B*cos(d*x+c)^4*a^3*b-15*A*cos(d*x+c)^2*a*b^3-264*B*co
s(d*x+c)^2*a^2*b^2-264*B*cos(d*x+c)*a*b^3+184*A*cos(d*x+c)^5*a^3*b+254*A*co
s(d*x+c)^4*a^2*b^2+15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^4*sin(d*x+c)-144*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+128*B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+720*A*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^
2*b^2+284*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*b+284*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+15*A*cos(d*x+c)*b^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a+72*A*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b-644*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellip

```
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+118*A*cos(d*x+c)*b^3*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a+960*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a^3*b+240*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^3+128*B*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^3*b+264*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b^2+264*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-608*B*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b+208*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^2-384*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+288*A*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*a^4-30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,((a-b)/(a+b))^(1/2))*b^4+15*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-144*A*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+
128*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^4+720*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+284*A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-15*A*cos(d*x+c)*b^4)*
(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Bb^2\cos(dx+c)^4\sec(dx+c)^3 + Aa^2\cos(dx+c)^4 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^4\sec(dx+c)^2 + \left(Ba^2 + 2\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (2*B
*a*b + A*b^2)*cos(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x
)
```

3.371 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=329

$$\frac{2\sqrt{a+b}\left(-8a^2B+2ab(5A+B)+b^2(5A-9B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{15b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) + (2*Sqrt[a + b]*(b^2*(5*A - 9*B) - 8*a^2*B + 2*a*b*(5*A + B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*b^3*d) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(15*b^2*d) + (2*B*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b*d)
```

Rubi [A] time = 0.617519, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4033, 4082, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(-8a^2B+2ab(5A+B)+b^2(5A-9B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{15b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) + (2*Sqrt[a + b]*(b^2*(5*A - 9*B) - 8*a^2*B + 2*a*b*(5*A + B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*b^3*d) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(15*b^2*d) + (2*B*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
```

```
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{2\int \frac{\sec(c+dx)\left(aB+\frac{3}{2}bB\sec(c+dx)+\frac{1}{2}(5\ldots\right.}{\sqrt{a+b\sec(c+dx)}}}{5b}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+d\ldots}}{5bd}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+d\ldots}}{5bd}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(10aAb-8a^2B-9b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\ldots\right)}{15b^4d}$$

Mathematica [B] time = 23.224, size = 3000, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sin[
c + d*x])/(15*b^3) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] - 4*a*B*Sin[c + d*
x]))/(15*b^2) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(5*b)))/(d*Sqrt[a + b*Sec[c
 + d*x]]) - (2*((2*a*A)/(3*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (3*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^2*B)/(15*b^2*
```

Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (7*a*B*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (3*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-10*a*A*b + 8*a^2*B + 9*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(

```
c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(15*b^3*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-10*a*A*b + 8*a^2*B
+ 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*
a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.756, size = 2499, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(5*A*cos(d*x+c)^3*b^3-10*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+10*A*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b+10*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^2-8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b-9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*A*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2
+10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b+10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+
2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^2-8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*B*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-3*B
*b^3-5*A*cos(d*x+c)*b^3+8*B*cos(d*x+c)^4*a^3-8*B*cos(d*x+c)^3*a^3+9*B*cos(d
*x+c)^3*b^3-6*B*cos(d*x+c)^2*b^3+5*A*cos(d*x+c)^4*a*b^2-4*B*cos(d*x+c)^4*a^
2*b+9*B*cos(d*x+c)^4*a*b^2+10*A*cos(d*x+c)^3*a^2*b-10*A*cos(d*x+c)^3*a*b^2+
```

```
8*B*cos(d*x+c)^3*a^2*b-10*B*cos(d*x+c)^3*a*b^2+5*A*cos(d*x+c)^2*a*b^2-4*B*c
os(d*x+c)^2*a^2*b+B*cos(d*x+c)*a*b^2-10*A*cos(d*x+c)^4*a^2*b+5*A*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^3+9*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^3-8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^3+9*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*B*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\sec{(c+dx)})\sec^{3}{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.372 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=261

$$-\frac{2\sqrt{a+b}(3Ab-B(2a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2(a-b}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b - 2*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) - (2*Sqrt[a
 + b]*(3*A*b - (2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*B*Sqrt[a + b*Sec[c
 + d*x]]*Tan[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.39711, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4010, 4005, 3832, 4004}

$$-\frac{2\sqrt{a+b}(3Ab-B(2a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2(a-b)\sqrt{a+b}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b - 2*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) - (2*Sqrt[a
 + b]*(3*A*b - (2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*B*Sqrt[a + b*Sec[c
 + d*x]]*Tan[c + d*x])/(3*b*d)
```


Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{bB}{2}+\frac{1}{2}(3Ab-2aB)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b}$$

$$= \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} + \frac{(3Ab-2aB)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b} + \frac{(-}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}(3Ab-2aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{3b^3d}$$

Mathematica [A] time = 16.407, size = 372, normalized size = 1.43

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2b(B(b-2a)+3Ab)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-3*
A*b + 2*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 2*b*(3*A*b + (-2*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*A*b - 2*a*B)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^2*d*Sqrt[Sec[(c + d
*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((
2*(3*A*b - 2*a*B)*Sin[c + d*x])/(3*b^2) + (2*B*Tan[c + d*x])/(3*b)))/(d*Sqr
t[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.495, size = 1567, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/b^2*(-1+cos(d*x+c))^2*(B*b^2+2*B*cos(d*x+c)^3*a^2-2*B*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^2+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*B*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2-3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^2+3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*A*c
os(d*x+c)^3*a*b-B*cos(d*x+c)^3*a*b+3*A*cos(d*x+c)^2*a*b+2*B*cos(d*x+c)^2*a*
b-B*cos(d*x+c)*a*b+3*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*B*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*B*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*c
os(d*x+c)^2*b^2-2*B*cos(d*x+c)^2*a^2+3*A*cos(d*x+c)*b^2-B*cos(d*x+c)^2*b^2)
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(
d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\sec(dx+c)^3+A\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec{(c + dx)})\sec^{2}{(c + dx)}}{\sqrt{a + b\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec{(dx + c)} + A)\sec{(dx + c)}^{2}}{\sqrt{b\sec{(dx + c)} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.373 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=210

$$\frac{2\sqrt{a+b}(A-B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2B(a-b)\sqrt{a+b}\,\text{co}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(A - B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d)
```

Rubi [A] time = 0.205972, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(A-B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2B(a-b)\sqrt{a+b}\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(A - B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d)
```


Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = (A-B)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + B\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1-}{}}}{b^2d}$$

Mathematica [A] time = 14.5161, size = 356, normalized size = 1.7

$$\frac{2B\sin(c+dx)(a\cos(c+dx)+b)(A+B\sec(c+dx))}{bd\sqrt{a+b\sec(c+dx)}(A\cos(c+dx)+B)} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(A+B\sec(c+dx))\left(-2b(A+B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*B*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*Sin[c + d*x])/(b*d*(B + A*Co
s[c + d*x])*Sqrt[a + b*Sec[c + d*x]]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(A + B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + B*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b*d*(B + A*Cos[c + d*x])*
Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.403, size = 829, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/b*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^2*(A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b+B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+B*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
```

```
*b-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b*sin(d*x+c)+B*cos(d*x+c)^2*a-B*cos(d*x+c)*a+B*cos(d*x+c)*b-B*b)/sin(d*x+c)
^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\sec(dx+c)^2+A\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.374 $\int \frac{A+B\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\cdots}}{\cdots}$$

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*d)
```

Rubi [A] time = 0.124049, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3921, 3784, 3832}

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+\cdots}{a+b}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*d)
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = A\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + B\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a+b}B\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} - \frac{2A\sqrt{a+}}{}$$

Mathematica [A] time = 2.23165, size = 147, normalized size = 0.71

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left((A-B)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2A\Pi\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((A - B)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*A*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.347, size = 215, normalized size = 1.

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d(b+a\cos(dx+c))(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(A\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*
x+c))*(A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*A*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-B*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)
```

3.375 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=348

$$\frac{A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{\sqrt{a+b}(Ab-2aB)\cot(c+d}{}$$

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (A*Sqrt[a + b]*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (Sqrt[a + b]*(A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (A*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.404588, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4034, 4059, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(Ab-2aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{A\sin(c+dx)\sqrt{a+b\,\mathrm{se}}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (A*Sqrt[a + b]*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (Sqrt[a + b]*(A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (A*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)+\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)-\frac{1}{2}Ab\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - \frac{(Ab)\int \frac{\sec(c\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\cdots}{\cdots}}\cdots}{abd}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\cdots}{\cdots}}\cdots}{abd}$$

Mathematica [C] time = 17.0238, size = 1027, normalized size = 2.95

$$\frac{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(-aA\sqrt{\frac{b-a}{a+b}}\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\tan^3\left(\frac{1}{2}(c\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^
(-1)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] - a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + (2*I)*A*b*EllipticPi[-((a + b)/(a -
 b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)
*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] + (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2
]^2*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*A*(a - b)*EllipticE[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - (2*I)*(A*b - a*B)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/(a + b)
]*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.39, size = 1028, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(-1+cos(d*x+c))^2*(2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a+2*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+2*A*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-4*B*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a
+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a-A*cos(d*x+c)^3*a+A*a*cos(d*x+c)^2-A*cos(d*x+c)^2*b+A*b*cos(d*x+c))*
```

```
(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos(dx+c)\sec(dx+c)+A\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))/sqrt(b*sec(d*x + c)
 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.376 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=435

$$-\frac{\sqrt{a+b}(3Ab-2a(A+2B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4a^2d}-\frac{\sqrt{a+b}\,(4\cdots}{\cdots}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b - 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*b*d) - (Sqrt[a +
 b]*(3*A*b - 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2*A
 + 3*A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*d) - ((3*A*b - 4*a*B
)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (A*Cos[c + d*x]*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.723814, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4034, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2A-4abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}-\frac{(3Ab-4\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b - 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*b*d) - (Sqrt[a +
 b]*(3*A*b - 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2*A
 + 3*A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*d) - ((3*A*b - 4*a*B
)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (A*Cos[c + d*x]*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(3Ab-4aB)-aA\sec(c+dx)\right.}{\sqrt{a+b\sec(c+dx)}}}{2a}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx}}{2ad}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx}}{2ad}$$

$$= -\frac{(a-b)\sqrt{a+b}(3Ab-4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{4a^2bd}$$

$$= -\frac{(a-b)\sqrt{a+b}(3Ab-4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{4a^2bd}$$

Mathematica [C] time = 15.9062, size = 1639, normalized size = 3.77

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (A*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Se
c[c + d*x]]) + (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a
*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*A*b^2*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2] + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a
*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 6*a*A*b*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]^3 - 8*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
 3*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*A*b^2*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2]^5 - 4*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(-3*A*b + 4*a*B)*EllipticE[I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*a^2*A + 3*A*b^2 - a*b*(A + 4*
B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(
```

```
a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt[(-a +
b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 +
Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2)
- b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.388, size = 1885, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a^2*(-1+cos(d*x+c))^2*(4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*A*cos(d*x+c)^4*a^2-8*A*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B
*cos(d*x+c)^3*a^2+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+A*cos(d*x+c)^3*a*
b-3*A*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)^2*a*b+4*B*cos(d*x+c)*a*b-2*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*B*cos(d*x+c)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+8*B
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*s
in(d*x+c)+2*A*cos(d*x+c)^2*a^2+2*A*cos(d*x+c)*a*b-6*A*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*B*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d
*x+c)+3*A*cos(d*x+c)^2*b^2+4*B*cos(d*x+c)^2*a^2-3*A*cos(d*x+c)*b^2+4*A*cos(
d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2+3*A*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-2*A*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a
*b-4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
```

```
(d*x+c)*a*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.377 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=525

$$\frac{\sqrt{a+b}\left(16a^2A+12a^2B-10aAb-18abB+15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right) \ldots\right.}{24a^3d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 15*A*b^2 - 18*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^3
*b*d) + (Sqrt[a + b]*(16*a^2*A - 10*a*A*b + 15*A*b^2 + 12*a^2*B - 18*a*b*B)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(24*a^3*d) + (Sqrt[a + b]*(4*a^2*A*b + 5*A*b^3 - 8*a^3*B - 6
*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^4*d) + ((16*a^2*A + 15*A*b^2 - 18*
a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a^3*d) - ((5*A*b - 6*a*B)
*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*a^2*d) + (A*Cos[c
+ d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 1.16764, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4034, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2A-18abB+15Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24a^3d}+\frac{\sqrt{a+b}\left(16a^2A+12a^2B-10aAb-18abB+15Ab^2\right)\cot(c \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 15*A*b^2 - 18*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^3
*b*d) + (Sqrt[a + b]*(16*a^2*A - 10*a*A*b + 15*A*b^2 + 12*a^2*B - 18*a*b*B)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(24*a^3*d) + (Sqrt[a + b]*(4*a^2*A*b + 5*A*b^3 - 8*a^3*B - 6
*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^4*d) + ((16*a^2*A + 15*A*b^2 - 18*
a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a^3*d) - ((5*A*b - 6*a*B)
*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*a^2*d) + (A*Cos[c
+ d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
```

```
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)\left(\frac{1}{2}(5Ab-6aB)-2aA\sec\right.}{\sqrt{a+b\sec(c}}}{3a}$$

$$= -\frac{(5Ab-6aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12a^2d} + \frac{A\cos^2(c+dx)\sqrt{}}{}$$

$$= \frac{\left(16a^2A+15Ab^2-18abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^3d} - \frac{(5Ab-6aB)\cos}{}$$

$$= \frac{\left(16a^2A+15Ab^2-18abB\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^3d} - \frac{(5Ab-6aB)\cos}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A+15Ab^2-18abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{24a^3bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2A+15Ab^2-18abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{24a^3bd}$$

Mathematica [B] time = 19.8698, size = 1585, normalized size = 3.02

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((A*Sin[c + d*x])/(12*a) + ((-5*A*b + 6*
a*B)*Sin[2*(c + d*x)])/(24*a^2) + (A*Sin[3*(c + d*x)])/(12*a)))/(d*Sqrt[a +
 b*Sec[c + d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 -
Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b
*Tan[(c + d*x)/2] + 15*a*A*b^2*Tan[(c + d*x)/2] + 15*A*b^3*Tan[(c + d*x)/2]
 - 18*a^2*b*B*Tan[(c + d*x)/2] - 18*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan
[(c + d*x)/2]^3 - 30*a*A*b^2*Tan[(c + d*x)/2]^3 + 36*a^2*b*B*Tan[(c + d*x)/
2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 15*a*A
*b^2*Tan[(c + d*x)/2]^5 - 15*A*b^3*Tan[(c + d*x)/2]^5 - 18*a^2*b*B*Tan[(c +
 d*x)/2]^5 + 18*a*b^2*B*Tan[(c + d*x)/2]^5 + 24*a^2*A*b*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - 48*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)] - 36*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b*EllipticPi[-1, -ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] + 30*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*a^3*B*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 - 36*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
```

```
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A + 15*A*b^2 - 18*a
*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(5*A*b^2 + 2*a*b*(A - 3*B) + 12*a^2*B)
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)]))/(24*a^3*d*Sqrt[a + b*Sec[c + d*x]]*Sqrt[1 + Tan
[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2))
)
```

Maple [B] time = 0.471, size = 2954, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d/a^3*(-1+cos(d*x+c))^2*(16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-18*B*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-18*B*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+12*B*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
b+36*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*a+16*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+15*A*cos(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-30*A*cos(d*x+c)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3+48*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+16*A*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+15*A*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*
x+c)-4*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*b-10*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*a-24*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+15*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-12*B*a^3*cos(d*x+c)^2
+8*A*cos(d*x+c)^5*a^3+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+15*A*cos(d
*x+c)^2*b^3-30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
```

```
b))^(1/2))*b^3*sin(d*x+c)-24*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+48*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-15*A*cos(d*x+c)*b^3+12*B*
cos(d*x+c)^4*a^3+18*A*cos(d*x+c)^2*a^2*b-18*B*cos(d*x+c)^2*a*b^2-16*A*cos(d
*x+c)*a^2*b+10*A*cos(d*x+c)*a*b^2-12*B*cos(d*x+c)*a^2*b+12*B*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b+36*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,((a-b)/(a+b))^(1/2))*a*b^2+16*A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+15*A*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
2-4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*b-10*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-24*A*cos(d*x+c)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-1
8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*b-18*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*A*cos(d*x+c)^3*a*b^2-6*B*cos
(d*x+c)^3*a^2*b-15*A*cos(d*x+c)^2*a*b^2+18*B*cos(d*x+c)^2*a^2*b+18*B*cos(d*
x+c)*a*b^2-2*A*cos(d*x+c)^4*a^2*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d
*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos(dx+c)^3\sec(dx+c)+A\cos(dx+c)^3}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)/sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.378 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(2a+b)(3Ab-B(4a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}-\frac{2a}{b^2d\,(a}$$

```
[Out] (-2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a
 + b]*d) - (2*(2*a + b)*(3*A*b - (4*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a +
 b]*d) - (2*a^2*(A*b - a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*B*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.719653, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4028, 4082, 4005, 3832, 4004}

$$-\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(6a^2Ab-8a^3B+5ab^2B-3Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{s}\right.}{3b^4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a
 + b]*d) - (2*(2*a + b)*(3*A*b - (4*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a +
 b]*d) - (2*a^2*(A*b - a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*B*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```


Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
```

```
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}ab(Ab-aB)-\frac{1}{2}\left(2a^2-b^2\right)(Ab-aB)\sec(c}{\sqrt{a+b\sec(c+dx)}}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2d} - \frac{4\int \frac{\text{se}}{}}{}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2d} + \frac{\left(6a^2A}{}$$

$$= -\frac{2\left(6a^2Ab-3Ab^3-8a^3B+5ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{3b^4\sqrt{a+b}d}$$

Mathematica [B] time = 24.7908, size = 3460, normalized size = 10.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B
- 5*a*b^2*B)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) + (2*(a^2*A*b*Sin[c + d*x]
- a^3*B*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*B*Tan[c
 + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])
```

```
*((2*a^2*A)/(b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(A*b)/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*B)/
(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(3
*b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqr
t[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Sqrt[Se
c[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*B*Sqrt[Se
c[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (7*a^2*B*Sqrt[
Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (b*B*Sqrt[Sec[
c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (a*A*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x
]]) - (8*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqr
t[b + a*Cos[c + d*x]]) + (5*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b
*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2
*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*A*b + 3*A*b^3 + 8
*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[
c + d*x])^(3/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*
(a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2
)*(3*A*b + (-4*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)] + (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Cos[c
+ d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-
a^2 + b^2)*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-6*a^2*A*b + 3*A
*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (
-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*(((-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B
 - 5*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x]
)/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x
])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])] + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))] - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-6*a^2*A*b + 3*A*b^3 + 8*
a^3*B - 5*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*
```

```
x)/2] - (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*a^2*A*b + 3*A*b^3 + 8
*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]^2 - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B
 - 5*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^3*(-a^2 + b^2)
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-6*a^2*A
*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a
+ b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c +
 d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3
*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Co
s[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.743, size = 3333, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d/(a-b)/(a+b)/b^3*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*b^4-8
*B*cos(d*x+c)^3*a^4-3*A*cos(d*x+c)^2*b^4+8*B*cos(d*x+c)^2*a^4-B*a^2*b^2+5*B
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^3+8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*B*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-5*B*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^3+6*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*A*s
in(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b-6*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^3*b-2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
```

```
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+6*A*cos(d*x+c)^3*a^3*b-3*A*cos(
d*x+c)^3*a*b^3+5*B*cos(d*x+c)^3*a^2*b^2-6*A*cos(d*x+c)^2*a^3*b+6*A*cos(d*x+
c)^2*a^2*b^2-8*B*cos(d*x+c)^2*a^3*b+5*B*cos(d*x+c)^2*a*b^3-3*A*cos(d*x+c)*a
^2*b^2+4*B*cos(d*x+c)*a^3*b-3*A*cos(d*x+c)^3*a^2*b^2+4*B*cos(d*x+c)^3*a^3*b
-B*cos(d*x+c)^3*a*b^3+3*A*cos(d*x+c)^2*a*b^3-4*B*cos(d*x+c)^2*a^2*b^2-4*B*c
os(d*x+c)*a*b^3-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-B*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^4+8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^4-3*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-6*A*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3*b-6*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+3*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+6*A*cos(d*x+c)*a^
2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))+3*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-
5*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^3-8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-2*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*B*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^3+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^4+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-B*cos(d*x+c)^2*b^4+3
*A*cos(d*x+c)*b^4)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
```

```
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.379 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{2(Ab-B(2a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\frac{2a(Ab-aB)}{bd\left(a^2-b^2\right)\sqrt{a}}$$

```
[Out] (2*(a*A*b - 2*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(A*b -
(2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*a*(A*b - a*B)*Tan[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.464035, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4009, 4005, 3832, 4004}

$$\frac{2a(Ab-aB)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(-2a^2B+aAb+b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(a*A*b - 2*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(A*b -
(2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*a*(A*b - a*B)*Tan[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}b(Ab-aB)-\frac{1}{2}\left(aAb-2a^2B+b^2B\right)\sec(c+dx}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(aAb-2a^2B+b^2B\right)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{2\left(aAb-2a^2B+b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{b^3\sqrt{a+b}d}$$

Mathematica [A] time = 18.7172, size = 467, normalized size = 1.7

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)\left(2b(a+b)(B(b-2a)+Ab)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(a*A*b - 2*a^2*B + b^2*B)*Sin[c
+ d*x])/(b^2*(-a^2 + b^2)) - (2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/
(b*(-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2
*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(2*(a + b)*(-(a*A*b) + 2*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(A*b + (-2*a + b)*B)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
(-(a*A*b) + 2*a^2*B - b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
 b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.469, size = 2276, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/b^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^3+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^3+2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-B*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-2*B*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-A*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-A*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+
A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)*a+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3*sin(d*x+c)+2*B*cos(d*x+c)*a^3-2*B*a^3*cos(d*x+c)^2+B*a^2*b+B*cos(d*x+c
)*b^3+2*B*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))-B*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-B*b^3-B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+A*cos
(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2-A*cos(d*x+c)*a^2*b+A*cos(d*x+c)*a*b^2-
2*B*cos(d*x+c)*a^2*b+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3*sin(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-2*B*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-A*cos(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^2+A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+2*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-B*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^2-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
```

```
))^(1/2))*a*b^2*sin(d*x+c)-A*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)^2*a^2*b-B*cos(
d*x+c)*a*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.380 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=254

$$\frac{2(A+B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2(Ab-aB)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx}}$$

```
[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*(A + B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rubi [A] time = 0.348156, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4003, 4005, 3832, 4004}

$$-\frac{2(Ab-aB)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*(A + B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```


Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{(A+B)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b} + \frac{(Ab-aB)\int \frac{s}{}}{}$$

$$= -\frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}}{}}}{b^2\sqrt{a+b}d}$$

Mathematica [A] time = 15.5771, size = 468, normalized size = 1.84

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)^2(A+B\sec(c+dx))\left(\frac{2(Ab\sin(c+dx)-aB\sin(c+dx))}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}-\frac{2(Ab-aB)\sin(c+dx)}{b\left(b^2-a^2\right)}\right)}{d(a+b\sec(c+dx))^{3/2}(A\cos(c+dx)+B)} - \frac{2\sqrt{\sec(c+dx)}\sqrt{\mathrm{co}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*(A*b - a*B)*
Sin[c + d*x])/(b*(-a^2 + b^2)) + (2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/
((-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c
+ d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*(-(A*b) + a*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a
 + b)*(A - B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - (A*b - a*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/((-(a^2*b) + b^3)*d*(B + A*Cos[c + d*x])*Sqrt[Sec[(c
+ d*x)/2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.381, size = 1634, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/b/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^2-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^2+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+A*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-A*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
a*b-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*
sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+A*cos(d*x+c)^2*a*b-A*cos(d*x+c)^2*b^
2-B*cos(d*x+c)^2*a^2+B*cos(d*x+c)^2*a*b-A*cos(d*x+c)*a*b+A*cos(d*x+c)*b^2+B
*cos(d*x+c)*a^2-B*cos(d*x+c)*a*b)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^2*
sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.381 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}+\frac{2b(Ab-aB)\tan(c+\ldots}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c\ldots}}$$

```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*
d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]])
```

Rubi [A] time = 0.433259, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(Ab-aB)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*
d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]])
```


Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A\left(a^2-b^2\right)+\frac{1}{2}a(Ab-aB)\sec(c+dx)+\frac{1}{2}b(Ab-aB)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A\left(a^2-b^2\right)+\left(\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(Ab-aB)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} -$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab\sqrt{a+b}d} +$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab\sqrt{a+b}d} -$$

Mathematica [C] time = 14.7124, size = 1491, normalized size = 3.97

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(A*b) + a*B
)*Sin[c + d*x])/(a*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c +
 d*x]))/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a +
 b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*
(A + B*Sec[c + d*x])*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2
]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2] + A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - a^2*Sqrt[(-a + b)/(a +
 b)]*B*Tan[(c + d*x)/2] - a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 2
*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 2*a^2*Sqrt[(-a + b)/(a +
 b)]*B*Tan[(c + d*x)/2]^3 + a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
 - A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - a^2*Sqrt[(-a + b)/(a +
 b)]*B*Tan[(c + d*x)/2]^5 + a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5
 - (2*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*A*b^2
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*A*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*A*b^2*Elliptic
Pi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(-(A*
b) + a*B)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(2
*A*b + a*(A - B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*S
qrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x
])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(
c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.385, size = 2009, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*A*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin
(d*x+c)-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^2+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+A*cos(d*x+c)^2*a*b+B*cos(d*x+c)
^2*a*b-B*cos(d*x+c)*a*b+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
```

```
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+B*cos(d*x+c)*a^2-B*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-A*cos
(d*x+c)*a*b+2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-A*cos(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^
2+A*cos(d*x+c)*b^2+A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*b^2-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*a*b+A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*a*b+B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*a*b)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx + c) + A}{(b\sec(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.382 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=427

$$\frac{(a(A-2B)+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}+\frac{b\left(a^2A+2abB\cdots\right.}{a^2d\left(a^2-b^2\right)\cdots}$$

```
[Out] ((a^2*A - 3*A*b^2 + 2*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((3*A*b
+ a*(A - 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*Sqrt[a + b]*d) + (Sqrt[a + b]*(3*A*b - 2*
a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[c + d*x])/(a*d*Sqrt[a + b*S
ec[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Tan[c + d*x])/(a^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.700514, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4034, 4061, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2A+2abB-3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\cdots}{\cdots}\right.\right.}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((a^2*A - 3*A*b^2 + 2*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((3*A*b
+ a*(A - 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*Sqrt[a + b]*d) + (Sqrt[a + b]*(3*A*b - 2*
a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[c + d*x])/(a*d*Sqrt[a + b*S
ec[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Tan[c + d*x])/(a^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{2}(3Ab-2aB)-\frac{1}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{a}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2A-3Ab^2+2abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{4}\left(a^2-b^2\right)(3}{}}{}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2A-3Ab^2+2abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{4}\left(a^2-b^2\right)(3}{}}{}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+b}d}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+b}d}$$

Mathematica [B] time = 19.7011, size = 1613, normalized size = 3.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b*(A*b - a*B)*Sin[c + d*x])/(a^
2*(-a^2 + b^2)) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(
a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - ((b + a
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]*(a^3*A*Tan[(c + d*x)/2] + a^2*A*b*Tan[(c + d*x)/2] - 3*a*A*b^2*T
an[(c + d*x)/2] - 3*A*b^3*Tan[(c + d*x)/2] + 2*a^2*b*B*Tan[(c + d*x)/2] + 2
*a*b^2*B*Tan[(c + d*x)/2] - 2*a^3*A*Tan[(c + d*x)/2]^3 + 6*a*A*b^2*Tan[(c +
 d*x)/2]^3 - 4*a^2*b*B*Tan[(c + d*x)/2]^3 + a^3*A*Tan[(c + d*x)/2]^5 - a^2*
A*b*Tan[(c + d*x)/2]^5 - 3*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*A*b^3*Tan[(c + d*
x)/2]^5 + 2*a^2*b*B*Tan[(c + d*x)/2]^5 - 2*a*b^2*B*Tan[(c + d*x)/2]^5 + 6*a
^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*b^2*B*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*A*b*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^3*B*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + 4*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(a^2*A -
```

```
 3*A*b^2 + 2*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(a + b)*(-(A*b) + a*B)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*Sqr
t[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*
x)/2]^2)))
```

Maple [B] time = 0.372, size = 2871, normalized size = 6.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+
2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*b+2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*a-2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)*b-4*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+A*cos(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*cos(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+6*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+4*B*cos(d*x+c)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
b*sin(d*x+c)-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2*sin(d*x+c)+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-6*A*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+A*cos
(d*x+c)^3*a^3-A*cos(d*x+c)^2*a^3-3*A*cos(d*x+c)^2*b^3+6*A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-2*B*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
```

```
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)
+4*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a
^3*sin(d*x+c)+3*A*cos(d*x+c)*b^3+A*cos(d*x+c)^2*a^2*b+2*B*cos(d*x+c)^2*a*b^
2-A*cos(d*x+c)*a^2*b-2*A*cos(d*x+c)*a*b^2+2*B*cos(d*x+c)*a^2*b-2*B*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2+A*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-3*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^2+2*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*b+2*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-6*A*cos(d*x+c)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*
B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-A*cos(d*x+c)^3*a*b^2+3*A*cos(d*x+
c)^2*a*b^2-2*B*cos(d*x+c)^2*a^2*b-2*B*cos(d*x+c)*a*b^2)/(b+a*cos(d*x+c))/si
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\cos(dx + c)}{(b\sec(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.383 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{\left(-2a^2(A+2B)+ab(5A-12B)+15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{4a^3d\sqrt{a+b}}$$

```
[Out] -((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*b*Sqrt
[a + b]*d) - ((15*A*b^2 + a*b*(5*A - 12*B) - 2*a^2*(A + 2*B))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2*A + 15*A*b^2 - 12*a*b*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(4*a^4*d) - ((5*A*b - 4*a*B)*Sin[c + d*x])/(4*a^2*d*Sq
rt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*S
ec[c + d*x]]) - (b*(7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Tan[c + d*
x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.13925, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4034, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\tan(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-2a^2(A+2B)+ab(5A-12B)+15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{4a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] -((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*b*Sqrt
[a + b]*d) - ((15*A*b^2 + a*b*(5*A - 12*B) - 2*a^2*(A + 2*B))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2*A + 15*A*b^2 - 12*a*b*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(4*a^4*d) - ((5*A*b - 4*a*B)*Sin[c + d*x])/(4*a^2*d*Sq
rt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*S
ec[c + d*x]]) - (b*(7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Tan[c + d*
x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
```

```
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
```

```
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(5Ab-4aB)-aA\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,d}{2a}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \int \frac{1}{4}\left(4a^2A+15Ab^2-12a\right.$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b\left(7a^2Ab-15Ab^3\right.}{4a^3\left(a^2-b\right.}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b\left(7a^2Ab-15Ab^3\right.}{4a^3\left(a^2-b\right.}$$

$$= -\frac{\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right.}{4a^3b\sqrt{a+b}d}$$

$$= -\frac{\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right.}{4a^3b\sqrt{a+b}d}$$

Mathematica [C] time = 18.5604, size = 2667, normalized size = 5.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^2*(A*b - a*B)*Sin[c + d*x])/(a
^3*(-a^2 + b^2)) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(a^3*(a^
2 - b^2)*(b + a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(4*a^2)))/(d*(a + b*S
ec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^
2)]*(-7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 7*a^2*A*b^2*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] + 15*A*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 4*a^4*Sqr
t[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*T
an[(c + d*x)/2] - 12*a^2*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 12
*a*b^3*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 14*a^3*A*b*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]^3 - 8*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 24*a^2*b^2*S
qrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - 7*a^3*A*b*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]^5 + 7*a^2*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
^5 + 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*A*b^4*Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 4*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c
 + d*x)/2]^5 - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 12*a^2
*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + 12*a*b^3*Sqrt[(-a + b)/(
a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^4*A*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
```

```
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (22*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + (30*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (24*I
)*a^3*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (24*I)*a*b^3*B*E
llipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^4*A*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (22*I)*a^2*A*b^2*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*A*b^4
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (24*I)
*a^3*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- (24*I)*a*b^3*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] - I*(a - b)*(-7*a^2*A*b + 15*A*b^3 + 4*a^3*B - 12*a*b^2*B)*Elliptic
E[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*(2*a^3*A + 15*A*
b^3 + a^2*b*(A - 8*B) + 2*a*b^2*(5*A - 6*B))*EllipticF[I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)]))/(4*a^3*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/
(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*
x)/2]^2)))
```

Maple [B] time = 0.525, size = 3980, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d/a^3/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(8*A*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d
*x+c)+2*A*a^4*cos(d*x+c)^4-2*A*a^4*cos(d*x+c)^2+4*B*cos(d*x+c)^3*a^4+15*A*c
os(d*x+c)^2*b^4-4*B*cos(d*x+c)^2*a^4-30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-7*A*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+15*A*(cos(d
```

```
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)
+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b
*sin(d*x+c)-4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^2*sin(d*x+c)-10*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-24*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+24*B*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+4*B*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*
x+c)-12*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2*sin(d*x+c)-12*B*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a+8*B*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+8*B*a^2*b^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-5*A*cos(d*x+c)^3*a^3
*b+5*A*cos(d*x+c)^3*a*b^3-4*B*cos(d*x+c)^3*a^2*b^2+7*A*cos(d*x+c)^2*a^3*b-5
*A*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)^2*a^3*b-12*B*cos(d*x+c)^2*a*b^3-2*A*
cos(d*x+c)*a^3*b+7*A*cos(d*x+c)*a^2*b^2+10*A*cos(d*x+c)*a*b^3-4*B*cos(d*x+c
)*a^3*b-8*B*cos(d*x+c)*a^2*b^2-15*A*cos(d*x+c)^2*a*b^3+12*B*cos(d*x+c)^2*a^
2*b^2+12*B*cos(d*x+c)*a*b^3-2*A*cos(d*x+c)^4*a^2*b^2+15*A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-4*A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+4*B
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d
*x+c)+22*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2-7*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-7*A*cos(d*x+c)*a^2
*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))+15*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-4*A*cos(d*x+c)*a^2*b^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-
10*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b+24*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^
3+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b-12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
```

```
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-12*B*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*B
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4-30*A*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*b^4+15*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+4*B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^4+22*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*a^2*b^2*sin(d*x+c)-7*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*b*sin(d*x+c)-15*A*cos(d*x+c)*b^4)/(b+a*cos(d*x+c))/sin(d
*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos(dx+c)^2\sec(dx+c)+A\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\cos(dx + c)^2}{(b\sec(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.384 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=630

$$\frac{\left(-6a^2b(A+5B)+4a^3(4A+3B)+5ab^2(7A-18B)+105Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-}\right.}{24a^4d\sqrt{a+b}}$$

```
[Out] ((16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a^3*b*B + 90*a*b^3*B)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(24*a^4*b*Sqrt[a + b]*d) + ((105*A*b^3 + 5*a*b^2*(7*A - 18*B) + 4*a^3*(
4*A + 3*B) - 6*a^2*b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^4*Sqrt[a + b]*d) + (Sqr
t[a + b]*(12*a^2*A*b + 35*A*b^3 - 8*a^3*B - 30*a*b^2*B)*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(8*a^5*d) + ((16*a^2*A + 35*A*b^2 - 30*a*b*B)*Sin[c + d*x])/(24*a^3*
d*Sqrt[a + b*Sec[c + d*x]]) - ((7*A*b - 6*a*B)*Cos[c + d*x]*Sin[c + d*x])/(
12*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d
*Sqrt[a + b*Sec[c + d*x]]) + (b*(16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a
^3*b*B + 90*a*b^3*B)*Tan[c + d*x])/(24*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]])
```

Rubi [A] time = 1.66949, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4034, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2A-30abB+35Ab^2\right)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}}+\frac{b\left(41a^2Ab^2+16a^4A-42a^3bB+90ab^3B-105Ab^4\right)\tan(c+dx)}{24a^4d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(-6a^2b(A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a^3*b*B + 90*a*b^3*B)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(24*a^4*b*Sqrt[a + b]*d) + ((105*A*b^3 + 5*a*b^2*(7*A - 18*B) + 4*a^3*(
4*A + 3*B) - 6*a^2*b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^4*Sqrt[a + b]*d) + (Sqr
t[a + b]*(12*a^2*A*b + 35*A*b^3 - 8*a^3*B - 30*a*b^2*B)*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(8*a^5*d) + ((16*a^2*A + 35*A*b^2 - 30*a*b*B)*Sin[c + d*x])/(24*a^3*
d*Sqrt[a + b*Sec[c + d*x]]) - ((7*A*b - 6*a*B)*Cos[c + d*x]*Sin[c + d*x])/(
12*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d
*Sqrt[a + b*Sec[c + d*x]]) + (b*(16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a
^3*b*B + 90*a*b^3*B)*Tan[c + d*x])/(24*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]])
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)\left(\frac{1}{2}(7Ab-6aB)-2aA\sec(c+dx)-\frac{5}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,d}{3a}$$

$$= -\frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a+b\sec(c+dx)}} + \int \frac{\cos(c+d}{}$$

$$= \frac{\left(16a^2A+35Ab^2-30abB\right)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{\left(16a^2A+35Ab^2-30abB\right)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{\left(16a^2A+35Ab^2-30abB\right)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{\left(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{24a^4b\sqrt{a+b}d}$$

$$= \frac{\left(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{24a^4b\sqrt{a+b}d}$$

Mathematica [B] time = 22.3852, size = 2343, normalized size = 3.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(-((a^4*A - a^2*A*b^2 + 24*A*b^4 - 2
4*a*b^3*B)*Sin[c + d*x])/(12*a^4*(-a^2 + b^2)) - (2*(A*b^5*Sin[c + d*x] - a
*b^4*B*Sin[c + d*x]))/(a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])) + ((-11*A*b +
6*a*B)*Sin[2*(c + d*x)])/(24*a^3) + (A*Sin[3*(c + d*x)])/(12*a^2)))/(d*(a +
 b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sq
rt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^5*A*Tan[(c + d*x)/2] + 16
*a^4*A*b*Tan[(c + d*x)/2] + 41*a^3*A*b^2*Tan[(c + d*x)/2] + 41*a^2*A*b^3*Ta
n[(c + d*x)/2] - 105*a*A*b^4*Tan[(c + d*x)/2] - 105*A*b^5*Tan[(c + d*x)/2]
- 42*a^4*b*B*Tan[(c + d*x)/2] - 42*a^3*b^2*B*Tan[(c + d*x)/2] + 90*a^2*b^3*
B*Tan[(c + d*x)/2] + 90*a*b^4*B*Tan[(c + d*x)/2] - 32*a^5*A*Tan[(c + d*x)/2
]^3 - 82*a^3*A*b^2*Tan[(c + d*x)/2]^3 + 210*a*A*b^4*Tan[(c + d*x)/2]^3 + 84
```

```
*a^4*b*B*Tan[(c + d*x)/2]^3 - 180*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 16*a^5*A*T
an[(c + d*x)/2]^5 - 16*a^4*A*b*Tan[(c + d*x)/2]^5 + 41*a^3*A*b^2*Tan[(c + d
*x)/2]^5 - 41*a^2*A*b^3*Tan[(c + d*x)/2]^5 - 105*a*A*b^4*Tan[(c + d*x)/2]^5
 + 105*A*b^5*Tan[(c + d*x)/2]^5 - 42*a^4*b*B*Tan[(c + d*x)/2]^5 + 42*a^3*b^
2*B*Tan[(c + d*x)/2]^5 + 90*a^2*b^3*B*Tan[(c + d*x)/2]^5 - 90*a*b^4*B*Tan[(
c + d*x)/2]^5 + 72*a^4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 138*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 210*A*b^5*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
48*a^5*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 132*a^3*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a*b^4*B*EllipticPi[-1, -ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a^4*A*b*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + 138*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 210*A*b^5*El
lipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 48*a^5*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 132*a^3*b^2*B*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + 180*a*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(
16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a^3*b*B + 90*a*b^3*B)*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 2*a*(a + b)*(-35*A*b^3 + 12*a^3*B - 2*a^2*b*(5*A + 9*B) + 3*
a*b^2*(7*A + 10*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*a^4*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]
^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.717, size = 5086, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3\sec(dx+c)+A\cos(dx+c)^3\right)\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.385 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=510

$$\frac{2\left(-2a^2b^2(3A+8B)-a^3(8Ab-12bB)+16a^4B+9ab^3(A-B)+b^4(3A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+}{a-b}}}{3b^4d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (-2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)*d) + (2*(9*a*b^3*(A - B) + b
^4*(3*A - B) + 16*a^4*B - 2*a^2*b^2*(3*A + 8*B) - a^3*(8*A*b - 12*b*B))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*Sec[c + d*x
]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(
3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*a*b^2*B)*Tan[c + d*x])/(3*b^3*(a^2 - b^2
)^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 1.58814, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4029, 4090, 4082, 4005, 3832, 4004}

$$\frac{2a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2Ab-6a^3B+10ab^2B-7Ab^3\right)\tan(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)*d) + (2*(9*a*b^3*(A - B) + b
^4*(3*A - B) + 16*a^4*B - 2*a^2*b^2*(3*A + 8*B) - a^3*(8*A*b - 12*b*B))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*Sec[c + d*x
]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(
3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*a*b^2*B)*Tan[c + d*x])/(3*b^3*(a^2 - b^2
)^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
```

```
, 1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec^2(c+dx)\left(2a(Ab-aB)-\frac{3}{2}b(Ab-aB)\sec(c\cdots}{(a+b\sec(c\cdots}}{3b\left(a^2-\cdots\right.}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2\cdots\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c\cdots}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2\cdots\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c\cdots}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+10ab^2\cdots\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c\cdots}}$$

$$= -\frac{2\left(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B\right)\cot(c+dx)E\left(\mathrm{si}\cdots\right.}{3(a-b)b^5(a+b)^{3/2}\cdots}$$

Mathematica [B] time = 26.9215, size = 4342, normalized size = 8.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*
b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*B)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2
)^2) + (2*(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (2*(-4*a^4*A*b*Sin[c + d*x] + 8*a^2*A*b^3*Sin[c +
 d*x] + 7*a^5*B*Sin[c + d*x] - 11*a^3*b^2*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b
^2)^2*(b + a*Cos[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c
+ d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*((5*a^2*A)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*A)/(3*b^2*(-a^2 + b^2)^2*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^2)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^5*B)/(3*b^3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (28*a^3*B)/(3*b*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a*b*B)/(3*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*A*Sqrt[Sec[c + d*x
]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a^3*A*Sqrt[Sec[c
+ d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*b*Sqrt[Sec[
c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c
+ d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*B*Sqrt[Sec[c +
 d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (32*a^4*B*Sqrt[Se
c[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*B*Sqrt[
Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*A*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*
x]]) + (5*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt
[b + a*Cos[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2
+ b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (8*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c +
 d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*B*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) -
 (28*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
```

```
x]]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2
*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4
*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b + 15*a^2*A*b^3 - 3*
A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(
c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sin[c + d*x]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*
a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) +
 b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b +
 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b
^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A
*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B
 - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A
+ 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + (-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a
*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]
) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-8*a^4*A*b + 15*a^2*A*b^3 -
 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^
5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
 + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-16*a^4*B -
9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8
*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Co
s[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])] + ((a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 2
8*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
 d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a
 + b)*(-16*a^4*B - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) +
2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-8*
a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos
[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-8*a^4*A*b +
15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*(b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-8*a^4*A*b + 15
*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-16*
a^4*B - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(
-3*A + 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
```

```
)/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-8*a^4*A
*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3
*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b
^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (
-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*
Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(C
os[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c +
 d*x]*Tan[c + d*x]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 1.589, size = 8044, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.386 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{2\left(2a^2b(A-3B)-8a^3B+3ab^2(A+3B)-3b^3(A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}\right.\right.}{3b^3d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*(a - b)*b^4*(a + b)^(3/2)*d) + (2*(2*a^2*b*(A - 3*B) - 3*b^3*(A - B) -
8*a^3*B + 3*a*b^2*(A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d
) - (2*a^2*(A*b - a*B)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Tan[c + d*x])
/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.01573, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4028, 4080, 4005, 3832, 4004}

$$-\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2b(A-3B)-8a^3B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*(a - b)*b^4*(a + b)^(3/2)*d) + (2*(2*a^2*b*(A - 3*B) - 3*b^3*(A - B) -
8*a^3*B + 3*a*b^2*(A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d
) - (2*a^2*(A*b - a*B)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Tan[c + d*x])
/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
```

```
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}ab(Ab-aB)-\frac{1}{2}\left(2a^2-3b^2\right)(Ab-aB)\cdots}{(a+b\sec(c+dx\cdots}}{3b^2\left(a^2-b\cdots\right.}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\tan\cdots}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+d\cdots}}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9ab^2B\right)\tan\cdots}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+d\cdots}}$$

$$= \frac{2\left(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\cdots\right.\right.}{3(a-b)b^4(a+b)^{3/2}d}$$

Mathematica [B] time = 26.4528, size = 3920, normalized size = 9.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*
B - 15*a^2*b^2*B + 3*b^4*B)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)^2) - (2*(a*A*
b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos[c + d*x]
)^2) - (2*(-(a^3*A*b*Sin[c + d*x]) + 5*a*A*b^3*Sin[c + d*x] + 4*a^4*B*Sin[c
 + d*x] - 8*a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d
*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) - (2*(b + a*Cos[c + d*x])^2*((2*a^3*
A)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*
A*b)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*
B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*B)
/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*
B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (5*a^2*A*
Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*A*
Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (A*b^
2*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*
Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a
^3*B*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3
*a*b*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a
^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]) + (2*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]
])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^3*B*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*b*
B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d
*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*
(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a
*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b +
6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[
(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a
^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B)
 - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b
^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))/(3*b^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]
) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-2*
a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2
*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*
A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(((-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-2*a^3*A*b + 6*a
*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos
[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(3*a*b^2*(A
 - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[(b + a*Cos[c +
```

```
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d
*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(
-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^
3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
 a*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d*x]
*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-2*a^3*A*b + 6*a*A*b^3
 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^
2*b^2*B + 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]^2 - (b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) -
 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-
2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1
 - Tan[(c + d*x)/2]^2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*
a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B
) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*
b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*
Sec[c + d*x]*Tan[c + d*x]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.794, size = 6455, normalized size = 15.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.387 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=387

$$\frac{2\left(2a^2B+ab(A+3B)-3b^2(A+B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d\sqrt{a+b}\left(a^2-b^2\right)}+$$

```
[Out] (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) + (2*(2*a^2*B - 3*b^2*(A + B) + a*b*(A + 3*B))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*Tan[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B
- 6*a*b^2*B)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.693662, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4009, 4003, 4005, 3832, 4004}

$$\frac{2\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(2a^2B+ab(A+3B)-3b^2(A+B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) + (2*(2*a^2*B - 3*b^2*(A + B) + a*b*(A + 3*B))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*Tan[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B
- 6*a*b^2*B)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
```

```
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}b(Ab-aB)+\frac{1}{2}\left(aAb+2a^2B-3b^2B\right)\right.}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\tan(}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx}}$$

$$= \frac{2a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\tan(}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx}}$$

$$= \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{3(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [B] time = 24.887, size = 3514, normalized size = 9.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B -
6*a*b^2*B)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) + (2*(A*b*Sin[c + d*x] - a*
B*Sin[c + d*x]))/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(2*a^2*A*b*Si
n[c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c +
d*x]))/(3*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^
(5/2)) + (2*(b + a*Cos[c + d*x])^2*((a^2*A)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*b^2)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*B)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*b*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (a^3*A*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^
2*Sqrt[b + a*Cos[c + d*x]]) - (a*A*b*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) - (5*a^2*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*
b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a*A*b*Cos[2*(c + d*x)]*Sqrt[S
ec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*B*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2
*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b +
a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3
*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(
a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(a^2*A*b + 3*A
*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a
^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*(b + a*Co
s[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^
2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3
 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((a^2*A*b +
3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^4)/2 + ((a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
 Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] -
(b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(a^
2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
```

```
*x]))] - (b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x]
)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))] - a*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)
*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (a^2*A*b +
 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin
[c + d*x]*Tan[(c + d*x)/2] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a
 + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c +
 d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)]) + ((a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B
 - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*A*b
+ 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)
/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*(-(a^2*b) + b^3)^2
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]])))
```

Maple [B] time = 0.42, size = 5170, normalized size = 13.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B \sec\left(dx+c\right)^{3} + A \sec\left(dx+c\right)^{2}\right)\sqrt{b \sec\left(dx+c\right)+a}}{b^{3} \sec\left(dx+c\right)^{3} + 3\,ab^{2} \sec\left(dx+c\right)^{2} + 3\,a^{2}b \sec\left(dx+c\right) + a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\sec(dx + c)^2}{(b\sec(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.388 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=353

$$\frac{2(3aA+aB-Ab-3bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}-\frac{2\left(a^2(-B)\right.}{3d\left(a^2\right.}$$

```
[Out] (-2*(4*a*A*b - a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*
d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2
)*d) - (2*(A*b - a*B)*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(
3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.598099, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4003, 4005, 3832, 4004}

$$-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(4*a*A*b - a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*
d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2
)*d) - (2*(A*b - a*B)*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(
3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```


Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

```
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int}{}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(4a}{}$$

$$= -\frac{2\left(4aAb-a^2B-3b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [B] time = 22.6848, size = 3225, normalized size = 9.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((-2*(-4*a*A*b
+ a^2*B + 3*b^2*B)*Sin[c + d*x])/(3*b*(-a^2 + b^2)^2) + (2*(A*b^2*Sin[c + d
*x] - a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (2*(-
5*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b^
2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos
[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*((-4*a*A
*b)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*B
)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2*B)/
((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*A*Sqrt[
Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (A*b^2*Sqrt[Se
c[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a^3*B*Sqrt[Sec[
c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*b*B*Sqrt[Sec[
```

```
c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*a^2*A*Cos[2*(c
+ d*x)]*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (
a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*C
os[c + d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2
*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*(-4*a*A*b + a^2*B + 3*b^2*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(
a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)] + (-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2
)^2*d*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5
/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-4*a
*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] + 2*b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-4*a*A*b + a
^2*B + 3*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*b*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*
x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a +
 b)*(-4*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-4*
a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-4*a*A*b + a
^2*B + 3*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (
(a + b)*(-4*a*A*b + a^2*B + 3*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Co
s[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d
*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(3*a*A + A*b - a*
B - 3*b*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(
1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])] + ((a + b)*(-4*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c
+ d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c
+ d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-4*a*A*b + a^2*B
 + 3*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d
*x)/2] + (-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-4*a*A*b + a^2*B + 3
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b*(a^2 - b^2)^2*Sqrt[b
 + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-4*a*A*b + a^2*
B + 3*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
```

```
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 2*b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-4*a*A*b + a^2*B + 3*b^
2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Se
c[c + d*x]*Tan[c + d*x]))/(3*b*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[
Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.387, size = 4213, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2/b*4^(1/2)*(-3*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+A*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-B*cos(d*x+c)^3*a^4+B*cos
(d*x+c)^2*a^4-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+3*B*sin(d*x+c)*
cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+A*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^3-4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*b-4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-B*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^3*b-4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b^2+3*B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-4*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+3*A*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c
)+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
3*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
```

```
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^3*b*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+3*B*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-B*a^2*b^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-4*B*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+3*A*sin(d
*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^3*b+4*A*cos(d*x+c)^3*a^3*b-3*B*cos(d*x+c)^3*a^2*b^2-4*A*cos(d*x+
c)^2*a^3*b+8*A*cos(d*x+c)^2*a^2*b^2-2*B*cos(d*x+c)^2*a^3*b-6*B*cos(d*x+c)^2
*a*b^3-3*A*cos(d*x+c)*a^2*b^2+4*A*cos(d*x+c)*a*b^3-B*cos(d*x+c)*a^2*b^2-5*A
*cos(d*x+c)^3*a^2*b^2+2*B*cos(d*x+c)^3*a^3*b+2*B*cos(d*x+c)^3*a*b^3-4*A*cos
(d*x+c)^2*a*b^3+4*B*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)*a*b^3+B*sin(d*x+c)*
cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^4+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^4-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-4*A*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-8*A*cos(
d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))-4*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+3*A*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+7*A*cos(d*x+c)*
a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))+5*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a-3*B*cos(d*x+c)*b^4+2*B*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+4*B
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^2+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*B*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^2-7*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+3*B*sin(d*x+c)*cos(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+3*B
*cos(d*x+c)^2*b^4+A*cos(d*x+c)^3*b^4-A*cos(d*x+c)*b^4)*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*
sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.389 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$-\frac{2\left(6a^2Ab+a^2bB-3a^3B-aAb^2-3Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{3a^2bd(a-b)(a+b)^{3/2}}$$

```
[Out] (2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*d) - (2*(6*a^2*A*b - a*A*b^2 - 3*A*b^3 - 3*a^3*B + a^2*b*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(a^3*d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Tan[c + d*x])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.766911, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b\left(7a^2Ab-4a^3B-3Ab^3\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2Ab+a^2bB-3a^3B-aAb^2-3A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*d) - (2*(6*a^2*A*b - a*A*b^2 - 3*A*b^3 - 3*a^3*B + a^2*b*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(a^3*d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Tan[c + d*x])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}A\left(a^2-b^2\right)+\frac{3}{2}a(Ab-aB)\sec(c+dx)-\frac{1}{2}b(Ab-aB)\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{3}{4}A(}{}$$

$$= \frac{2b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{3}{4}A(}{}$$

$$= \frac{2\left(7a^2Ab-3Ab^3-4a^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{3a^2(a-b)b(a+b)^{3/2}d}$$

$$= \frac{2\left(7a^2Ab-3Ab^3-4a^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{3a^2(a-b)b(a+b)^{3/2}d}$$

Mathematica [C] time = 17.1595, size = 2083, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*(-7*a^2*A*b
 + 3*A*b^3 + 4*a^3*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) - (2*(A*b^3*Sin[c
 + d*x] - a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2)
 - (2*(-8*a^2*A*b^2*Sin[c + d*x] + 4*A*b^4*Sin[c + d*x] + 5*a^3*b*B*Sin[c +
 d*x] - a*b^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))
/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*A*b*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 7*a^2*A*b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] - 3*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*A*b^
4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 4*a^4*Sqrt[(-a + b)/(a + b)]*B*
Tan[(c + d*x)/2] - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 14*a
^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 6*a*A*b^3*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^3 + 8*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2]^3 + 7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a^2*A*b^2*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*a*A*b^3*Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]^5 + 3*A*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 4*
a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + 4*a^3*b*Sqrt[(-a + b)/(a
+ b)]*B*Tan[(c + d*x)/2]^5 - (6*I)*a^4*A*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] + (12*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - (6*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*a^
4*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
```

```
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (12
*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] - (6*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] + I*(a - b)*(-7*a^2*A*b + 3*A*b^3 + 4*a^3*B)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(-4*a*A*b^2 - 6*A*b^3 + 3*a^3*(A
 - B) + a^2*b*(9*A + B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x
)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
))/(3*a^2*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d*(B + A*Cos[c + d*x])*(a +
b*Sec[c + d*x])^(5/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^
2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c +
 d*x)/2]^2)))
```

Maple [B] time = 0.414, size = 5710, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3
+ 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx + c) + A}{(b\sec(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.390 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=582

$$-\frac{\left(-a^2b(21A+2B)-3a^3(A-4B)+ab^2(5A-6B)+15Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{3a^3d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] ((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) - ((15*A*b^3 + a*b^2*(5*A - 6*B) - 3*a^3
*(A - 4*B) - a^2*b*(21*A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*Sqrt[a + b]*(a^2 - b^
2)*d) + (Sqrt[a + b]*(5*A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + (A
*Sin[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2*A - 5*A*b^2 + 2
*a*b*B)*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b
*(3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Tan[c + d*x])
/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.2142, antiderivative size = 582, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4034, 4061, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(-26a^2Ab^2+3a^4A+14a^3bB-6ab^3B+15Ab^4\right)\tan(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b\left(3a^2A+2abB-5Ab^2\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\left(-a^2b(21A+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) - ((15*A*b^3 + a*b^2*(5*A - 6*B) - 3*a^3
*(A - 4*B) - a^2*b*(21*A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*Sqrt[a + b]*(a^2 - b^
2)*d) + (Sqrt[a + b]*(5*A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + (A
*Sin[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2*A - 5*A*b^2 + 2
*a*b*B)*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b
*(3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Tan[c + d*x])
/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
```

```
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
```

```
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}dx = \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{2}(5Ab-2aB)-\frac{3}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{a}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2A-5Ab^2+2abB\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{3}{4}\left(a^2-b^2\right)(5}{}}{}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2A-5Ab^2+2abB\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a\right.}{3}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2A-5Ab^2+2abB\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a\right.}{3}$$

$$= \frac{\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{3a^3(a-b)b(a+b)^{3/2}d}$$

$$= \frac{\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{3a^3(a-b)b(a+b)^{3/2}d}$$

Mathematica [B] time = 22.0462, size = 2390, normalized size = 4.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*b*(-10*a^2*A*b + 6*A*b^3 + 7*a^
3*B - 3*a*b^2*B)*Sin[c + d*x])/(3*a^3*(-a^2 + b^2)^2) + (2*(A*b^4*Sin[c + d
*x] - a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (
2*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*b^5*Sin[c + d*x] + 8*a^3*b^2*B*Sin[c +
d*x] - 4*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])))
)/(d*(a + b*Sec[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]
^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a^5*A*Tan[(c + d*x)
/2] + 3*a^4*A*b*Tan[(c + d*x)/2] - 26*a^3*A*b^2*Tan[(c + d*x)/2] - 26*a^2*A
*b^3*Tan[(c + d*x)/2] + 15*a*A*b^4*Tan[(c + d*x)/2] + 15*A*b^5*Tan[(c + d*x
)/2] + 14*a^4*b*B*Tan[(c + d*x)/2] + 14*a^3*b^2*B*Tan[(c + d*x)/2] - 6*a^2*
b^3*B*Tan[(c + d*x)/2] - 6*a*b^4*B*Tan[(c + d*x)/2] - 6*a^5*A*Tan[(c + d*x)
/2]^3 + 52*a^3*A*b^2*Tan[(c + d*x)/2]^3 - 30*a*A*b^4*Tan[(c + d*x)/2]^3 - 2
8*a^4*b*B*Tan[(c + d*x)/2]^3 + 12*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 3*a^5*A*Ta
n[(c + d*x)/2]^5 - 3*a^4*A*b*Tan[(c + d*x)/2]^5 - 26*a^3*A*b^2*Tan[(c + d*x
)/2]^5 + 26*a^2*A*b^3*Tan[(c + d*x)/2]^5 + 15*a*A*b^4*Tan[(c + d*x)/2]^5 -
15*A*b^5*Tan[(c + d*x)/2]^5 + 14*a^4*b*B*Tan[(c + d*x)/2]^5 - 14*a^3*b^2*B*
Tan[(c + d*x)/2]^5 - 6*a^2*b^3*B*Tan[(c + d*x)/2]^5 + 6*a*b^4*B*Tan[(c + d*
x)/2]^5 + 30*a^4*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
```

```
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] - 60*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^5*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^5*B
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 24*a^3*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] - 12*a*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^4*A*b*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - 60*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^5*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] - 12*a^5*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^3*b^2*B*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 12*a*b^4*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(3*a^4*A - 26*a
^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*
a*(a + b)*(5*A*b^3 + 3*a^3*B + 3*a^2*b*(-2*A + B) - a*b^2*(3*A + 2*B))*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*Sq
rt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d
*x)/2]^2)))
```

Maple [B] time = 0.677, size = 8545, normalized size = 14.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{b\sec(dx+c)+a}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) +
 a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.391 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=686

$$\frac{\left(-5a^2b^2(27A+4B)-a^3(27Ab-84bB)+6a^4(A+2B)+5ab^3(7A-12B)+105Ab^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{s}}{}}}{12a^4d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] -((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a
*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))]])/(12*a^4*(a - b)*b*(a + b)^(3/2)*d) + ((105*A*b^4 + 5*
a*b^3*(7*A - 12*B) + 6*a^4*(A + 2*B) - 5*a^2*b^2*(27*A + 4*B) - a^3*(27*A*b
 - 84*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))]])/(12*a^4*Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a + b]
*(4*a^2*A + 35*A*b^2 - 20*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(4*a^5*d) - ((7*
A*b - 4*a*B)*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (A*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(27*a^2*A*b -
35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a +
b*Sec[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^
5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```

Rubi [A] time = 2.05047, antiderivative size = 686, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4034, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\left(-170a^2Ab^3+33a^4Ab+104a^3b^2B-12a^5B-60ab^4B+105Ab^5\right)\tan(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{b\left(27a^2Ab-12a^3B+20ab^2B-35\right.}{12a^3d\left(a^2-b^2\right)(a+b\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a
*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))]])/(12*a^4*(a - b)*b*(a + b)^(3/2)*d) + ((105*A*b^4 + 5*
a*b^3*(7*A - 12*B) + 6*a^4*(A + 2*B) - 5*a^2*b^2*(27*A + 4*B) - a^3*(27*A*b
 - 84*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))]])/(12*a^4*Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a + b]
*(4*a^2*A + 35*A*b^2 - 20*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(4*a^5*d) - ((7*
A*b - 4*a*B)*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (A*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(27*a^2*A*b -
35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a +
b*Sec[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^
5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(7Ab-4aB)-aA\sec(c+dx)-\frac{5}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,d}{2a}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{1}{4}\left(4a^2A+35Ab^2-2\right.}{}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(27a^2Ab-35A\right.}{12a^3\left(a^2-\right.}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(27a^2Ab-35A\right.}{12a^3\left(a^2-\right.}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(27a^2Ab-35A\right.}{12a^3\left(a^2-\right.}$$

$$= -\frac{\left(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B\right)\cot(c+dx}{12a^4(a-b)b(a+b}$$

$$= -\frac{\left(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B\right)\cot(c+dx}{12a^4(a-b)b(a+b}$$

Mathematica [A] time = 14.7169, size = 823, normalized size = 1.2

$$\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{2\left(10Ba^3-13Aba^2-6b^2Ba+9Ab^3\right)\sin(c+dx)b^2}{3a^4\left(b^2-a^2\right)^2} - \frac{2\left(Ab^5\sin(c+dx)-ab^4B\sin(c+dx)\right)}{3a^4\left(a^2-b^2\right)(b+a\cos(c+dx))^2} - \frac{2\left(10A\sin(c+dx)b^6-7a\right.}{}\right.}{d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^2*(-13*a^2*A*b + 9*A*b^3 + 10*
a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^4*(-a^2 + b^2)^2) - (2*(A*b^5*Sin[c +
 d*x] - a*b^4*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) -
 (2*(-14*a^2*A*b^4*Sin[c + d*x] + 10*A*b^6*Sin[c + d*x] + 11*a^3*b^3*B*Sin[
c + d*x] - 7*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x
```

```
])) + (A*Sin[2*(c + d*x)])/(4*a^3)))/(d*(a + b*Sec[c + d*x])^(5/2)) - ((b +
 a*Cos[c + d*x])^2*Sec[c + d*x]*(-(a*(a + b)*(-33*a^4*A*b + 170*a^2*A*b^3 -
 105*A*b^5 + 12*a^5*B - 104*a^3*b^2*B + 60*a*b^4*B)*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(105*A*b^5 + 6*a^5*(A + 2*B) - 3
0*a*b^4*(7*A + 2*B) + 4*a^3*b^2*(57*A + 10*B) - 3*a^4*b*(13*A + 48*B) + 2*a
^2*b^3*(-29*A + 60*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
 + 3*(a - b)^2*(a + b)^2*(4*a^2*A + 35*A*b^2 - 20*a*b*B)*((a - b)*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(-33*a^4*A*b + 170*a^2*A*b^3 - 105*A
*b^5 + 12*a^5*B - 104*a^3*b^2*B + 60*a*b^4*B)*(b + a*Cos[c + d*x])*(Cos[c +
 d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(12*a^5*(a^
2 - b^2)^2*d*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*(a + b*Sec[c + d*x])^(
5/2))
```

Maple [B] time = 0.957, size = 10322, normalized size = 15.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2\sec(dx+c)+A\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x +
```

```
c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.392 $\int \frac{\sec(e+fx)(A+A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2A(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2f}$$

```
[Out] (-2*A*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b^2*f)
```

Rubi [A] time = 0.0812832, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {4004}

$$-\frac{2A(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(A + A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*A*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b^2*f)
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(A+A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx=-\frac{2A(a-b)\sqrt{a+b}\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\ldots}{\ldots}}}{b^2f}$$

Mathematica [B] time = 10.5517, size = 248, normalized size = 2.36

$$\frac{A(\sec(e+fx)+1)\left(2\tan\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx)+b)+\frac{\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}\left(\tan\left(\frac{1}{2}(e+fx)\ldots\right.\right.}{\sqrt{\sec\ldots}}\right.}{bf\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(A + A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (A*(1 + Sec[e + f*x])*(2*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2] + (Sqrt[Sec[
(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((Sqrt[(a - b)/(a + b
)]*(a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Elliptic
E[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)])/Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])] + (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1
 + Tan[(e + f*x)/2]^2))/Sqrt[Sec[e + f*x]]))/(b*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] time = 0.432, size = 642, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*A/f/b*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x
+e))^2*(2*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*b-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+
b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*Elliptic
E((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b+2*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+
cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*sin(f*x+e)-(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*
EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Elliptic
E((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2
*a-a*cos(f*x+e)+b*cos(f*x+e)-b)/sin(f*x+e)^5/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A\sec\left(fx+e\right)+A\right)\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(f*x + e) + A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{A\sec\left(fx+e\right)^2+A\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((A*sec(f*x + e)^2 + A*sec(f*x + e))/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$A\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] A*(Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)), x) + Integral(sec(e + f*
x)**2/sqrt(a + b*sec(e + f*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A\sec\left(fx+e\right)+A\right)\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((A*sec(f*x + e) + A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

3.393 $\int \frac{\sec(e+fx)(A-A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{2A\sqrt{a-b}(a+b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)|\frac{a-b}{a+b}\right)}{b^2f}$$

```
[Out] (2*A*Sqrt[a - b]*(a + b)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f
*x]]/Sqrt[a - b]], (a - b)/(a + b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b^2*f)
```

Rubi [A] time = 0.0849392, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {4004}

$$\frac{2A\sqrt{a-b}(a+b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)|\frac{a-b}{a+b}\right)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(A - A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (2*A*Sqrt[a - b]*(a + b)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f
*x]]/Sqrt[a - b]], (a - b)/(a + b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b^2*f)
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(A-A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{2A\sqrt{a-b}(a+b)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-}}{b^2f}$$

Mathematica [A] time = 8.09658, size = 211, normalized size = 1.97

$$\frac{A(a+b)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\sec(e+fx)+1}E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\right.\right.}{bf\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(A - A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (A*(a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Sec[(e +
 f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Ellipt
icE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 + Sec[e + f*x]] - Sqr
t[(1 + Cos[e + f*x])^(-1)]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e +
f*x]))]*Sqrt[Sec[e + f*x]]*Sin[e + f*x]))/(b*f*((1 + Cos[e + f*x])^(-1))^(3
/2)*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] time = 0.427, size = 457, normalized size = 4.3

$$-2\,\frac{A\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}}{fb\left(\sin\left(fx+e\right)\right)^{5}\left(a\cos\left(fx+e\right)+b\right)}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)}{(a+b)\left(1+\cos\left(\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*A/f/b*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x
+e))^2*(cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*a+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)
*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e)
)/sin(f*x+e),((a-b)/(a+b))^(1/2))*b+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a
+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1
/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin
(f*x+e),((a-b)/(a+b))^(1/2))*b*sin(f*x+e)-cos(f*x+e)^2*a+a*cos(f*x+e)-b*cos
(f*x+e)+b)/sin(f*x+e)^5/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(A\sec\left(fx+e\right)-A\right)\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] -integrate((A*sec(f*x + e) - A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{A\sec\left(fx+e\right)^{2}-A\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-(A*sec(f*x + e)^2 - A*sec(f*x + e))/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-A\left(\int -\frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] -A*(Integral(-sec(e + f*x)/sqrt(a + b*sec(e + f*x)), x) + Integral(sec(e +
f*x)**2/sqrt(a + b*sec(e + f*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(A\sec\left(fx+e\right)-A\right)\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(-(A*sec(f*x + e) - A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

3.394 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(5aA+3bB)\text{si}}{}$$

```
[Out] (-2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(5*a*A + 3*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
b*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.182551, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(5aA+3bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(5*a*A + 3*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
b*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(5aA\right.$$

$$= \frac{2bB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (Ab+aB)\int \sec^{\frac{5}{2}}(c+dx)$$

$$= \frac{2(5aA+3bB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(Ab+aB)\sec}{}$$

$$= \frac{2(5aA+3bB)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(Ab+aB)\sec}{}$$

$$= -\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}$$

Mathematica [A] time = 1.87147, size = 132, normalized size = 0.73

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20(aB+Ab)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-12(5aA+3bB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\,\text{si}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(5*a*A + 3*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(5*a*A + 3*b*B)*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 5.766, size = 663, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*B*b/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+
2*A*a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^
2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec\left(dx+c\right)^{3}+Aa\sec\left(dx+c\right)+\left(Ba+Ab\right)\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^3 + A*a*sec(d*x + c) + (B*a + A*b)*sec(d*x + c)^
2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.395 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2(aB+Ab)\sqrt{\cos(c+dx)}}{}$$

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*b*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.14663, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2(aB+Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*b*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{2bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}(3aA\right.\\ &= \frac{2bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+(Ab+aB)\int \sec^{\frac{3}{2}}(c+dx\\ &= \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2bB\sec^{\frac{3}{2}}(c+dx)}{3d}\\ &= \frac{2(3aA+bB)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\\ &= -\frac{2(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.913275, size = 104, normalized size = 0.73

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((3aA+bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-3(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)(3(aB+Ab)\cos(c}{\cos^{\frac{3}{2}}(c+dx)}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*x
)/2, 2] + (3*a*A + b*B)*EllipticF[(c + d*x)/2, 2] + ((b*B + 3*(A*b + a*B)*C
os[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 4.188, size = 428, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*b
```

```
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(sec(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.396 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=111

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

```
[Out] (2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d + (2*b*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.138849, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3997, 3787, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d + (2*b*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bB\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+2\int \frac{\frac{1}{2}(aA-bB)+\frac{1}{2}(Ab+aB)\sec(c+}{\sqrt{\sec(c+dx)}}$$
$$= \frac{2bB\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+(Ab+aB)\int \sqrt{\sec(c+dx)}\,dx+(aA-$$
$$= \frac{2bB\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\left((Ab+aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$
$$= \frac{2(aA-bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(Ab+aB)\sqrt{}}{}$$

Mathematica [A] time = 0.295481, size = 84, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left((aB+Ab)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(aA-bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B*Sin[c
 + d*x]))/d
```

Maple [A] time = 1.928, size = 244, normalized size = 2.2

$$-2\,\frac{A\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)b-A\sqrt{2\,(\sin(1/2\,dx+c/2)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*(A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*b-A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+B*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a+B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-2*B*b*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\sec(dx+c)^2 + Aa + (Ba+Ab)\sec(dx+c)}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sqrt(sec(d*x
 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c+dx))(a + b\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c) + A)(b\sec(dx+c) + a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.397 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|$$

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.147281, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3996, 3787, 3771, 2639, 2641}

$$\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{1}{2}(aA+3bB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - (-Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{3}(-aA-3bB)\int \sqrt{s$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \left((-Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+d$$

$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(aA+3bB)\sqrt{c}}{}$$

Mathematica [A] time = 0.25468, size = 90, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2(aA+3bB)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6(aB+Ab)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+aA\sin(2(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*S
in[2*(c + d*x)]))/(3*d)
```

Maple [B] time = 1.785, size = 326, normalized size = 2.8

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,Aa\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+Aa\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b-2*A*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sec(d*x + c)
^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))(a+b\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.398 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\sqrt{}}{5d}$$

```
[Out] (2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.163442, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{1}{2}(3aA+5bB)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx - \frac{1}{5}(-3aA-5bB)\int$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-Ab-aB)\int \sqrt{\sec(c}$$

$$= \frac{2(3aA+5bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2aA\sin(c}{5d\sec^{\frac{3}{2}}(c}$$

$$= \frac{2(3aA+5bB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(Ab+a}{}$$

Mathematica [A] time = 0.604255, size = 108, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(10(aB+Ab)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))(3aA\cos(c+dx)+5aB+5Ab)+6(}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5
*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] time = 1.806, size = 371, normalized size = 2.5

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,Aa\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(24\,Aa+20\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a+20*A*b+20*B*a)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-9*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5
*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*a-15*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sec(d*x + c)
^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))(a+b\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.399 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+7bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a*A + 7*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.181352, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+7bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a*A + 7*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(5aA+7bB)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx - \frac{1}{7}(-5aA-7bB)\int \\
&= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5aA+7bB)\sin(c+dx}{21d\sqrt{\sec(c+dx)}} \\
&= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5aA+7bB)\sin(c+dx}{21d\sqrt{\sec(c+dx)}} \\
&= \frac{6(Ab+aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(5aA+7bB}{}
\end{aligned}$$

Mathematica [A] time = 1.05471, size = 125, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(20(5aA+7bB)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))(42(aB+Ab)\cos(c+dx)+15aA\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 20*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
(65*a*A + 70*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*a*A*Cos[2*(c + d*x)])*S
in[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 1.948, size = 413, normalized size = 2.3

$$-\frac{2}{105\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,Aa\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^8+(-360\,Aa-16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a-168*A*b-168*B*a)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+25*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+35*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sec(d*x + c)
^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.400 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=263

$$\frac{2\left(7a(aB+2Ab)+5b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2\left(5a^2A+6abB+3Ab^2\right)\sin(c+dx)\sqrt{\sec}}{5d}$$

```
[Out] (-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*b^2*B + 7*a*(2*A*b + a*B))*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(5*a^2*A
 + 3*A*b^2 + 6*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(5*b^2*B
+ 7*a*(2*A*b + a*B))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(7*A*b
+ 9*a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b*B*Sec[c + d*x]^(5/2
)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.37276, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4026, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\left(5a^2A+6abB+3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(5a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*b^2*B + 7*a*(2*A*b + a*B))*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(5*a^2*A
 + 3*A*b^2 + 6*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(5*b^2*B
+ 7*a*(2*A*b + a*B))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(7*A*b
+ 9*a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b*B*Sec[c + d*x]^(5/2
)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \mathrm{se}$$

$$= \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \mathrm{se}$$

$$= \frac{2\left(5b^2B+7a(2Ab+aB)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{2b(}{}$$

$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(5}{}$$

$$= \frac{2\left(5b^2B+7a(2Ab+aB)\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{21d}$$

$$= -\frac{2\left(5a^2A+3Ab^2+6abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Mathematica [A] time = 4.60238, size = 221, normalized size = 0.84

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(40\left(7a^2B+14aAb+5b^2B\right)\cos^{\frac{7}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-168\left(5a^2A+6abB+3Ab^2\right)\cos^{\frac{7}{2}}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]^(7/2)*
EllipticE[(c + d*x)/2, 2] + 40*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Cos[c + d*x]^
(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(140*a*A*b + 70*a^2*B + 110*b^2*B + 21*
(15*a^2*A + 13*A*b^2 + 26*a*b*B)*Cos[c + d*x] + 10*(14*a*A*b + 7*a^2*B + 5*
b^2*B)*Cos[2*(c + d*x)] + 105*a^2*A*Cos[3*(c + d*x)] + 63*A*b^2*Cos[3*(c +
d*x)] + 126*a*b*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] time = 7.429, size = 859, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(-1/56*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b*(A
*b+2*B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)*(-1/6*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c
)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+2*a^2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2 \sec(dx+c)^4 + Aa^2 \sec(dx+c) + \left(2\,Bab + Ab^2\right)\sec(dx+c)^3 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)^2\right)\sqrt{\sec(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^4 + A*a^2*sec(d*x + c) + (2*B*a*b + A*b^2)*sec
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

3.401 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=221

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(5a(aB+2Ab)+3b^2B\right)\sin(c+dx)\sqrt{\sec(}}{5d}$$

```
[Out] (-2*(3*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*b^2*B +
5*a*(2*A*b + a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(5*A*b + 7
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*b*B*Sec[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.31499, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4026, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aB+2Ab)+3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(3*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*b^2*B +
5*a*(2*A*b + a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(5*A*b + 7
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*b*B*Sec[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{}$$

$$= \frac{2bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{}$$

$$= \frac{2\left(3b^2B+5a(2Ab+aB)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2b}{}$$

$$= \frac{2\left(3b^2B+5a(2Ab+aB)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2b}{}$$

$$= -\frac{2\left(3b^2B+5a(2Ab+aB)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Mathematica [A] time = 2.69002, size = 171, normalized size = 0.77

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20\left(3a^2A+2abB+Ab^2\right)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-12\left(5a^2B+10aAb+3b^2B\right)\cos^{\frac{5}{2}}(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[c + d*x]^(5/2)*
EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*A + A*b^2 + 2*a*b*B)*Cos[c + d*x]^(5/
2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(2*a*A*b + a^2*B + b^2*B) + 10*b*(A*b
+ 2*a*B)*Cos[c + d*x] + 3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[2*(c + d*x)])*
```

```
Sin[c + d*x]))/(30*d)
```

Maple [B] time = 6.353, size = 750, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5
*B*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b+2*B*a)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+2*a*(2*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*
c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\sec(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

3.402 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=177

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{d}$$

```
[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*(3*A*b + 5*a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*B*Sqrt[Sec[c + d*x]]*(a + b*Sec
[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.270419, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4026, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*(3*A*b + 5*a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*B*Sqrt[Sec[c + d*x]]*(a + b*Sec
[c + d*x])*Sin[c + d*x])/(3*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bB\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}a(3aA-bB)\cdots}{\cdots}$$

$$= \frac{2bB\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}a(3aA-bB)\cdots}{\cdots}$$

$$= \frac{2b(3Ab+5aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2bB\sqrt{\sec(c+dx)}(a+b\,\mathrm{se}\cdots}{3d}$$

$$= \frac{2\left(6aAb+3a^2B+b^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\cdots$$

$$= \frac{2\left(a^2A-Ab^2-2abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\cdots$$

Mathematica [A] time = 1.26277, size = 125, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(3a^2B+6aAb+b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\cdots\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(3*(a^2*A - A*b^2 - 2*a*b*B)*Ellip
ticE[(c + d*x)/2, 2] + (6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2
] + (b*(b*B + 3*(A*b + 2*a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2
)))/(3*d)
```

Maple [B] time = 4.855, size = 677, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*A*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
B*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b+2*B*a)*(-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Bb^2\sec(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\sec(dx+c)}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

3.403 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2\left(b^2B-a(aB+2Ab)\right)}{}$$

```
[Out] (-2*(b^2*B - a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*A*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*b^2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.247762, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4024, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2\left(b^2B-a(aB+2Ab)\right)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(b^2*B - a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*A*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*b^2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)+\left(A\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)-3abB\right)\mathrm{s}}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)-\frac{3}{2}b^2B\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{3}$$

$$= \frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2b^2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(b^2B-a(2Ab+\right.$$

$$= \frac{2\left(a^2A+3Ab^2+6abB\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} +$$

$$= -\frac{2\left(b^2B-a(2Ab+aB)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.757449, size = 124, normalized size = 0.77

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)\left(a^2A\cos(c+dx)+3b^2B\right)\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2] + 2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + 2*(3*b^2*B + a^2*A*Cos[c + d*x])*Sin[c + d*x]))/(3*d
)
```

Maple [B] time = 2.187, size = 404, normalized size = 2.5

$$-\frac{2}{3d}\left(4Aa^2\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^4+a^2A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\text{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
-2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-6*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb^2\sec(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\sec(dx+c)}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\,(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

3.404 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(3a^2A+10abB+5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{5d}$$

```
[Out] (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.260082, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4024, 4047, 3771, 2639, 4045, 2641}

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^2A+10abB+5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```


Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB)+\left(A\left(-\frac{3a^2}{2}-\frac{5b^2}{2}\right)-5abB\right)s}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB)-\frac{5}{2}b^2B\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx - \frac{1}{5}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(-2aAb-a^2B-3\right.$$

$$= \frac{2\left(3a^2A+5Ab^2+10abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

$$= \frac{2\left(3a^2A+5Ab^2+10abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A] time = 0.971852, size = 128, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(10\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6\left(3a^2A+10abB+5Ab^2\right)\sqrt{\cos(c+d}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + a*(10*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[2*
(c + d*x)]))/(15*d)
```

Maple [B] time = 1.87, size = 487, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^2+40*A*a*b+20*B*a^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^2-20*A*a*b-10*B*a^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+10*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+5*B*a^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+15*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb^2\sec\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x
)
```

3.405 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\left(5a^2A+14abB+7Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2\left(5a^2A+14abB+7Ab^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+$$

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2)) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.288722, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4024, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(5a^2A+14abB+7Ab^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(5a^2A+14abB+7Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(3a^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2)) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```


Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(2Ab+aB)+\left(A\left(-\frac{5a^2}{2}-\frac{7b^2}{2}\right)-7abB\right)\ldots}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(2Ab+aB)-\frac{7}{2}b^2B\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx - \frac{1}{7}\ldots$$

$$= \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2A+7Ab^2+1\ldots\right.}{21d\sqrt{\sec(\ldots}}$$

$$= \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2A+7Ab^2+1\ldots\right.}{21d\sqrt{\sec(\ldots}}$$

$$= \frac{2\left(6aAb+3a^2B+5b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\ldots$$

Mathematica [A] time = 1.44826, size = 161, normalized size = 0.76

$$\frac{\sqrt{\sec(c+dx)}\left(20\left(5a^2A+14abB+7Ab^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(5\left(3a^2A\cos(2(c+\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 20*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (42*a*(2*A*b + a*B)*Cos[c + d*x] + 5*(13*a^
```

```
2*A + 14*A*b^2 + 28*a*b*B + 3*a^2*A*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(
210*d)
```

Maple [B] time = 2.148, size = 548, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a
^2+280*B*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-84*A*a*b-7
0*A*b^2-42*B*a^2-140*B*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-126*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b+25*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*A*b^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-105*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\sec\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

3.406 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$\frac{2\left(5a(aB+2Ab)+7b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2\left(7a^2A+18abB+9Ab^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*S
in[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(7
*d*Sec[c + d*x]^(5/2)) + (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sin[c + d*x])/(4
5*d*Sec[c + d*x]^(3/2)) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sin[c + d*x])/(2
1*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.337205, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4024, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2\left(7a^2A+18abB+9Ab^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(7a^2A+18abB+9Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2A\sin}{9d\sec}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*S
in[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(7
*d*Sec[c + d*x]^(5/2)) + (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sin[c + d*x])/(4
5*d*Sec[c + d*x]^(3/2)) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sin[c + d*x])/(2
1*d*Sqrt[Sec[c + d*x]])
```


Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{9}{2}a(2Ab+aB)+\left(A\left(-\frac{7a^2}{2}-\frac{9b^2}{2}\right)-9abB\right)\text{s}}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{9}{2}a(2Ab+aB)-\frac{9}{2}b^2B\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx - \frac{1}{9}$$

$$= \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(7a^2A+9Ab^2+1\right.}{45d\sec^{\frac{3}{2}}(}$$

$$= \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(7a^2A+9Ab^2+1\right.}{45d\sec^{\frac{3}{2}}(}$$

$$= \frac{2\left(7a^2A+9Ab^2+18abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{15d}$$

$$= \frac{2\left(7a^2A+9Ab^2+18abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{15d}$$

Mathematica [A] time = 1.94013, size = 189, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(120\left(5a^2B+10aAb+7b^2B\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(7\left(43a^2A+72abB\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2] + 120*(10*a*A*b + 5*a^2*B + 7*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(43*a^2*A + 36*A*b^2 + 72*a*b*B)*Cos[
c + d*x] + 5*(156*a*A*b + 78*a^2*B + 84*b^2*B + 18*a*(2*A*b + a*B)*Cos[2*(c
 + d*x)] + 7*a^2*A*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] time = 1.985, size = 610, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^2+1440*A*a*b+720*B*a^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^2-2160*A*a*b-504*A*b^2-
1080*B*a^2-1008*B*a*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*A*a^2+1
680*A*a*b+504*A*b^2+840*B*a^2+1008*B*a*b+420*B*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-168*A*a^2-480*A*a*b-126*A*b^2-240*B*a^2-252*B*a*b-210*B*
b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-147*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+150*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
378*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+75*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*B*
b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\sec\left(dx+c\right)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sec\left(dx+c\right)^2 + \left(Ba^2 + 2\,Aab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x
)
```

3.407 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=345

$$\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2b\left(22a^2B+27aAb+7b^2B\right)\text{s}}{45d}$$

```
[Out] (-2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3
+ 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(21*d) + (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B
+ 15*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22
*a^2*B + 7*b^2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b +
 13*a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*b*B*Sec[c + d*x]^(5/2
)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.572322, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4026, 4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3
+ 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(21*d) + (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B
+ 15*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22
*a^2*B + 7*b^2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b +
 13*a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*b*B*Sec[c + d*x]^(5/2
)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
```

```
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{9d} + \frac{2}{9}\int \sec$$

$$= \frac{2b^2(9Ab+13aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB\sec^{\frac{5}{2}}(c+d}{}$$

$$= \frac{2b^2(9Ab+13aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB\sec^{\frac{5}{2}}(c+d}{}$$

$$= \frac{2\left(21a^2Ab+5Ab^3+7a^3B+15ab^2B\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{21d}$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)\sqrt{\sec(c+dx)}\sin(c+}{15d}$$

$$= \frac{2\left(21a^2Ab+5Ab^3+7a^3B+15ab^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

$$= -\frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [A] time = 6.55319, size = 452, normalized size = 1.31

$$\frac{\cos^4(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\left(2\left(105a^2Ab+35a^3B+75ab^2B+25Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{105d(a\cos(c+dx)+b)^3(A\cos(c+dx)+B)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^4*((2*(-105*a^3*A - 189*a*A*b^2 - 189*a^2*b*B - 49*b^3*B)*Ell
ipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(105*a^
2*A*b + 25*A*b^3 + 35*a^3*B + 75*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x])
)/(105*d*(b + a*Cos[c + d*x])^3*(B + A*Cos[c + d*x])) + ((a + b*Sec[c + d*x
])^3*(A + B*Sec[c + d*x])*((2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B
)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^3*(A*b^3*Sin[c + d*x] + 3*a*b^2*B*Sin[
c + d*x]))/7 + (2*Sec[c + d*x]*(21*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d
*x] + 7*a^3*B*Sin[c + d*x] + 15*a*b^2*B*Sin[c + d*x]))/21 + (2*Sec[c + d*x]
^2*(27*a*A*b^2*Sin[c + d*x] + 27*a^2*b*B*Sin[c + d*x] + 7*b^3*B*Sin[c + d*x
]))/45 + (2*b^3*B*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^
3*(B + A*Cos[c + d*x])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 9.93, size = 1193, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6/5*a*b*(A*b+B
*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-
```

```
1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-2
4*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1
/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b+3*B*a)*(-1/56*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(3*A*b+B*a)*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*B*b^3*(-1/144*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\sec(dx+c)^5 + Aa^3\sec(dx+c) + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^4 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^3 + \left(Ba^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^5 + A*a^3*sec(d*x + c) + (3*B*a*b^2 + A*b^3)*s
ec(d*x + c)^4 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^3 + (B*a^3 + 3*A*a^2*b)*
```

```
sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

3.408 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=295

$$\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2b\left(18a^2B+21aAb+5b^2\ldots\right.}{2\ldots}$$

```
[Out] (-2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 +
21*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*b*B*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.504427, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4026, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(15a^2Ab+5a^3B+9ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 +
21*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*b*B*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
```

```
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int$$

$$= \frac{2b^2(7Ab+11aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\sec^{\frac{3}{2}}(c$$

$$= \frac{2b^2(7Ab+11aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\sec^{\frac{3}{2}}(c$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\sec(c+dx)}\sin(c}{5d}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\sec(c+dx)}\sin(c}{5d}$$

$$= -\frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{5d}$$

Mathematica [A] time = 3.88042, size = 225, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(5\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+21\left(15a^2Ab+5a^3B+9\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(21*a^3*A + 21*a*A*b^2 + 21*a
^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(15*a^2
*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 5*b*(21*a*A*b + 21*a^2
*B + 5*b^2*B)*Tan[c + d*x] + 21*b^2*(A*b + 3*a*B)*Sec[c + d*x]*Tan[c + d*x]
 + 15*b^3*B*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

Maple [B] time = 8.33, size = 944, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5
*b^2*(A*b+3*B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*b^3*(-1/56*cos(1/2*d*x+1/2
```

```
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c
)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b*(A*b+B*a)*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(3*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\sec\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

3.409 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\left(14a^2B+15aAb+3b^2B\right)\sin(c}{5d}$$

```
[Out] (2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^
3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (2*b*(15*a*A*b + 14*a^2*B + 3*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*b^2*(5*A*b + 9*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
15*d) + (2*b*B*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d
)
```

Rubi [A] time = 0.483027, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4026, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\left(14a^2B+15aAb+3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^
3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (2*b*(15*a*A*b + 14*a^2*B + 3*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*b^2*(5*A*b + 9*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
15*d) + (2*b*B*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d
)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bB\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a+b\sec(c\cdots}{\cdots}$$

$$= \frac{2b^2(5Ab+9aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2bB\sqrt{\sec(c+dx)}(a+b\,\mathrm{s}\cdots}{5\cdots}$$

$$= \frac{2b^2(5Ab+9aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2bB\sqrt{\sec(c+dx)}(a+b\,\mathrm{s}\cdots}{5\cdots}$$

$$= \frac{2b\left(15aAb+14a^2B+3b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b^2(5Ab+9\cdots}{\cdots}$$

$$= \frac{2\left(9a^2Ab+Ab^3+3a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec\cdots}}{3d}$$

$$= \frac{2\left(5a^3A-15aAb^2-15a^2bB-3b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\cdots}}{5d}$$

Mathematica [A] time = 2.45572, size = 190, normalized size = 0.78

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)\cos^{\frac{5}{2}}(c+dx)\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+12\left(5a^3A-15a^2bB-15aAb^{\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(12*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Cos[c
 + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*B +
 3*a*b^2*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*b*(15*(3*a*A*b
 + 3*a^2*B + b^2*B) + 10*b*(A*b + 3*a*B)*Cos[c + d*x] + 9*(5*a*A*b + 5*a^2*
B + b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 6.648, size = 997, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a^2*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2/5*B*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b+3*B*a)*(-1/6*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))+6*a*b*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x
+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\sec(dx+c)^4 + Aa^3 + (3\,Bab^2 + Ab^3)\sec(dx+c)^3 + 3\,(Ba^2b + Aab^2)\sec(dx+c)^2 + (Ba^3 + 3\,Aa^2b)\sec(dx+c)}{\sqrt{\sec(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c) + A)(b\sec(dx+c) + a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

3.410 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2b\left(2a^2A-9abB-3Ab^2\right)\sin(c+}{3d}$$

```
[Out] (2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^
3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 - (2*b*(2*a^2*A - 3*A*b^2 - 9*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
) - (2*b^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*(a +
 b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.510185, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4025, 4076, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b\left(2a^2A-9abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^
3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 - (2*b*(2*a^2*A - 3*A*b^2 - 9*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
) - (2*b^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*(a +
 b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{(a+b\sec(c+dx))\left(-\frac{1}{2}a(\right.}{}$$

$$= -\frac{2b^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA(a+b\sec(c+dx))^2\sin}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA(a+b\sec(c+dx))^2\sin}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b\left(2a^2A-3Ab^2-9abB\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(aA-bB)}{}$$

$$= \frac{2\left(a^3A+9aAb^2+9a^2bB+b^3B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3d}$$

$$= \frac{2\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{d}$$

Mathematica [A] time = 1.98562, size = 166, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6\left(3a^2Ab+a^3B-3ab^2B-Ab\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*
b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B
)*EllipticF[(c + d*x)/2, 2] + ((a^3*A + 2*b^3*B + 6*b^2*(A*b + 3*a*B)*Cos[c
 + d*x] + a^3*A*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 5.698, size = 1212, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*A*a^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*
x+1/2*c)^2+18*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-18*A*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+6*A*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*b^3*sin(1/2*d*x+1/2*c)^2-8*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
4-12*A*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+18*B*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+2*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d
*x+1/2*c)^2-6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+18*B*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-36*B*a*b^2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-9*A*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*a*b^2+9*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-3*A*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*b^3+2*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*A*b^3*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-B*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*b^3+3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-9*B*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*a*b^2+18*B*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*B*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3 \sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\text{s}}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c) + A)(b\sec(dx+c) + a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

3.411 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(3a^3A+15a^2bB+15aAb^2-5b\right.}{}$$

```
[Out] (2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^
3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]
]) - (2*b^2*(a*A - 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*A*(
a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.461241, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4025, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)\sqrt{c}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^
3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]
]) - (2*b^2*(a*A - 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*A*(
a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(-\frac{1}{2}a(9\right.}{}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2(aA-5bB)\sqrt{\sec(c+dx)}\sin(c+dx}{5d}$$

$$= \frac{2\left(3a^2Ab+3Ab^3+a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3d}$$

$$= \frac{2\left(3a^3A+15aAb^2+15a^2bB-5b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}$$

Mathematica [A] time = 1.58586, size = 172, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(20\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)\left(3\left(a^3A\cos(2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*A*b + 3*A*b^3 + a^3*B +
 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(10*a^2*(3*A*b
 + a*B)*Cos[c + d*x] + 3*(a^3*A + 10*b^3*B + a^3*A*Cos[2*(c + d*x)]))*Sin[c
 + d*x]))/(30*d)
```

Maple [B] time = 2.298, size = 867, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-24*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^2*(6*A*a+15*A*b+5*B*a)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*a^3+15*A*a
^2*b+5*B*a^3+15*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*a^2*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+15*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)-9*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3-45*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+5*B*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+45*B*a*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-45*B*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
*b+15*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb^3\sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\sec(dx+c)}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c+dx))\,(a + b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c) + A)(b\sec(dx+c) + a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x
)
```

3.412 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\left(5a^2A+21abB+18Ab^2\right)}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 21
*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```

Rubi [A] time = 0.466728, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4025, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 21
*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{(a+b\sec(c+dx))\left(-\frac{1}{2}a(\right.}{}$$

$$= \frac{2a^2(11Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(11Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(11Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2A+18Ab^2+21abB\right)\sin(c+}{21d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{5d}$$

$$= \frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{5d}$$

Mathematica [A] time = 1.35764, size = 180, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(20\left(5a^3A+21a^2bB+21aAb^2+21b^3B\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a\sin(2(c+dx))\left(5\left(3a^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(5*a^3*A + 21*a*A*b^2 + 21*a^2*
b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(42*a*(3*A
*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 42*A*b^2 + 42*a*b*B + 3*a^2*A*Cos[2*
(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] time = 2.01, size = 664, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^3-504*A*a^2*b-168*B*a^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^3+504*A*a^2*b+420*A*a*b^2+1
68*B*a^3+420*B*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^3-12
6*A*a^2*b-210*A*a*b^2-42*B*a^3-210*B*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)+25*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3+105*A*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a
*b^2-189*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-105*A*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^
3+105*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b+105*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-6
3*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-315*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3\sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\sec(}{\sec(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x
)
```

3.413 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=295

$$\frac{2\left(15a^2Ab+5a^3B+21ab^2B+7Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\left(7a^2A+27abB+22Ab^2\right)s}{45d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(15*a^2*A*b + 7*A*b^3 +
 5*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*a^2*(13*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*a*(7*a^2*A + 22*A*b^2 + 27*a*b*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(15*a^2*A*b + 7*A*b^3 + 5*a^3*B + 21*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.538318, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4025, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a\left(7a^2A+27abB+22Ab^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(15a^2Ab+5a^3B+21ab^2B+7Ab^3\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(15a^2Ab+5a^3B+21ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(15*a^2*A*b + 7*A*b^3 +
 5*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*a^2*(13*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*a*(7*a^2*A + 22*A*b^2 + 27*a*b*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(15*a^2*A*b + 7*A*b^3 + 5*a^3*B + 21*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{(a+b\sec(c+dx))\left(-\frac{1}{2}a(13A\right.}{}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4}{6}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4}{6}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\left(7a^2A+22Ab^2+27abB\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\left(7a^2A+22Ab^2+27abB\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(7a^3A+27aAb^2+27a^2bB+15b^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{15d}$$

Mathematica [A] time = 2.09568, size = 219, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(120\left(15a^2Ab+5a^3B+21ab^2B+7Ab^3\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(7a\left(43a^2A\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(15*a^2*A*b + 7*A*b^3 + 5*a
^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*a*(43*
a^2*A + 108*A*b^2 + 108*a*b*B)*Cos[c + d*x] + 5*(234*a^2*A*b + 84*A*b^3 + 7
8*a^3*B + 252*a*b^2*B + 18*a^2*(3*A*b + a*B)*Cos[2*(c + d*x)] + 7*a^3*A*Cos
[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] time = 2.046, size = 745, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*a^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^3+2160*A*a^2*b+720*B*a^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^3-3240*A*a^2*b-1512*A
*a*b^2-1080*B*a^3-1512*B*a^2*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(95
2*A*a^3+2520*A*a^2*b+1512*A*a*b^2+420*A*b^3+840*B*a^3+1512*B*a^2*b+1260*B*a
*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a^3-720*A*a^2*b-378*A
*a*b^2-210*A*b^3-240*B*a^3-378*B*a^2*b-630*B*a*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a*b^2+225*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-315*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3+75*B*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*B*a*b^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bb^3\sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\mathrm{s}}{\sec(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x
)
```

3.414 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=345

$$\frac{2\left(45a^3A+165a^2bB+165aAb^2+77b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{2a\left(9a^2A+33abB+\right.}{77d\sec^{\frac{5}{2}}(}$$

```
[Out] (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(45*a^3*A + 165*a*A*b^2
+ 165*a^2*b*B + 77*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(231*d) + (2*a^2*(15*A*b + 11*a*B)*Sin[c + d*x])/(99*d*Sec[
c + d*x]^(7/2)) + (2*a*(9*a^2*A + 26*A*b^2 + 33*a*b*B)*Sin[c + d*x])/(77*d*
Sec[c + d*x]^(5/2)) + (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sin[
c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*(45*a^3*A + 165*a*A*b^2 + 165*a^2*
b*B + 77*b^3*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Se
c[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```

Rubi [A] time = 0.57392, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4025, 4074, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a\left(9a^2A+33abB+26Ab^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{2\left(21a^2Ab+7a^3B+27ab^2B+9Ab^3\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(45a^3A+165a^2bB+\right.}{231}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x]
```


```
[Out] (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(45*a^3*A + 165*a*A*b^2
+ 165*a^2*b*B + 77*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(231*d) + (2*a^2*(15*A*b + 11*a*B)*Sin[c + d*x])/(99*d*Sec[
c + d*x]^(7/2)) + (2*a*(9*a^2*A + 26*A*b^2 + 33*a*b*B)*Sin[c + d*x])/(77*d*
Sec[c + d*x]^(5/2)) + (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sin[
c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*(45*a^3*A + 165*a*A*b^2 + 165*a^2*
b*B + 77*b^3*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Se
c[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{2}{11}\int \frac{(a+b\sec(c+dx))\left(-\frac{1}{2}a\right.}{}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a\left(9a^2A+26Ab^2+33abB\right)\sin(c+}{77d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a\left(9a^2A+26Ab^2+33abB\right)\sin(c+}{77d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(21a^2Ab+9Ab^3+7a^3B+27ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

$$= \frac{2\left(21a^2Ab+9Ab^3+7a^3B+27ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

Mathematica [A] time = 3.09557, size = 256, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(240\left(45a^3A+165a^2bB+165aAb^2+77b^3B\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(18\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(3696*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 240*(45*a^3*A + 165*a*A*b^2 + 1
65*a^2*b*B + 77*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (154*
(129*a^2*A*b + 36*A*b^3 + 43*a^3*B + 108*a*b^2*B)*Cos[c + d*x] + 180*a*(16*
a^2*A + 33*A*b^2 + 33*a*b*B)*Cos[2*(c + d*x)] + 770*a^2*(3*A*b + a*B)*Cos[3
*(c + d*x)] + 15*(531*a^3*A + 1716*a*A*b^2 + 1716*a^2*b*B + 616*b^3*B + 21*
a^3*A*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(27720*d)
```

Maple [B] time = 1.988, size = 825, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a^3-36960*A*a^2*b-1232
0*B*a^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^3+73920*A*a^2*
b+23760*A*a*b^2+24640*B*a^3+23760*B*a^2*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x
```

```
+1/2*c)+(-34920*A*a^3-68376*A*a^2*b-35640*A*a*b^2-5544*A*b^3-22792*B*a^3-35
640*B*a^2*b-16632*B*a*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A
*a^3+31416*A*a^2*b+27720*A*a*b^2+5544*A*b^3+10472*B*a^3+27720*B*a^2*b+16632
*B*a*b^2+4620*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2790*A*a^3-5
544*A*a^2*b-7920*A*a*b^2-1386*A*b^3-1848*B*a^3-7920*B*a^2*b-4158*B*a*b^2-23
10*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+675*A*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^3+2475*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2-4851*A*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^2*b-2079*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3+2475*B*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^2*b+1155*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-1617*B*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^3-6237*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^3 \sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\sec(\right.}{\sec(dx+c)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2),
x)
```

3.415 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d}-\frac{2\left(-5a^2B+5aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5b^3d}+\frac{2}{\ldots}$$

```
[Out] (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*d) + (2*a^2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a + b)*d) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*b^3*d) + (2*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b
^2*d) + (2*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 1.01417, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4033, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2\left(-5a^2B+5aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5b^3d}+\frac{2\left(-5a^2B+5aAb-3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\ldots\right.}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*d) + (2*a^2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a + b)*d) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*b^3*d) + (2*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b
^2*d) + (2*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aB}{2}+\frac{3}{2}bB\sec(c+dx)+\frac{5}{2}(Ab-aB)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}}{5b}$$

$$= \frac{2(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2d} + \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{4\int \frac{\sqrt{\sec(\ldots}}{\ldots}}{\ldots}$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(Ab-aB)\sec^{\frac{3}{2}}(c+dx)}{3b^2d}$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(Ab-aB)\sec^{\frac{3}{2}}(c+dx)}{3b^2d}$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(Ab-aB)\sec^{\frac{3}{2}}(c+dx)}{3b^2d}$$

$$= \frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} - \frac{2\left(5aAb-5a^2\ldots\right.}{\ldots}$$

$$= \frac{2\left(5aAb-5a^2B-3b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^3d} + \frac{2(Ab-\ldots}{\ldots}$$

Mathematica [B] time = 6.96241, size = 669, normalized size = 2.42

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2\left(5a^2B-5aAb+3b^2B\right)\sin(c+dx)}{5b^3}+\frac{2\sec(c+dx)(Ab\sin(c+dx)-aB\sin(c+dx))}{3b^2}+\frac{2B\tan(c+dx)\sec(c+dx)}{5b}\right)}{d} - \frac{2\left(-45a^2Ab+45a^3B+19ab^2B\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((-2*(-40*a*A*b^2 + 40*a^2*b*B + 18*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a
), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d
*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4
5*a^2*A*b - 10*A*b^3 + 45*a^3*B + 19*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[Arc
Sin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]
]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b
+ a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(-15*a^2*A*b + 15*a^3*B + 9*a*
b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2
+ 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*
b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(30*b^3*d) + (Sqrt[
Sec[c + d*x]]*((2*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[c + d*x])/(5*b^3) + (2
*Sec[c + d*x]*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*b^2) + (2*B*Sec[c +
 d*x]*Tan[c + d*x])/(5*b)))/d
```

Maple [B] time = 6.75, size = 785, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*B/b/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*(A*b-B*a)/b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))-2*(A*b-B*a)*a^3/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*(A*b-B*a)/b^3*a*(-(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

3.416 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}+\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}-\frac{2(Ab-aB)\sqrt{\cos(c+}\cdots}{\cdots}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(b^2*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) + (2*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d) + (2*B*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*b*d)
```

Rubi [A] time = 0.71332, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4033, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(b^2*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) + (2*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d) + (2*B*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}+\frac{1}{2}bB\sec(c+dx)+\frac{3}{2}(Ab-aB)\sec^2(c+dx\right.}{a+b\sec(c+dx)}}{3b}$$

$$= \frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int -}{}$$

$$= \frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int -}{}$$

$$= \frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{B\int \sqrt{}}{}$$

$$= -\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} + \frac{2(Ab-aB}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}}{}$$

Mathematica [A] time = 3.65854, size = 229, normalized size = 1.09

$$-\frac{\cot(c+dx)\left(-2\left(3a^2B+3ab(B-A)+b^2(B-3A)\right)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)-6a^2B\sqrt{-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]*(-(b^2*B*Sec[c + d*x]^(5/2)) + b^2*B*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) - 6*b*(A*b - a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[-Tan[c + d*x]^2] - 2*(3*a^2*B + b^2*(-3*A + B) + 3*a*b*(-A + B))*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a*A*b*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a^2*B
*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])
)/(3*b^3*d)
```

Maple [A] time = 5.356, size = 466, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+2*(A*b-B*a)/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2
*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.417 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c-}{bd}$$

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(b*d)
```

Rubi [A] time = 0.400632, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4033, 4106, 3849, 2805, 12, 3771, 2639}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c-}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{2\int \frac{-\frac{aB}{2}-\frac{1}{2}bB\sec(c+dx)+\frac{1}{2}(Ab-aB)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{b}$$

$$= \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{2\int -\frac{a^2B}{2\sqrt{\sec(c+dx)}}\,dx}{a^2b} + \frac{(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} - \frac{B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} + \frac{\left((Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2B\sqrt{\sec(c+dx)}}{bd}$$

$$= -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\right.}{b(a}$$

Mathematica [A] time = 1.33576, size = 125, normalized size = 0.99

$$-\frac{2\cos(2(c+dx))\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)\left((Ab-B(a+b))\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+(Ab-\right.}{b^2d\left(\sec^2(c+dx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] + (A*b - (a + b)*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b
- a*B)*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*Sq
```

```
rt[-Tan[c + d*x]^2])/(b^2*d*(-2 + Sec[c + d*x]^2))
```

Maple [A] time = 3.801, size = 325, normalized size = 2.6

$$-\frac{1}{d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-2\,\frac{(Ab-Ba)\,a\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2}}{b\left(a^2-ab\right)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a)/b/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+2*B/b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*
c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.418 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d)
```

Rubi [A] time = 0.19823, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4038, 3771, 2641, 3849, 2805}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d)
```


Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx &= \frac{A\int \sqrt{\sec(c+dx)}\,dx}{a} - \frac{(Ab-aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{\left((Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}\\ &= \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2}{a}\right.}{a(a}\end{aligned}$$

Mathematica [A] time = 0.581318, size = 78, normalized size = 0.77

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(aB\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+(aB-Ab)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(a*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(A*b) +
a*B)*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x
]^2])/(a*b*d)
```

Maple [A] time = 2.02, size = 217, normalized size = 2.2

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{a\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(A\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec{(c + dx)})\sqrt{\sec{(c + dx)}}}{a + b\sec{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec{(dx + c)} + A)\sqrt{\sec{(dx + c)}}}{b\sec{(dx + c)} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.419 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=149

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+a\right.}{a^2d(a+b)}$$

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a^2*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rubi [A] time = 0.257062, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4038, 3771, 2639, 3848, 2803, 2641, 2805}

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a^2*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx}{a}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{\left((Ab-aB)\sqrt{\cos(c+dx)}\right.}{}$$
$$= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\left((Ab-aB)\sqrt{\cos(c+dx)}\sqrt{s\right.}{a}$$
$$= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}F\left(\right.}{a^2}$$

Mathematica [A] time = 6.94094, size = 224, normalized size = 1.5

$$\frac{\cot(c+dx)\left(2aA\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+2Ab\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*(-(a*A*Sec[c + d*x]^(3/2)) - a*A*Cos[2*(c + d*x)]*Sec[c + d*x
]^(3/2) + a*A*Sec[c + d*x]^(7/2) + a*A*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2)
- 2*a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2
*a*A*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*A*
b*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
 - 2*a*B*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c +
d*x]^2]))/(a^2*d)
```

Maple [A] time = 2.069, size = 295, normalized size = 2.

$$2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{\left(a-b\right)a^{2}\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(A\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b)/a^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

3.420 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=196

$$\frac{2\left(a^2A-3abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^3d}-\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\right.}{a^3d(a+b)}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a^2*d) + (2*(a^2*A + 3*A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.467163, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4034, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(a^2A-3abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\right.}{a^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a^2*d) + (2*(a^2*A + 3*A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{2}(Ab-aB)-\frac{1}{2}aA\sec(c+dx)-\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{2}a(Ab-aB)-\left(\frac{a^2A}{2}+\frac{3}{2}b(Ab-aB)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^3} - \frac{\left(b^2(Ab-aB)\right)}{}$$

$$= \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{\left(a^2A+3Ab^2-3abB\right)\int \sqrt{\mathrm{se}}}{3a^3}$$

$$= -\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d} + \frac{2A\sin(c}{3ad\sqrt{\sec(c}}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\left(a^2A+3Ab^2-\right.}{}$$

Mathematica [A] time = 6.52404, size = 282, normalized size = 1.44

$$\frac{2\csc(c+dx)\left(a(aA+3aB-3Ab)\sqrt{-\tan^2(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+a^2A\sin(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*Csc[c + d*x]*(3*a*A*b - 3*a^2*B - 3*a*A*b*Sec[c + d*x]^2 + 3*a^2*B*Sec[c
 + d*x]^2 + a^2*A*Sin[c + d*x]*Tan[c + d*x] - 3*a*(-(A*b) + a*B)*EllipticE[
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] +
a*(a*A - 3*A*b + 3*a*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*A*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 3*a*b*B*Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan
[c + d*x]^2]))/(3*a^3*d*Sec[c + d*x]^(3/2))
```

Maple [B] time = 2.248, size = 786, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*A*a^3-4*A*
a^2*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^3+2*A*a^2*b)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3-A*a^2*b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+3*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2-3*A*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^3-3*B*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^2*b+3*B*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))*a*b^2)/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

3.421 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=242

$$-\frac{2\left(a^2+3b^2\right)(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d}+\frac{2\left(3a^2A-5abB+5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{5a^3d}$$

```
[Out] (2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(a^2 + 3*b^2)*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*d) + (2*b^3*(A
*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a^4*(a + b)*d) + (2*A*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3
/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.755326, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4034, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2\left(a^2+3b^2\right)(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2\left(3a^2A-5abB+5Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(a^2 + 3*b^2)*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*d) + (2*b^3*(A
*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a^4*(a + b)*d) + (2*A*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3
/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{5}{2}(Ab-aB)-\frac{3}{2}aA\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{4\int \frac{\frac{3}{4}\left(3a^2A+5Ab^2-5abB\right)+\frac{1}{4}a(4Ab+5aB)\,\mathrm{se}}{\sqrt{\sec(c+dx)}(a+b}}{15a^2}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{4\int \frac{\frac{3}{4}a\left(3a^2A+5Ab^2-5abB\right)-\left(-\frac{1}{4}a^2(4Ab+5a\right.}{\sqrt{\sec(}}}{15a}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{\left(\left(a^2+3b^2\right)(Ab-aB)\right)\int \sqrt{\sec(c+}}{3a^4}$$

$$= \frac{2b^3(Ab-aB)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^4(a+b)d} + \frac{2A\sin(c+dx}{5ad\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2\left(3a^2A+5Ab^2-5abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^3d} - \frac{2\left(a^2+3\right.}{}$$

Mathematica [B] time = 6.93873, size = 617, normalized size = 2.55

$$\frac{2\left(9a^2A-5abB+5Ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)} - \frac{2\left(9a^2A-1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((-2*(8*a*A*b + 10*a^2*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x
])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(9*a^2*A + 5*A*b^2 -
5*a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellipt
icPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) - (2*(9*a^2*A + 15*A*b^2 - 15*a*b*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x
])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a
^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[
c + d*x]^2)))/(30*a^2*d) + (Sqrt[Sec[c + d*x]]*((A*Sin[c + d*x])/(10*a) + (
(-(A*b) + a*B)*Sin[2*(c + d*x)])/(3*a^2) + (A*Sin[3*(c + d*x)])/(10*a)))/d
```

Maple [B] time = 2.22, size = 1074, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*A*a^4+2
4*A*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^4-44*A*a^3*b+20*
A*a^2*b^2+20*B*a^4-20*B*a^3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*
A*a^4+16*A*a^3*b-10*A*a^2*b^2-10*B*a^4+10*B*a^3*b)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b
-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-
15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^4+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2
-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)*a*b^3)/a^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

3.422 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=406

$$-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))} -$$

```
[Out] -(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) - ((3*a*A*b - 5*a
^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B
)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/((a - b)*b^3*(a + b)^2*d) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b
^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((3*a*A*b - 5*
a^2*B + 2*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (
a*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x]))
```

Rubi [A] time = 1.16166, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d\left(a^2-b^2\right)}+\frac{\left(3a^2Ab-5a^3B+4ab^2B-\right.}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) - ((3*a*A*b - 5*a
^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B
)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/((a - b)*b^3*(a + b)^2*d) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b
^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((3*a*A*b - 5*
a^2*B + 2*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (
a*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}dx = \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(Ab-aB)-b(Ab-aB)\sec(c+dx}{a+b\sec(c+}}{b(a^2-b}$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)d} + \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx}{b(a^2-b^2)d(a+b\sec}$$

$$= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} - \frac{(3aAb-5a^2B}{}$$

$$= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} - \frac{(3aAb-5a^2B}{}$$

$$= \frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} - \frac{(3aAb-5a^2B}{}$$

$$= -\frac{a(3a^2Ab-5Ab^3-5a^3B+7ab^2B)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{sec}}}{(a-b)b^3(a+b)^2d}$$

$$= -\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{b^3(a^2-b^2)d}$$

Mathematica [A] time = 7.14499, size = 738, normalized size = 1.82

$$\frac{2(-27a^3Ab-44a^2b^2B+45a^4B+30aAb^3-4b^4B)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\mathrm{sec}}\right.\right.\right.}{b(1-\cos^2(c+dx))(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((-2*(-24*a^2*A*b^2 + 12*A*b^4 + 40*a^3*b*B - 28*a*b^3*B)*Cos[c + d*x]^2*El
lipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt
[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)) + (2*(-27*a^3*A*b + 30*a*A*b^3 + 45*a^4*B - 44*a^2*b^2*B - 4*b^4*B)*C
os[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a
), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c +
d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(-
9*a^3*A*b + 6*a*A*b^3 + 15*a^4*B - 12*a^2*b^2*B)*Cos[2*(c + d*x)]*(a + b*Se
c[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
 d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*
(2 - Sec[c + d*x]^2)))/(12*(a - b)*b^3*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((
-3*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a*b^2*B)*Sin[c + d*x])/(b^3*(-a^2 + b^2)
) + (a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(b + a*Co
s[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^2)))/d
```

Maple [B] time = 9.448, size = 1024, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a)*a/
b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*(A*b-2*B*a)/b^3/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))+2/b^2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b
-2*B*a)/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x
)
```

3.423 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=315

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\left(-3a^2B+aAb+2\right.}{}$$

```
[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((a^2*A*b -
 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((a*A*b -
 3*a^2*B + 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) +
(a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x]))
```

Rubi [A] time = 0.836707, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{bd\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((a^2*A*b -
 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((a*A*b -
 3*a^2*B + 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) +
(a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-aB)-b(Ab-aB)\sec(c+dx)-\frac{1}{2}\right.}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin}{b\left(a^2-b^2\right)d(a+b\sec(c}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin}{b\left(a^2-b^2\right)d(a+b\sec(c}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin}{b\left(a^2-b^2\right)d(a+b\sec(c}$$

$$= \frac{\left(a^2Ab-3Ab^3-3a^3B+5ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{(a-b)b^2(a+b)^2d}$$

$$= \frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} + \frac{(Ab-aB)}{}$$

Mathematica [B] time = 6.98402, size = 685, normalized size = 2.17

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{b^2\left(b^2-a^2\right)}+\frac{a^2B\sin(c+dx)-aAb\sin(c+dx)}{b\left(b^2-a^2\right)(a\cos(c+dx)+b)}\right)}{d} - \frac{2\left(-3a^2Ab+9a^3B-10ab^2B+4Ab^3\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+d}}{b(1-co}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] -((-2*(-4*a*A*b^2 + 8*a^2*b*B - 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]
^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-3*a^
2*A*b + 4*A*b^3 + 9*a^3*B - 10*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sq
rt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1
])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(-(a^2*A*b) + 3*a^3*B - 2*a*b^2*B)*C
os[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*b^2*(a + b)*d) +
 (Sqrt[Sec[c + d*x]]*(((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*(-a^2
 + b^2)) + (-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(b
+ a*Cos[c + d*x]))))/d
```

Maple [B] time = 6.326, size = 877, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b*(
a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3
/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*B*a^2/b^2/(a^2-a*b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1
/2))+2*B/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x
)
```

3.424 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=257

$$-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{(Ab-aB)\sqrt{c}}{}$$

```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B
- 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.527313, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4029, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{(Ab-aB)\sqrt{\cos(c+dx}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B
- 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)-b(Ab-aB)\sec(c+dx)+\frac{1}{2}\left(aAb+a^2B-\right.}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a^2(Ab-aB)-\frac{1}{2}ab(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2b\left(a^2-b^2\right)} + \frac{(a}{}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{(Ab-aB)\int\sqrt{\sec(c+dx)}\,dx}{2a\left(a^2-b^2\right)} - \frac{(Ab-a}{2}$$

$$= \frac{\left(a^2Ab+Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)b(a+b)^2d}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d} - \frac{(Ab-aB)\sqrt{\cos(c+d}{a}$$

Mathematica [B] time = 6.88949, size = 643, normalized size = 2.5

$$\frac{\frac{2\left(-3a^2B-aAb+4b^2B\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)}-\frac{2\left(aAb\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((-2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(a*A*b) - 3*a^2*B +
4*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellipt
icPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) - (2*(a*A*b - a^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*
b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[
c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4
*b*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-(((A*b - a*B)*Sin[c + d*x])/
(b*(-a^2 + b^2))) + (A*b*Sin[c + d*x] - a*B*Sin[c + d*x])/((-a^2 + b^2)*(b
+ a*Cos[c + d*x]))))/d
```

Maple [B] time = 5.18, size = 715, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*
(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+
3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*A/(a^2-a*b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec{(c + dx)})\sec^{\frac{3}{2}}{(c + dx)}}{(a + b\sec{(c + dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2,
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec{(dx + c)} + A)\sec{(dx + c)}^{\frac{3}{2}}}{(b\sec{(dx + c)} + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x
)
```

3.425 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=263

$$\frac{\left(2a^2A-abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(Ab\cdots}{\cdots}$$

```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b^2 - a*b*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b -
 A*b^3 - a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.506196, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4027, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2A-abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b^2 - a*b*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b -
 A*b^3 - a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}(-Ab+aB)-(aA-bB)\sec(c+dx)+\frac{1}{2}(Ab-aB)\sec}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{-a^2+b^2}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}a(-Ab+aB)-\left(\frac{1}{2}b(-Ab+aB)-a(-aA+bB)\right)\sec(c}{\sqrt{\sec(c+dx)}}}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\left(a^2-b^2\right)} + \frac{\left(2a^2A-A\right.}{}$$

$$= -\frac{\left(3a^2Ab-Ab^3-a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d}$$

$$= \frac{(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} + \frac{\left(2a^2A-Ab^2-abB\right)\surd}{}$$

Mathematica [B] time = 6.90542, size = 727, normalized size = 2.76

$$\sec(c+dx)(a\cos(c+dx)+b)^2(A+B\sec(c+dx))\left(-\frac{2(Ab-aB)\sin(c+dx)\cos(2(c+dx))(a+b\sec(c+dx))\left(a(a-2b)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*(4*a*A - 4*b
*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) + (2*(-(A*b) + a*B)*Cos[c + d*x]^2*(EllipticF[Ar
cSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x
]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b
 + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(A*b - a*B)*Cos[2*(c + d*x)]*
(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(
a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
)*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]]*(2 - Sec[c + d*x]^2))))/(4*(a - b)*(a + b)*d*(B + A*Cos[c + d*x])*
(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*(A + B
*Sec[c + d*x])*(((-(A*b) + a*B)*Sin[c + d*x])/(a*(a^2 - b^2)) + (A*b^2*Sin[
c + d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(B
 + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 5.786, size = 802, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b-B*a)/a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-2*A*b+B*a)/a
```

```
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sqrt{\sec(c + dx)}}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\sqrt{\sec(dx + c)}}{(b\sec(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

3.426 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=283

$$-\frac{\left(4a^2Ab-2a^3B+ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c}}{ad\left(a^2-b^2\right)(a+b\sec(c+}$$

```
[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B +
a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^
3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.569037, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4030, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\left(4a^2Ab-2a^3B+ab^2B-3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|2\right)}{a^3d\left(a^2-b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B +
a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^
3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-2a^2A+3Ab^2-abB\right)+a(Ab-aB)\sec(c+dx)-\frac{1}{2}b(\cdots}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}a\left(-2a^2A+3Ab^2-abB\right)-\left(-a^2(Ab-aB)+\frac{1}{2}b\left(-2a^2\cdots\right.\right.}{\sqrt{\sec(c+dx)}}}{a^3\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2a^2A-3Ab^2+abB\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a^2\left(a^2-b^2\right)} - \cdots$$

$$= \frac{b\left(5a^2Ab-3Ab^3-3a^3B+ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+\cdots}}{a^3(a-b)(a+b)^2d}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} - \frac{\left(4a^2Ab-\cdots\right.}{}$$

Mathematica [B] time = 6.97451, size = 657, normalized size = 2.32

$$\frac{2\left(-2a^2A+abB+Ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)}-\frac{2\left(-2a^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((-2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-2*a^2*A + A*b^2 + a*
b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi
[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Se
c[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) -
 (2*(-2*a^2*A + 3*A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a
*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2
*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b
 + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x
]^2)))/(4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((b*(A*b - a*B)*Sin
[c + d*x])/(a^2*(-a^2 + b^2))) + (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d
*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/d
```

Maple [B] time = 6.439, size = 843, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2*b^2*(A*b-B*a)/a^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^
2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
-2/a^2*b*(3*A*b-2*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
```

```
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

3.427 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=365

$$\frac{\left(16a^2Ab^2+2a^4A-12a^3bB+9ab^3B-15Ab^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d\left(a^2-b^2\right)}+\frac{b(Ab-}{ad\left(a^2-b^2\right)\sqrt{\sec}}$$

```
[Out] -(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4*A + 16*
a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*
b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*
x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a
+ b*Sec[c + d*x]))
```

Rubi [A] time = 0.859338, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4030, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{\left(16a^2Ab^2+2a^4A-12a^3bB+9a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] -(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4*A + 16*
a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*
b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*
x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a
+ b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-2a^2A+5Ab^2-3abB\right)+a(Ab-aB)}{\sec^{\frac{3}{2}}(c+dx)(a}}{a\left(a^2\right.}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c}$$

$$= -\frac{b^2\left(7a^2Ab-5Ab^3-5a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{a^4(a-b)(a+b)^2d}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{a^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.14556, size = 704, normalized size = 1.93

$$\frac{2\left(-8a^2Ab+6a^3B-3ab^2B+5Ab^3\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((-2*(4*a^3*A + 8*a*A*b^2 - 12*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^
2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-8*a^2
*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt
[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])
*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[
c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(-12*a^2*A*b + 15*A*b^3 + 6*a^3*B - 9*
a*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^
2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] -
2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(12*a^2*(a - b)*(
a + b)*d) + (Sqrt[Sec[c + d*x]]*((b^2*(A*b - a*B)*Sin[c + d*x])/(a^3*(-a^2
+ b^2)) - (-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x])/(a^3*(a^2 - b^2)*(
b + a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(3*a^2)))/d
```

Maple [B] time = 7.161, size = 1059, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*
A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*b^3*(A*b-B*a)/a^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2
/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a
^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^
3*b^2*(4*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

3.428 $\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=583

$$-\frac{\left(15a^3Ab+61a^2b^2B-35a^4B-33aAb^3-8b^4B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12b^3d\left(a^2-b^2\right)^2}+\frac{a(Ab-aB)\sin(c}{2bd\left(a^2-b^2\right)(a}$$

```
[Out] -((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4
*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8
*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
2*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*
B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b),
 (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + ((15*a
^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*a
*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(12*b^3*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*
a^3*B + 13*a*b^2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.77941, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {4029, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(3a^2Ab-7a^3B+13ab^2B-9Ab^3\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{\left(15a^3Ab+61a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] -((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4
*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8
*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
2*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*
B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b),
 (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + ((15*a
^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*a
*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(12*b^3*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*
a^3*B + 13*a*b^2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
```

```
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(Ab-aB)-2b(Ab-aB)\sec(c+dx)-\frac{1}{2}\right.}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(3a^2Ab-9Ab^3-7a^3B+13ab^2B\right)\sec}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d} + \frac{a(}{2}$$

$$= \frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\sec(c+dx)}\sin(c}{4b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\sec(c+dx)}\sin(c}{4b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\sec(c+dx)}\sin(c}{4b^4\left(a^2-b^2\right)^2d}$$

$$= -\frac{a\left(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B\right)\sqrt{\cos(c+dx)}\Pi}{4(a-b)^2b^4(a+b)^3d}$$

$$= -\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{4b^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.45455, size = 902, normalized size = 1.55

$$-\frac{2\left(-48Ab^6+160aBb^5+240a^2Ab^4-512a^3Bb^3-120a^4Ab^2+280a^5Bb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + 2($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((-2*(-120*a^4*A*b^2 + 240*a^2*A*b^4 - 48*A*b^6 + 280*a^5*b*B - 512*a^3*b^3
*B + 160*a*b^5*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*
x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b
 + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-135*a^5*A*b + 285*a^3*A*b^3
 - 168*a*A*b^5 + 315*a^6*B - 641*a^4*b^2*B + 328*a^2*b^4*B + 16*b^6*B)*Cos[
c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a),
-ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(-45*
a^5*A*b + 87*a^3*A*b^3 - 24*a*A*b^5 + 105*a^6*B - 195*a^4*b^2*B + 72*a^2*b^
4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 +
2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^
2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*(a - b)^2*b^4*(a
+ b)^2*d) + (Sqrt[Sec[c + d*x]]*(((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35
*a^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2) +
(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(2*b^2*(-a^2 + b^2)*(b + a*Cos[
c + d*x])^2) + (-5*a^4*A*b*Sin[c + d*x] + 11*a^2*A*b^3*Sin[c + d*x] + 9*a^5
*B*Sin[c + d*x] - 15*a^3*b^2*B*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^2*(b + a*C
os[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^3)))/d
```

Maple [B] time = 15.662, size = 2178, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(A*b-2*B*a
)/b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*B/b^3*(-1/6*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))-2*a*(A*b-B*a)/b^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*
```

```
a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b
)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b
^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2
-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1
/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*(A*b-
3*B*a)/b^4/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*(A*b-3*B*a)/b^4*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x
)
```

3.429 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=480

$$\frac{\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

```
[Out] ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d)
+ ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b -
 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a
 - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*
B - 8*b^4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*d) + (a*
(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.37823, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {4029, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{\left(3a^3Ab+29a^2b^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d)
+ ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b -
 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a
 - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*
B - 8*b^4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*d) + (a*
(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
```

```
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(Ab-aB)-2b(Ab-aB)\sec(c+dx)-\frac{1}{2}\right.}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2B\right)\sec^{\frac{3}{2}}(}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} + \frac{a(A}{2b}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} + \frac{a(A}{2b}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} + \frac{a(A}{2b}$$

$$= \frac{\left(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\right.}{4(a-b)^2b^3(a+b)^3d}$$

$$= \frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{4b^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.27139, size = 847, normalized size = 1.76

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(15Ba^4-3Aba^3-29b^2Ba^2+9Ab^3a+8b^4B\right)\sin(c+dx)}{4b^3\left(b^2-a^2\right)^2} + \frac{a^2B\sin(c+dx)-aAb\sin(c+dx)}{2b\left(b^2-a^2\right)(b+a\cos(c+dx))^2} + \frac{-5B\sin(c+dx)a^4+Ab\sin(c+dx)a^3+11b^2B\sin(c+dx}{4b^2\left(b^2-a^2\right)^2(b+a\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] -((-2*(-8*a^3*A*b^2 + 32*a*A*b^4 + 40*a^4*b*B - 80*a^2*b^3*B + 16*b^5*B)*Co
s[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + (2*(-9*a^4*A*b + 19*a^2*A*b^3 - 16*A*b^5 + 45*a^5*B
- 95*a^3*b^2*B + 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c +
```

```
d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*S
ec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) - (2*(-3*a^4*A*b + 9*a^2*A*b^3 + 15*a^5*B - 29*a^3*b
^2*B + 8*a*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[
c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b
/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*(a -
 b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((-3*a^3*A*b + 9*a*A*b^3 + 15
*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^2) + (-(
a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(2*b*(-a^2 + b^2)*(b + a*Cos[c +
d*x])^2) + (a^3*A*b*Sin[c + d*x] - 7*a*A*b^3*Sin[c + d*x] - 5*a^4*B*Sin[c +
 d*x] + 11*a^2*b^2*B*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x
]))))/d
```

Maple [B] time = 10.555, size = 2024, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*a/b^2*(a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(1/2*a^2/b/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
```

```
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b
^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
+2*B*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*B/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin
(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x
)
```

3.430 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

```
[Out] ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*
A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((a^4*A*b - 10*a^2*A*b^3 - 3*A*b
^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3
*d) + (a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
 + b*Sec[c + d*x])^2) - (a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.914029, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4029, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(3a^2Ab+a^3B-7ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*
A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((a^4*A*b - 10*a^2*A*b^3 - 3*A*b
^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3
*d) + (a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
 + b*Sec[c + d*x])^2) - (a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-aB)-2b(Ab-aB)\sec(c+dx)+\ldots\right.}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)\ldots}$$

$$= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\sec(\ldots}}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c\ldots}$$

$$= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\sec(\ldots}}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c\ldots}$$

$$= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\sec(\ldots}}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c\ldots}$$

$$= \frac{\left(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\ldots\right.}{4a(a-b)^2b^2(a+b)^3d}$$

$$= \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2d} + \ldots$$

Mathematica [A] time = 7.01683, size = 800, normalized size = 1.99

$$-\frac{\frac{2\left(16Ab^4-32aBb^3+8a^2Ab^2+8a^3Bb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(9Ba^4+3Aba^3-19b^2Ba^2-9Ab^3\ldots\right.}{\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((-2*(8*a^2*A*b^2 + 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) + (2*(3*a^3*A*b - 9*a*A*b^3 + 9*a^4*B - 19*a^2*b^2*B + 16*b^4*B)*Cos[c +
 d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -Ar
cSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2
]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(a^3*A*b
 + 5*a*A*b^3 + 3*a^4*B - 9*a^2*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])
*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
+ a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2
*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c
+ d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-((a^2*A*
b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2) + (
A*b*Sin[c + d*x] - a*B*Sin[c + d*x])/(2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2
) + (3*a^2*A*b*Sin[c + d*x] + 3*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7
*a*b^2*B*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/d
```

Maple [B] time = 8.45, size = 1768, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(a^2/b/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*(-A*b+B*a)/a*(1/2*a^2/b/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/
b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x
)
```

3.431 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right.}{4bd\left(a^2-b^2\right)^2(a+}$$

```
[Out] -((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b
^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^
5 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b)^3*d)
 + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a +
b*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.914072, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4029, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\left(7a^2Ab-3a^3B-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b
^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^
5 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b)^3*d)
 + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a +
b*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)-2b(Ab-aB)\sec(c+dx)+\frac{1}{2}\left(3aAb\right.}{\sqrt{\sec(c+dx)}(a+b\sec(c+d}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)\sqrt{\text{se}}}{4b\left(a^2-b^2\right)^2 d(a+b\sec}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)\sqrt{\text{se}}}{4b\left(a^2-b^2\right)^2 d(a+b\sec}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2B\right)\sqrt{\text{se}}}{4b\left(a^2-b^2\right)^2 d(a+b\sec}$$

$$= \frac{\left(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}\right.}{4a^2(a-b)^2b(a+b)^3d}$$

$$= -\frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 7.01503, size = 887, normalized size = 2.21

$$\frac{\sec^2(c+dx)(A+B\sec(c+dx))\left(-\frac{2\left(16Bb^3-24aAb^2+8a^2Bb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((-2*(-24*a*A*b
^2 + 8*a^2*b*B + 16*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[S
ec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*
x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(a^2*A*b + 5*A*b^3 +
 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c +
d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - C
os[c + d*x]^2)) - (2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[2*(c + d*
x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
 a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a
), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x
]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[S
ec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(16*(a - b)^2*b*(a + b)^2*d*(B + A*Cos
[c + d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(
5/2)*(A + B*Sec[c + d*x])*(((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sin[c +
 d*x])/(4*a*b*(-a^2 + b^2)^2) - (-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]
)/(2*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (-7*a^2*A*b*Sin[c + d*x] + A*b
^3*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x] + 3*a*b^2*B*Sin[c + d*x])/(4*a*(a^2
- b^2)^2*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x
```

```
])^3)
```

Maple [B] time = 8.338, size = 1872, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-2*A*b+B*a)/
a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*b*(A*b-B*a)/a^2*(1/2*a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)
^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a
^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/
(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/
(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2)))-2*A/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x
)
```

3.432 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$\frac{\left(-5a^2Ab^2+8a^4A-7a^3bB+ab^3B+3Ab^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{\left(7a^2Ab-3a^3B-3ab^2B\right.}{4ad\left(a^2-b^2\right.}$$

```
[Out] ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^
2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a
^2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)
^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.865092, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4027, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\left(-5a^2Ab^2+8a^4A-7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^
2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a
^2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)
^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
```

```
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}dx = -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(-Ab+aB)-2(aA-bB)\sec(c+dx)+\frac{3}{2}(Ab-aB)\sec}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}}{2(a^2-b^2)}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2B)\sqrt{\sec(c}}{4a(a^2-b^2)^2d(a+b\sec(c}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2B)\sqrt{\sec(c}}{4a(a^2-b^2)^2d(a+b\sec(c}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B-3ab^2B)\sqrt{\sec(c}}{4a(a^2-b^2)^2d(a+b\sec(c}$$

$$= -\frac{(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}\right.}{4a^3(a-b)^2(a+b)^3d}$$

$$= \frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2d} + \cdots$$

Mathematica [B] time = 6.98584, size = 890, normalized size = 2.21

$$\frac{\sec^2(c+dx)(A+B\sec(c+dx))\left(-\frac{2(16Aa^3-24bBa^2+8Ab^2a)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{a(b+a\cos(c+dx))(1-\cos^2(c+dx))}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((-2*(16*a^3*A
+ 8*a*A*b^2 - 24*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x
])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-5*a^2*A*b - A*b^3 +
 a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*
x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos
[c + d*x]^2)) - (2*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Cos[2*(c + d*x
)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a)
, -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Se
c[c + d*x]]*(2 - Sec[c + d*x]^2))))/(16*a*(a - b)^2*(a + b)^2*d*(B + A*Cos[
c + d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5
/2)*(A + B*Sec[c + d*x])*(((-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c
 + d*x])/(4*a^2*(-a^2 + b^2)^2) - (A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x
])/(2*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (11*a^2*A*b^2*Sin[c + d*x]
- 5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c + d*x] + a*b^3*B*Sin[c + d*x])/(4*
a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a + b*Se
```

```
c[c + d*x])^3)
```

Maple [B] time = 8.849, size = 1959, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a
^3*b*(3*A*b-2*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b^2*(A*b-B
*a)/a^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b
^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)
/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^
2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-3*A*b+B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x
)
```

3.433 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=427

$$-\frac{\left(-33a^2Ab^3+24a^4Ab+5a^3b^2B-8a^5B-3ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b(1}{}$$

```
[Out] ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d)
- ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*
(a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*
a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2
) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.998036, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4030, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\left(-33a^2Ab^3+24\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d)
- ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*
(a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*
a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2
) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(-4a^2A+5Ab^2-abB\right)+2a(Ab-aB)\sec(c+dx}{\sqrt{\sec(c+dx)}(a+b\sec(c+}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)\sqrt{}}{4a^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)\sqrt{}}{4a^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)\sqrt{}}{4a^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}}$$

$$= \frac{b\left(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B\right)\sqrt{\cos(c+dx)}\Pi}{4a^4(a-b)^2(a+b)^3d}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.38786, size = 823, normalized size = 1.93

$$-\frac{2\left(16Ba^4-32Aba^3+8b^2Ba^2+8Ab^3a\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(8Aa^4-5bBa^3-7Ab^2a^2-b^3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((-2*(-32*a^3*A*b + 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)) + (2*(8*a^4*A - 7*a^2*A*b^2 + 5*A*b^4 - 5*a^3*b*B - a*b^3*B)*Cos[c + d*
x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*S
in[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(8*a^4*A -
29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Cos[2*(c + d*x)]*(a + b*Se
c[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
 d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*
(2 - Sec[c + d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]
*(-(b*(-13*a^2*A*b + 7*A*b^3 + 9*a^3*B - 3*a*b^2*B)*Sin[c + d*x])/(4*a^3*(-
a^2 + b^2)^2) - (-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x])/(2*a^3*(a^2
- b^2)*(b + a*Cos[c + d*x])^2) + (-15*a^2*A*b^3*Sin[c + d*x] + 9*A*b^5*Sin[
c + d*x] + 11*a^3*b^2*B*Sin[c + d*x] - 5*a*b^4*B*Sin[c + d*x])/(4*a^3*(a^2
- b^2)^2*(b + a*Cos[c + d*x])))))/d
```

Maple [B] time = 9.803, size = 2000, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2/a^4*b^2*(4*A*b-3*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^
2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2)))+2*b^3*(A*b-B*a)/a^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2
+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(
a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-6/a^3*b*(2*A*b-B*a)/(a^2-a*b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

3.434 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=521

$$\frac{\left(128a^4Ab^2-223a^2Ab^4+8a^6A+99a^3b^3B-72a^5bB-45ab^5B+105Ab^6\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{12a^5d\left(a^2-b^2\right)^2}$$

```
[Out] -((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4
*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 -
 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b
- 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*
a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b
*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
+ (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b
*Sec[c + d*x])^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c +
 d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.44398, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {4030, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b\left(13a^2Ab-9a^3B+3ab^2B-7Ab^3\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}+\frac{\left(-61a^2Ab^2+8a^4A+\right.}{12a^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] -((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4
*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 -
 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b
- 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*
a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b
*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
+ (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b
*Sec[c + d*x])^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c +
 d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
```

```
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(-4a^2A+7Ab^2-3abB\right)+2a(Ab-aB)}{\sec^{\frac{3}{2}}(c+dx)(a+}}{2a\left(a^2\right.}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} + \frac{b\left(13a^2Ab-7Ab^3-9a^3B+3\right.}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}(}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b(Ab}{2a\left(a^2-b^2\right)d\sqrt{s}}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b(Ab}{2a\left(a^2-b^2\right)d\sqrt{s}}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b(Ab}{2a\left(a^2-b^2\right)d\sqrt{s}}$$

$$= -\frac{b^2\left(63a^4Ab-86a^2Ab^3+35Ab^5-35a^5B+38a^3b^2B-15ab^4B\right)\sqrt{\cos(c+dx)}\Pi}{4a^5(a-b)^2(a+b)^3d}$$

$$= -\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.35287, size = 868, normalized size = 1.67

$$-\frac{2\left(16Aa^5-96bBa^4+112Ab^2a^3+24b^3Ba^2-56Ab^4a\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+dx)}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)} + \frac{2\left(24Ba^5-56Aba^4-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((-2*(16*a^5*A + 112*a^3*A*b^2 - 56*a*A*b^4 - 96*a^4*b*B + 24*a^2*b^3*B)*Co
s[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + (2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35*A*b^5 + 24*a^5*B
```

```
 - 21*a^3*b^2*B + 15*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c +
 d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*
Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x]
)*(1 - Cos[c + d*x]^2)) - (2*(-72*a^4*A*b + 195*a^2*A*b^3 - 105*A*b^5 + 24*
a^5*B - 87*a^3*b^2*B + 45*a*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2
*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a
^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*
(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d
*x]^2)))/(48*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((b^2*(-17*a^
2*A*b + 11*A*b^3 + 13*a^3*B - 7*a*b^2*B)*Sin[c + d*x])/(4*a^4*(-a^2 + b^2)^
2) - (A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*
Cos[c + d*x])^2) + (19*a^2*A*b^4*Sin[c + d*x] - 13*A*b^6*Sin[c + d*x] - 15*
a^3*b^3*B*Sin[c + d*x] + 9*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(b +
a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(3*a^3)))/d
```

Maple [B] time = 11.085, size = 2216, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^5*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18
*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+2/a^5*b^3*(5*A*b-4*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^
2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
-2*b^4*(A*b-B*a)/a^5*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4
*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2
-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
```

```
2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a
^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/
8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)
/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4/a^4*b^2*(5*A*b-3*B*a)/(a^2-a*b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

3.435 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=336

$$\frac{(3aB+4Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{4bd\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((4*a*A
*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]
) - ((4*A*b + a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
 d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((4
*A*b + a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*
d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.10781, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4031, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((4*a*A
*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]
) - ((4*A*b + a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
 d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((4
*A*b + a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*
d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{\sec(c}}{}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4bd} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4bd} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4bd} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4bd} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{\left(4aAb-a^2B+4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{sec}}}{4bd\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(4Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \cdot$$

Mathematica [C] time = 5.3401, size = 422, normalized size = 1.26

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2i(aB+4Ab)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}\right),\frac{b-a}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh\right.\right.\right.}{ab^2\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((8*a*B*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((
a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(4*a*A*b - 3*a^2*B + 8*b^2*
B)*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*(a + b)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]) - ((2*I)*(4*A*b + a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a
+ b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*Elli
pticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x
]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b^2*
Sqrt[b + a*Cos[c + d*x]]) + (4*(4*A*b + a*B)*Tan[c + d*x])/b + 8*B*Sec[c +
d*x]*Tan[c + d*x]))/(16*d*Sqrt[Sec[c + d*x]])
```

Maple [C] time = 0.492, size = 2521, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/((a-b)/(a+b))^(1/2)/b*(4*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+
c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+
b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-2*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-B*sin
(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+c)
*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-B*sin(d
*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b+2*B*((a-b)/(a+b))^(1/2)*b^2-B*cos(d*x+c)^3*((a-b)/
(a+b))^(1/2)*a^2-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+B*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^2+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-2*B*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*b^2-4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-2*B*cos
```

```
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-
B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
*b+4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*B*si
n(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*b^2-4*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-2*B*sin(d*x+c)*cos(d*x+c)^
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2+4*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2
*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+c)*cos(d*x+c)^3*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^2-4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-2*B*sin(d*x+c)*cos(d*
x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a
*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

3.436 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=253

$$\frac{(2aA+bB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.782507, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {4031, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2aA+bB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d
```


Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
```

```
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{-\frac{aB}{2}+aA \cdots}{\cdots}$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(2Ab+aB \cdots$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}B\int \frac{\sqrt{a+\cdots}}{\sqrt{s\cdots}}$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{((2aA+bB\cdots}{\cdots}$$

$$= \frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} \cdots$$

$$= \frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \cdots$$

Mathematica [C] time = 6.09633, size = 377, normalized size = 1.49

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{8aA\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}} - \frac{2iB\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}\right),\frac{b-a}{a+b}\cdots\right.\right.\right.}{ab\sqrt{\frac{1}{a-b}}\sqrt{a}\cdots}\cdots\right)}{4d\sqrt{\sec\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((8*a*A*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((
a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(4*A*b + a*B)*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
- ((2*I)*B*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*
x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(
-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ell
```

```
ipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-
a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*B*
Tan[c + d*x]))/(4*d*Sqrt[Sec[c + d*x]])
```

Maple [C] time = 0.503, size = 1431, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)^2*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b+4*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)^2
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a+B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*B*cos(d*x+c)^2*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*a+2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b+4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+B
*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b+2*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+B*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+B*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*b-B*((a-b)/(a+b))^(1/2)*b)*(1/cos(d*x+c))^(1/2)*(
(b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

3.437 $\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2aB\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2bB\sqrt{\sec(c+a}}{}$$

```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```

Rubi [A] time = 0.54336, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {4037, 3854, 3858, 2663, 2661, 3859, 2807, 2805, 3856, 2655, 2653}

$$\frac{2A\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aB\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```


Rule 4037

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx = A\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx + B\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

$$= (aB)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx + (bB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx + \frac{(A\sqrt{}}{}$$

$$= \frac{\left(aB\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(bB\sqrt{b+a}\right.}{}$$

$$= \frac{2AE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{\left(aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+}\right.}{\sqrt{a+b\sec}}$$

$$= \frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2bB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{d}$$

Mathematica [A] time = 2.54861, size = 122, normalized size = 0.59

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(B\left(a\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)+A(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{d(a+b)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (2*(A*(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + B*(a*EllipticF[(c + d
*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]))*Sqrt[
a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec
[c + d*x]])
```

Maple [C] time = 0.491, size = 1549, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*(A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b-A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-B*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
```

```
c)+1))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-2*B*sin(d*x+c)*cos(d*x+c)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*b+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*sin(d*x+c)-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-A*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+A*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*B*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-A*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b+A*b*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

3.438 $\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{2A\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

```
[Out] (2*A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A
*b + 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.479203, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4032, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A
*b + 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int\frac{\frac{1}{2}(Ab+3aB)+\frac{1}{2}(aA+3bB)$$

$$=\frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{\left(A\left(a^2-b^2\right)\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{3a}$$

$$=\frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{\left(A\left(a^2-b^2\right)\sqrt{b+a\cos(c+dx)}\right.}{3a\sqrt{a+b\sec}}$$

$$=\frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{\left(A\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(\right.}{3a\sqrt{a+b\sec}}$$

$$=\frac{2A\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab+3}{}$$

Mathematica [A] time = 0.753537, size = 165, normalized size = 0.82

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(A\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+(a+b)(3aB+Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}E\left(\frac{1}{2}(c+dx)\right|}{3ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*((a + b)*(A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + A*(a^2 - b^2)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*A*(b + a
*Cos[c + d*x])*Sin[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]
])
```

Maple [B] time = 0.36, size = 1926, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)/a*(-A*((a-b)/(a+b))^(1/2)*a*b-3*B*((a-b)/(a+b))^
(1/2)*a*b-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*b^2*sin(d*x+c)-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2-A*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+A*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-A*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b*sin(d*x+c)+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+A*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b-A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-A*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b-A*((a-b)/(a+b))^(1/2)*b^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^
```

```
(1/2)*a^2+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-3*B*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-A*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^2+A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*B*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+2
*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/
sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

3.439 $\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=267

$$-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+5abB-2Ab^2\right)\sqrt{a+b\sec}}{15a^2d\sqrt{\sec(c+dx)}\sqrt{}}$$

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[
c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d
*Sec[c + d*x]^(3/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.747935, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+5abB-2Ab^2\right)\sqrt{a+b\sec(c+dx)}E}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[
c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d
*Sec[c + d*x]^(3/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
```

```
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5aB)+\frac{1}{2}(3aA+5b\ldots}{\sec^{\frac{3}{2}}(c+dx)\ldots}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}\ldots}{15ad\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}\ldots}{15ad\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}\ldots}{15ad\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}\ldots}{15ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}\ldots}{15a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.18059, size = 200, normalized size = 0.75

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(\left(a^2-b^2\right)(5aB-2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+(a+b)\left(9a^2A+5abB-2Ab^2\right)\ldots}{15a^2d\sqrt{\sec(c+dx)}(a\cos(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2
),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*((a + b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b
^2)*(-2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(A*b + 5*a*B + 3*a*A*Cos[c + d*
x])*Sin[c + d*x]))/(15*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.494, size = 2739, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d/((a-b)/(a+b))^(1/2)/a^2*(9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
```

```
1))^(1/2)*sin(d*x+c)+7*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+2*A*cos(d*x+c)*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a*b^2-9*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-2*A*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^2-5*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+5*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-
5*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*A*a^2*b*((a-b)/(a+b))^(1/2)-A*a*b^2*((a
-b)/(a+b))^(1/2)-5*B*a^2*b*((a-b)/(a+b))^(1/2)-5*B*a*b^2*((a-b)/(a+b))^(1/2
)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+10*B*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+5*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+5
*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*a^3+7*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)-2*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*B*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+5*B*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)-5*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*b^3*((a-b)/(a+b))^(1/2)+5*B*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*cos(d*x+c)-
9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b
^3+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*A*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^3-9*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+9*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+2
*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*b^3+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-5*B
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b^2+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b)*((b+a*cos(d*x+c))/cos(d*x+c
))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
 x)
```

3.440 $\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=343

$$\frac{2\left(a^2-b^2\right)\left(25a^2A-14abB+8Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2A+7abB-4Ab^2\right)\text{s}}{105a^2d\sqrt{\text{s}}}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^
3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(A*b + 7*a*B)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*x]^(3/2)) + (2*(25*
a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^2*
d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.03491, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2\left(a^2-b^2\right)\left(25a^2A-14abB+8Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}{105a^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^
3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(A*b + 7*a*B)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*x]^(3/2)) + (2*(25*
a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^2*
d*Sqrt[Sec[c + d*x]])
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7aB)+\frac{1}{2}(5aA+7bB)\mathrm{s}}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A+8Ab^2-14abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec}}{105a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.3112, size = 208, normalized size = 0.61

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{4\left((a-b)\left(25a^2A-14abB+8Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(19a^2Ab+63a^3B-14ab^2B+8Ab^3\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(\left(115a^2A+28a\right.\right.\right.}{210a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2
),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((4*((19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*
B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*(25*a^2*A + 8*A*b^2 - 14
*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/Sqrt[(b + a*Cos[c + d*x])/(
a + b)] + a*((115*a^2*A - 16*A*b^2 + 28*a*b*B)*Sin[c + d*x] + 3*a*(2*(A*b +
 7*a*B)*Sin[2*(c + d*x)] + 5*a*A*Sin[3*(c + d*x)]))))/(210*a^3*d*Sqrt[Sec[c
 + d*x]])
```

Maple [B] time = 0.582, size = 3778, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d/((a-b)/(a+b))^(1/2)/a^3*(25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(c
```

os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-19*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-25*A*a^3*b*((a-b)/(a+b))^(1/2)-19*A*a^
2*b^2*((a-b)/(a+b))^(1/2)+4*A*a*b^3*((a-b)/(a+b))^(1/2)-63*B*a^3*b*((a-b)/(
a+b))^(1/2)-7*B*a^2*b^2*((a-b)/(a+b))^(1/2)+14*B*a*b^3*((a-b)/(a+b))^(1/2)+
14*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-14*
B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+14*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+25*A*s
in(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*a^4-8*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-63*B*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-19*A*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2
*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+19*A*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21*B
*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a^4+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-63*B*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*A*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^4-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-19*A*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+8*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+49*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+14*B*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-14*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+14*B*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+18*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-A*cos(d*x+c)^3*((a-b)/(a+
b))^(1/2)*a^2*b^2+28*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+26*A*cos(d*x+

```
c)^2*((a-b)/(a+b))^(1/2)*a^3*b+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-7
*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-19*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^3*b+20*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-8*A*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a*b^3+35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+14*B*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-14*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-8
*A*b^4*((a-b)/(a+b))^(1/2)+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-8*A*sin(d*x+
c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*a*b^3+19*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-19*A*sin(d*x+c)*co
s(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+49*B*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a^3*b-63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*
x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
 x)
```

3.441 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=421

$$\frac{\left(17a^2B+42aAb+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)}{24bd}$$

```
[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c
+ d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(8*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((30*a*A*b + 3*a^2*B + 16*
b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d) +
 ((6*A*b + 7*a*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(12*d) + (b*B*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*d)
```

Rubi [A] time = 1.59673, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24bd}+\frac{\left(17a^2B+42aAb+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{24d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c
+ d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(8*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((30*a*A*b + 3*a^2*B + 16*
b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d) +
 ((6*A*b + 7*a*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(12*d) + (b*B*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
```

```
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int$$

$$= \frac{(6Ab+7aB)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{12d}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{24bd}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{24bd}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{24bd}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{24bd}$$

$$= \frac{\left(6a^2Ab+8Ab^3-a^3B+12ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(\right.}{8bd\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(42aAb+17a^2B+16b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{24d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.7827, size = 673, normalized size = 1.6

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{\sec(c+dx)\left(3a^2B\sin(c+dx)+30aAb\sin(c+dx)+16b^2B\sin(c+dx)\right)}{24b}+\frac{1}{12}\sec^2(c+dx)(7aB\sin(c+dx)+6Ab\sin(c\right.}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] -((a + b*Sec[c + d*x])^(3/2)*((2*(-24*a*A*b^2 - 28*a^2*b*B)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c
 + d*x]] + (2*(-6*a^2*A*b - 48*A*b^3 + 9*a^3*B - 56*a*b^2*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*
Cos[c + d*x]] + ((2*I)*(30*a^2*A*b + 3*a^3*B + 16*a*b^2*B)*Sqrt[(a - a*Cos[
c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*
b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
 (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
 a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt
[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])
/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x
]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])
^2))))/(96*b*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((a + b*Sec
[c + d*x])^(3/2)*((Sec[c + d*x]^2*(6*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x])
)/12 + (Sec[c + d*x]*(30*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x] + 16*b^2
*B*Sin[c + d*x]))/(24*b) + (b*B*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*(b + a*
```

```
Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [C] time = 0.575, size = 4051, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/24/d/((a-b)/(a+b))^(1/2)/b*(30*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+8*B*((a-
b)/(a+b))^(1/2)*b^3-3*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-16*B*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*b^3+8*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-12*A*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3+42*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
*b^2+17*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+3*B*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^3+12*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+22*B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a*b^2+30*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-3*B*cos(d
*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b+16*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-14*B*cos(d*
x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b+20*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-72*B*cos(d*x
+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+30*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
-30*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-12*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-
12*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-36*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*a^2*b-3*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+16*B*cos(d*x+c)^3*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^2-14*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+20*B*cos(d*x+c)^3*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b^2-72*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2-30*A*cos(d*x+c
```

```
)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b^2-12*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*A*cos(d*x+c)
^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2-36*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-30
*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b-12*A*cos(d*x+c)^4*((a-b)/(a+b))^(
1/2)*a*b^2-14*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b-16*B*cos(d*x+c)^4*((
a-b)/(a+b))^(1/2)*a*b^2-30*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-3*B*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-6*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*
b^2+24*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-48*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*b^3+3*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-16*B*cos(d*x+c)^4*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*b^3-6*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*a^3+24*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-48*A*cos(d*x+c)^3*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*b^3+3*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-16*B*cos(d*x+c)^3*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b^3-6*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*B*cos(d*x+c)^3*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*a^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))
^(3/2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
```

```
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

3.442 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=339

$$\frac{\left(8a^2A+7abB+4Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2B+12aAb+4b^2B\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec}}$$

```
[Out] ((8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*
x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a +
b*Sec[c + d*x]]) - ((4*A*b + 5*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]) + ((4*A*b + 5*a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(4*d) + (b*B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rubi [A] time = 1.2072, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(8a^2A+7abB+4Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2B+12aAb+4b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{4d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*
x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a +
b*Sec[c + d*x]]) - ((4*A*b + 5*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]) + ((4*A*b + 5*a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(4*d) + (b*B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}+\frac{1}{2}\int$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{\left(12aAb+3a^2B+4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(8a^2A+4Ab^2+7abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.72021, size = 595, normalized size = 1.76

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{1}{4}\sec(c+dx)(5aB\sin(c+dx)+4Ab\sin(c+dx))+\frac{1}{2}bB\tan(c+dx)\sec(c+dx)\right)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}+\frac{(a+b\sec(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((2*(16*a^2*A + 4*a*b*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]
] + (2*(20*a*A*b + a^2*B + 8*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-
4*a*A*b - 5*a^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d
*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a -
b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*
ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a
*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]]
, (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos
[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)) + ((a + b*Sec[c + d*x])^(3/2)*((Sec[c + d*x]*(4*A*b*Sin
[c + d*x] + 5*a*B*Sin[c + d*x]))/4 + (b*B*Sec[c + d*x]*Tan[c + d*x])/2))/(d
*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [C] time = 0.402, size = 2947, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(-4*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*cos(d*x+
c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b-8*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+24*A*sin(d*x+c)*cos
(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+5*B*sin(d*x
+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b-4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+24*A*sin(d*x+c)*co
s(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
```

```
-b),I/((a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+5*B*sin(d*
x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b-2*B*((a-b)/(a+b))^(1/2)*b^2+5*B*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^2+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2-5*B*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^2-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+2*B*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*b^2+8*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+8*A*cos(d*x
+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+2*B*cos(d*x
+c)^3*((a-b)/(a+b))^(1/2)*a*b-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+5*B*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-7*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b
-4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*B*si
n(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*b^2+4*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+2*B*sin(d*x+c)*cos(d*x+c)^
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2-4*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
+6*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)^
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*b^2+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+2*B*sin(d*x+c)*cos(
d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b
+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

3.443 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=272

$$\frac{\left(2a^2B+2aAb+b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2aA-bB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] ((2*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
 + (b*(2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) +
((2*a*A - b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b*B*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.867908, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {4026, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^2B+2aAb+b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2aA-bB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] ((2*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
 + (b*(2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) +
((2*a*A - b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b*B*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
```

```
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}dx = \frac{bB\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{\frac{1}{2}a(2aA-bB)\ \cdots}{\cdots}$$

$$= \frac{bB\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(b(2Ab+3aB))\ \cdots$$

$$= \frac{bB\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(2aA-bB)\int \cdots$$

$$= \frac{bB\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(\left(-2aAb-2a^2B-\cdots\right.\right.}{\cdots}$$

$$= \frac{b(2Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{bB\cdots}{\cdots}$$

$$= \frac{\left(2aAb+2a^2B+b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}\cdots}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.68484, size = 554, normalized size = 2.04

$$\frac{bB\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{d\sqrt{\sec(c+dx)}(a\cos(c+dx)+b)} + \frac{(a+b\sec(c+dx))^{3/2}\left(\frac{2i\left(2a^2A-abB\right)\sin(c+dx)\cos(2(c+dx))\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a\cos(c+dx)+a}{a-b}}\left(a\left(2b\cdots\right.\right.}{b\sqrt{\frac{1}{a-b}}\sqrt{\cdots}}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])*Sqrt[
Sec[c + d*x]]) + ((a + b*Sec[c + d*x])^(3/2)*((2*(8*a*A*b + 4*a^2*B)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
+ a*Cos[c + d*x]] + (2*(2*a^2*A + 4*A*b^2 + 5*a*b*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
d*x]] + ((2*I)*(2*a^2*A - a*b*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a
```

```
 + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*
b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b
)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqr
t[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 -
4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(4*d*(b + a*Cos[c +
 d*x])^(3/2)*Sec[c + d*x]^(3/2))
```

Maple [C] time = 0.502, size = 2595, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(2*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+6*B*sin(
d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-2*A*sin(d*x+c)*cos(d*x+c)^2*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2+
4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+2*A*sin(d*x+c)*cos(d*x+c)^2
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2+B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-2*A*sin(d*x+c)*cos(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2+4*A*
sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+2*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-B
*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*B*cos(d*x+c)
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*b-2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b+4*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
```

```
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-B*
sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b-B*((a-b)/(a+b))^(1/2)*b^2-2*A*((a-b)/(a+b))^(
1/2)*cos(d*x+c)^2*a^2+B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-2*A*cos(d*x+c)^2
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2-2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a*b+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-B*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a*b+2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2)*((b+a*cos(d*x+c))/cos(d
*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

3.444 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=276

$$\frac{2\left(a^2A+3abB-Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+4Ab)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}$$

```
[Out] (2*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*b^2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b + 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/
(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.917958, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {4025, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A+3abB-Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+4Ab)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*b^2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b + 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/
(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{1}{2}a(4Ab+3aB) - \frac{1}{2}\left(a^2\right.}{\sqrt{s}}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{1}{2}a(4Ab+3aB) + \frac{1}{2}\left(-\right.}{\sqrt{\sec(c+dx}}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-4Ab-3aB)\int \frac{\sqrt{a+b\sec}}{\sqrt{\sec(c}}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left(\left(-a^2A+Ab^2-3abB\right)\sqrt{b+}\right.}{3}$$

$$= \frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2aA\sqrt{a+b}}{3d}$$

$$= \frac{2\left(a^2A-Ab^2+3abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 4.43076, size = 437, normalized size = 1.58

$$(a+b\sec(c+dx))^{3/2}\left(\frac{4\left(a^2A+6abB+3Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^2} + \frac{2i(3aB+4Ab)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((4*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d
*x])^2 + (2*(4*a*A*b + 3*a^2*B + 6*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + ((2*I
)*(4*A*b + 3*a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos
[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a
```

```
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF
[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]
+ a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x
]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b*(b + a*Cos[c + d*x])^(3/2
)) + (4*a*A*Sin[c + d*x])/(b + a*Cos[c + d*x])))/(6*d*Sec[c + d*x]^(3/2))
```

Maple [C] time = 0.391, size = 2552, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)*(-A*((a-b)/(a+b))^(1/2)*a*b-3*B*((a-b)/(a+b))^(1
/2)*a*b+3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-4*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-3*B*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^2-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^2+3*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2+A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*cos(d*x+c)*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2+4*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-4*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+
6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b*sin(d*x+c)+6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b-4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a*b-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^2*sin(d*x+c)+6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*A*((a-b)/(a
+b))^(1/2)*b^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+4*A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*b^2-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
```

```
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+A*
cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+5*A*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-3*B*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^2+6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2)*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*
x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

3.445 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{2\left(a^2-b^2\right)(5aB+3Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+20abB+3Ab^2\right)\sqrt{a+b\sec}}{15ad\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}$$

```
[Out] (2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*S
ec[c + d*x]^(3/2)) + (2*(6*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.791623, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)(5aB+3Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+20abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}E}{15ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*S
ec[c + d*x]^(3/2)) + (2*(6*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(15*d*Sqrt[Sec[c + d*x]])
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
```

```
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{1}{2}a(6Ab+5aB)-\frac{1}{2}\left(3a\right.}{}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{a+b\sec(c+}}{15d\sqrt{\sec(c+dx}}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{a+b\sec(c+}}{15d\sqrt{\sec(c+dx}}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{a+b\sec(c+}}{15d\sqrt{\sec(c+dx}}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{a+b\sec(c+}}{15d\sqrt{\sec(c+dx}}$$

$$= \frac{2\left(a^2-b^2\right)(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx}}{15ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.57805, size = 201, normalized size = 0.76

$$\frac{2(a+b\sec(c+dx))^{3/2}\left(\left(a^2-b^2\right)(5aB+3Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+(a+b)\left(9a^2A+20abB+3Ab\right.}{15ad\sec^{\frac{3}{2}}(c+dx)(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(3/2)*((a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2
- b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(6*A*b + 5*a*B + 3*a*A*Cos[c
+ d*x])*Sin[c + d*x]))/(15*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.441, size = 2915, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d/a/((a-b)/(a+b))^(1/2)*(9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+5*
B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
```

)^(1/2)*sin(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+15*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-3*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-9*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+3*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-20*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+20*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-20*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*A*a^2*b*((a-b)/(a+b))^(1/2)-6*A*a*b^2*((a-b)/(a+b))^(1/2)-5*B*a^2*b*((a-b)/(a+b))^(1/2)-20*B*a*b^2*((a-b)/(a+b))^(1/2)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+15*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+9*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+25*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+5*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+12*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-20*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+20*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-20*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*b^3*((a-b)/(a+b))^(1/2)+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*cos(d*x+c)-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+3*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+9*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-3*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-3*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+9*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2

```
*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/s
in(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2
), x)
```

3.446 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=342

$$\frac{2\left(a^2-b^2\right)\left(25a^2A+21abB-6Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2A+42abB+3A\right.}{105a}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2
*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(8*A*b + 7*a*
B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*(2
5*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a
*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.12856, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{2\left(a^2-b^2\right)\left(25a^2A+21abB-6Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{105a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2
*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(8*A*b + 7*a*
B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*(2
5*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a
*d*Sqrt[Sec[c + d*x]])
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{1}{2}a(8Ab+7aB)-\frac{1}{2}\left(5a\right.}{}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\sec(c+}}{35d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\sec(c+}}{35d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\sec(c+}}{35d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\sec(c+}}{35d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\sec(c+}}{35d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A-6Ab^2+21abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.9355, size = 255, normalized size = 0.75

$$\frac{(a+b\sec(c+dx))^{3/2}\left(4\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a^2\left(25a^2A+84abB+51Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(82a^2Ab+63a^3B+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(4*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(25*
a^2*A + 51*A*b^2 + 84*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + (82*a^
2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2
*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(b + a*Cos[c +
 d*x])*((115*a^2*A + 12*A*b^2 + 168*a*b*B)*Sin[c + d*x] + 3*a*(2*(8*A*b + 7
*a*B)*Sin[2*(c + d*x)] + 5*a*A*Sin[3*(c + d*x)]))))/(210*a^2*d*(b + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.568, size = 3752, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d/a^2/((a-b)/(a+b))^(1/2)*(25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*A*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-82*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-25*A*a^3*b*((a-b)/(a+b))^(1/2)-82*A*a^
2*b^2*((a-b)/(a+b))^(1/2)-3*A*a*b^3*((a-b)/(a+b))^(1/2)-63*B*a^3*b*((a-b)/(
a+b))^(1/2)-42*B*a^2*b^2*((a-b)/(a+b))^(1/2)-21*B*a*b^3*((a-b)/(a+b))^(1/2)
-21*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+21
*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-21*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+25*A*
sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*a^4+6*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-63*B*sin(d*x+c)
*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*a^4+63*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-82*A*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
51*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+82*A*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21
*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/
2)*a^4-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-63*B*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*A*cos(d*x+c)^3*((a-
b)/(a+b))^(1/2)*a^4-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-82*A*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-6*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+84*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-21*B*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)-63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-21*B*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
```

```
(d*x+c)+39*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+27*A*cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*a^2*b^2+63*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+68*A*cos
(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*
b^3+63*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-82*A*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^3*b+55*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+6*A*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a*b^3-21*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+21*B
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+6*A*b^4*((a-b)/(a+b))^(1/2)+51*A*sin(
d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a^2*b^2+6*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^3+82*A*sin(d*x+
c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3*b-82*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*A*sin(d*x+c)*c
os(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^3+84*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-63*B*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/
cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bb\sec(dx+c)^{2}+Aa+(Ba+Ab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2
), x)
```

3.447 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=427

$$\frac{2\left(a^2-b^2\right)\left(39a^2Ab+75a^3B-18ab^2B+8Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(49a^2A+7\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(315*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 +
8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c
+ d*x]^(7/2)) + (2*(10*A*b + 9*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sec[c + d*x]^(3/2)) + (2*(88*a^2*A*b -
4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315
*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.49522, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{a+b}}{315a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(315*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 +
8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c
+ d*x]^(7/2)) + (2*(10*A*b + 9*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sec[c + d*x]^(3/2)) + (2*(88*a^2*A*b -
4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315
*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{1}{2}a(10Ab+9aB)-\frac{1}{2}(7$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\sec(c+}}{63d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\sec(c+}}{63d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\sec(c+}}{63d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\sec(c+}}{63d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\sec(c+}}{63d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\sec(c+}}{63d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2(a^2-b^2)(39a^2Ab+8Ab^3+75a^3B-18ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{315a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.07189, size = 313, normalized size = 0.73

$$\frac{(a+b\sec(c+dx))^{3/2}\left(8\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a^2\left(186a^2Ab+75a^3B+153ab^2B+2Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(33a^2A\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(8*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(186
*a^2*A*b + 2*A*b^3 + 75*a^3*B + 153*a*b^2*B)*EllipticF[(c + d*x)/2, (2*a)/(
a + b)] + (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(
(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])) + a*(b + a*Cos[c + d*x])*((804*a^2*A*b - 32*A*b^3 + 690*a^3*
B + 72*a*b^2*B)*Sin[c + d*x] + a*(2*(133*a^2*A + 6*A*b^2 + 144*a*b*B)*Sin[2
*(c + d*x)] + 5*a*(2*(10*A*b + 9*a*B)*Sin[3*(c + d*x)] + 7*a*A*Sin[4*(c + d
*x)])))))/(1260*a^3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.763, size = 4846, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)

[Out] 2/315/d/a^3/((a-b)/(a+b))^(1/2)*(147*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-186*A*sin(d*x+c)*co
s(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a^4*b-147*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-34*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^2*b^3+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+246*B*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*a^4*b-165*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2-18*B*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+18*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-5
3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2-117*B*cos(d*x+c)^4*((a-b)/(a+b
))^(1/2)*a^4*b-52*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+A*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a^2*b^3-81*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2-85
*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b-68*A*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^3*b^2-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-204*B*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^4*b+9*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3-10*A
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b+33*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
^3*b^2+33*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2-2*A*sin(d*x+c)*cos(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2
*b^3+8*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4+147*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-
33*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+33*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-8
*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+246*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b-153*B*
sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*a^3*b^2-18*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3-246*B*
sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^4*b+246*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
x+c))((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*A*b^5
*((a-b)/(a+b))^(1/2)+8*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-75*B*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-35*A*cos(d*
x+c)^6*((a-b)/(a+b))^(1/2)*a^5-14*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5-45
*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^5-30*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/
2)*a^5+75*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-98*A*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^5+147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-8*A*cos(d*x+c)*((a-b

```
)/(a+b))^(1/2)*b^5+147*A*a^4*b*((a-b)/(a+b))^(1/2)+88*A*a^3*b^2*((a-b)/(a+b
))^(1/2)+33*A*a^2*b^3*((a-b)/(a+b))^(1/2)-4*A*a*b^4*((a-b)/(a+b))^(1/2)+75*
B*a^4*b*((a-b)/(a+b))^(1/2)+246*B*a^3*b^2*((a-b)/(a+b))^(1/2)+9*B*a^2*b^3*(
(a-b)/(a+b))^(1/2)-18*B*a*b^4*((a-b)/(a+b))^(1/2)+18*B*sin(d*x+c)*cos(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^3-18*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+33*A*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*
A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+147*A*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-33*A
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+33*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+246*
B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-153*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-18*B*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2
46*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+246*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+18*B*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-18*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+147*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5-147*A*sin(d*x+c
)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^5+8*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5-75*B*sin(d*x+c)*cos(d*x+
c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*a^5-186*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(
d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

3.448 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=513

$$\frac{(472a^2Ab+133a^3B+356ab^2B+128Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{192d\sqrt{a+b\sec(c+dx)}}+\frac{(59a^2B+104aAb}{}$$

```
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(192
*d*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a
^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(64*b*d*Sqrt[a + b*Sec[c + d*x]]
) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B +
284*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192
*b*d) + ((104*a*A*b + 59*a^2*B + 36*b^2*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(96*d) + (b*(8*A*b + 11*a*B)*Sec[c + d*x]^(5/2)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (b*B*Sec[c + d*x]^(5/2)*(a +
 b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 1.99846, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(59a^2B+104aAb+36b^2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{(264a^2Ab+15a^3B+284ab^2B+128Ab^3)\text{s}}{192}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(192
*d*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a
^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(64*b*d*Sqrt[a + b*Sec[c + d*x]]
) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B +
284*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192
*b*d) + ((104*a*A*b + 59*a^2*B + 36*b^2*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(96*d) + (b*(8*A*b + 11*a*B)*Sec[c + d*x]^(5/2)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (b*B*Sec[c + d*x]^(5/2)*(a +
 b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
```

```
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{1}{4}\int \text{se}$$

$$= \frac{b(8Ab+11aB)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$

$$= \frac{\left(104aAb+59a^2B+36b^2B\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx}}{96d}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{192bd}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{192bd}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{192bd}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{192bd}$$

$$= \frac{\left(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B\right)\sqrt{\frac{b+a\cos}{a+}}}{64bd\sqrt{a+b\sec(c+d}}$$

$$= \frac{\left(472a^2Ab+128Ab^3+133a^3B+356ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F}{192d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.90957, size = 768, normalized size = 1.5

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{96}\sec^2(c+dx)\left(59a^2B\sin(c+dx)+104aAb\sin(c+dx)+36b^2B\sin(c+dx)\right)+\frac{\sec(c+dx)\left(264a^2Ab\sin\right.}{}\right.}{d\sec^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] -((a + b*Sec[c + d*x])^(5/2)*((2*(-416*a^2*A*b^2 - 236*a^3*b*B - 144*a*b^3*
B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/Sqrt[b + a*Cos[c + d*x]] + (2*(24*a^3*A*b - 832*a*A*b^3 + 45*a^4*B - 436*
a^2*b^2*B - 288*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
+ d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(264*a^3*A*b +
128*a*A*b^3 + 15*a^4*B + 284*a^2*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*
Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE
[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]
+ a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
(-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)
]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2
*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(768*b*d*(b
```

```
+ a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*(
(Sec[c + d*x]^3*(8*A*b^2*Sin[c + d*x] + 17*a*b*B*Sin[c + d*x]))/24 + (Sec[c
 + d*x]^2*(104*a*A*b*Sin[c + d*x] + 59*a^2*B*Sin[c + d*x] + 36*b^2*B*Sin[c
+ d*x]))/96 + (Sec[c + d*x]*(264*a^2*A*b*Sin[c + d*x] + 128*A*b^3*Sin[c + d
*x] + 15*a^3*B*Sin[c + d*x] + 284*a*b^2*B*Sin[c + d*x]))/(192*b) + (b^2*B*S
ec[c + d*x]^3*Tan[c + d*x])/4))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2
))
```

Maple [C] time = 0.791, size = 5392, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B \sec (dx + c) + A)(b \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

3.449 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=422

$$\frac{\left(48a^3A+59a^2bB+66aAb^2+16b^3B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}+\frac{(33a^2B+54aAb+16b}{}$$

```
[Out] ((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sq
rt[a + b*Sec[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((54*a*A*b + 33*a^2*B
+ 16*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((54*a*A*b
+ 33*a^2*B + 16*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(24*d) + (b*(2*A*b + 3*a*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(4*d) + (b*B*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.59404, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\left(48a^3A+59a^2bB+66aAb^2+16b^3B\right)\sqrt{\sec}}{24d\sqrt{a+b\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sq
rt[a + b*Sec[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((54*a*A*b + 33*a^2*B
+ 16*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((54*a*A*b
+ 33*a^2*B + 16*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(24*d) + (b*(2*A*b + 3*a*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(4*d) + (b*B*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(3*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
```

```
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
```

```
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int \sqrt{}$$

$$= \frac{b(2Ab+3aB)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{24d}$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{24d}$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{24d}$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{24d}$$

$$= \frac{\left(30a^2Ab+8Ab^3+5a^3B+20ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(\right.}{8d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(48a^3A+66aAb^2+59a^2bB+16b^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{24d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.92307, size = 678, normalized size = 1.61

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{24}\sec(c+dx)\left(33a^2B\sin(c+dx)+54aAb\sin(c+dx)+16b^2B\sin(c+dx)\right)+\frac{1}{12}\sec^2(c+dx)\left(13a\right.\right.}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(96*a^3*A + 24*a*A*b^2 + 52*a^2*b*B)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
+ a*Cos[c + d*x]] + (2*(126*a^2*A*b + 48*A*b^3 - 3*a^3*B + 104*a*b^2*B)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-54*a^2*A*b - 33*a^3*B - 16*a*b^2*B)*Sqr
t[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c
 + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Co
s[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(
-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I
*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*
Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^
2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*
Cos[c + d*x])^2))))/(96*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) +
((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]^2*(6*A*b^2*Sin[c + d*x] + 13*a*b
*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(54*a*A*b*Sin[c + d*x] + 33*a^2*B*Sin[
c + d*x] + 16*b^2*B*Sin[c + d*x]))/24 + (b^2*B*Sec[c + d*x]^2*Tan[c + d*x])
```

```
/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] time = 0.515, size = 4258, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d/((a-b)/(a+b))^(1/2)*(54*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+8*B*((a-b)
/(a+b))^(1/2)*b^3-33*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-16*B*cos(d*x+c)
^3*((a-b)/(a+b))^(1/2)*b^3+8*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-12*A*co
s(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3+66*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*
b^2+59*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-48*A*cos(d*x+c)^4*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^3-48*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+33*B*cos(d*x+c)^3*((a-b)/(a+b))^
(1/2)*a^3+12*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+34*B*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a*b^2+54*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-33*B*cos(d*x+c
)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b+16*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-26*B*cos(d*x+c)
^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b+44*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-120*B*cos(d*x+c)
^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+54*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-54
*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+36*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*
A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-180*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*a^2*b-33*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+16*B*cos(d*x+c)^3*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b^2-26*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+44*B*cos(d*x+c)^3*sin(d*x+c)
```

```
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^2-120*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2-54*A*cos(d*x+c
)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b^2+36*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*A*cos(d*x+c)
^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2-180*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-5
4*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b-12*A*cos(d*x+c)^4*((a-b)/(a+b))^
(1/2)*a*b^2-26*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b-16*B*cos(d*x+c)^4*(
(a-b)/(a+b))^(1/2)*a*b^2-54*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-33*B*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-18*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a*b^2+24*A*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-48*A*cos(d*x+c)^4*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*b^3+33*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-16*B*cos(d*x+c)^4*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^3-18*B*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*B*cos(d*x+c)^4*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*a^3+24*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-48*A*cos(d*x+c)^3*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*b^3+33*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-16*B*cos(d*
x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*b^3-18*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*B*cos(d*x+c)^
3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*a^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/co
s(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

3.450 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=359

$$\frac{\left(16a^2Ab+8a^3B+11ab^2B+4Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\left(8a^2A-9abB-4Ab^2\right)\sqrt{a+}}{4d\sqrt{\sec(c+d}}$$

```
[Out] ((16*a^2*A*b + 4*A*b^3 + 8*a^3*B + 11*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[
a + b*Sec[c + d*x]]) + (b*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b*(4*A*b + 7*a*B)*Sqrt[Sec[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b*B*Sqrt[Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.24905, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4026, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(16a^2Ab+8a^3B+11ab^2B+4Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\left(8a^2A-9abB-4Ab^2\right)\sqrt{a+b\sec(c+}}{4d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] ((16*a^2*A*b + 4*A*b^3 + 8*a^3*B + 11*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[
a + b*Sec[c + d*x]]) + (b*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b*(4*A*b + 7*a*B)*Sqrt[Sec[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b*B*Sqrt[Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{bB\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{a+b\sec(c-}}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{bB\sqrt{\sec(c}}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{bB\sqrt{\sec(c}}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{bB\sqrt{\sec(c}}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{bB\sqrt{\sec(c}}{}$$

$$= \frac{b\left(20aAb+15a^2B+4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{4d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(16a^2Ab+4Ab^3+8a^3B+11ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}}}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 7.00194, size = 628, normalized size = 1.75

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{4}\sec(c+dx)\left(9abB\sin(c+dx)+4Ab^2\sin(c+dx)\right)+\frac{1}{2}b^2B\tan(c+dx)\sec(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}+\frac{(a+b\,\mathrm{s}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(48*a^2*A*b + 16*a^3*B + 4*a*b^2*B)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b +
 a*Cos[c + d*x]] + (2*(8*a^3*A + 36*a*A*b^2 + 21*a^2*b*B + 8*b^3*B)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt
[b + a*Cos[c + d*x]] + ((2*I)*(8*a^3*A - 4*a*A*b^2 - 9*a^2*b*B)*Sqrt[(a - a
*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]
*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqr
t[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c +
d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*
Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(4*A*b^2*Sin[c + d*x] + 9*a*b*B*Sin[c +
d*x]))/4 + (b^2*B*Sec[c + d*x]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])^2*
Sec[c + d*x]^(5/2))
```

Maple [C] time = 0.523, size = 3939, normalized size = 11.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(40*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*
b^2-8*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+4*A*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*b^3-8*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*A*cos(d*x+c)^2*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2+30*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-6*B*cos(d*x+c)^2
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b+2*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*B*cos(d*x+c)^2*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
```

```
b))^(1/2))*a^2*b+9*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+40*A*cos(d*x+c)^3*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a*b^2+30*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a
^2*b-2*B*((a-b)/(a+b))^(1/2)*b^3+24*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*A*
cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b^2+2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-4*
A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-9*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/
2)*a^2*b-8*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+9*B*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a*b^2-8*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-4*A*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*b^3+8*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-11*B*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a*b^2+8*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-8*A*co
s(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*b-4*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+24*A*cos(
d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b-16*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*B*cos(d*
x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b+9*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-6*B*cos(d*x+c
)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b+2*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+8*A*cos(d*x+c)^3
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3+4*A*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+8*B*cos(d*x+c)^3*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/
((a-b)/(a+b))^(1/2))*b^3-4*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-8*A*cos(d*x+c)^2
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3+8*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+4*A*cos(d*x+c)^2*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^3+8*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
```

```
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+4*A*cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*a*b^2+9*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+2*B*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2+8*B*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+8*B
*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3-4*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

3.451 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=349

$$\frac{\left(2a^3A+12a^2bB+4aAb^2+3b^3B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{\left(6a^2B+14aAb-3b^2B\right)\sqrt{\ldots}}{3d\sqrt{\sec(c\ldots}}$$

```
[Out] ((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a
 + b*Sec[c + d*x]]) + (b^2*(2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
+ b*Sec[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]) - (b*(2*a*A - 3*b*B)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.2503, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4025, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^3A+12a^2bB+4aAb^2+3b^3B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{\left(6a^2B+14aAb-3b^2B\right)\sqrt{a+b\sec\ldots}}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\ldots}{\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```


```
[Out] ((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a
 + b*Sec[c + d*x]]) + (b^2*(2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
+ b*Sec[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]) - (b*(2*a*A - 3*b*B)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{3}{2}\right.}{}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(}{}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(}{}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(}{}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2aA(}{}$$

$$= \frac{b^2(2Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{b(}{}$$

$$= \frac{\left(2a^3A+4aAb^2+12a^2bB+3b^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{}}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.94982, size = 599, normalized size = 1.72

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2}{3}a^2A\sin(c+dx)+b^2B\tan(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2\left(4a^3A+36a^2bB+36aAb^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Ellipt}}{\sqrt{a\cos(c+dx)+b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(4*a^3*A + 36*a*A*b^2 + 36*a^2*b*B)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b +
 a*Cos[c + d*x]] + (2*(14*a^2*A*b + 12*A*b^3 + 6*a^3*B + 27*a*b^2*B)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqr
t[b + a*Cos[c + d*x]] + ((2*I)*(14*a^2*A*b + 6*a^3*B - 3*a*b^2*B)*Sqrt[(a -
 a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x
)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c
+ d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[
c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c
+ d*x])^2))))/(12*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a +
b*Sec[c + d*x])^(5/2)*((2*a^2*A*Sin[c + d*x])/3 + b^2*B*Tan[c + d*x]))/(d*(
b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] time = 0.412, size = 3663, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/3/d/((a-b)/(a+b))^(1/2)*(-14*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-12*B*cos(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*a*b^2+14*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-14*A*cos(d*x+c
)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b^2+18*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*B*cos(d*x+c)
^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2-6*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-3*B*cos(d*x+c)^2*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
```

a-b))^(1/2))*a*b^2+18*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+14*A*cos(d*x+c)*sin(d
x+c)(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b-14*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+18*B*cos(d*x+c)*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*a^2*b-6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2-3*B*((a-b)/(a+b))^(1/2)*b^3-14*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+18*A*cos
(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a*b^2+14*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+6*
B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^2*b-6*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-14*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a^2*b+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+30*B*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*a*b^2+30*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2
+6*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+2*A*cos(d*x+c)^4*((a-b)/(a+b))^(1
/2)*a^3-2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+2*A*cos(d*x+c)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*a^3-14*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-6*B*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^2*b-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+16*A*cos(d*x+c)^3*((
a-b)/(a+b))^(1/2)*a^2*b+2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6*B*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^3+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-6*B*cos(d*x+
c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3-6*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+12*A*cos(d*x+c)^2*s
in(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*b^3+6*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
x+c))((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+3*B*cos(d*x
+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^3-6*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+12*A*cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+

```
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+3*B*cos(d*x+c)*s
in(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(1/cos(d*x+
c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

3.452 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=342

$$\frac{2\left(8a^2Ab+5a^3B+10ab^2B-8Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+35abB+23Ab^2\right)}{15d\sqrt{\sec(c}}$$

```
[Out] (2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqr
t[a + b*Sec[c + d*x]]) + (2*b^3*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 1.22294, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4025, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(8a^2Ab+5a^3B+10ab^2B-8Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+35abB+23Ab^2\right)\sqrt{a+b\,\text{se}}}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqr
t[a + b*Sec[c + d*x]]) + (2*b^3*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{1}{2}a(\right.}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx)}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2b^3B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2a(8Ab+5aB)}{1}$$

$$= \frac{2\left(8a^2Ab-8Ab^3+5a^3B+10ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{s}}{15d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.96606, size = 616, normalized size = 1.8

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{5}a^2A\sin(2(c+dx))+\frac{2}{15}a(5aB+11Ab)\sin(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(34a^2Ab+10a^3B+90a\ldots}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(34*a^2*A*b + 30*A*b^3 + 10*a^3*B + 90*a*b^
2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/Sqrt[b + a*Cos[c + d*x]] + (2*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B + 30*b
^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(
a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b
*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b
 + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 -
 a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a
^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*
(b + a*Cos[c + d*x])^2))))/(30*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5
/2)) + ((a + b*Sec[c + d*x])^(5/2)*((2*a*(11*A*b + 5*a*B)*Sin[c + d*x])/15
+ (a^2*A*Sin[2*(c + d*x)])/5))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)
)
```

Maple [C] time = 0.466, size = 3564, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d/((a-b)/(a+b))^(1/2)*(9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+30*B*EllipticPi((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-23*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+17*A*cos(d*x+c)*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a^2*b+15*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-15*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-15*B*sin(d*x+c)*c
os(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
```

```
-b))^(1/2))*b^3+30*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+45*B*cos(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*a*b^2-23*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-9*A*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b+23*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-35*B*cos(d*x+c)*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a^2*b+35*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-35*B*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^2-9*A*a^2*b*((a-b)/(a+b))^(1/2)-11*A*a*b^2*((a-b)/(a+b))^(1/2)-5*B*a^2
*b*((a-b)/(a+b))^(1/2)-35*B*a*b^2*((a-b)/(a+b))^(1/2)-9*A*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
45*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b^2*sin(d*x+c)+34*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+40*B*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-5*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2
*b+5*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+17*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-23*A*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+23*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-35*B*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+35*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-35*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-23*A*b^3*((a-b)/(a+b))^
(1/2)+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*
cos(d*x+c)-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+23*A*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*b^3+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*A*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^3-9*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+9*A*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3-23*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-23*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a*b^2-35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+35*B*cos(d*x+c)*((a-b)
```

```
/(a+b))^(1/2)*a*b^2+14*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+15*A*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/
cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

3.453 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=340

$$\frac{2\left(a^2-b^2\right)\left(25a^2A+56abB+15Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2A+77abB+45\right.}{10}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a*d*
Sqrt[a + b*Sec[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*
a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(10*A*b +
 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) +
(2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 1.15505, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105d\sqrt{\sec(c+dx)}}+\frac{2\left(a^2-b^2\right)\left(25a^2A+56abB+15Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a}{-}}}{105ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a*d*
Sqrt[a + b*Sec[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*
a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(10*A*b +
 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) +
(2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{\sqrt{a+b\sec(c+dx)}\left(-\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+}{7d\sec^{\frac{5}{2}}}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2A+45Ab\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2A+45Ab\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2A+45Ab\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2A+45Ab\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A+15Ab^2+56abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{105ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.79112, size = 257, normalized size = 0.76

$$\frac{(a+b\sec(c+dx))^{5/2}\left(2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a\left(25a^3A+119a^2bB+135aAb^2+105b^3B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(145a^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(25*a^
3*A + 135*a*A*b^2 + 119*a^2*b*B + 105*b^3*B)*EllipticF[(c + d*x)/2, (2*a)/(
a + b)] + (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*((a + b)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))
 + a*(b + a*Cos[c + d*x])*(65*a^2*A + 90*A*b^2 + 154*a*b*B + 6*a*(15*A*b +
7*a*B)*Cos[c + d*x] + 15*a^2*A*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*a*d*(b
 + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.557, size = 3980, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d/a/((a-b)/(a+b))^(1/2)*(25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+105*B*sin(d*x+c)*cos(
d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*a*b^3-15*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-145*A*sin(d*x+c)*cos(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-25
*A*a^3*b*((a-b)/(a+b))^(1/2)-145*A*a^2*b^2*((a-b)/(a+b))^(1/2)-45*A*a*b^3*(
(a-b)/(a+b))^(1/2)-63*B*a^3*b*((a-b)/(a+b))^(1/2)-77*B*a^2*b^2*((a-b)/(a+b)
)^(1/2)-161*B*a*b^3*((a-b)/(a+b))^(1/2)-161*B*sin(d*x+c)*cos(d*x+c)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b
^2-63*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+161*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
-161*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+25*A*sin(d*x+c)*cos(d*x+c)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4-15*A*
sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^4-63*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4+63*B*sin(d*x+c
)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^4-145*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+135*A*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-15*A
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+145*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21*B*cos(d*x+c)^4*((a-b)/(a+b)
)^(1/2)*a^4+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4+15*A*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*b^4-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+15*A*cos(d*x+c)^
5*((a-b)/(a+b))^(1/2)*a^4+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-25*A*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-145*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+15*A*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+119*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-161*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-63*B*EllipticE
```

```
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+161*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-161*B*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+60*A*cos(d*x+
c)^4*((a-b)/(a+b))^(1/2)*a^3*b+90*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^
2+98*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+110*A*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^3*b+60*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+238*B*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-145*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+
55*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-15*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a*b^3-35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-161*B*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^2*b^2+161*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-15*A*b^4*
((a-b)/(a+b))^(1/2)+135*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-15*A*sin(d*x+c)*c
os(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a*b^3+145*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b^2+15*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a^3*b-63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+105*B*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(
d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/
2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Bb^{2}\sec\left(dx+c\right)^{3}+Aa^{2}+\left(2\,Bab+Ab^{2}\right)\sec\left(dx+c\right)^{2}+\left(Ba^{2}+2\,Aab\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)(b\sec\left(dx+c\right)+a)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2
), x)
```

3.454 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=425

$$\frac{2\left(a^2-b^2\right)\left(114a^2Ab+75a^3B+45ab^2B-10Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(49a^2A+\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
d*x]])/(315*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
Sqrt[Sec[c + d*x]]) + (2*a*(4*A*b + 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(163*a^
2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(315*a*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 1.51814, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{}}{315ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
d*x]])/(315*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
Sqrt[Sec[c + d*x]]) + (2*a*(4*A*b + 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(163*a^
2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(315*a*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{3}{2}\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+}{9d\sec^{\frac{7}{2}}(}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2A+75Ab^2\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(114a^2Ab-10Ab^3+75a^3B+45ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{315a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.49694, size = 313, normalized size = 0.74

$$\frac{(a+b\sec(c+dx))^{5/2}\left(8\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a^2\left(261a^2Ab+75a^3B+405ab^2B+155Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(279a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(8*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(261
*a^2*A*b + 155*A*b^3 + 75*a^3*B + 405*a*b^2*B)*EllipticF[(c + d*x)/2, (2*a)
/(a + b)] + (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*
B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])) + a*(b + a*Cos[c + d*x])*(2*(747*a^2*A*b + 20*A*b^3 + 34
5*a^3*B + 540*a*b^2*B)*Sin[c + d*x] + a*((266*a^2*A + 300*A*b^2 + 540*a*b*B
```

```
)*Sin[2*(c + d*x)] + 5*a*(2*(19*A*b + 9*a*B)*Sin[3*(c + d*x)] + 7*a*A*Sin[4
*(c + d*x)]))))))/(1260*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.744, size = 4847, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x)
```

```
[Out] 2/315/d/a^2/((a-b)/(a+b))^(1/2)*(147*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-261*A*sin(d*x+c)*co
s(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a^4*b-147*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-199*A*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a^2*b^3-10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+435*B*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a^4*b-165*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+45*B*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3-45*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4
-170*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2-180*B*cos(d*x+c)^4*((a-b)/(
a+b))^(1/2)*a^4*b-82*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b-80*A*cos(d*x+
c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3-270*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3
*b^2-130*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b-272*A*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a^3*b^2+5*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-330*B*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b-180*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^2*b^3+65*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b+279*A*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^3*b^2+279*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2-155*A*sin(d*x+c)*c
os(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a^2*b^3-10*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4+147*A*sin(d*x+c)*cos
(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^4*b-279*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+279*A*sin(d*x+c)*cos(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b^3+10*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+435*B*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a^4*b-405*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+45*B*sin(d*x+c)*cos(d*x+
c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*a^2*b^3-435*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+435*B*sin(d*x+c)*cos(d*x+c
```

```
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^3*b^2-10*A*b^5*((a-b)/(a+b))^(1/2)-10*A*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-75*B*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-35*A*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5-14*A*cos(d*x+c)^4*((a-b
)/(a+b))^(1/2)*a^5-45*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^5-30*B*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^5+75*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-98*A*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5+147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5
+10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5+147*A*a^4*b*((a-b)/(a+b))^(1/2)+16
3*A*a^3*b^2*((a-b)/(a+b))^(1/2)+279*A*a^2*b^3*((a-b)/(a+b))^(1/2)+5*A*a*b^4
*((a-b)/(a+b))^(1/2)+75*B*a^4*b*((a-b)/(a+b))^(1/2)+435*B*a^3*b^2*((a-b)/(a
+b))^(1/2)+135*B*a^2*b^3*((a-b)/(a+b))^(1/2)+45*B*a*b^4*((a-b)/(a+b))^(1/2)
-45*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+45*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+27
9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)-155*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-10*A*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+147*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-279*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+279*A*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+10*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+435*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-405*B*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+45*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)-435*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+435*B*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)-45*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+45*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+147*A*sin(d*x+c)
*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*a^5-147*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-10*A*sin(d*x+c)*cos(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
```

```
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^5-75*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5-261*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*((b+a*
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)/
(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\sec(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(9/2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2
), x)
```

3.455 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=519

$$\frac{2\left(a^2-b^2\right)\left(285a^2Ab^2+675a^4A+1254a^3bB-110ab^3B+40Ab^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3465a^3d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a
*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(3465*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*
a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^
4*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*
a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(14*A*b
 + 11*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
 + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(693*d*Sec[c + d*x]^(5/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B +
 825*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a*d*Sec[c + d*x]
^(3/2)) + (2*(675*a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b
^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Sqrt[Sec[c + d*x]]
) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2
))
```

Rubi [A] time = 1.95957, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(1145a^2Ab+539a^3B+825ab^2B+15Ab^3\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3465ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x
]
```


```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a
*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(3465*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*
a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^
4*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*
a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(14*A*b
 + 11*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
 + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(693*d*Sec[c + d*x]^(5/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B +
 825*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a*d*Sec[c + d*x]
^(3/2)) + (2*(675*a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b
^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Sqrt[Sec[c + d*x]]
) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2
))
```


Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
```

```
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{2}{11}\int \frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{1}{2}a\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+d}{11d\sec^{\frac{9}{2}}(}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\left(81a^2A+113Ab^2\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(675a^4A+285a^2Ab^2+40Ab^4+1254a^3bB-110ab^3B\right)\sqrt{\frac{b+}{}}}{3465a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 3.5526, size = 380, normalized size = 0.73

$$\frac{(a+b\sec(c+dx))^{5/2}\left(16\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a^2\left(3315a^2Ab^2+675a^4A+2871a^3bB+1705ab^3B+10Ab^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(1
1/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(16*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(67
5*a^4*A + 3315*a^2*A*b^2 + 10*A*b^4 + 2871*a^3*b*B + 1705*a*b^3*B)*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)] + (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 +
1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2,
(2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(b + a*Cos[c
 + d*x])*(2*(6525*a^4*A + 9330*a^2*A*b^2 - 160*A*b^4 + 16434*a^3*b*B + 440*
a*b^3*B)*Sin[c + d*x] + a*(4*(3095*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a
*b^2*B)*Sin[2*(c + d*x)] + 5*a*((513*a^2*A + 452*A*b^2 + 836*a*b*B)*Sin[3*(
c + d*x)] + 7*a*((46*A*b + 22*a*B)*Sin[4*(c + d*x)] + 9*a*A*Sin[5*(c + d*x)
]))))))/(27720*a^3*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 1.022, size = 5946, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/
2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2\sec(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(11/
```

```
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/
2), x)
```

3.456 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=344

$$\frac{(4Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}{4b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((4*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((4*a*A
*b - 3*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d
*x]]) - ((4*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + ((4*A*b - 3*a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(4*b^2*d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*b*d)
```

Rubi [A] time = 1.11121, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4033, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] ((4*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((4*a*A
*b - 3*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d
*x]]) - ((4*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + ((4*A*b - 3*a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(4*b^2*d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*b*d)
```


Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}+bB\sec(c+dx)+\frac{1}{2}\right.}{\sqrt{a+b\sec(c+}}}{2b}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{}$$

$$= -\frac{\left(4aAb-3a^2B-4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4b^2d\sqrt{a+b\sec(c+dx)}} + \frac{(}{}$$

$$= \frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-3a^2B-\right.}{}$$

Mathematica [C] time = 3.99187, size = 451, normalized size = 1.31

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2i(3aB-4Ab)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)+b}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}\right),\frac{b-a}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\,\mathrm{s}\right.\right.\right.}{ab^3\sqrt{\frac{1}{a-b}}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((8*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)])/b + (2*(-12*a*A*b + 9*a^2*B + 8*b^2*B)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/b^2 +
((2*I)*(-4*A*b + 3*a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1
 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*x]*(-2*b*(a +
 b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos
[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a -
b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-
1)]*b^3) - (4*a*(-4*A*b + 3*a*B)*Sin[c + d*x])/b^2 + (8*a*B*Tan[c + d*x])/b
 + (4*(4*A*b - 3*a*B)*Tan[c + d*x])/b + 8*B*Sec[c + d*x]*Tan[c + d*x]))/(16
*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.46, size = 2737, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)/b^2*(-4*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+8*A*cos(
d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b+8*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+c)*
cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*sin(
d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b-4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+c)*
cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*sin(
```

```
d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b-2*B*((a-b)/(a+b))^(1/2)*b^2-3*B*cos(d*x+c)^3*((a-
b)/(a+b))^(1/2)*a^2+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+3*B*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a^2-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+2*B*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+2*
B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a*b-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+B*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a*b-4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
+6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*b^2+4*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-6*B*sin(d*x+c)*cos(
d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2+6*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(
d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*b^2+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-6*B*sin(d*x+
c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

3.457 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=256

$$\frac{B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(b*d)
```

Rubi [A] time = 0.731077, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {4033, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}+\frac{B\sqrt{\sec(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(b*d)
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
```

```
d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\int \frac{-\frac{aB}{2}+\frac{1}{2}(2Ab-aB)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{b}$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} - \frac{(aB)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{1}{2}B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,d$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\left(B\sqrt{b+a\cos(c+dx)}\sqrt{\sec(\dots}\right.}{2\sqrt{a+b\,\text{se}\dots}}$$

$$= \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{B\sqrt{\sec(c+dx\dots}}{}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\dots}{bd\sqrt{a+\dots}}$$

Mathematica [C] time = 6.7537, size = 339, normalized size = 1.32

$$\sqrt{\sec(c+dx)}\left(-\frac{2iB\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)+b}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}\right),\frac{b-a}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh\dots\right.\right.\right.}{ab\sqrt{\frac{1}{a-b}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(4*A*b - 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)] - ((2*I)*B*Sqrt[-((a*(-1 + Cos[c +
 d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d
*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1
 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a
 + b)])))/(a*Sqrt[(a - b)^(-1)]*b) + 4*B*(b + a*Cos[c + d*x])*Tan[c + d*x])
```

```
)/(4*b*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.43, size = 1440, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/b*(4*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-2
*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
a-B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a+B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*B*sin(d*x+c
)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a+4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+
c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b-2*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a+B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*B*sin(d*x+c)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+B*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+B*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*b-B*((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)*(1/cos(d*
x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

3.458 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.391646, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4036, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4036

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (
A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = A\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + B\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{b+a\cos(c+\right.}{}$$

$$= \frac{\left(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec\right.}{\sqrt{a+}}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(\right.}{d\sqrt{a+b\,s}}$$

Mathematica [A] time = 0.258445, size = 91, normalized size = 0.66

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(A\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+B\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(A*EllipticF[(c + d*x)/2, (2*a)/(a +
b)] + B*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/(d*S
qrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.395, size = 283, normalized size = 2.1

$$2\frac{\cos(dx+c)\left(\sin(dx+c)\right)^{2}\sqrt{\left(\cos(dx+c)\right)^{-1}}\sqrt{\left(\cos(dx+c)+1\right)^{-1}}}{d\left(-1+\cos(dx+c)\right)\left(b+a\cos(dx+c)\right)}\left(A\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*cos(d*x+c)*sin(
d*x+c)^2*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/(-1+cos(d*x
+c))/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec(dx+c)+A\right)\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

3.459 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=150

$$\frac{2A\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.310366, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$
$$= -\frac{\left((Ab-aB)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{a+\right.}{a\cdots}$$
$$= -\frac{\left((Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{a+b\operatorname{se}\right.}{a\sqrt{}}$$
$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2AE\left(\frac{1}{2}(c+dx)\right.}{ad\sqrt{\frac{b+a\cos}{a\cdots}}}$$

Mathematica [A] time = 3.65105, size = 103, normalized size = 0.69

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left((aB-Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+A(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(A*(a + b)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)] + (-(A*b) + a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Se
c[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.382, size = 940, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)/a*(A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*sin(d*x+c)*cos(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a+A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+A*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d
*x+c)-A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-B*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-A*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*b+A*b*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/
(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec (dx + c) + A}{\sqrt{b \sec (dx + c) + a}\sqrt{\sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B \sec (dx + c) + A)\sqrt{b \sec (dx + c) + a}\sqrt{\sec (dx + c)}}{b \sec (dx + c)^2 + a \sec (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b*sec(d*x + c)^2 + a*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{\sqrt{a + b\sec(c + dx)}\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx + c) + A}{\sqrt{b\sec(dx + c) + a}\sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

3.460 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=212

$$\frac{2\left(a^2A-3abB+2Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(2*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d
*x]])
```

Rubi [A] time = 0.479502, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4034, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A-3abB+2Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(2*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d
*x]])
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(2Ab-3aB)-\frac{1}{2}aA\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(2Ab-3aB)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a^2} + \frac{1}{3}\Big(A \ldots$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(A+\frac{b(2Ab-3aB)}{a^2}\right)\sqrt{b+a\cos(c+dx)}\sqrt{s\ldots}\right.}{3\sqrt{a+b\sec(c\ldots}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(A+\frac{b(2Ab-3aB)}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c\ldots}\right.}{3\sqrt{a+b\sec(c+\ldots}}$$

$$= \frac{2\left(A+\frac{b(2Ab-3aB)}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} - \frac{2(2Ab-3\ldots}{3\ldots}$$

Mathematica [A] time = 0.877595, size = 161, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(\left(a^2A-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+(a+b)(3aB-2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}E\left(\frac{1}{2}(c\right.\right.}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a + b)*(-2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2*A + 2*A*b^2 - 3*a*b*B)
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] +
 a*A*(b + a*Cos[c + d*x])*Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.401, size = 1731, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/a^2/((a-b)/(a+b))^(1/2)*(A*((a-b)/(a+b))^(1/2)*a*b+3*B*((a-b)/(a+b))^
(1/2)*a*b-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^2*sin(d*x+c)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2+A*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^2-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-A*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-2*A*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b*sin(d*x+c)+2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*A*((a-b)/(a+b))^(1/2)
*b^2-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+2*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b^2+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*sin(d*x+c)-2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-A*cos(d*x+c)
```

```
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
*b-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^{3}+a\sec(dx+c)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

3.461 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=280

$$-\frac{2\left(7a^2Ab-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A-10abB+8Ab\right.}{15a^3d\sqrt{s}}$$

```
[Out] (-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*
d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*EllipticE[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(4*A*b - 5*a*B)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.750023, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4034, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(7a^2Ab-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A-10abB+8Ab^2\right)\sqrt{a+}}{15a^3d\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```


```
[Out] (-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*
d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*EllipticE[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(4*A*b - 5*a*B)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
```

```
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{1}{2}(4Ab-5aB)-\frac{3}{2}aA\sec(c+dx)-Ab\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}d}{5a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\left(7a^2Ab+8Ab^3-5a^3B-10ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{15a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.25912, size = 198, normalized size = 0.71

$$\frac{2\sqrt{\sec(c+dx)}\left(\left(-7a^2Ab+5a^3B+10ab^2B-8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+(a+b)\left(9a^2A-10abB\right.\right.}{15a^3d\sqrt{a+b\sec(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (-7*a^2*A*b -
 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3
*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.46, size = 2739, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a^3/((a-b)/(a+b))^(1/2)*(9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
```

```
^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-8*A*cos(d*x+c)*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a*b^2-9*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+8*A*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^2+10*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-10*B*cos(d*x+c)*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b+10*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*A*a^2*b*((a-b)/(a+b))^(1/2)+4*A*a*b^
2*((a-b)/(a+b))^(1/2)-5*B*a^2*b*((a-b)/(a+b))^(1/2)+10*B*a*b^2*((a-b)/(a+b)
)^(1/2)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-5*B
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^2*b+5*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+2*A*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-9*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+10*B*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-10*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+10*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*b^3*((a-b)/(a+b))^
(1/2)+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*
cos(d*x+c)-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+8*A*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*b^3+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*A*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a^3-9*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+9*A*cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3-8*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a*b^2+10*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-10*B*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a*b^2-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b)*((b+a*cos(d*x+c)
)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d
*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B \sec(dx + c) + A)\sqrt{b \sec(dx + c) + a}\sqrt{\sec(dx + c)}}{b \sec(dx + c)^4 + a \sec(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b*sec(d*x + c)^4 + a*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx + c) + A}{\sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

3.462 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=371

$$\frac{B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+2aAb+b^2B\right)}{}$$

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - 3*a*B)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a*A*b - 3*a^2*B +
b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2
*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*
a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*
Sec[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.26909, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4029, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{\left(-3a^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - 3*a*B)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a*A*b - 3*a^2*B +
b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2
*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*
a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*
Sec[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(Ab-aB)\sec(c+d}{\sqrt{a+b\sec(c+}}}{b\left(a^2-b^2\right.}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{b^2\left(a^2-b^2\right)c}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{b^2\left(a^2-b^2\right)c}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{b^2\left(a^2-b^2\right)c}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{\sec(c+dx)}\sqrt{a}}{b^2\left(a^2-b^2\right)c}$$

$$= \frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sec}{b\left(a^2-b^2\right)d}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(\right.}{b^2d\sqrt{a+b}}$$

Mathematica [C] time = 5.7458, size = 518, normalized size = 1.4

$$\sec^{\frac{3}{2}}(c+dx)\left(\frac{4\tan(c+dx)(a\cos(c+dx)+b)\left(a\left(-3a^2B+2aAb+b^2B\right)\cos(c+dx)+bB\left(b^2-a^2\right)\right)}{b^4-a^2b^2}-\frac{(a\cos(c+dx)+b)^{3/2}\left(\frac{2i\left(3a^2B-2aAb-b^2B\right)\csc(c+dx)\sqrt{-\frac{a(\cos(c+\cdots}{a+b}}\cdots}{\cdots}\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-(((b + a*Cos[c + d*x])^(3/2)*((8*a*b*(-(A*b) + a*B)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sq
rt[b + a*Cos[c + d*x]] + (2*(-6*a^2*A*b + 4*A*b^3 + 9*a^3*B - 7*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-2*a*A*b + 3*a^2*B - b^2*B)*Sqrt[-((a*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c +
d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*Arc
Sinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*
Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))) + (4*(b + a*Cos[c + d*x])*(b
*(-a^2 + b^2)*B + a*(2*a*A*b - 3*a^2*B + b^2*B)*Cos[c + d*x])*Tan[c + d*x])
/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.359, size = 2655, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/(a+b)/b^2*(-B*((a-b)/(a+b))^(1/2)*a*b-3*B*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a^2-6*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-6*
B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*cos(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2
-2*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*b^2+B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-2*A*sin(d*x+c)*cos(d*x+c)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2+4*A
*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
```

```
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+4*A*cos(d*x+c)*sin(d*x+c)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*B*cos(d*x+c)*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2+4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*cos(
d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b-4*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*A*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a*b+2*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*sin(d*x+c)*cos(
d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*a*b+4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-B*((a-b)/(a+
b))^(1/2)*b^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+B*((a-b)/(a+b))^(1/2
)*cos(d*x+c)*b^2-2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+B*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a*b-6*B*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-3*B*sin
(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+
6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(
d*x+c)^2*(1/cos(d*x+c))^(5/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.463 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{bd\sqrt{a+b\sec(c}}$$

```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(A*b -
a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]])
```

Rubi [A] time = 0.625313, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4029, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{bd\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(A*b -
a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]])
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(Ab-aB)\sec(c+dx)+\frac{1}{2}\left(a^2-b\right.}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)} + \cdots$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(Ab-aB)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{b\left(a^2-b^2\right)} + \frac{\left(B\sqrt{b+\cdots}\right.}{\cdots}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{\sec\cdots}{\sqrt{\frac{b}{a+b}+\cdots}}}{b\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\cdots}{b\left(a^2-b^2\right)d\sqrt{a+b\sec\cdots}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|\frac{2\cdots}{a\cdots}\right.}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(\cdots}{a+b}}}$$

Mathematica [C] time = 4.58367, size = 464, normalized size = 2.11

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{4a(Ab-aB)\sin(c+dx)(a\cos(c+dx)+b)}{a^2-b^2} + \frac{(a\cos(c+dx)+b)^{3/2}\left(\frac{4b(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{a\cos(c+dx)+b}} - \frac{2i(aB-Ab)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-\cdots}{a+b}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*b*(A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b +
a*Cos[c + d*x]] + (2*(a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]
] - ((2*I)*(-(A*b) + a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*
(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*E
llipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(
a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos
[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b)))/((-a + b)*(a +
 b)) + (4*a*(A*b - a*B)*(b + a*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)))/(2
*b*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.405, size = 1585, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d/b/(a+b)/((a-b)/(a+b))^(1/2)*(A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-A*sin(d*x+c)*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*b-2*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-B*sin(d*x+c)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+B*
cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a+2*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+2*B*s
in(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-A*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*
B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+2*B*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*si
n(d*x+c)+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-A*b*((a-b)/(a+b))^(1/2)+B*a*((a-b)/(a+
b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^
(3/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.464 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cdots}}{ad\left(a^2-b^2\right)\sqrt{\cdots}}$$

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b - a*B)*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.571867, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4027, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+\cdots}{a+b}}}{ad\sqrt{a+b\sec\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b - a*B)*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{1}{2}(aA-bB)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{A\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} + \frac{(Ab-aB)\int \sqrt{}}{a\left(a^2\right.}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\frac{b}{a+b}}}}{a\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

Mathematica [A] time = 0.69517, size = 161, normalized size = 0.75

$$\frac{2\sqrt{\sec(c+dx)}\left(A\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+a(aB-Ab)\sin(c+dx)-(a+b)(aB-Ab)\sqrt{\frac{a\cos}{}}\right.}{ad(a-b)(a+b)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a + b)*(-(A*b) + a*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]) + A*(a^2 - b^2)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-(A*b) +
a*B)*Sin[c + d*x]))/(a*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.392, size = 941, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d/a/(a+b)/((a-b)/(a+b))^(1/2)*(A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+A*sin(d*x+c)*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*b+B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+A*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*sin(d*x+c)+A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-B*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-A*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*b+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+A*b*((a-b)/(a+b)
)^(1/2)-B*a*((a-b)/(a+b))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.465 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{2(2Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A\right.}{}$$

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(
a^2*A - 2*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.579098, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4030, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(2Ab-aB)\sqrt{\mathrm{s}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(
a^2*A - 2*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2A+2Ab^2-abB\right)+\frac{1}{2}a(Ab-aB)\sec(c+d\ldots}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2} + \frac{\left(a^2\ldots\right.}{\ldots}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{b+a\cos(c+dx)}\sqrt{\sec\ldots}\right.}{a^2\sqrt{a+b\sec(c\ldots}}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+\ldots}\right.}{a^2\sqrt{a+b\sec(c+\ldots}}$$

$$= -\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-2Ab^2\ldots\right.}{a^2\left(a^2\ldots\right.}$$

Mathematica [A] time = 1.00067, size = 178, normalized size = 0.76

$$-\frac{2\sqrt{\sec(c+dx)}\left(-\left(a^2-b^2\right)(aB-2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-(a+b)\left(a^2A+abB-2Ab^2\right)\sqrt{\frac{a\cos(}{}}\right.}{a^2d(a-b)(a+b)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (-2*Sqrt[Sec[c + d*x]]*(-((a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]) - (a^2 - b^2)*(-
2*A*b + a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a
)/(a + b)] + a*b*(-(A*b) + a*B)*Sin[c + d*x]))/(a^2*(a - b)*(a + b)*d*Sqrt[
a + b*Sec[c + d*x]])
```

Maple [B] time = 0.421, size = 1452, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)/(a+b)/a^2*(A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b-A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^2-B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+A*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-B
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*sin(d*x+c)-B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2
-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
2-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
*b+A*((a-b)/(a+b))^(1/2)*a*b+2*A*((a-b)/(a+b))^(1/2)*b^2-B*((a-b)/(a+b))^(1
```

```
/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

3.466 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$\frac{2\left(a^2A-6abB+8Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+}}$$

```
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c +
d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a
^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^
2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.835639, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A-6ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c +
d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a
^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^
2 - b^2)*d*Sqrt[Sec[c + d*x]])
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
```

```
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2A+4Ab^2-3abB\right)+\frac{1}{2}a(Ab}{\sec^{\frac{3}{2}}(c+d}}{a\left(\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a}}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a}}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a}}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a}}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2\left(a^2A+8Ab^2-6abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(\right.}{}$$

Mathematica [A] time = 1.57627, size = 252, normalized size = 0.77

$$\frac{2\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a^2\left(a^2A-3abB+2Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(-5a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*(a^2*(a^2*A + 2*A*b^2 - 3*a*b*B)*EllipticF[(c + d*x)/2, (2*
a)/(a + b)] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*((a + b)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))
+ a*(a - b)*(a + b)*(b*(a^2*A - 4*A*b^2 + 3*a*b*B) + a*A*(a^2 - b^2)*Cos[c
+ d*x])*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.37, size = 2285, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/a^3/(a+b)/((a-b)/(a+b))^(1/2)*(8*A*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
```

```
x+c)+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*sin(d*x+c)+6*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+A*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^3+8*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-5*A*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b-6*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-6*B*cos(d*x+c)*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b^2-A*a^2*b*((a-b)/(a+b))^(1/2)+4*A*a*b^2*((a-b)/(a+b))^(1/2)-3*B*a^2
*b*((a-b)/(a+b))^(1/2)-6*B*a*b^2*((a-b)/(a+b))^(1/2)+A*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a^2*b+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-3*B*cos(d*x+c)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*a^3+6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*A*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
6*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-6*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*b^3*((a-b)/(a+b))^(1/2)-4
*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-3*B*a^3*((a-b)/(a+b))^(1/2)*cos(d
*x+c)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/
2)*b^3+A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+8*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+6*B*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b
+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3)*
((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d
*x+c)/(b+a*cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

3.467 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{2\left(12a^2Ab-5a^3B-40ab^2B+48Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^4d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+5abB-6Ab^2\right)\text{s}}{5a^2d\left(a^2-\right.}$$

```
[Out] (-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^
4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*
a^3*b*B + 40*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]
^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2
*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(15*a^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.22216, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A+5abB-6Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(9a^2Ab-5a^3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^
4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*
a^3*b*B + 40*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]
^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2
*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(15*a^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2A+6Ab^2-5abB\right)+\frac{1}{2}a(Ab-aB}{\sec^{\frac{5}{2}}(c+dx)\sqrt{}}}{a\left(a^2\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-6Ab^2+5abB\right)\sqrt{a+b}}{5a^2\left(a^2-b^2\right)d\,\mathrm{se}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-6Ab^2+5abB\right)\sqrt{a+b}}{5a^2\left(a^2-b^2\right)d\,\mathrm{se}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-6Ab^2+5abB\right)\sqrt{a+b}}{5a^2\left(a^2-b^2\right)d\,\mathrm{se}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-6Ab^2+5abB\right)\sqrt{a+b}}{5a^2\left(a^2-b^2\right)d\,\mathrm{se}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-6Ab^2+5abB\right)\sqrt{a+b}}{5a^2\left(a^2-b^2\right)d\,\mathrm{se}}$$

$$= -\frac{2\left(12a^2Ab+48Ab^3-5a^3B-40ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(}}{15a^4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.17327, size = 316, normalized size = 0.75

$$-\frac{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(a^2\left(3a^2Ab-5a^3B-10ab^2B+12Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] -((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(2*(a^2 - b^2)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*(a^2*(3*a^2*A*b + 12*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)] - (9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^
3*b*B + 40*a*b^3*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(a - b)*(a + b)*(30*b^3*(-(A*b) + a
*B)*Sin[c + d*x] + 2*(a^2 - b^2)*(9*A*b - 5*a*B)*(b + a*Cos[c + d*x])*Sin[c
 + d*x] - 3*a*A*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[2*(c + d*x)])))/(15*a^
4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.458, size = 3156, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)

[Out] -2/15/d/a^4/(a+b)/((a-b)/(a+b))^(1/2)*(-9*A*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*A*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-48*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-12*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-9*A*a^3*b*(
(a-b)/(a+b))^(1/2)-24*A*a*b^3*((a-b)/(a+b))^(1/2)-5*B*a^3*b*((a-b)/(a+b))^(
1/2)+20*B*a^2*b^2*((a-b)/(a+b))^(1/2)+40*B*a*b^3*((a-b)/(a+b))^(1/2)+3*A*co
s(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4
+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4+40*B*sin(d*x+c)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*
b^2-25*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+40*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-9
*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*a^4-48*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+5*B*sin(d*x
+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*a^4-12*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-36*A*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-48*A
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-6*A*cos(d
*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-20*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3
*b+48*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-5*B*cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a^4-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+9*A*sin(d*x+c)*cos(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^4+24*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+30*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+40*B*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)-25*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+40*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A*cos(d*x+c)^4*((a-b)
/(a+b))^(1/2)*a^3*b-6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+5*B*cos(d*
x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b
+24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-20*B*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a^2*b^2+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-18*A*cos(d*x+c)*((
```

```
a-b)/(a+b))^(1/2)*a^2*b^2+20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-40*B*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-48*A*b^4*((a-b)/(a+b))^(1/2)-36*A*sin(d*
x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a^2*b^2-48*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^3+24*A*sin(d*x+c
)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^2+30*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b+5*B*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/
sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^5+2\,ab\sec(dx+c)^4+a^2\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2)), x)
```

3.468 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=399

$$\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a\left(3a^3B-7\ldots\right.}{3b^2\ldots}$$

```
[Out] (2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*S
ec[c + d*x]])
```

Rubi [A] time = 1.37446, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {4029, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\ldots}{3bd\left(a^2-b^2\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*S
ec[c + d*x]])
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-aB)-\frac{3}{2}b(Ab-aB)\sec\cdots\right.}{(a+b\sec(c+dx\cdots}}{3b\left(a^2-b\cdots\right.}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7ab^2B\right)\sqrt{\sec(c\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec\cdots}}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7ab^2B\right)\sqrt{\sec(c\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec\cdots}}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7ab^2B\right)\sqrt{\sec(c\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec\cdots}}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7ab^2B\right)\sqrt{\sec(c\cdots}}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec\cdots}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+\cdots}{3b\left(a^2-b^2\right)d(a+b\,\mathrm{s}\cdots}$$

$$= \frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\cdots}{b^2\cdots}$$

Mathematica [C] time = 6.82696, size = 726, normalized size = 1.82

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{2\left(aAb\sin(c+dx)-a^2B\sin(c+dx)\right)}{3b\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}-\frac{2\left(-7a^2b^2B\sin(c+dx)+3a^4B\sin(c+dx)+4aAb^3\sin(c+dx)\right)}{3b^2\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^{5/2}} + \frac{\sec^{\frac{5}{2}}(c\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*((2*(2*a^2*A*b^2 + 6*A*b^4 +
 4*a^3*b*B - 12*a*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(4*a*A*b^3 + 9*a^4*B
- 19*a^2*b^2*B + 6*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(4*a*A*b^3 +
 3*a^4*B - 7*a^2*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[
c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ellipti
cF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)
] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b +
a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(6*(a - b)^2*b^2*(a + b)^2*d
*(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*(
(-2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos
[c + d*x])^2) - (2*(4*a*A*b^3*Sin[c + d*x] + 3*a^4*B*Sin[c + d*x] - 7*a^2*b
^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b
```

```
*Sec[c + d*x])^(5/2))
```

Maple [C] time = 0.408, size = 5195, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.469 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2Ab+a^3B-5ab^2B+2Ab^3\right)\sin(c+dx)\sqrt{\sec(c}}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(2*a^2*A*b + 2*A*b^
3 + a^3*B - 5*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*
d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.842217, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4029, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(2a^2Ab+a^3B-5ab^2B+2Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\sec(c}}{3ad\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(2*a^2*A*b + 2*A*b^
3 + a^3*B - 5*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*
d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{3}{2}b(Ab-aB)\sec(c+dx)+\frac{1}{2}(2aAb}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx}}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-5ab^2B\right)\sqrt{\sec}}{3b\left(a^2-b^2\right)^2d\sqrt{a+b\sec(}}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-5ab^2B\right)\sqrt{\sec}}{3b\left(a^2-b^2\right)^2d\sqrt{a+b\sec(}}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-5ab^2B\right)\sqrt{\sec}}{3b\left(a^2-b^2\right)^2d\sqrt{a+b\sec(}}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-5ab^2B\right)\sqrt{\sec}}{3b\left(a^2-b^2\right)^2d\sqrt{a+b\sec(}}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(3a^2A+Ab^2-4\right.}{3a\left(a^2-\right.}$$

Mathematica [A] time = 2.2445, size = 217, normalized size = 0.66

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{2\sin(c+dx)(a\cos(c+dx)+b)\left(a\left(3a^2A-4abB+Ab^2\right)\cos(c+dx)+2a^2Ab+a^3B-5ab^2B+2Ab^3\right)}{\left(a^2-b^2\right)^2} - \frac{2(a+b)\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{5/2}\left(\left(3a^2A-4abB+Ab^2\right)E\left(\frac{1}{2}\right.\right.}{a(}\right.}{3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*a
^2*A + A*b^2 - 4*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(-(
A*b) + a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b +
 a*Cos[c + d*x])*(2*a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B + a*(3*a^2*A + A*
b^2 - 4*a*b*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*d*(a + b*Sec[
c + d*x])^(5/2))
```

Maple [B] time = 0.401, size = 3138, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*(-A*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*x
+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
```

```
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*a^2*b-3*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-B*a^3*((a-b)/(a+b)
)^(1/2)-A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
*b+4*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-A*cos(d*x+c)*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-3*A*
cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*b-A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-2*B*cos(d*x+c
)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*a^2*b+4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+4*B*cos(d*x+c)*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b^2-2*A*a^2*b*((a-b)/(a+b))^(1/2)+A*a*b^2*((a-b)/(a+b))^(1/2)-B*a
^2*b*((a-b)/(a+b))^(1/2)+4*B*a*b^2*((a-b)/(a+b))^(1/2)-A*cos(d*x+c)^2*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^2*b+3*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+B*cos(d*x+c)*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3
+3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+B*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+4*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*b^3*((a-b)/(a+b))^(1/2)-A*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a^2*b-3*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+A*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*A*
cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*a^3-3*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-A*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*b^3-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+4*B*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a^2*b-4*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+3*A*cos(d*x+c)
```

```
^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3-3*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+B*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a^3+B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3)*((b+a*cos(d*x+c))/cos(d*
x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(B\sec(dx+c)^2+A\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(
sec(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*
x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.470 $\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$\frac{2\left(3a^2A-abB-2Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(5a^2Ab-2a^3B-2ab^2B-Ab^3\right)\sin(c+\ldots}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c\ldots}}$$

```
[Out] (2*(3*a^2*A - 2*A*b^2 - a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)
^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3
/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.821523, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4027, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(5a^2Ab-2a^3B-2ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(3a^2A-abB-2\ldots\right.}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(3*a^2*A - 2*A*b^2 - a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)
^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3
/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
```

```
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{3}{2}(aA-bB)\sec(c+dx)+(Ab-aB)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}}{3(a^2-b^2)}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2B)\sqrt{\mathrm{se}}}{3a(a^2-b^2)^2\,d\sqrt{a+b\sec}}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2B)\sqrt{\mathrm{se}}}{3a(a^2-b^2)^2\,d\sqrt{a+b\sec}}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2B)\sqrt{\mathrm{se}}}{3a(a^2-b^2)^2\,d\sqrt{a+b\sec}}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B-2ab^2B)\sqrt{\mathrm{se}}}{3a(a^2-b^2)^2\,d\sqrt{a+b\sec}}$$

$$= \frac{2(3a^2A-2Ab^2-abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(6a^2}{}$$

Mathematica [A] time = 2.04244, size = 245, normalized size = 0.71

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{a\sin(c+dx)\left(a\left(-6a^2Ab+3a^3B+ab^2B+2Ab^3\right)\cos(c+dx)+b\left(-5a^2Ab+2a^3B+2ab^2B+Ab^3\right)\right)}{(a^2-b^2)^2} - \frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left((-6a\right.}{}\right.}{3a^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(-((((b + a*Cos[c + d*x])/(a + b
))^(3/2)*((-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)] - (a - b)*(3*a^2*A - 2*A*b^2 - a*b*B)*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]))/(a - b)^2) + (a*(b*(-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b
^2*B) + a*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c +
d*x])/(a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.411, size = 3865, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^2*(-B*cos(d*x+c)^2*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
a^3*b-2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
```

```
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-2*A*cos(d*x+c)^2*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2
*b^2-B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-2*A*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+5*A*a^2*b^2*(
(a-b)/(a+b))^(1/2)-A*a*b^3*((a-b)/(a+b))^(1/2)-2*B*a^3*b*((a-b)/(a+b))^(1/2
)+B*a^2*b^2*((a-b)/(a+b))^(1/2)-B*a*b^3*((a-b)/(a+b))^(1/2)-B*sin(d*x+c)*co
s(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a^2*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-B*sin(d*x+c)*cos(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^2-B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+3*A*sin(d*x+c)*cos(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4
-2*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+3*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4-3*B*sin(d*
x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^4+3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-B*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^3*b+6*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-3*A*sin(d*x+
c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a^3*b+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4+2*A*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b^4-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+6*A*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-B*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b
+3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+6*A*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a^3*b-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-2*A*cos(d*x+c)*((a-b)/
```

```
(a+b))^(1/2)*a*b^3+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-B*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^2*b^2+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-2*A*b^4*((
a-b)/(a+b))^(1/2)-5*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-2*A*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a*b^3+6*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+6*A*sin(d*x+c)*cos(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*b^2-2*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+2*B*sin(d*x+c)*cos(d*x+c)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b
+3*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4-3*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*B*co
s(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*a^4)*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+
c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.471 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{2\left(9a^2Ab-3a^3B+2ab^2B-8Ab^3\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(8a^2Ab-5a^3B+ab^2B-\right.}{3a^2d\left(a^2-b^2\right)^2}$$

```
[Out] (-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a
^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^
4 + 6*a^3*b*B - 2*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*
a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.936826, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4030, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(9a^2Ab-3a^3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a
^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^
4 + 6*a^3*b*B - 2*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*
a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx = \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3a^2A+4Ab^2-abB)+\frac{3}{2}a(Ab-aB)\sec(c+}{\sqrt{\sec(c+dx)}(a+b\sec(c+}}{3a(a^2-b^2)}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-5a^3B+ab^2B)\sqrt{}}{3a^2(a^2-b^2)^2d\sqrt{a+b\mathrm{s}}}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-5a^3B+ab^2B)\sqrt{}}{3a^2(a^2-b^2)^2d\sqrt{a+b\mathrm{s}}}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-5a^3B+ab^2B)\sqrt{}}{3a^2(a^2-b^2)^2d\sqrt{a+b\mathrm{s}}}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-5a^3B+ab^2B)\sqrt{}}{3a^2(a^2-b^2)^2d\sqrt{a+b\mathrm{s}}}$$

$$- \frac{2(9a^2Ab-8Ab^3-3a^3B+2ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{3a^3(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.35227, size = 297, normalized size = 0.81

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(-\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(a^2\left(-\left(-6a^2Ab+3a^3B+ab^2B+2Ab^3\right)\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-\left(-15a^2Ab^2+3a^4A+6a^3bB-2ab^3\right.\right.}{(a-b)^2(a+b)}\right.}{3a^3d(a+b\sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(-((((b + a*Cos[c + d*x])/(a + b
))^(3/2)*(-(a^2*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticF[(c + d
*x)/2, (2*a)/(a + b)]) - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*
a*b^3*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c +
d*x)/2, (2*a)/(a + b)])))/((a - b)^2*(a + b))) - (a*b*(b*(-8*a^2*A*b + 4*A*
b^3 + 5*a^3*B - a*b^2*B) + a*(-9*a^2*A*b + 5*A*b^3 + 6*a^3*B - 2*a*b^2*B)*C
os[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5
/2))
```

Maple [B] time = 0.464, size = 5169, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^4+3\,ab^2\sec(dx+c)^3+3\,a^2b\sec(dx+c)^2+a^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*sec(d*x + c)^4 + 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*
sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

3.472 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=472

$$\frac{2\left(16a^2Ab^2+a^4A-9a^3bB+8ab^3B-16Ab^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(-13a^2Ab^2+a^4\right.}{}$$

```
[Out] (2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(3*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2
*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c +
d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (
2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2
- b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^4*A - 13*a^
2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.40857, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{2b\left(10a^2Ab-7a^3B+3ab^2B-6Ab^3\right)\sin}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```



```
[Out] (2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(3*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2
*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c +
d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (
2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2
- b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^4*A - 13*a^
2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
```

```
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2A-2Ab^2+abB\right)+\frac{3}{2}a(Ab-}{\sec^{\frac{3}{2}}(c+dx}}{3a\,(}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+}}$$

$$= \frac{2\left(a^4A+16a^2Ab^2-16Ab^4-9a^3bB+8ab^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.90866, size = 353, normalized size = 0.75

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(a^2\left(7a^2Ab^2+a^4A-6a^3bB+2ab^3B-4Ab^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(28a^2Ab^3-8a^4Ab-15a^3b^2B+3\right.\right.}{(a-b)^2(a+b)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*(a^2*(a^4*A + 7*a^2*A*b^2 - 4*A*b^4 - 6*a^3*b*B + 2*a*b^3*B)*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)] + (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*
```

```
a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])))/((a - b)^2*(a + b)) + (a
*(a^6*A - 25*a^2*A*b^4 + 16*A*b^6 + 16*a^3*b^3*B - 8*a*b^5*B + 2*a*b*(2*a^4
*A - 16*a^2*A*b^2 + 10*A*b^4 + 9*a^3*b*B - 5*a*b^3*B)*Cos[c + d*x] + A*(a^3
 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a
 + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.541, size = 6745, normalized size = 14.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^5+3\,ab^2\sec(dx+c)^4+3\,a^2b\sec(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*sec(d*x + c)^5 + 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*
sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

3.473 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2\left(116a^2Ab^3+17a^4Ab-80a^3b^2B-5a^5B+80ab^4B-128Ab^5\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) +
 (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*
a^3*b^3*B - 80*a*b^5*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c
+ d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a
^3*B + 5*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)*S
qrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A*b^4 + 50*a^3*b
*B - 30*a*b^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)
^2*d*Sec[c + d*x]^(3/2)) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5
*B + 65*a^3*b^2*B - 40*a*b^4*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*
a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.87864, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(-71a^2Ab^2+3a^4A+50a^3bB-30ab^3B+48Ab^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(12a^2Ab-9a^3B+5ab^2B-8A\ldots\right.}{3a^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) +
 (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*
a^3*b^3*B - 80*a*b^5*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c
+ d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a
^3*B + 5*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)*S
qrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A*b^4 + 50*a^3*b
*B - 30*a*b^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)
^2*d*Sec[c + d*x]^(3/2)) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5
*B + 65*a^3*b^2*B - 40*a*b^4*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*
a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
```

```
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3a^2A+8Ab^2-5abB\right)+\cdots}{\sec^{\frac{5}{2}}\cdots}}{\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sec^{\frac{3}{2}}(c\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sec^{\frac{3}{2}}(c\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sec^{\frac{3}{2}}(c\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sec^{\frac{3}{2}}(c\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sec^{\frac{3}{2}}(c\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sec^{\frac{3}{2}}(c\cdots}$$

$$= -\frac{2\left(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B\right)\sqrt{\frac{b+a\cos(c\cdots}{a+b}}\cdots}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 3.74211, size = 392, normalized size = 0.67

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(a\left(\frac{10b^4(Ab-aB)\sin(c+dx)}{b^2-a^2}-\frac{10b^3\left(-15a^2Ab+12a^3B-8ab^2B+11Ab^3\right)\sin(c+dx)(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^2}-2(14Ab\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((-2*((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*(a^2*(8*a^4*A*b + 44*a^2*A*b^3 - 32*A*b^5 - 5*a^5*B - 35*a^3*b^2*B
+ 20*a*b^4*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - (9*a^6*A + 55*a^4*A*b
^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(
(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])))/((a - b)^2*(a + b)) + a*((10*b^4*(A*b - a*B)*Sin[c + d*x])/
(-a^2 + b^2) - (10*b^3*(-15*a^2*A*b + 11*A*b^3 + 12*a^3*B - 8*a*b^2*B)*(b +
 a*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 2*(14*A*b - 5*a*B)*(b + a*Co
s[c + d*x])^2*Sin[c + d*x] + 3*a*A*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])
))/(15*a^5*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.81, size = 8251, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^6+3\,ab^2\sec(dx+c)^5+3\,a^2b\sec(dx+c)^4+a^3\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*sec(d*x + c)^6 + 3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*
sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

3.474 $\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$A\text{Unintegrable}\left((a+b\sec(c+dx))^{2/3},x\right)+\frac{\sqrt{2}B\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\ldots}{\ldots}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Unintegrable[(a + b*S
ec[c + d*x])^(2/3), x]
```

Rubi [A] time = 0.157717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Defer[Int][(a + b*Sec
[c + d*x])^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = A\int (a+b\sec(c+dx))^{2/3}\,dx+B\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx-\frac{(B\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\ldots\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+\ldots}}$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx-\frac{\left(B(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Su}\ldots}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec\ldots}}$$

$$= \frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{\ldots}}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] time = 21.1818, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]
```

Maple [A] time = 0.145, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^{\frac{2}{3}} (A + B\sec(dx + c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx + c) + A)(b\sec(dx + c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec(c + dx))\,(a + b\sec(c + dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx + c) + A)(b\sec(dx + c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3), x)
```

3.475 $\int \sqrt[3]{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$A\text{Unintegrable}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)+\frac{\sqrt{2}B\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-s}{}\right.}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Unintegrable[(a + b*S
ec[c + d*x])^(1/3), x]
```

Rubi [A] time = 0.145576, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Defer[Int][(a + b*Sec
[c + d*x])^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = A\int \sqrt[3]{a+b\sec(c+dx)}\,dx + B\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$$

$$= A\int \sqrt[3]{a+b\sec(c+dx)}\,dx - \frac{(B\tan(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+}}$$

$$= A\int \sqrt[3]{a+b\sec(c+dx)}\,dx - \frac{\left(B\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\text{Subs}}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(}}$$

$$= \frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+d}}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [A] time = 17.8291, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]), x]
```

Maple [A] time = 0.139, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(dx+c)}\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3), x)
```

3.476 $\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$A\text{Unintegrable}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)+\frac{\sqrt{2}B\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Unintegrable[(a + b*Se
c[c + d*x])^(-1/3), x]
```

Rubi [A] time = 0.176441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Defer[Int][(a + b*Sec[
c + d*x])^(-1/3), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx &= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx+B\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx\\
&= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx-\frac{(B\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\\
&= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx-\frac{\left(B\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+}}\\
&= \frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}+A\int \frac{}{\sqrt[3]{a+b}}
\end{aligned}$$

Mathematica [A] time = 3.33825, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3), x]
```

Maple [A] time = 0.165, size = 0, normalized size = 0.

$$\int (A + B\sec(dx + c))\frac{1}{\sqrt[3]{a + b\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx + c) + A}{(b\sec(dx + c) + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{\sqrt[3]{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(1/3), x)
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3), x)
```

3.477 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=125

$$A\text{Unintegrable}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)+\frac{\sqrt{2}B\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Unintegrable[(a + b*Se
c[c + d*x])^(-2/3), x]
```

Rubi [A] time = 0.16203, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Defer[Int][(a + b*Sec[
c + d*x])^(-2/3), x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx+B\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx-\frac{(B\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx,x,\sec(c-\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx-\frac{\left(B\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x}\sqrt{1+x}(-}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}(a+}$$

$$= \frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}+A\int$$

Mathematica [A] time = 3.33016, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3), x]
```

Maple [A] time = 0.181, size = 0, normalized size = 0.

$$\int (A + B\sec(dx+c))\,(a + b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c) + A}{(b\sec(dx+c) + a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c+dx)}{(a + b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3), x)
```

3.478 $\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx$

Optimal. Leaf size=35

$$\text{Unintegrable}\left((A+B\sec(e+fx))(c\sec(e+fx))^n(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x])
, x]
```

Rubi [A] time = 0.0929956, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),
x]
```

Rubi steps

$$\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx = \int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx)$$

Mathematica [A] time = 4.26979, size = 0, normalized size = 0.

$$\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] Integrate[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]), x
]
```

Maple [A] time = 1.197, size = 0, normalized size = 0.

$$\int \left(c\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^m\left(A+B\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

```
[Out] int((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(fx+e)+A)(b\sec(fx+e)+a)^m(c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\sec(fx+e)+A)(b\sec(fx+e)+a)^m(c\sec(fx+e))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))**n*(a+b*sec(f*x+e))**m*(A+B*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(fx+e)+A)(b\sec(fx+e)+a)^m(c\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

3.479 $\int \sec^m(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=544

$$-\frac{\sin(c+dx)\left(6a^2Ab^2m(m+3)+a^4A\left(m^2+4m+3\right)+4a^3bBm(m+3)+4ab^3Bm(m+2)+Ab^4m(m+2)\right)\sec^{m-1}(c+d}{d(1-m)(m+1)(m+3)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (b*(A*b^3*(8 + 6*m + m^2) + 4*a*b^2*B*(8 + 6*m + m^2) + 2*a^3*B*(19 + 8*m +
 m^2) + a^2*A*b*(68 + 37*m + 5*m^2))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*
(1 + m)*(3 + m)*(4 + m)) + (b^2*(b^2*B*(3 + m)^2 + 2*a*A*b*(4 + m)^2 + a^2*
B*(26 + 9*m + m^2))*Sec[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)*(
4 + m)) + (b*(A*b*(4 + m) + a*B*(7 + m))*Sec[c + d*x]^(1 + m)*(a + b*Sec[c
+ d*x])^2*Sin[c + d*x])/(d*(3 + m)*(4 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a
+ b*Sec[c + d*x])^3*Sin[c + d*x])/(d*(4 + m)) - ((A*b^4*m*(2 + m) + 4*a*b^3
*B*m*(2 + m) + 6*a^2*A*b^2*m*(3 + m) + 4*a^3*b*B*m*(3 + m) + a^4*A*(3 + 4*m
 + m^2))*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m)*(1 + m)*(3 + m)*Sqrt[Sin[c + d*x]
^2]) + ((b^4*B*(3 + 4*m + m^2) + 4*a*A*b^3*(4 + 5*m + m^2) + 6*a^2*b^2*B*(4
 + 5*m + m^2) + 4*a^3*A*b*(8 + 6*m + m^2) + a^4*B*(8 + 6*m + m^2))*Hypergeo
metric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x]
)/(d*m*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 1.63038, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4026, 4096, 4076, 4047, 3772, 2643, 4046}

$$-\frac{\sin(c+dx)\left(6a^2Ab^2m(m+3)+a^4A\left(m^2+4m+3\right)+4a^3bBm(m+3)+4ab^3Bm(m+2)+Ab^4m(m+2)\right)\sec^{m-1}(c+d}{d(1-m)(m+1)(m+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*(A*b^3*(8 + 6*m + m^2) + 4*a*b^2*B*(8 + 6*m + m^2) + 2*a^3*B*(19 + 8*m +
 m^2) + a^2*A*b*(68 + 37*m + 5*m^2))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*
(1 + m)*(3 + m)*(4 + m)) + (b^2*(b^2*B*(3 + m)^2 + 2*a*A*b*(4 + m)^2 + a^2*
B*(26 + 9*m + m^2))*Sec[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)*(
4 + m)) + (b*(A*b*(4 + m) + a*B*(7 + m))*Sec[c + d*x]^(1 + m)*(a + b*Sec[c
+ d*x])^2*Sin[c + d*x])/(d*(3 + m)*(4 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a
+ b*Sec[c + d*x])^3*Sin[c + d*x])/(d*(4 + m)) - ((A*b^4*m*(2 + m) + 4*a*b^3
*B*m*(2 + m) + 6*a^2*A*b^2*m*(3 + m) + 4*a^3*b*B*m*(3 + m) + a^4*A*(3 + 4*m
 + m^2))*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m)*(1 + m)*(3 + m)*Sqrt[Sin[c + d*x]
^2]) + ((b^4*B*(3 + 4*m + m^2) + 4*a*A*b^3*(4 + 5*m + m^2) + 6*a^2*b^2*B*(4
 + 5*m + m^2) + 4*a^3*A*b*(8 + 6*m + m^2) + a^4*B*(8 + 6*m + m^2))*Hypergeo
metric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x]
)/(d*m*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{d(4+m)} + \frac{\int \sec^m(\ldots}{\ldots}$$

$$= \frac{b(Ab(4+m)+aB(7+m))\sec^{1+m}(c+dx)(a+b\sec(c+dx)\ldots}{d(3+m)(4+m)}$$

$$= \frac{b^2\left(b^2B(3+m)^2+2aAb(4+m)^2+a^2B\left(26+9m+m^2\right)\right)\sec\ldots}{d(2+m)\left(12+7m+m^2\right)}$$

$$= \frac{b^2\left(b^2B(3+m)^2+2aAb(4+m)^2+a^2B\left(26+9m+m^2\right)\right)\sec\ldots}{d(2+m)\left(12+7m+m^2\right)}$$

$$= \frac{b\left(Ab^3\left(8+6m+m^2\right)+4ab^2B\left(8+6m+m^2\right)+2a^3B\left(19+\ldots\right.\right.}{d(1+m)\left(\ldots\right.}$$

$$= \frac{b\left(Ab^3\left(8+6m+m^2\right)+4ab^2B\left(8+6m+m^2\right)+2a^3B\left(19+\ldots\right.\right.}{d(1+m)\left(\ldots\right.}$$

$$= \frac{b\left(Ab^3\left(8+6m+m^2\right)+4ab^2B\left(8+6m+m^2\right)+2a^3B\left(19+\ldots\right.\right.}{d(1+m)\left(\ldots\right.}$$

Mathematica [A] time = 4.42286, size = 365, normalized size = 0.67

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\left(\frac{a^3(aB+4Ab)\cos^4(c+dx)\text{Hypergeometric2F1}\left(\ldots\right.}{m+1}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*((a^4*A*Cos[c + d*x]^5*Hypergeometric2F1[1/2, m/2, (2 + m)/2,
 Sec[c + d*x]^2])/m + (a^3*(4*A*b + a*B)*Cos[c + d*x]^4*Hypergeometric2F1[1
/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2])/(1 + m) + b*((2*a^2*(3*A*b + 2*a
*B)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x
]^2])/(2 + m) + b*((2*a*(2*A*b + 3*a*B)*Cos[c + d*x]^2*Hypergeometric2F1[1/
2, (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2])/(3 + m) + b*(((A*b + 4*a*B)*Cos[c
 + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Sec[c + d*x]^2])/(4 +
m) + (b*B*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Sec[c + d*x]^2])/(5
+ m)))))*Sec[c + d*x]^(-1 + m)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*
Sqrt[-Tan[c + d*x]^2])/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))
```

Maple [F] time = 0.804, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (a+b\sec(dx+c))^4 (A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^4\sec(dx+c)^5 + Aa^4 + \left(4\,Bab^3 + Ab^4\right)\sec(dx+c)^4 + 2\left(3\,Ba^2b^2 + 2\,Aab^3\right)\sec(dx+c)^3 + 2\left(2\,Ba^3b + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b^4*sec(d*x + c)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*sec(d*x + c)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec(c+dx))\,(a + b\sec(c+dx))^4\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c) + A)(b\sec(dx+c) + a)^4\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*sec(d*x + c)^m, x)
```

3.480 $\int \sec^m(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=366

$$-\frac{\sin(c+dx)\left(a^3A\left(m^2+4m+3\right)+3a^2bBm(m+3)+3aAb^2m(m+3)+b^3Bm(m+2)\right)\sec^{m-1}(c+dx)\text{Hypergeometri}}{d(m+3)\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (b*(b^2*B*(2 + m) + 3*a*A*b*(3 + m) + 2*a^2*B*(4 + m))*Sec[c + d*x]^(1 + m)
*Sin[c + d*x])/(d*(1 + m)*(3 + m)) + (b^2*(A*b*(3 + m) + a*B*(5 + m))*Sec[c
 + d*x]^(2 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (b*B*Sec[c + d*x]^(1 +
m)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(d*(3 + m)) - ((b^3*B*m*(2 + m) + 3
*a*A*b^2*m*(3 + m) + 3*a^2*b*B*m*(3 + m) + a^3*A*(3 + 4*m + m^2))*Hypergeom
etric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*S
in[c + d*x])/(d*(3 + m)*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b^3*(1 + m) +
 3*a*b^2*B*(1 + m) + 3*a^2*A*b*(2 + m) + a^3*B*(2 + m))*Hypergeometric2F1[1
/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*m*(2 +
 m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.786254, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4026, 4076, 4047, 3772, 2643, 4046}

$$-\frac{\sin(c+dx)\left(a^3A\left(m^2+4m+3\right)+3a^2bBm(m+3)+3aAb^2m(m+3)+b^3Bm(m+2)\right)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\right.}{d(m+3)\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```


```
[Out] (b*(b^2*B*(2 + m) + 3*a*A*b*(3 + m) + 2*a^2*B*(4 + m))*Sec[c + d*x]^(1 + m)
*Sin[c + d*x])/(d*(1 + m)*(3 + m)) + (b^2*(A*b*(3 + m) + a*B*(5 + m))*Sec[c
 + d*x]^(2 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (b*B*Sec[c + d*x]^(1 +
m)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(d*(3 + m)) - ((b^3*B*m*(2 + m) + 3
*a*A*b^2*m*(3 + m) + 3*a^2*b*B*m*(3 + m) + a^3*A*(3 + 4*m + m^2))*Hypergeom
etric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*S
in[c + d*x])/(d*(3 + m)*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b^3*(1 + m) +
 3*a*b^2*B*(1 + m) + 3*a^2*A*b*(2 + m) + a^3*B*(2 + m))*Hypergeometric2F1[1
/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*m*(2 +
 m)*Sqrt[Sin[c + d*x]^2])
```


Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{d(3+m)} + \frac{\int \text{se}}{}$$

$$= \frac{b^2(Ab(3+m)+aB(5+m))\sec^{2+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} +$$

$$= \frac{b^2(Ab(3+m)+aB(5+m))\sec^{2+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} +$$

$$= \frac{b\left(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m)\right)\sec^{1+m}(c-}{d(1+m)(3+m)}$$

$$= \frac{b\left(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m)\right)\sec^{1+m}(c-}{d(1+m)(3+m)}$$

$$= \frac{b\left(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m)\right)\sec^{1+m}(c-}{d(1+m)(3+m)}$$

Mathematica [A] time = 2.20831, size = 307, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\left(\frac{a^2(aB+3Ab)\cos^3(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\ldots\right)}{m+1}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*((a^3*A*Cos[c + d*x]^4*Hypergeometric2F1[1/2, m/2, (2 + m)/2,
 Sec[c + d*x]^2])/m + (a^2*(3*A*b + a*B)*Cos[c + d*x]^3*Hypergeometric2F1[1
/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2])/(1 + m) + b*((3*a*(A*b + a*B)*Co
s[c + d*x]^2*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2])/
(2 + m) + b*(((A*b + 3*a*B)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2,
(5 + m)/2, Sec[c + d*x]^2])/(3 + m) + (b*B*Hypergeometric2F1[1/2, (4 + m)/2
, (6 + m)/2, Sec[c + d*x]^2])/(4 + m))))*Sec[c + d*x]^(-1 + m)*(a + b*Sec[c
 + d*x])^3*(A + B*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2])/(d*(b + a*Cos[c + d*
x])^3*(B + A*Cos[c + d*x]))
```

Maple [F] time = 0.623, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (a+b\sec(dx+c))^3 (A+B\sec(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^3\sec(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\sec(dx+c)^2 + \left(Ba^3 + 3\,Aa^2b\right)\sec\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sec (c + dx)) (a + b\sec (c + dx))^3 \sec^m (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx + c) + A)(b\sec (dx + c) + a)^3 \sec (dx + c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

3.481 $\int \sec^m(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=261

$$-\frac{\sin(c+dx)\left(a^2A(m+1)+2abBm+Ab^2m\right)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(c+dx)\right)}{d\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}+\frac{\sin(c+}{}$$

```
[Out] (b*(A*b*(2 + m) + a*B*(3 + m))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m
)*(2 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(
d*(2 + m)) - ((A*b^2*m + 2*a*b*B*m + a^2*A*(1 + m))*Hypergeometric2F1[1/2,
(1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(
d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((b^2*B*(1 + m) + a*(2*A*b + a*B)*(2 +
m))*Hypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*
Sin[c + d*x])/(d*m*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.406208, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4026, 4047, 3772, 2643, 4046}

$$-\frac{\sin(c+dx)\left(a^2A(m+1)+2abBm+Ab^2m\right)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}+\frac{\sin(c+dx)\left(a(m+2)(aB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*(A*b*(2 + m) + a*B*(3 + m))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m
)*(2 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(
d*(2 + m)) - ((A*b^2*m + 2*a*b*B*m + a^2*A*(1 + m))*Hypergeometric2F1[1/2,
(1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(
d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((b^2*B*(1 + m) + a*(2*A*b + a*B)*(2 +
m))*Hypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*
Sin[c + d*x])/(d*m*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{d(2+m)} + \frac{\int \sec}{}$$

$$= \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{d(2+m)} + \frac{\int \sec}{}$$

$$= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)} +$$

$$= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)} +$$

$$= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)} +$$

Mathematica [A] time = 0.963626, size = 239, normalized size = 0.92

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m+2}(c+dx)\left(a^2A\left(m^3+6m^2+11m+6\right)\cos^3(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{m}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(a^2*A*(6 + 11*m + 6*m^2 + m^3)*Cos[c + d*x]^3*Hypergeometric
2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2] + a*(2*A*b + a*B)*m*(6 + 5*m + m^2
)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^
2] + b*m*(1 + m)*((A*b + 2*a*B)*(3 + m)*Cos[c + d*x]*Hypergeometric2F1[1/2,
 (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2] + b*B*(2 + m)*Hypergeometric2F1[1/2,
 (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2]))*Sec[c + d*x]^(2 + m)*Sqrt[-Tan[c +
 d*x]^2])/(d*m*(1 + m)*(2 + m)*(3 + m))
```

Maple [F] time = 1.058, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{m} (a+b\sec (dx+c))^{2} (A+B\sec (dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)(b\sec (dx+c)+a)^{2} \sec (dx+c)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^{2} \sec (dx+c)^{3}+Aa^{2}+\left(2\,Bab+Ab^{2}\right)\sec (dx+c)^{2}+\left(Ba^{2}+2\,Aab\right)\sec (dx+c)\right)\sec (dx+c)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec (c+dx))\,(a+b\sec (c+dx))^{2} \sec^{m} (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)(b\sec (dx+c)+a)^{2} \sec (dx+c)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

3.482 $\int \sec^m(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=177

$$-\frac{\sin(c+dx)(aA(m+1)+bBm)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(c+dx)\right)}{d\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}+\frac{(aB+Ab)\sin(c+d$$

```
[Out] (b*B*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m)) - ((b*B*m + a*A*(1 + m)
)*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]
^(-1 + m)*Sin[c + d*x])/(d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b + a*B)*H
ypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c
 + d*x])/(d*m*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.201102, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3997, 3787, 3772, 2643}

$$-\frac{\sin(c+dx)(aA(m+1)+bBm)\sec^{m-1}(c+dx)\,_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}+\frac{(aB+Ab)\sin(c+dx)\sec^m(c+dx)\,_2F_1}{dm\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*B*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m)) - ((b*B*m + a*A*(1 + m)
)*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]
^(-1 + m)*Sin[c + d*x])/(d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b + a*B)*H
ypergeometric2F1[1/2, -m/2, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c
 + d*x])/(d*m*Sqrt[Sin[c + d*x]^2])
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} + \frac{\int \sec^m(c+dx)(bBm+a\ldots}{}$$
$$= \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} + (Ab+aB)\int \sec^{1+m}(c+\ldots$$
$$= \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} + ((Ab+aB)\cos^m(c+dx)\ldots$$
$$= \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} - \frac{\left(aA+\frac{bBm}{1+m}\right){}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.387213, size = 168, normalized size = 0.95

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m+1}(c+dx)\left(m(m+2)(aB+Ab)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sec\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(a*A*(2 + 3*m + m^2)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, m/
2, (2 + m)/2, Sec[c + d*x]^2] + (A*b + a*B)*m*(2 + m)*Cos[c + d*x]*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2] + b*B*m*(1 + m)*Hyperg
eometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^(1 + m
)*Sqrt[-Tan[c + d*x]^2])/(d*m*(1 + m)*(2 + m))
```

Maple [F] time = 0.559, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (a+b\sec(dx+c))(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sec(d*x + c)
^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\sec^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^m, x)
```

3.483 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{2a(5A+7B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(5A+$$

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.229494, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(5A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(5A+7B)\sin(}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(B+A\cos(c+dx))\,dx \\
&= \int \cos^{\frac{3}{2}}(c+dx)\left(aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+d\right. \\
&= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(5A+\right. \\
&= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+(a(A+B))\int \cos^{\frac{5}{2}}(c+dx)\,d \\
&= \frac{2a(5A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a(A+B)\cos^{\frac{3}{2}}(c+}{5d} \\
&= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+
\end{aligned}$$

Mathematica [C] time = 6.28382, size = 872, normalized size = 6.61

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{3(A+B)\cot(c)}{5d}+\frac{(23A+28B)\cos(dx)\sin(c)}{84d}+\frac{(A+B)\cos(2dx)\sin(2c)}{10d}+\frac{A\cos(3\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)*
Cot[c])/(5*d) + ((23*A + 28*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d*x
```

```
]*Sin[2*c])/(10*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) + ((23*A + 28*B)*Cos[c]
*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (A*Cos[3*c]*Sin[
3*d*x])/(28*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*
d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2
]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(10*d))
```

Maple [B] time = 1.923, size = 383, normalized size = 2.9

$$-\frac{2a}{105d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Bigg(240\,A\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8+(-528\,A-168$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(448*A+308*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-12
2*A-112*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)^3\sec(dx+c)^2+(A+B)a\cos(dx+c)^3\sec(dx+c)+Aa\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^3*sec(d*x + c)^2 + (A + B)*a*cos(d*x + c)^3*sec(
d*x + c) + A*a*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.484 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(c}{}$$

```
[Out] (2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.209603, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(c+dx)\,c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))(B+A\cos(c+dx))\,dx$$
$$= \int \sqrt{\cos(c+dx)}\left(aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}a(3A+\right.$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \cos^{\frac{3}{2}}(c+dx)\,d$$
$$= \frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sqrt{\cos(c+dx)}\sin($$
$$= \frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} +$$

Mathematica [C] time = 6.20442, size = 830, normalized size = 8.22

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(3A+5B)\cot(c)}{5d}+\frac{(A+B)\cos(dx)\sin(c)}{3d}+\frac{A\cos(2dx)\sin(2c)}{10d}+\frac{(A+B)\cos(c)\sin\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((3*A + 5*B
)*Cot[c])/(5*d) + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[2*d*x]*Sin[2*c])
/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (A*Cos[2*c]*Sin[2*d*x])/(10*d))
 - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
```

```
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^
2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[
c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) -
(B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.14, size = 355, normalized size = 3.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(44\,A+20\,B\right)\left(s\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-16*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)^2\sec(dx+c)^2+(A+B)a\cos(dx+c)^2\sec(dx+c)+Aa\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2*sec(d*x + c)^2 + (A + B)*a*cos(d*x + c)^2*sec(
d*x + c) + A*a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.485 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=70

$$\frac{2a(A+3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.185505, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2954, 2968, 3023, 2748, 2641, 2639}

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a(A+3B)+\frac{3}{2}a(A+}{\sqrt{\cos(c+d}}$$
$$= \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + (a(A+B))\int \sqrt{\cos(c+dx)}$$
$$= \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + $$

Mathematica [C] time = 5.91764, size = 309, normalized size = 4.41

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(-4(A+3B)\sin(c)\sqrt{\csc^2(c)}\sqrt{\sec^2(c)}\cos(c+dx)\sqrt{\cos^2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-6*(A + B)*HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c
]]] + (9*(A + B)*Cos[c - d*x - ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*A*Cos[c +
d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*B*Cos[c + d*x + ArcTan[Tan[c]]]*Csc
[c]*Sec[c] - 12*A*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 12*B*Cos[c + d*x]*Co
t[c]*Sqrt[Sec[c]^2] - 4*(A + 3*B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]
]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 4*A*Cos[c + d
*x]*Sqrt[Sec[c]^2]*Sin[c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Maple [B] time = 2.037, size = 321, normalized size = 4.6

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-2*A*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\cos(dx+c)\sec(dx+c)^2+(A+B)a\cos(dx+c)\sec(dx+c)+Aa\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)*sec(d*x + c)^2 + (A + B)*a*cos(d*x + c)*sec(d*x
+ c) + A*a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.486 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=66

$$\frac{2a(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*x
)/2, 2])/d + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.194115, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2954, 2968, 3021, 2748, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*x
)/2, 2])/d + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{1}{2}a(A+B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,d\\ &= \frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+(a(A-B))\int \sqrt{\cos(c+dx)}\,dx+(a(A+B\\ &= \frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2}{d}\end{aligned}$$

Mathematica [C] time = 6.074, size = 252, normalized size = 3.82

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2(A-B)\sec(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right)}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}}-4(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((Csc[c]*(3*(A - B)*Cos[c - d*x -
ArcTan[Tan[c]]]*Sec[c] + (A - B)*Cos[c + d*x + ArcTan[Tan[c]]]*Sec[c] - 2*(
(A - 2*B)*Cos[d*x] + A*Cos[2*c + d*x])*Sqrt[Sec[c]^2]))/Sqrt[Sec[c]^2] - 4*
(A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcT
an[Cot[c]]]*Sin[c] - (2*(A - B)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(Sqrt[Sec[c]^2]*
Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 2.013, size = 240, normalized size = 3.6

$$-2\frac{a\left(A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -2*a*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)*sqrt(cos(d*x +
 c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sqrt{\cos(c+dx)}\,dx+\int A\sqrt{\cos(c+dx)}\sec(c+dx)\,dx+\int B\sqrt{\cos(c+dx)}\sec(c+dx)\,dx+\int B\sqrt{\cos(c+dx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(cos(c + d*x)), x) + Integral(A*sqrt(cos(c + d*x))*sec(c
+ d*x), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(B*sq
rt(cos(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.487 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2a(3A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.217369, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}a(A+B)+\frac{1}{2}a(3A+B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(a(3A+B))\int \frac{}{\sqrt{\mathrm{co}}} \\
&= \frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \\
&= -\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aB\sin(c+}{3d\cos^{\frac{3}{2}}(c+}
\end{aligned}$$

Mathematica [C] time = 6.33821, size = 813, normalized size = 8.56

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{B\sec(c)\sin(dx)\sec^2(c+dx)}{3d} + \frac{\sec(c)(B\sin(c)+3A\sin(dx)+3B\sin(dx))\sec(c+dx)}{3d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((A + B)*Csc
[c]*Sec[c])/d + (B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c +
```

```
d*x]*(B*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c + d*x
])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*
x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (B*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 4.806, size = 426, normalized size = 4.5

$$-4\,\frac{\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)(\sin(1/2\,dx+c/2))^2}a}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(1/2\,\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)*(-(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\sec\left(dx+c\right)^2+(A+B)a\sec\left(dx+c\right)+Aa}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/sqrt(cos(d*x +
 c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{A\sec\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{B\sec\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{B\sec^2\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(A*sec(c + d*x)/sqrt(cos(c +
 d*x)), x) + Integral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(B*se
c(c + d*x)**2/sqrt(cos(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.488 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{2a(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.232189, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aB\ldots}{5d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\
&= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\
&= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\frac{5}{2}a(A+B)+\frac{1}{2}a(5A+3B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+(a(A+B))\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{5}(a(5A+3B))\int \frac{}{\text{co}}\\
&= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{1}{3}\\
&= -\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aB\sin(c}{5d\cos^{\frac{5}{2}}(c}
\end{aligned}$$

Mathematica [C] time = 6.38727, size = 865, normalized size = 6.55

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{B\sec(c)\sin(dx)\sec^3(c+dx)}{5d}+\frac{\sec(c)(3B\sin(c)+5A\sin(dx)+5B\sin(dx))\sec^2(c+d}{15d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 3*B)
*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*B*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec[c]
*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 15*A*Sin[d*x] + 9*B*Sin[d*x]))/(15
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot
[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
Cot[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sq
rt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
+ (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin
[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) +
 (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-
1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*T
an[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan
[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c
]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sq
rt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 5.993, size = 661, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*((1/2*A+1/2*
B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*B/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2
*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x + c) + A*a)/cos(d*x + c)^(
3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.489 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(5A+6B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(11A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+$$

```
[Out] (4*a^2*(8*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 6*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(8*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(11*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*A*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.403935, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(11A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(8A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (4*a^2*(8*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 6*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(8*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(11*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*A*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(B+A\cos(c+dx))\,dx$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d} + \frac{2}{9}\int \mathrm{co}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d} + \frac{2}{9}\int \mathrm{co}$$

$$= \frac{2a^2(11A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= \frac{2a^2(11A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= \frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4a^2(8A+9B)\cos}{4}$$

$$= \frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|\right.}{21d}$$

Mathematica [C] time = 6.34127, size = 1086, normalized size = 5.6

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-((8*A + 9*B)*Cot[c])/(15*d) + ((46*A + 51*B)*Cos[d*x]*Sin[c])/(
168*d) + ((37*A + 36*B)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((2*A + B)*Cos[3*d*x
]*Sin[3*c])/(56*d) + (A*Cos[4*d*x]*Sin[4*c])/(144*d) + ((46*A + 51*B)*Cos[c
]*Sin[d*x])/(168*d) + ((37*A + 36*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((2*A +
 B)*Cos[3*c]*Sin[3*d*x])/(56*d) + (A*Cos[4*c]*Sin[4*d*x])/(144*d)))/(B + A*
Cos[c + d*x]) - (5*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])
^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c
]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A +
B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (
4*A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A +
B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]))/(15*d*(B + A*Cos[c + d*x])) - (3*B*Cos[c + d*x]^3*Csc[
c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(1
0*d*(B + A*Cos[c + d*x]))
```

Maple [A] time = 1.789, size = 413, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1568*A+1134*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-351*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
```

```
(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ba^2\cos(dx+c)^4\sec(dx+c)^3 + (A+2B)a^2\cos(dx+c)^4\sec(dx+c)^2 + (2A+B)a^2\cos(dx+c)^4\sec(dx+c\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^4*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^
4*sec(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)^4*sec(d*x + c) + A*a^2*cos(d*
x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x
)
```

3.490 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=161

$$\frac{4a^2(6A+7B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(9A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+$$

```
[Out] (4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.362177, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(9A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(6A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2(B+A\cos(c+dx))\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{}$$

$$= \frac{2a^2(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\left(\right.}{}$$

$$= \frac{2a^2(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\left(\right.}{}$$

$$= \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7B)\sqrt{\cos(c+dx}}{21d}$$

$$= \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

Mathematica [C] time = 6.2486, size = 1040, normalized size = 6.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-((3*A + 4*B)*Cot[c])/(5*d) + ((51*A + 56*B)*Cos[d*x]*Sin[c])/(1
68*d) + ((2*A + B)*Cos[2*d*x]*Sin[2*c])/(20*d) + (A*Cos[3*d*x]*Sin[3*c])/(5
6*d) + ((51*A + 56*B)*Cos[c]*Sin[d*x])/(168*d) + ((2*A + B)*Cos[2*c]*Sin[2*
d*x])/(20*d) + (A*Cos[3*c]*Sin[3*d*x])/(56*d)))/(B + A*Cos[c + d*x]) - (2*A
*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*
x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
an[Cot[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*
x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (3*A*Cos[c + d*x]^3*Csc[
c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(1
0*d*(B + A*Cos[c + d*x])) - (2*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(B + A*Cos[c + d*x])
)
```

Maple [A] time = 1.744, size = 385, normalized size = 2.4

$$-\frac{4a^2}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+(-348\,A-84\,B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(378*A+224*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
17*A-91*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
```

```
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-84*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Ba^2\cos(dx+c)^3\sec(dx+c)^3 + (A+2B)a^2\cos(dx+c)^3\sec(dx+c)^2 + (2A+B)a^2\cos(dx+c)^3\sec(dx+c\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^
3*sec(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)^3*sec(d*x + c) + A*a^2*cos(d*
x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x
)
```

3.491 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=126

$$\frac{4a^2(A+2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2A}{}$$

```
[Out] (4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```

Rubi [A] time = 0.347456, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2A\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```



```
[Out] (4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}}{}$$

$$= \frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A\sqrt{\cos(c+dx)}\left(\right.}{}$$

$$= \frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2A\sqrt{\cos(c+dx)}\left(\right.}{}$$

$$= \frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

Mathematica [C] time = 6.30096, size = 994, normalized size = 7.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-((4*A + 5*B)*Cot[c])/(5*d) + ((2*A + B)*Cos[d*x]*Sin[c])/(6*d)
```

```
+ (A*Cos[2*d*x]*Sin[2*c])/(20*d) + ((2*A + B)*Cos[c]*Sin[d*x])/(6*d) + (A*C
os[2*c]*Sin[2*d*x])/(20*d)))/(B + A*Cos[c + d*x]) - (A*Cos[c + d*x]^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B
 + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(B + A*Cos[c + d
*x])) - (B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^
2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 1.776, size = 357, normalized size = 2.8

$$-\frac{4a^2}{15d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\,A\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6+(32\,A+10\,B)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(32*A+10*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-13*A-5*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^2\sec(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2\sec(dx+c)^2 + (2A+B)a^2\cos(dx+c)^2\sec(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^2*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^
2*sec(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)^2*sec(d*x + c) + A*a^2*cos(d*
x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

3.492 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{4a^2(2A+3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+$$

```
[Out] (4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a^2*(A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.336789, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)\left(a^2 c\right.}{d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a^2*(A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+d}{}$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a^2(A+3B)+\Big(}{}$$

$$= \frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B\left(a^2+a^2\cos(c+\right.}{d\sqrt{\cos(c}}$$

$$= \frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B\left(a^2+a^2\cos(c+\right.}{d\sqrt{\cos(c}}$$

$$= \frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a}{}$$

Mathematica [C] time = 6.37429, size = 735, normalized size = 6.34

$$-\frac{A\csc(c)\cos^3(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2(A+B\sec(c+dx))\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{Hypergeometri}}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\text{ta}\right.}}\right.}{2d(A\cos(c+dx)+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-((2*A - B + 2*A*Cos[2*c] + B*Cos[2*c])*Csc[c]*Sec[c])/(4*d) + (
A*Cos[d*x]*Sin[c])/(6*d) + (A*Cos[c]*Sin[d*x])/(6*d) + (B*Sec[c]*Sec[c + d*
x]*Sin[d*x])/(2*d)))/(B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*C
os[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]) - (A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*
Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 1.988, size = 388, normalized size = 3.3

$$-\frac{4a^2}{3d}\left(2A\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4-\sqrt{-2\left(\sin\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -4/3*a^2*(2*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*A*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\cos(dx+c)\sec(dx+c)^3 + (A+2B)a^2\cos(dx+c)\sec(dx+c)^2 + (2A+B)a^2\cos(dx+c)\sec(dx+c) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)*sec(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)*se
c(d*x + c)^2 + (2*A + B)*a^2*cos(d*x + c)*sec(d*x + c) + A*a^2*cos(d*x + c)
)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c) + A)(a\sec(dx+c) + a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

3.493 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=120

$$\frac{4a^2(3A+2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)\left(a^2\cos(c\right.}{3d\cos^{\frac{3}{2}}(c+d}$$

```
[Out] (-4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(3*A + 5*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.350451, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)\left(a^2\cos(c+dx)+\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(3*A + 5*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+}{}$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a^2(3A+5B)}{}$$

$$= \frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} +$$

Mathematica [C] time = 6.42647, size = 736, normalized size = 6.13

$$\frac{B\csc(c)\cos^3(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2(A+B\sec(c+dx))\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricF}}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(\right.}}\right.}{2d(A\cos(c+dx)+B)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-((-A - 4*B + A*Cos[2*c])*Csc[c]*Sec[c])/(4*d) + (B*Sec[c]*Sec[c
 + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] +
 6*B*Sin[d*x]))/(6*d)))/(B + A*Cos[c + d*x]) - (A*Cos[c + d*x]^3*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x
])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 2.253, size = 513, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+2*B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(3*A+7*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ba^2\sec(dx+c)^3 + (A+2B)a^2\sec(dx+c)^2 + (2A+B)a^2\sec(dx+c) + Aa^2\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c) + A)(a\sec(dx+c) + a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

3.494 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{4a^2(2A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(5*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.380816, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(5*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+dx))\left(\frac{1}{2}a(5\right.}{\text{c}}$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(5A+7B)+\left(\frac{1}{2}a^2(5A\right.}{}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{1}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}$$

$$= \frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A}{5d\sqrt{}}$$

$$= -\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a}{}$$

Mathematica [C] time = 6.51973, size = 1025, normalized size = 6.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(((5*A + 4*B)*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^3*Sin
[d*x])/(10*d) + (Sec[c]*Sec[c + d*x]^2*(3*B*Sin[c] + 5*A*Sin[d*x] + 10*B*Si
n[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*A*Sin[c] + 10*B*Sin[c] + 30*A*Sin
[d*x] + 24*B*Sin[d*x]))/(30*d)))/(B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*
Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x
)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c +
 d*x])) + (2*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*
x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
```

```
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(B + A*Cos[c + d*x]))
```

Maple [B] time = 6.256, size = 741, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/20*B/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+(1/2*A+1/4*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2 \sec(dx+c)^3 + (A+2B)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{A}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{2A\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{A\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{2B\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(2*A*sec(c + d*x)/sqrt(co
s(c + d*x)), x) + Integral(A*sec(c + d*x)**2/sqrt(cos(c + d*x)), x) + Integ
ral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(2*B*sec(c + d*x)**2/sq
rt(cos(c + d*x)), x) + Integral(B*sec(c + d*x)**3/sqrt(cos(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

3.495 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(7A+6B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(7A+9B)\sin(}{35d\cos^{\frac{5}{2}}(c+}$$

```
[Out] (-4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(4*A + 3*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*B*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.415113, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(4*A + 3*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*B*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+dx))\left(\frac{1}{2}a(7A\right.}{\cos}$$

$$= \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a^2(7A+9B)+\left(\frac{1}{2}a^2(7A+\right.}{}$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4}{35}\int$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{5}\left(2\right.$$

$$= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(4A+3B)\sin}{5d\sqrt{\cos(c}}$$

$$= -\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^2(}{3}$$

Mathematica [C] time = 6.55533, size = 1067, normalized size = 5.5

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(((4*A + 3*B)*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^4*Sin
[d*x])/(14*d) + (Sec[c]*Sec[c + d*x]^3*(5*B*Sin[c] + 7*A*Sin[d*x] + 14*B*Si
n[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]^2*(21*A*Sin[c] + 42*B*Sin[c] + 70*A*
Sin[d*x] + 60*B*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 30
*B*Sin[c] + 84*A*Sin[d*x] + 63*B*Sin[d*x]))/(105*d)))/(B + A*Cos[c + d*x])
- (A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c
+ d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[
c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (2*A*Cos[c + d*x]^
3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(
(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
 Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]))/(5*d*(B + A*Cos[c + d*x])) + (3*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)
/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2
```

```
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(B + A*Cos[c +
 d*x]))
```

Maple [B] time = 7.114, size = 851, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(-1
/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2
*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/4*A+1/2*B)/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+1/4*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ba^2\sec(dx+c)^3+(A+2B)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+Aa^2}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*sec(d*x + c)^3 + (A + 2*B)*a^2*sec(d*x + c)^2 + (2*A + B)*a
^2*sec(d*x + c) + A*a^2)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

3.496 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$-\frac{5(A-B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A-5B)\sin}{}$$

```
[Out] (3*(7*A - 5*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*(A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a*d) - (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d
) + ((7*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A - B)*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.26462, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A-5B)\sin(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```


```
[Out] (3*(7*A - 5*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*(A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a*d) - (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d
) + ((7*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A - B)*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a(7A-5\right.}{a^2}$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(7A-5B)\int \cos^{\frac{5}{2}}(c+dx)\,dx}{2a} - \frac{(5(A-B)}{}$$

$$= -\frac{5(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{(7A-5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{(A}{}$$

$$= \frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5(A-B)\sqrt{\cos(c+d}}{3ad}$$

Mathematica [C] time = 6.60087, size = 1292, normalized size = 8.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((21*I)/20)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*
((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
+ I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d
*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*S
in[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Si
n[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Si
n[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(
(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (((3*I)/4)*B*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
```

```
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[
c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*
((2*(-5*A + 5*B - 16*A*Cos[c] + 10*B*Cos[c])*Csc[c])/(5*d) + (4*(-A + B)*Co
s[d*x]*Sin[c])/(3*d) + (2*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*(-A + B)*Cos[c]*S
in[d*x])/(3*d) + (2*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((B + A*Cos[c + d*x])*(a
 + a*Sec[c + d*x])) + (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x
])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3
*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 1.886, size = 282, normalized size = 1.8

$$-\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/2*d*x+1/2*c
)^6+(-30*A+90*B)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B)*sin(1/2*d*x+1/2*c)^2)/a/c
os(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(cos(d*x +
c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.497 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{(5A-3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c$$

```
[Out] (-3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
 - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.244492, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2748, 2639, 2635, 2641}

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sqrt{c}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
 - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(5A-3\right.}{a^2}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(5A-3B)\int\cos^{\frac{3}{2}}(c+dx)\,dx}{2a} - \frac{(3(A-B)}{}$$

$$= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(A-B)\cos}{d(a}$$

$$= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-3B)\sqrt{\cos(c+}}{3ad}$$

Mathematica [C] time = 6.53272, size = 1239, normalized size = 9.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((
B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
```

```
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c
 + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(
(-2*(-A + B)*(1 + 2*Cos[c])*Csc[c])/d + (4*A*Cos[d*x]*Sin[c])/(3*d) - (2*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*A*Co
s[c]*Sin[d*x])/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (5*A*C
os[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos
[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 +
 Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.127, size = 262, normalized size = 2.1

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B)*sin(1/2*d*x+1/2*c)^4+(-7*A
+3*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.498 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{(A-B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] ((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - B)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rubi [A] time = 0.220289, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2954, 2977, 2748, 2641, 2639}

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```


```
[Out] ((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - B)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= \int \frac{\sqrt{\cos(c+dx)}(B+A\cos(c+dx))}{a+a\cos(c+dx)}\,dx\\ &= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(3A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}\\ &= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{(3A-B)\int \sqrt{\cos(c+dx)}\,dx}{2a}\\ &= \frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [C] time = 6.4759, size = 1208, normalized size = 13.73

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((
2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin
[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[
c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B
 + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - ((I/4)*B*Cos[c/2 + (d*x)/2]^2*Cs
c[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*
x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*((-2*(
A - B + 2*A*Cos[c])*Csc[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*
x)/2]) + B*Sin[(d*x)/2]))/d))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) +
 (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*
```

```
Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (B*Cos[c/2 + (d*x)
/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 1.775, size = 244, normalized size = 2.8

$$\frac{1}{ad}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\cos(c+dx)}}{\sec(c+dx)+1}\,dx + \int \frac{B\sqrt{\cos(c+dx)}\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sqrt(cos
(c + d*x))*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.499 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=83

$$\frac{(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] -(((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A + B)*EllipticF[(c + d*x)
/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
 d*x]))
```

Rubi [A] time = 0.222453, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2954, 2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A + B)*EllipticF[(c + d*x)
/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
 d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$$
$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A+B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$
$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A-B)\int \sqrt{\cos(c+dx)}\,dx}{2a} + \frac{(A+B)\int \frac{1}{\sqrt{\text{co}}}}{2a}$$
$$= -\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\text{si}}{d(a+a\cos(c+d}$$

Mathematica [C] time = 6.45122, size = 1204, normalized size = 14.51

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((-I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B +
A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + ((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c
/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)
*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Co
s[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] +
d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])
) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*((-2*(-A
+ B)*Csc[c])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[
(d*x)/2]))/d))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (A*Cos[c/2 + (
d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*S
```

```
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(
a + a*Sec[c + d*x]))
```

Maple [A] time = 1.912, size = 243, normalized size = 2.9

$$-\frac{1}{ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)\sec\left(dx+c\right)+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)*sec(d*x +
c) + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx + \int \frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) + In
tegral(B*sec(c + d*x)/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))
), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

3.500 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=113

$$\frac{(A-B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+$$

```
[Out] ((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A - B)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A - 3*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.239635, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2978, 2748, 2636, 2639, 2641}

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] ((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A - B)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A - 3*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{(A-3B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(A-B)\int \frac{1}{\sqrt{\cos(c}}}{2a}$$

$$= \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+}$$

$$= \frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{}{d}$$

Mathematica [C] time = 6.6631, size = 1240, normalized size = 10.97

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^
((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Si
n[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
- (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^
2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])
/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((
-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Cs
c[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
```

```
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*
x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(((2*B
 - A*Cos[c] + B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 +
 (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*B*Sec[c]*Sec[c + d*x
]*Sin[d*x])/d))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (A*Cos[c/2 +
(d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]
*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*
(a + a*Sec[c + d*x]))
```

Maple [A] time = 4.232, size = 318, normalized size = 2.8

$$-\frac{1}{ad}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{-2\,(\sin(1/2\,dx+c/2))^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-3*B)*sin(1/2*d*x+1/2*c)^4-(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-5*B)*sin(1/2*d*x+1/2*c)^
2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2\sec(dx+c)+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^2*sec(d*x
+ c) + a*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

3.501 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=152

$$-\frac{(3A-5B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(}{3ad\cos^{\frac{3}{2}}(c}$$

```
[Out] (-3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c +
d*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
 + (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)*Sin[c + d*x
])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.25962, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2978, 2748, 2636, 2641, 2639}

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c +
d*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
 + (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)*Sin[c + d*x
])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(3A-5B)+\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(3A-5B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx}{2a} + \frac{(3(A-B))\int -}{2a}$$

$$= -\frac{(3A-5B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+}$$

$$= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{(3A-5B)\sin(c+dx}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 6.98705, size = 1277, normalized size = 8.4

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((
B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
```

```
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c
 + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(
-(((-A + B)*(2 + Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*B*Sec[c]*Sec[c + d
*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] - 3
*B*Sin[d*x]))/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (A*Cos[
c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (5*B*Cos[c/2 + (d*x)/2]^2*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 +
Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 5.406, size = 493, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*B*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-A+B)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+(2*A-2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3\sec(dx+c)+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^3*sec(d*x
+ c) + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

3.502 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=204

$$-\frac{5(3A-2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{7(8A-5}{}$$

```
[Out] (7*(8*A - 5*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A - 2*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*a^2*d) + (7*(8*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d
) - ((3*A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])
) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.418915, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{7(8A-5B)\sin(}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (7*(8*A - 5*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A - 2*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*a^2*d) + (7*(8*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d
) - ((3*A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])
) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{7}{2}a(A-B)+\frac{1}{2}a(11A-5B)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,d}{3a^2}$$

$$= -\frac{(3A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int c}{}$$

$$= -\frac{(3A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(7(8}{}$$

$$= -\frac{5(3A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{7(8A-5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} -$$

$$= \frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(3A-2B)\sqrt{\cos(}}{3a}$$

Mathematica [C] time = 6.83331, size = 1396, normalized size = 6.84

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (((28*I)/5)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Se
c[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2
*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
```

```
d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))
*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (((7*I)/2)*B*Cos
[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B +
A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt
[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqr
t[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[
c + d*x])*((4*(-20*A + 15*B - 36*A*Cos[c] + 20*B*Cos[c])*Csc[c])/(5*d) + (8
*(-2*A + B)*Cos[d*x]*Sin[c])/(3*d) + (4*A*Cos[2*d*x]*Sin[2*c])/(5*d) - (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-4*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/d +
(8*(-2*A + B)*Cos[c]*Sin[d*x])/(3*d) + (4*A*Cos[2*c]*Sin[2*d*x])/(5*d) - (2
*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 2.219, size = 465, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*A*cos(1/2
*d*x+1/2*c)^10-352*A*cos(1/2*d*x+1/2*c)^8+80*B*cos(1/2*d*x+1/2*c)^8+120*A*c
os(1/2*d*x+1/2*c)^6-150*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-33
6*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+60*B*cos(1/2*d*x+1/2*c)^
6+100*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+210*B*cos(1/2*d*x+1/
2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+266*A*cos(1/2*d*x+1/2*c)^4-240*B*cos(1/2*d
*x+1/2*c)^4-135*A*cos(1/2*d*x+1/2*c)^2+105*B*cos(1/2*d*x+1/2*c)^2+5*A-5*B)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos(dx+c)^2\sec(dx+c)+A\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(cos(d*x +
c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x
)
```

3.503 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=171

$$\frac{5(2A-B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{5(2A-B)\sin}{}$$

```
[Out] -(((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (5*(2*A - B)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(3*a^2*d) - ((7*A - 4*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Co
s[c + d*x])) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c
+ d*x])^2)
```

Rubi [A] time = 0.400595, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2748, 2639, 2635, 2641}

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -(((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (5*(2*A - B)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(3*a^2*d) - ((7*A - 4*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Co
s[c + d*x])) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c
+ d*x])^2)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{3}{2}a(3A-B)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(7A-4B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \sqrt{}}{}$$

$$= -\frac{(7A-4B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(7A}{}$$

$$= -\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{(7A-4B}{3a^2}$$

$$= -\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2A-B)\sqrt{\cos(c}}{3a^2}$$

Mathematica [C] time = 6.71788, size = 1352, normalized size = 7.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (((-7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Se
c[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2
*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
```

```
d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))
*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + ((2*I)*B*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^2) - (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*S
ec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1
 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c
+ d*x])*((-4*(-3*A + 2*B - 4*A*Cos[c] + 2*B*Cos[c])*Csc[c])/d + (8*A*Cos[d*
x]*Sin[c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*
Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-3*A*Sin[(d*x)/2] +
2*B*Sin[(d*x)/2]))/d + (8*A*Cos[c]*Sin[d*x])/(3*d) + (2*(-A + B)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a +
a*Sec[c + d*x])^2)
```

Maple [B] time = 2.084, size = 435, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*cos(1/2*
d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+42*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*co
s(1/2*d*x+1/2*c)^6-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*A*cos(1/2*d*x+1/2*c)^4+3
8*B*cos(1/2*d*x+1/2*c)^4+21*A*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{\cos(dx+c)}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x
)
```

3.504 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{(5A-2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(5A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(}{3d(a\,\text{co}}$$

```
[Out] ((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - 2*B)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a
^2*d*(1 + Cos[c + d*x])) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.374605, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2954, 2977, 2748, 2641, 2639}

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(5A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\,\text{c}}{3d(a\cos(c+d\text{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - 2*B)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a
^2*d*(1 + Cos[c + d*x])) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(7A-B)\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(5A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{(5A-2B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - (5A$$

$$= \frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(5A-2B)\sqrt{\cos(c+a}}{3a^2d(1+\cos(c}$$

Mathematica [C] time = 6.6319, size = 1318, normalized size = 9.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - ((I/2)*B*Cos[c/2 + (d
*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^2) + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
```

```
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
+ d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
+ d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x]
)*((-4*(2*A - B + 2*A*Cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-
2*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-
(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c +
 d*x])^2)
```

Maple [B] time = 1.839, size = 421, normalized size = 3.1

$$\frac{1}{6da^2}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+10\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,(\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d
*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*cos(1/
2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4+15*A
*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^
3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sqrt{\cos(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{B\sqrt{\cos(c+dx)}\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) +
 Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*
x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

3.505 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(2A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+a}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -((A*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A + B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rubi [A] time = 0.345724, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2978, 2748, 2641, 2639}

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] -((A*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A + B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(5A+B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a^2(2A+B)-}{\sqrt{\cos}}}{3}$$

$$= \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{A\int\sqrt{\cos(c}}{2a^2}$$

$$= -\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))}$$

Mathematica [C] time = 6.51782, size = 921, normalized size = 7.61

$$-\frac{iA\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec(c+dx)(A+B\sec(c+dx))\left(\frac{2e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i(-\right.}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}\right.}{2(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((-I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c
+ d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*
d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c
```

```
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin
[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (4*A*Cos[c/2 + (d*x)
/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (2*B*Cos[c/2 + (d*x)
/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^
4*(A + B*Sec[c + d*x])*((4*A*Csc[c])/d + (4*A*Sec[c/2]*Sec[c/2 + (d*x)/2]*S
in[(d*x)/2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Si
n[(d*x)/2]))/(3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sq
rt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.116, size = 350, normalized size = 2.9

$$-\frac{1}{6\,d a^2}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\mathrm{cc}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3-20*A*cos(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+9*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)\sec(dx+c)^2+2\,a^2\cos(dx+c)\sec(dx+c)+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)*sec(d*x
+ c)^2 + 2*a^2*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

3.506 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(A+2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+d}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A + 2*B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.355074, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2954, 2978, 2748, 2641, 2639}

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A + 2*B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A+5B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a^2(A+2B)\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{B\int\sqrt{\cos(c\cdots}}{2a^2\cdots}$$

$$= \frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \cdots$$

Mathematica [C] time = 6.51282, size = 921, normalized size = 7.61

$$\frac{iB\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec(c+dx)(A+B\sec(c+dx))\left(\frac{2e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+\cdots\right.\right.}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}\cdots\right.}{2(B\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (2*A*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (4*B*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c
```

```
 + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4
*(A + B*Sec[c + d*x])*((-4*B*Csc[c])/d - (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*S
in[(d*x)/2])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Si
n[(d*x)/2]))/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sq
rt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 1.891, size = 350, normalized size = 2.9

$$-\frac{1}{6\,da^2}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}\mathrm{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+16*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2*sec(d*
x + c)^2 + 2*a^2*cos(d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

3.507 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{(2A-5B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+d$$

```
[Out] ((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A - 5*B)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) - ((A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]
) + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x
])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2
)
```

Rubi [A] time = 0.39664, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2978, 2748, 2636, 2639, 2641}

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A - 5*B)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) - ((A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]
) + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x
])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2
)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-7B)+\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{(2}{}$$

$$= \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(2A-5B)\sin(c+d}{3a^2d\sqrt{\cos(c+dx)}(1+\cos}$$

$$= \frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.72682, size = 1351, normalized size = 8.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
```

```
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - ((2*I)*B*Cos[c/2 + (d
*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^2) - (4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c +
 d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot
[c]^2]*(a + a*Sec[c + d*x])^2) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
+ d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x]
)*((2*(2*B - A*Cos[c] + 2*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) + (4*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + 2*B*Sin[(d*x)/2]))/d + (8*B*
Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2]
)/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.314, size = 492, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A-4*B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-37*B)*sin(1/2*d*x+1/2*c)^2)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3\sec(dx+c)^2+2a^2\cos(dx+c)^3\sec(dx+c)+a^2\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3*sec(d*
x + c)^2 + 2*a^2*cos(d*x + c)^3*sec(d*x + c) + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

3.508 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$-\frac{5(A-2B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}-\frac{5(A-2B)\,\text{s}}{3a^2d\cos^{\frac{3}{2}}}$$

```
[Out] -(((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - (5*(A - 2*B)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d
*x]^(3/2)) + ((4*A - 7*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) + ((4*A
- 7*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) + ((A
- B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.427585, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2978, 2748, 2636, 2641, 2639}

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c+}{3a^2d\cos^{\frac{3}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] -(((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - (5*(A - 2*B)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d
*x]^(3/2)) + ((4*A - 7*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) + ((4*A
- 7*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) + ((A
- B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{3}{2}a(A-3B)+\frac{5}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \ldots$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \ldots$$

$$= -\frac{5(A-2B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4A-7B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(\ldots}$$

$$= -\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(A-2B)\sin(c+\ldots}{3a^2d\cos^{\frac{3}{2}}(c+dx\ldots}$$

Mathematica [C] time = 7.23243, size = 1392, normalized size = 7.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c
+ d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*
```

```
d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin
[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (((7*I)/2)*B*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^2) + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*S
ec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1
 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c
+ d*x])*((-2*(-2*A + 4*B - 2*A*Cos[c] + 3*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c
])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]
))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x
)/2]))/d + (8*B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d
*x]*(B*Sin[c] + 3*A*Sin[d*x] - 6*B*Sin[d*x]))/(3*d) - (2*(-A + B)*Sec[c/2 +
 (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a +
 a*Sec[c + d*x])^2)
```

Maple [B] time = 6.874, size = 750, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-A
+B)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*
c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+
4*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-2*A+4*B)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(4*A-8*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
```

```
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/
2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4\sec(dx+c)^2+2\,a^2\cos(dx+c)^4\sec(dx+c)+a^2\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4*sec(d*
x + c)^2 + 2*a^2*cos(d*x + c)^4*sec(d*x + c) + a^2*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)),
 x)
```

3.509 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{(33A-13B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-1}{}$$

```
[Out] (-7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Ell
ipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*
Cos[c + d*x])^3) - ((2*A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a +
a*Cos[c + d*x])^2) - (7*(17*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.582192, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2748, 2639, 2635, 2641}

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Ell
ipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*
Cos[c + d*x])^3) - ((2*A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a +
a*Cos[c + d*x])^2) - (7*(17*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{7}{2}a(A-B)+\frac{1}{2}a(13A-3B)\cos(c+dx\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \int$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - 7($$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - 7($$

$$= -\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} - (A$$

$$= -\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + (33A-13$$

Mathematica [C] time = 6.90569, size = 1448, normalized size = 6.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (((-119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A +
 B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2
```

```
*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)
*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 +
E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*
x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
 E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((
2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*
d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (((49*I)/10
)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*
x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x)
)*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin
[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])
))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (22*A*Cos[c/2 + (d*x)/2]
^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c +
 d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (26*B*Cos[c/2 + (d*x)/2
]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^
6*(A + B*Sec[c + d*x])*((-4*(-59*A + 29*B - 60*A*Cos[c] + 20*B*Cos[c])*Csc[
c])/(5*d) + (16*A*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5
*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/
2]^3*(-19*A*Sin[(d*x)/2] + 14*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(-59*A*Sin[(d*x)/2] + 29*B*Sin[(d*x)/2]))/(5*d) + (16*A*Cos[c]*
Sin[d*x])/(3*d) + (4*(-19*A + 14*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) -
 (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B
+ A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.1, size = 465, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*cos(1/
2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
48*B*cos(1/2*d*x+1/2*c)^8-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x
+1/2*c)^6+578*B*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1
/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{\cos(dx+c)}}{a^3\sec(dx+c)^3+3a^3\sec(dx+c)^2+3a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x
)
```

3.510 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=188

$$-\frac{(13A-3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B)\sin}{5d(a\,c}$$

```
[Out] ((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*Ellipti
cF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.547795, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2954, 2977, 2748, 2641, 2639}

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B)\sin(c+dx}{5d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```



```
[Out] ((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*Ellipti
cF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a(11A-B)\cos(c+dx\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= \frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B)\cos^{\frac{5}{2}}(c}{5d(a+a}$$

Mathematica [C] time = 6.82387, size = 1415, normalized size = 7.53

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B
*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (((9*I)/10)*B
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])
*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
```

```
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*S
in[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/
((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (26*A*Cos[c/2 + (d*x)/2]^6*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^
6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A
 + B*Sec[c + d*x])*((-4*(29*A - 9*B + 20*A*Cos[c])*Csc[c])/(5*d) + (2*Sec[c
/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*S
ec[c/2]*Sec[c/2 + (d*x)/2]*(-29*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-14*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/
(15*d) - (4*(-14*A + 9*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A +
B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.265, size = 451, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos(1/2
*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8-30*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*B*cos(1/2*d*x+1/2*c)^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+1/2*
c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x
+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^
3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x
)
```

3.511 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$\frac{(3A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+}$$

```
[Out] -((9*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*
d*(a + a*Cos[c + d*x])^2) + ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.539013, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2978, 2748, 2641, 2639}

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] -((9*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*
d*(a + a*Cos[c + d*x])^2) + ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(9A+B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + $$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + $$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + $$

$$= -\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B)\cos^{\frac{3}{2}}(c+}{5d(a+a\cos}$$

Mathematica [C] time = 6.74588, size = 1407, normalized size = 7.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B
*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
```

```
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - ((I/10)*B*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2
*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B
+ A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*
Sec[c + d*x])*((4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]*(9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x
)/2]^3*(-9*A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2]))/(15*d) + (4*(-9*A + 4*B)*Sec
[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c
/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^
3)
```

Maple [B] time = 2.089, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8+10*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6-2*B*cos(1/2*d*x+1/2*c)^6+
114*A*cos(1/2*d*x+1/2*c)^4-24*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c
)^2+17*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)\sec(dx+c)^3+3\,a^3\cos(dx+c)\sec(dx+c)^2+3\,a^3\cos(dx+c)\sec(dx+c)+a^3\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)*sec(d*x
+ c)^3 + 3*a^3*cos(d*x + c)*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)*sec(d*x + c
) + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

3.512 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{(A+B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(4A+B)\sin(c+dx)\sqrt{\cdots}}{15ad(a\cos(c+dx\cdots}$$

```
[Out] -((A - B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B)*EllipticF[(c + d
*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a +
a*Cos[c + d*x])^3) + ((4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
 + a*Cos[c + d*x])^2) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^
3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.522084, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2977, 2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(4A+B)\sin(c+dx)\sqrt{\cos(c+\cdots}}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] -((A - B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B)*EllipticF[(c + d
*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a +
a*Cos[c + d*x])^3) + ((4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
 + a*Cos[c + d*x])^2) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^
3 + a^3*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx=\int \frac{\sqrt{\cos(c+dx)}(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$=-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(7A+3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$=-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+$$

$$=-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+$$

$$=-\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+$$

$$=-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos(c+dx}}{5d(a+a\cos(c}$$

Mathematica [C] time = 6.65469, size = 1406, normalized size = 7.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] ((-I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec
[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*
```

```
x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
 E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((
2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*
I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*
Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + ((I/10)*B*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E^(
(2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
 (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/
E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-
I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*
Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqr
t[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Se
c[c + d*x])*((-4*(-A + B)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-
(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(4*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(15*d) + (4*(4*A + B)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5
*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.285, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8+10*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+22*B*cos(1/2*d*x+1/2*c)^6-24*
A*cos(1/2*d*x+1/2*c)^4-6*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-7
*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^2\sec(dx+c)^3+3\,a^3\cos(dx+c)^2\sec(dx+c)^2+3\,a^3\cos(dx+c)^2\sec(dx+c)+a^3\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^2*sec(d*
x + c)^3 + 3*a^3*cos(d*x + c)^2*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^2*sec(d
*x + c) + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

3.513 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{(A+3B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-6B)\sin(c+d}{15ad(a\cos(c+}$$

```
[Out] ((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.532698, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2954, 2978, 2748, 2641, 2639}

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-6B)\sin(c+dx)\sqrt{\cos(}}{15ad(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```



```
[Out] ((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A+9B)+\frac{3}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{(A}{}$$

$$= \frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B)\sqrt{\cos(c+d}}{5d(a+a\cos(}$$

Mathematica [C] time = 6.70472, size = 1407, normalized size = 7.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[
c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (((9*I)/10)*B*Cos[
c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
```

```
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B +
 A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[
c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*S
ec[c + d*x])*((-4*(A + 9*B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(-(A*Sin[(d*x)/2]) + 6*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2
+ (d*x)/2]*(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) - (4*(-A + 6*B)*Sec[c
/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2
])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.002, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8-30*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6-138*B*cos(1/2*d*x+1/2*c)^6+
6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2
+3*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3\sec(dx+c)^3+3\,a^3\cos(dx+c)^3\sec(dx+c)^2+3\,a^3\cos(dx+c)^3\sec(dx+c)+a^3\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3*sec(d*
x + c)^3 + 3*a^3*cos(d*x + c)^3*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^3*sec(d
*x + c) + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

3.514 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{(3A-13B)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(3A-13B)\sin(c-}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c\right.}$$

```
[Out] ((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*Ellipti
cF[(c + d*x)/2, 2])/(6*a^3*d) - ((9*A - 49*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[
Cos[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^3) + ((3*A - 8*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a
*Cos[c + d*x])^2) + ((3*A - 13*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^
3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.577353, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2978, 2748, 2636, 2639, 2641}

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(3A-13B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] ((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*Ellipti
cF[(c + d*x)/2, 2])/(6*a^3*d) - ((9*A - 49*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[
Cos[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[
c + d*x])^3) + ((3*A - 8*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a
*Cos[c + d*x])^2) + ((3*A - 13*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^
3 + a^3*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-11B)+\frac{5}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} + \frac{(A-B)\sin}{5d\sqrt{\cos(c+dx)}(a}$$

$$= \frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(9A-49B)\sin}{10a^3d\sqrt{\cos(}}$$

Mathematica [C] time = 6.97275, size = 1447, normalized size = 6.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*
Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (((49*I)/10)*B
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])
*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*S
in[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/
((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*C
sc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x
])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (26*B*Cos[c/2 + (d*x)/2]^6*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A
 + B*Sec[c + d*x])*((2*(20*B - 9*A*Cos[c] + 29*B*Cos[c])*Csc[c/2]*Sec[c/2]*
Sec[c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin
[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-6*A*Sin[(d*x)/2] + 1
1*B*Sin[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-9*A*Sin[(d*x)/
2] + 29*B*Sin[(d*x)/2]))/(5*d) + (16*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4
*(-6*A + 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2
+ (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a
+ a*Sec[c + d*x])^3)
```

Maple [B] time = 2.533, size = 685, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
```

```
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A-49*B)*s
in(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(147*A-817*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(43*A-248*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(69*A-439*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d
*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^4\sec(dx+c)^3+3\,a^3\cos(dx+c)^4\sec(dx+c)^2+3\,a^3\cos(dx+c)^4\sec(dx+c)+a^3\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4*sec(d*
x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^4*sec(d
*x + c) + a^3*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)),
 x)
```

3.515 $\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=220

$$\frac{2a(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (32*a*(8*A + 9*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Sec[c + d*x]]) + (4*a*(8*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(7/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.475591, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4015, 3805, 3804}

$$\frac{2a(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (32*a*(8*A + 9*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Sec[c + d*x]]) + (4*a*(8*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(7/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec}{\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}\left((8A+9B)\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$= \frac{2a(8A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin}{9d\sqrt{a+a\sec(c}}$$

$$= \frac{4a(8A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(8A+9B)\cos^{\frac{5}{2}}(c}{63d\sqrt{a+a\,s}}$$

$$= \frac{16a(8A+9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{4a(8A+9B)\cos^{\frac{3}{2}}(}{105d\sqrt{a+a}}$$

$$= \frac{32a(8A+9B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a(8A+9B)\sqrt{\cos(}}{315d\sqrt{a+a\,s}}$$

Mathematica [A] time = 0.484972, size = 119, normalized size = 0.54

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}(94(8A+9B)\cos(c+dx)+4(83A+54B)\cos(2(c+dx))+80A\cos(3(c+d}{1260d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(1321*A + 1368*B + 94*(8*A + 9*B)*Cos[c + d*x] + 4*(83*
A + 54*B)*Cos[2*(c + d*x)] + 80*A*Cos[3*(c + d*x)] + 90*B*Cos[3*(c + d*x)]
+ 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(1260*d*(
1 + Cos[c + d*x]))
```

Maple [A] time = 0.34, size = 130, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(35A(\cos(dx+c))^4+40A(\cos(dx+c))^3+45B(\cos(dx+c))^3+48A(\cos(dx+c))^2+54B(c\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+40*A*cos(d*x+c)^3+45*B*cos(d*x+
c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+64*A*cos(d*x+c)+72*B*cos(d*x+c)+12
8*A+144*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.06703, size = 738, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(1890*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c)))*sin(9/2*d*x + 9/2*c) + 420*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 252*cos(4/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 45*cos(2/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 1890*c
os(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2
*c))) - 420*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) - 252*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) - 45*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*sin(9/2*d*x + 9/2*c) + 45*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) - 18*sqrt(2)*(7*(15*sin(3*
d*x + 3*c) + 5*sin(2*d*x + 2*c) + sin(d*x + c))*cos(7/2*arctan2(sin(d*x + c
), cos(d*x + c))) - (105*cos(3*d*x + 3*c) + 35*cos(2*d*x + 2*c) + 7*cos(d*x
 + c) + 10)*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 7*sin(5/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 35*sin(3/2*arctan2(sin(d*x + c), cos(d*x +
 c))) - 105*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.485765, size = 309, normalized size = 1.4

$$\frac{2\left(35\,A\cos\left(dx+c\right)^{4}+5\,(8\,A+9\,B)\cos\left(dx+c\right)^{3}+6\,(8\,A+9\,B)\cos\left(dx+c\right)^{2}+8\,(8\,A+9\,B)\cos\left(dx+c\right)+128\,A+\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*cos(d*x + c)^4 + 5*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8*A + 9*B)*c
os(d*x + c)^2 + 8*(8*A + 9*B)*cos(d*x + c) + 128*A + 144*B)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) +
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.516 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{2a(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+}}$$

```
[Out] (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.403348, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4015, 3805, 3804}

$$\frac{2a(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left((6A+7B)\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$= \frac{2a(6A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin}{7d\sqrt{a+a\sec(c}}$$

$$= \frac{8a(6A+7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(6A+7B)\cos^{\frac{3}{2}}(c}{35d\sqrt{a+a\,s}}$$

$$= \frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a(6A+7B)\sqrt{\cos(c}}{105d\sqrt{a+a\,s}}$$

Mathematica [A] time = 0.339418, size = 96, normalized size = 0.55

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}((141A+112B)\cos(c+dx)+6(6A+7B)\cos(2(c+dx))+15A\cos(3(c+dx}{210d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(228*A + 266*B + (141*A + 112*B)*Cos[c + d*x] + 6*(6*A
+ 7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]
*Sin[c + d*x])/(210*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.276, size = 108, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^3+18A(\cos(dx+c))^2+21B(\cos(dx+c))^2+24A\cos(dx+c)+28B\cos(d\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+48*A+56*B)*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.02598, size = 564, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) - 14*sqrt(2)*(5*(6*sin(2*d*x + 2*c) + sin(d*x + c))*
cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - (30*cos(2*d*x + 2*c) + 5*cos
(d*x + c) + 6)*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 5*sin(3/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 30*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.480417, size = 265, normalized size = 1.51

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{3}+3\left(6\,A+7\,B\right)\cos\left(dx+c\right)^{2}+4\left(6\,A+7\,B\right)\cos\left(dx+c\right)+48\,A+56\,B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^3 + 3*(6*A + 7*B)*cos(d*x + c)^2 + 4*(6*A + 7*B)*c
os(d*x + c) + 48*A + 56*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2),
 x)
```

3.517 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=130

$$\frac{2a(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]) + (2*a*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*S
ec[c + d*x]])
```

Rubi [A] time = 0.331671, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4015, 3805, 3804}

$$\frac{2a(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]) + (2*a*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*S
ec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}\left((4A+5B)\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$= \frac{2a(4A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin}{5d\sqrt{a+a\sec(c}}$$

$$= \frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a(4A+5B)\sqrt{\cos(c+}}{15d\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.164615, size = 79, normalized size = 0.61

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left((4A+5B)\cos(c+dx)+3A\cos^2(c+dx)+8A+10B\right)}{15d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(8*A + 10*B + (4*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c +
d*x]^2)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(15*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.3, size = 86, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(3A(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+c)+8A+10B\right)}{15d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+8*A
+10*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.98797, size = 400, normalized size = 3.08

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 - 10*(3*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(d*x + c)
- (3*sqrt(2)*cos(d*x + c) + 2*sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*
x + c))) - 3*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a
))/d
```

Fricas [A] time = 0.473478, size = 216, normalized size = 1.66

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{2}+\left(4\,A+5\,B\right)\cos\left(dx+c\right)+8\,A+10\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 8*A + 10*B)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

3.518 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=82

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.262272, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2955, 4013, 3804}

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}(A+B}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left((A+\right.$$

$$= \frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

Mathematica [A] time = 0.175153, size = 56, normalized size = 0.68

$$\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(A\cos(c+dx)+2A+3B)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(2*A + 3*B + A*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x]
)]*Tan[(c + d*x)/2])/(3*d)
```

Maple [A] time = 0.265, size = 65, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))(A\cos(dx+c)+2A+3B)}{3d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(A*cos(d*x+c)+2*A+3*B)*cos(d*x+c)^(1/2)*(a*(cos(d*x+
c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.94239, size = 190, normalized size = 2.32

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*
sqrt(a)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/d
```

Fricas [A] time = 0.470526, size = 171, normalized size = 2.09

$$\frac{2\left(A\cos\left(dx+c\right)+2\,A+3\,B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*(A*cos(d*x + c) + 2*A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

3.519 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.25894, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4015, 3801, 215}

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\,\mathrm{se}}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(2B\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= \frac{2\sqrt{a}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{}{d}$$

Mathematica [A] time = 0.311601, size = 94, normalized size = 0.98

$$\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(A\sqrt{1-\sec(c+dx)}-B\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*Sqrt[1 - Sec[c + d*x]] - B*ArcSin[Sqrt[Sec[c + d*x
]]]*Sqrt[Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqr
t[1 - Sec[c + d*x]])
```

Maple [B] time = 0.301, size = 169, normalized size = 1.8

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2}\left(2A\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}-B\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-B*2^(1/2)*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+B*2^(
1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1))
^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 1.91334, size = 354, normalized size = 3.69

$$\frac{4\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+B\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + B*sqrt(a)*(log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [A] time = 0.552341, size = 799, normalized size = 8.32

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(B\cos\left(dx+c\right)+B\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(a}}{\cos(dx+c)^3+\cos(dx+c}\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(d*cos(d*x + c) + d), (2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sqrt(cos(c + d*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

3.520 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{a}(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.256568, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4016, 3801, 215}

$$\frac{\sqrt{a}(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}(A+B\,\mathrm{se}$$

$$= \frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left((2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}\right.$$

$$= \frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left((2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{\sqrt{a}(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{}{d\cos^{\frac{3}{2}}(c}$$

Mathematica [A] time = 0.377472, size = 89, normalized size = 0.91

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(2A+B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2
*A + B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sec[c + d*x]*Sin[(c + d*x)/
2]))/(2*d)
```

Maple [B] time = 0.297, size = 275, normalized size = 2.8

$$\frac{-1+\cos(dx+c)}{2d(\sin(dx+c))^2}\left(2A\cos(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)-2A\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-2*A*cos(d*x+c)*2^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+B*cos(d*x+c)
*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x
+c)))-B*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))-2*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d
*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c
)^(1/2)
```

Maxima [B] time = 2.12408, size = 1222, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(2*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2)) - (4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))
/d
```

Fricas [A] time = 0.674699, size = 929, normalized size = 9.48

$$\left[\frac{4\,B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\left((2\,A+B)\cos(dx+c)^{2}+(2\,A+B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\ldots}{\ldots}\right.}{4\left(d\cos(dx+c)^{2}+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + ((2*A + B)*cos(d*x + c)^2 + (2*A + B)*cos(d*x + c))*sqrt(a)*log((a
*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
```

```
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(
2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
 + ((2*A + B)*cos(d*x + c)^2 + (2*A + B)*cos(d*x + c))*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d
*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sqrt(cos(c + d*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

3.521 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{a(4A+3B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\,\mathrm{s}}{2d\cos^{\frac{5}{2}}(c+d}$$

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*C
os[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(4*A + 3*B)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.331359, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4016, 3803, 3801, 215}

$$\frac{a(4A+3B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\,\mathrm{s}}{2d\cos^{\frac{5}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*C
os[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(4*A + 3*B)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
```

```
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\,\mathrm{se}$$

$$= \frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left((4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}\right.$$

$$= \frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(4A+3B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(4A+3B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{a}(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{}{2d\cos}$$

Mathematica [A] time = 0.622837, size = 106, normalized size = 0.7

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(4A+3B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(4
*A + 3*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 3*B + 2
*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.344, size = 342, normalized size = 2.3

$$-\frac{-1+\cos(dx+c)}{8d(\sin(dx+c))^2}\left(-4A(\cos(dx+c))^2\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(-4*A*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+4*A*cos(d*x+c)^2*2^(1/2)*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-3*B*co
s(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1-sin(d*x+c)))+3*B*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2)+6*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*B*(-2/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(-2/(
cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.27221, size = 2601, normalized size = 17.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*s
qrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) +
 (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*si
n(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos
(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log
(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d
```

```
*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 3*(
2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*
d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
s(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*cos
(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sqrt(
2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1))/d
```

Fricas [A] time = 0.681025, size = 1035, normalized size = 6.85

$$\left[\frac{4\left(\left(4A+3B\right)\cos\left(dx+c\right)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\left(4A+3B\right)\cos\left(dx+c\right)^{3}+\left(4A+3B\right)\cos\left(\right.\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A + 3*B)*cos(d*x + c)^3 + (4*A
 + 3*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*
x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*c
os(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*A + 3*B)*cos(d*x + c) + 2*B)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (
(4*A + 3*B)*cos(d*x + c)^3 + (4*A + 3*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos
(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{a\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

3.522 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{a(6A+5B)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{8d}$$

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*C
os[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c + d*x])/
(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c +
 d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.395795, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4016, 3803, 3801, 215}

$$\frac{a(6A+5B)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(6A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2),x]
```



```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*C
os[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c + d*x])/
(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c +
 d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
```

```
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}(A+B\\ &= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{6}\left((6A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec\right.\\ &= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{a(6A+5B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}\\ &= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{a(6A+5B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}\\ &= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{a(6A+5B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}\\ &= \frac{\sqrt{a}(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}+\frac{}{3d\,c}\end{aligned}$$

Mathematica [A] time = 1.04659, size = 131, normalized size = 0.67

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(6A+5B)\cos(c+dx)+3(6A+5B)\cos(2(c+dx))+18A+3\right.}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(6*A + 5*B)*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (18*A + 31*B + 4*(6*A + 5*B)*Co
s[c + d*x] + 3*(6*A + 5*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c
 + d*x]^(5/2))
```

Maple [B] time = 0.353, size = 404, normalized size = 2.1

$$-\frac{-1+\cos(dx+c)}{48\,d\,(\sin(dx+c))^2}\left(18\,A\,(\cos(dx+c))^3\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{2}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-18*A*cos(d*x+c)^3*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+15*B
*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*2^(1/2)-15*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+36*A*cos(d*x+c)^2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2)+30*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+24*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+20*B*cos(d*x+c)*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))
*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.6058, size = 4512, normalized size = 23.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sq
rt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d
*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
```

```
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqr
t(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1
2*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1
)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
+ 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*c
os(2*d*x + 2*c) + 1) + (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x
+ 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqr
t(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(
sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x
 + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*
d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5
/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*
sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*
cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 2) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c
) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*co
s(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
 + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2
) - 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) -
20*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(
sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
```

```
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt
(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*c
os(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*
d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(3*cos(4*d*
x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 +
6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(
2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.687832, size = 1131, normalized size = 5.77

$$\left[\frac{4\left(3\,(6\,A+5\,B)\cos\left(dx+c\right)^{2}+2\,(6\,A+5\,B)\cos\left(dx+c\right)+8\,B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left((6\,A+5\,B)\right.}{96\left(d\cos\left(dx+c\right)^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(4*(3*(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3
*((6*A + 5*B)*cos(d*x + c)^4 + (6*A + 5*B)*cos(d*x + c)^3)*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*
(2*(3*(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((6*A
 + 5*B)*cos(d*x + c)^4 + (6*A + 5*B)*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x
 + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

3.523 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=275

$$\frac{2a^2(12A+11B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(c+dx)}{1155d\sqrt{a\sec(c+dx)}}$$

```
[Out] (32*a^2*(168*A + 187*B)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(168*A + 187*B)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*A*Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11
*d)
```

Rubi [A] time = 0.713592, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(12A+11B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(c+dx)}{1155d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (32*a^2*(168*A + 187*B)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(168*A + 187*B)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*A*Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11
*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}(\ldots}{\sec^{\frac{11}{2}}(c+\ldots}$$

$$= \frac{2aA\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{11d} + \frac{1}{11}\ldots$$

$$= \frac{2a^2(12A+11B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{9}{2}}(\ldots}{\ldots}$$

$$= \frac{2a^2(168A+187B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(12A\ldots}{\ldots}$$

$$= \frac{4a^2(168A+187B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(168\ldots}{\ldots}$$

$$= \frac{16a^2(168A+187B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}} + \frac{4a^2(16\ldots}{\ldots}$$

$$= \frac{32a^2(168A+187B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(168A+\ldots}{346\ldots}$$

Mathematica [A] time = 0.547844, size = 131, normalized size = 0.48

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(35(21A+11B)\cos^4(c+dx)+(840A+935B)\cos^3(c+dx)+6(168A\ldots\right.}{3465d(\cos(c+dx)+1)\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x
]),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(2688*A + 2992*B + 8*(168*A + 187*B)*Cos[c + d*x] +
 6*(168*A + 187*B)*Cos[c + d*x]^2 + (840*A + 935*B)*Cos[c + d*x]^3 + 35*(21
*A + 11*B)*Cos[c + d*x]^4 + 315*A*Cos[c + d*x]^5)*Sqrt[a*(1 + Sec[c + d*x])
]*Sin[c + d*x])/(3465*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.303, size = 153, normalized size = 0.6

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(315\,A\,(\cos(dx+c))^{5}+735\,A\,(\cos(dx+c))^{4}+385\,B\,(\cos(dx+c))^{4}+840\,A\,(\cos(dx+c))^{3}+9\right.}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+735*A*cos(d*x+c)^4+385*B*co
s(d*x+c)^4+840*A*cos(d*x+c)^3+935*B*cos(d*x+c)^3+1008*A*cos(d*x+c)^2+1122*B
*cos(d*x+c)^2+1344*A*cos(d*x+c)+1496*B*cos(d*x+c)+2688*A+2992*B)*cos(d*x+c)
^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.13177, size = 949, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] 1/110880*(21*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(
11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*c
os(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d
*x + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x
+ 11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d
*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c)
, cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/
2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55
*a*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2
*d*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(
11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) - 44*sqrt(2)*(189*(10*
a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 7*(270*a*cos(4*d*x + 4*c) + 27*a*cos(2*d*x + 2*c) + 5*
a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 135*a*sin(7/4*arc
```

```
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 189*a*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 1050*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 1890*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*B*sqrt(a))/d
```

Fricas [A] time = 0.493559, size = 397, normalized size = 1.44

$$\frac{2\left(315\,Aa\cos\left(dx+c\right)^{5}+35\,(21\,A+11\,B)a\cos\left(dx+c\right)^{4}+5\,(168\,A+187\,B)a\cos\left(dx+c\right)^{3}+6\,(168\,A+187\,B)a\cos\left(\right.}{3465\,(d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(315*A*a*cos(d*x + c)^5 + 35*(21*A + 11*B)*a*cos(d*x + c)^4 + 5*(168
*A + 187*B)*a*cos(d*x + c)^3 + 6*(168*A + 187*B)*a*cos(d*x + c)^2 + 8*(168*
A + 187*B)*a*cos(d*x + c) + 16*(168*A + 187*B)*a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(11/
2), x)
```

3.524 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(10A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)\sqrt{\cos}}{315d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (16*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Se
c[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d
*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(34*A + 39*B)*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(10*A + 9*B)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^
(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.686358, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(10A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)\sqrt{\cos}}{315d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (16*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Se
c[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d
*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(34*A + 39*B)*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(10*A + 9*B)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^
(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}(A}{\sec^{\frac{9}{2}}(c+}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d} + \frac{1}{9}\big(2$$

$$= \frac{2a^2(10A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{7}{2}}(c}{}$$

$$= \frac{2a^2(34A+39B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(10A+}{63}$$

$$= \frac{8a^2(34A+39B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(34A+}{10}$$

$$= \frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a^2(34A+39B}{315d\sqrt{}}$$

Mathematica [A] time = 0.422066, size = 118, normalized size = 0.52

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(5(17A+9B)\cos^3(c+dx)+3(34A+39B)\cos^2(c+dx)+4(34A+3\right.}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(8*(34*A + 39*B) + 4*(34*A + 39*B)*Cos[c + d*x] + 3
*(34*A + 39*B)*Cos[c + d*x]^2 + 5*(17*A + 9*B)*Cos[c + d*x]^3 + 35*A*Cos[c
+ d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]
))
```

Maple [A] time = 0.315, size = 131, normalized size = 0.6

$$-\frac{2\,a\left(-1+\cos\left(dx+c\right)\right)\left(35\,A\left(\cos\left(dx+c\right)\right)^{4}+85\,A\left(\cos\left(dx+c\right)\right)^{3}+45\,B\left(\cos\left(dx+c\right)\right)^{3}+102\,A\left(\cos\left(dx+c\right)\right)^{2}+117\,B\right.}{315\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*
x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+136*A*cos(d*x+c)+156*B*cos(d*x
+c)+272*A+312*B)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d
*x+c)
```

Maxima [B] time = 2.09384, size = 753, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) - 6*sqrt(2)*(175*a*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(2*d*x + 2*c) - 5*(35*a*cos(2*d*x + 2*c) + 6*a)*sin(7/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 126*a*sin(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 175*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 1470*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a
))/d
```

Fricas [A] time = 0.48523, size = 332, normalized size = 1.46

$$\frac{2\left(35\,Aa\cos\left(dx+c\right)^{4}+5\,(17\,A+9\,B)a\cos\left(dx+c\right)^{3}+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^{2}+4\,(34\,A+39\,B)a\cos\left(dx+c\right)+8\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^4 + 5*(17*A + 9*B)*a*cos(d*x + c)^3 + 3*(34*A +
39*B)*a*cos(d*x + c)^2 + 4*(34*A + 39*B)*a*cos(d*x + c) + 8*(34*A + 39*B)*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.525 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+\dots}}$$

```
[Out] (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sqrt[a + a
*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.616517, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sqrt[a + a
*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}(A}{\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{7d} + \frac{1}{7}\big(2$$

$$= \frac{2a^2(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{2a^2(52A+63B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+7}{35}$$

$$= \frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+63B}{105d\sqrt{}}$$

Mathematica [A] time = 0.316271, size = 100, normalized size = 0.55

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(3(13A+7B)\cos^2(c+dx)+(52A+63B)\cos(c+dx)+2(52A+63B\right.}{105d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(2*(52*A + 63*B) + (52*A + 63*B)*Cos[c + d*x] + 3*(
13*A + 7*B)*Cos[c + d*x]^2 + 15*A*Cos[c + d*x]^3)*Sqrt[a*(1 + Sec[c + d*x])
]*Sin[c + d*x])/(105*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.273, size = 109, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+39A(\cos(dx+c))^2+21B(\cos(dx+c))^2+52A\cos(dx+c)+63B\,c\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*
x+c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+104*A+126*B)*cos(d*x+c)^(1/2)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.06328, size = 609, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) - 84*(10*sqrt(2)*a*cos(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 5*sqrt(2)*a*
sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 10*sqrt(2)*a*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (10*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sq
rt(a))/d
```

Fricas [A] time = 0.48177, size = 282, normalized size = 1.56

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{3}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)^{2}+(52\,A+63\,B)a\cos\left(dx+c\right)+2\,(52\,A+63\,B)a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\ldots}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^3 + 3*(13*A + 7*B)*a*cos(d*x + c)^2 + (52*A + 63
*B)*a*cos(d*x + c) + 2*(52*A + 63*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2
), x)
```

3.526 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c$$

```
[Out] (8*a^2*(3*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d)
```

Rubi [A] time = 0.383111, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4013, 3809, 3804}

$$\frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (8*a^2*(3*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
```

```
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}(A}{\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}($$

$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx}{15d}$$

$$= \frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a(3A+5B)\sqrt{c}}{}$$

Mathematica [A] time = 0.266995, size = 80, normalized size = 0.61

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left((9A+5B)\cos(c+dx)+3A\cos^2(c+dx)+18A+25B\right)}{15d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(18*A + 25*B + (9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c
 + d*x]^2)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(15*d*(1 + Cos[c + d*x]
))
```

Maple [A] time = 0.272, size = 87, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(3A(\cos(dx+c))^2+9A\cos(dx+c)+5B\cos(dx+c)+18A+25B\right)}{15d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\mathrm{co}}{\mathrm{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*x+c)+1
8*A+25*B)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.00264, size = 373, normalized size = 2.85

$$3\sqrt{2}\left(20a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x +
 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*
sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))))*A*sqrt(a) + 20*(sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 9*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*B*sqrt(a))/d
```

Fricas [A] time = 0.475222, size = 228, normalized size = 1.74

$$\frac{2\left(3\,Aa\cos\left(dx+c\right)^{2}+\left(9\,A+5\,B\right)a\cos\left(dx+c\right)+\left(18\,A+25\,B\right)a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + (18*A + 25*B)*a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*
cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2
), x)
```

3.527 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=145

$$\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos}}{}$$

```
[Out] (2*a^(3/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.444454, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3801, 215}

$$\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(3/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}(A+}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{c}\right.$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

$$= \frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [A] time = 0.429692, size = 101, normalized size = 0.7

$$\frac{2a^2\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(A\cos(c+dx)+5A+3B)+3B\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^2*((5*A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*B*ArcSin[Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[
c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.296, size = 201, normalized size = 1.4

$$-\frac{a}{6d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-3B\sin(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/6/d*a*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-3*B*sin(d*x
+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(
d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+3*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+4*A*cos(d*x+c)^2+16*A*cos(d*x+c)+12*B*cos(d*x+c)-20*A-12*B)/sin(d*x+c
)
```

Maxima [B] time = 2.05598, size = 787, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*A*sqrt(a) + 3*(2*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 40*sqrt(2)*a*sin(1/2*d*
x + 1/2*c) - 2*sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 20*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2) - 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 5*a*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*B*sqrt(a))/d
```

Fricas [A] time = 0.561161, size = 910, normalized size = 6.28

$$\left[\frac{4\,(Aa\cos\left(dx+c\right)+(5\,A+3\,B)a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\,(Ba\cos\left(dx+c\right)+Ba)\sqrt{a}\log\left(\frac{a\cos(dx+c}{\ }\right.}{6\,(d\cos\left(dx+c\right)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/6*(4*(A*a*cos(d*x + c) + (5*A + 3*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c) + B*a)*sqrt(a
```

```
)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(A*a*
cos(d*x + c) + (5*A + 3*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c) + B*a)*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

3.528 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\text{co}}}$$

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(2*A - B)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.433127, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4018, 4015, 3801, 215}

$$\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\text{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(2*A - B)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}(A+}{\sqrt{\sec(c+dx}}$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c+dx)}}{d\sqrt{\cos(c+}}$$

$$= \frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c+dx)}}{d\sqrt{\cos(c+}}$$

$$= \frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}$$

Mathematica [A] time = 0.766489, size = 133, normalized size = 0.92

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{1-\sec(c+dx)}(2A+B\sec(c+dx))+2A\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-}\right.\right.}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(2*A*ArcSin[Sqrt[1 - Sec[c
 + d*x]]]*Sqrt[Sec[c + d*x]] - 3*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[c +
d*x]] + Sqrt[1 - Sec[c + d*x]]*(2*A + B*Sec[c + d*x]))*Tan[(c + d*x)/2])/(d
*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.309, size = 306, normalized size = 2.1

$$-\frac{a(-1+\cos(dx+c))}{2d(\sin(dx+c))^2}\left(4A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}+2A\cos(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/2/d*a*(-1+cos(d*x+c))*(4*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)+2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))-2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+3*B*cos(d*x+c)*2^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-3*B*cos(d*x+
c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d
*x+c)))+2*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.15954, size = 1913, normalized size = 13.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
```

```
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.681821, size = 1018, normalized size = 7.07

$$\left[\frac{4\,(2\,Aa\cos\left(dx+c\right)+Ba)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left((2\,A+3\,B)a\cos\left(dx+c\right)^{2}+(2\,A+3\,B)a\cos\left(dx+\right.\right.}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(4*(2*A*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*
B)*a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c
) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*
x + c)^2 + d*cos(d*x + c)), 1/2*(2*(2*A*a*cos(d*x + c) + B*a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + 3*B)*a
*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

3.529 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=153

$$\frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sqrt{}}{2d\cos}$$

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
 d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.453554, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4018, 4016, 3801, 215}

$$\frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sqrt{}}{2d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
 d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
```

```
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}(A$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+}{2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+}{2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} +$$

Mathematica [A] time = 0.837058, size = 107, normalized size = 0.7

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(12A+7B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\mathrm{se}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*
(12*A + 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 7*B
+ 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.312, size = 343, normalized size = 2.2

$$-\frac{a(-1+\cos(dx+c))}{8d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(12A(\cos(dx+c))^2\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/8/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(12*A*cos(d*x+
c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin
(d*x+c)))-12*A*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+7*B*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-7*B*cos(d*x+c)^2*2^(
1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+14*B*cos(d*x+c)*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/s
in(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.45689, size = 4575, normalized size = 29.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
```

```
/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(
2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(
2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(
2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3
/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*
a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) -
 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*
cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7
*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sq
rt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt
(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(
```

```
3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)
*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a)/(2*(2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d
```

Fricas [A] time = 0.680888, size = 1062, normalized size = 6.94

$$\left[\frac{4\left((4A+7B)a\cos(dx+c)+2Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\left((12A+7B)a\cos(dx+c)^3+(12A+7B\right.}{16\left(d\cos(dx+c)^3+d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*((4*A + 7*B)*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c)^3
 + (12*A + 7*B)*a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*A + 7*B)*a*cos(d*x
+ c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c)^3 + (12*A + 7*B)*a*cos(d*x + c)^2
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*co
s(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

3.530 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{a^2(14A+11B)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{8d}$$

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c
+ d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(14*A + 1
1*B)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B
*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.540209, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(14A+11B)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c
+ d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(14*A + 1
1*B)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B
*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+}{3d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(14A+11B)\sin(c+d}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(14A+11B)\sin(c+d}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}$$

$$= \frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} +$$

Mathematica [A] time = 1.28155, size = 134, normalized size = 0.67

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(6A+11B)\cos(c+dx)+(42A+33B)\cos(2(c+dx))+7(6A\right.}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(6*A + 7*B) + 4*(6*A + 1
1*B)*Cos[c + d*x] + (42*A + 33*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*
d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.289, size = 405, normalized size = 2.

$$\frac{a\left(-1+\cos\left(dx+c\right)\right)}{48\,d\left(\sin\left(dx+c\right)\right)^{2}}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(42\,A\left(\cos\left(dx+c\right)\right)^{3}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\left(\cos\left(dx+c\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/48/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(42*A*cos(d*x+
c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)*2^(1/2)-42*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*2^(1/2)+33*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-33*B*cos(d*x+c)^3*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1
/2)-84*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-66*B*cos(d*x+c)^
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-24*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2)-44*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-16*B*
(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/cos(d*x+c)^(5/2)/sin(d*x+c)^2/(-2/(co
s(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.83197, size = 6218, normalized size = 31.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*
sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin
(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))))*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
```

```
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)
*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(2*(2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
```

```
*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (132*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x +
 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x +
6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x +
6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 +
9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*
a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d
*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
 + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2
*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log
(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c
)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^
2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*
a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*co
s(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*
x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x
 + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c)
+ 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6
*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)
*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*c
os(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*
(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*
```

```
sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x
+ 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*
a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(3*cos(4*d*x + 4*c)
+ 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2
*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.690498, size = 1177, normalized size = 5.88

$$\left[\frac{4\left(3\,(14\,A+11\,B)a\cos\left(dx+c\right)^{2}+2\,(6\,A+11\,B)a\cos\left(dx+c\right)+8\,Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(\right.\right.}{96\left(d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(4*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c)
+ 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) + 3*((14*A + 11*B)*a*cos(d*x + c)^4 + (14*A + 11*B)*a*cos(d*x + c)^3)
*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d
*x + c)^3), 1/48*(2*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*co
s(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + 3*((14*A + 11*B)*a*cos(d*x + c)^4 + (14*A + 11*B)*a*cos(
d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*
a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

3.531 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^2(88A+75B)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
 + d*x])/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(88*A +
75*B)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a
^2*(88*A + 75*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
d*x]]) + (a*B*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2
))
```

Rubi [A] time = 0.633667, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(88A+75B)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
 + d*x])/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(88*A +
75*B)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a
^2*(88*A + 75*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
d*x]]) + (a*B*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2
))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c+dx)}\sin(c+d}{4d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{}{2}$$

Mathematica [A] time = 1.8487, size = 153, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((1048A+1155B)\cos(c+dx)+4(88A+75B)\cos(2(c+dx))+\right.}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(88*A + 75*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (352*A + 492*B + (1048*A +
1155*B)*Cos[c + d*x] + 4*(88*A + 75*B)*Cos[2*(c + d*x)] + 264*A*Cos[3*(c +
d*x)] + 225*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^(7/2
))
```

Maple [B] time = 0.33, size = 467, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/384/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-264*A*cos(
d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))+264*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-225*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+225*B*cos(d*x
+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*2^(1/2)+528*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+450*B*si
n(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+352*A*cos(d*x+c)^2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)+300*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2)+128*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+240*B*cos(d
*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+96*B*(-2/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c))/sin(d*x+c)^2/cos(d*x+c)^(7/2)/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 3.81291, size = 7937, normalized size = 32.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/768*(8*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
```

```
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x
 + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c)
+ 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9
*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a
*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*
x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
+ 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*
c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)
^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2
 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a
*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos
(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x
 + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3
*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*
x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2
)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c
) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(
6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c)
+ sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sq
rt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2
*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*A*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9
*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*
x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2
*c) + 1) + 3*(300*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c
```

) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(15/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*a*sin(8*d*x + 8*
c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2
)*a*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1140*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt
(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqr
t(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*
d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqr
t(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*
d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6
*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*
cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*a*sin(8
*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) +
 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 300*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) +
 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*c
os(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*s
in(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*
a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(
6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x +
 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c
) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) +
 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a
*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a
*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 4
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*co
s(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x
 + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6
c) + 12(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c)
 + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16
*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 +
 a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 +
 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*
cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d
*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x +
 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*
c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*
d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 +
16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2
 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2
 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*
a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8

```
*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
 + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x +
2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x
+ 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sq
rt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(15/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x
+ 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt
(2)*a)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2
)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x
 + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2
)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)
*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c
) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)
*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 100*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
 + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)
*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(
8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c)
+ 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*co
s(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x +
6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)
^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*
sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x +
2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.829933, size = 1277, normalized size = 5.17

$$\left[\frac{4\left(3\left(88A+75B\right)a\cos\left(dx+c\right)^3+2\left(88A+75B\right)a\cos\left(dx+c\right)^2+8\left(8A+15B\right)a\cos\left(dx+c\right)+48Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/768*(4*(3*(88*A + 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c
)^2 + 8*(8*A + 15*B)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c
)^5 + (88*A + 75*B)*a*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
```

```
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(88*A + 75*B)*
a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*cos(
d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c)^5 + (88*A + 75*B)*a*cos(d
*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

3.532 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=275

$$\frac{2a^2(14A+11B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2a^3(194A+209B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(710A+\cdots}{11\cdots}$$

```
[Out] (16*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99
*d) + (2*a*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(1
1*d)
```

Rubi [A] time = 0.831167, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(14A+11B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2a^3(194A+209B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(710A+\cdots}{11\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (16*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99
*d) + (2*a*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(1
1*d)
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(\ldots}{\sec^{\frac{11}{2}}(c+\ldots}$$

$$= \frac{2aA\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d} + \frac{1\ldots}{1\ldots}$$

$$= \frac{2a^2(14A+11B)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c\ldots}{99d}$$

$$= \frac{2a^3(194A+209B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(14\ldots}{}$$

$$= \frac{2a^3(710A+803B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(194\ldots}{}$$

$$= \frac{8a^3(710A+803B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710\ldots}{}$$

$$= \frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a^3(710A+\ldots}{346\ldots}$$

Mathematica [A] time = 0.583902, size = 137, normalized size = 0.5

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(35(32A+11B)\cos^4(c+dx)+5(355A+286B)\cos^3(c+dx)+3(71\ldots\right.}{3465d(\cos(c+dx)+1\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
]),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(8*(710*A + 803*B) + 4*(710*A + 803*B)*Cos[c + d*
x] + 3*(710*A + 803*B)*Cos[c + d*x]^2 + 5*(355*A + 286*B)*Cos[c + d*x]^3 +
35*(32*A + 11*B)*Cos[c + d*x]^4 + 315*A*Cos[c + d*x]^5)*Sqrt[a*(1 + Sec[c +
 d*x])]*Sin[c + d*x])/(3465*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.34, size = 155, normalized size = 0.6

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(315\,A\left(\cos\left(dx+c\right)\right)^{5}+1120\,A\left(\cos\left(dx+c\right)\right)^{4}+385\,B\left(\cos\left(dx+c\right)\right)^{4}+1775\,A\left(\cos\left(dx+c\right)\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B
*cos(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+2130*A*cos(d*x+c)^2+2
409*B*cos(d*x+c)^2+2840*A*cos(d*x+c)+3212*B*cos(d*x+c)+5680*A+6424*B)*cos(d
*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.16263, size = 1018, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 34
65*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 318
78*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11
/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1
287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2
*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 1
1/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*
x + 11/2*c))))*A*sqrt(a) + 44*sqrt(2)*(225*a^2*sin(7/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 378*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2100*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
```

```
) + 4095*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 63*(65*
a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 7*(585*a^2*cos(4*d*x + 4*c) + 54*a^2*cos(2*d*x +
 2*c) + 5*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt
(a))/d
```

Fricas [A] time = 0.494805, size = 412, normalized size = 1.5

$$\frac{2\left(315\,Aa^2\cos\left(dx+c\right)^5+35\,(32\,A+11\,B)a^2\cos\left(dx+c\right)^4+5\,(355\,A+286\,B)a^2\cos\left(dx+c\right)^3+3\,(710\,A+803\,B)a^2}{3465\,(d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(315*A*a^2*cos(d*x + c)^5 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^4 + 5*
(355*A + 286*B)*a^2*cos(d*x + c)^3 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^2 +
 4*(710*A + 803*B)*a^2*cos(d*x + c) + 8*(710*A + 803*B)*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) +
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/
2), x)
```

3.533 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(4A+3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a^3(124A+135B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(292A+3}{315}$$

```
[Out] (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(124*A + 135*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Cos[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c + d
*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.75792, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3805, 3804}

$$\frac{2a^2(4A+3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a^3(124A+135B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(292A+3}{315}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(124*A + 135*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Cos[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c + d
*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(A}{\sec^{\frac{9}{2}}(c+}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d} + \frac{1}{9}$$

$$= \frac{2a^2(4A+3B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+d}{21d}$$

$$= \frac{2a^3(124A+135B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(4A}{}$$

$$= \frac{2a^3(292A+345B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(124}{}$$

$$= \frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A+34}{315d}$$

Mathematica [A] time = 0.457175, size = 116, normalized size = 0.51

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(5(26A+9B)\cos^3(c+dx)+3(73A+60B)\cos^2(c+dx)+(292A+\right.}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(584*A + 690*B + (292*A + 345*B)*Cos[c + d*x] + 3
*(73*A + 60*B)*Cos[c + d*x]^2 + 5*(26*A + 9*B)*Cos[c + d*x]^3 + 35*A*Cos[c
+ d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]
))
```

Maple [A] time = 0.293, size = 133, normalized size = 0.6

$$-\frac{2 a^2 (-1+\cos(dx+c)) \left(35 A (\cos(dx+c))^4 + 130 A (\cos(dx+c))^3 + 45 B (\cos(dx+c))^3 + 219 A (\cos(dx+c))^2 + 18\right.}{315 d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+45*B*cos
(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+292*A*cos(d*x+c)+345*B*cos(
d*x+c)+584*A+690*B)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/si
n(d*x+c)
```

Maxima [B] time = 2.10051, size = 805, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) - 30*sqrt(2)*(77*a^2*cos(7/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 42*a^2*sin(5/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 77*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 630*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (77*a^2*cos(2*d*x + 2*c) + 6*a^2)*sin(7/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.486682, size = 348, normalized size = 1.53

$$\frac{2\left(35 A a^2 \cos(dx+c)^4 + 5(26A+9B)a^2 \cos(dx+c)^3 + 3(73A+60B)a^2 \cos(dx+c)^2 + (292A+345B)a^2 \cos(dx+\right.}{315(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^4 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^3 + 3*(73*
A + 60*B)*a^2*cos(d*x + c)^2 + (292*A + 345*B)*a^2*cos(d*x + c) + 2*(292*A
+ 345*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.534 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=178

$$\frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(5A+7B)\sin(c$$

```
[Out] (64*a^3*(5*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (2*a*(5*A + 7*B)*Cos[c + d*x]^(3/2)*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.458249, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4013, 3809, 3804}

$$\frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(5A+7B)\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (64*a^3*(5*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (2*a*(5*A + 7*B)*Cos[c + d*x]^(3/2)*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
```

```
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(A}{\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}($$

$$= \frac{2a(5A+7B)\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+}{35d}$$

$$= \frac{16a^2(5A+7B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+}{105d}$$

$$= \frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(5A+7B)}{}$$

Mathematica [A] time = 0.350622, size = 99, normalized size = 0.56

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(3(20A+7B)\cos^2(c+dx)+(115A+98B)\cos(c+dx)+15A\cos^3(c\right.}{105d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(230*A + 301*B + (115*A + 98*B)*Cos[c + d*x] + 3*
(20*A + 7*B)*Cos[c + d*x]^2 + 15*A*Cos[c + d*x]^3)*Sqrt[a*(1 + Sec[c + d*x]
)]*Sin[c + d*x])/(105*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.274, size = 111, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+60A(\cos(dx+c))^2+21B(\cos(dx+c))^2+115A\cos(dx+c)+98B\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+60*A*cos(d*x+c)^2+21*B*cos(
d*x+c)^2+115*A*cos(d*x+c)+98*B*cos(d*x+c)+230*A+301*B)*cos(d*x+c)^(1/2)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.06843, size = 651, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*
cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^
2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin
(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) - 28*(75*s
qrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 25*sqrt(2)*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 75*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*
(25*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.481874, size = 294, normalized size = 1.65

$$\frac{2\left(15\,Aa^2\cos\left(dx+c\right)^3+3\,(20\,A+7\,B)a^2\cos\left(dx+c\right)^2+(115\,A+98\,B)a^2\cos\left(dx+c\right)+(230\,A+301\,B)a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a^2*cos(d*x + c)^3 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^2 + (115*A
 + 98*B)*a^2*cos(d*x + c) + (230*A + 301*B)*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2
), x)
```

3.535 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=192

$$\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a^{5/2}B\sqrt{\cos(c+dx)}\sqrt{}}{}$$

```
[Out] (2*a^(5/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sin[c + d*x])/(1
5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c
+ d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.618572, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4017, 4015, 3801, 215}

$$\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a^{5/2}B\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```



```
[Out] (2*a^(5/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sin[c + d*x])/(1
5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c
+ d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(A}{\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}$$

$$= \frac{2a^2(8A+5B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+d}{15d}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{}}{}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{}}{}$$

$$= \frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}$$

Mathematica [A] time = 0.634799, size = 118, normalized size = 0.61

$$\frac{2a^3\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}\left((14A+5B)\cos(c+dx)+3A\cos^2(c+dx)+43A+40B\right)+15B\sqrt{\sec(c+dx)}\sin^{-1}\right.}{15d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^3*((43*A + 40*B + (14*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c + d*x]^2)*Sqrt
[1 - Sec[c + d*x]] + 15*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]]
)*Sin[c + d*x])/(15*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.311, size = 225, normalized size = 1.2

$$-\frac{a^{2}}{30\,d\sin\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(-15\,B\sin\left(dx+c\right)\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\,(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/30/d*a^2*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-15*B*sin
(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+15*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+12*A*cos(d*x+c)^3+44*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+116*A*cos(
d*x+c)+140*B*cos(d*x+c)-172*A-160*B)/sin(d*x+c)
```

Maxima [B] time = 2.0021, size = 475, normalized size = 2.47

$$\left(3\sqrt{2}a^{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}a^{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}a^{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+5\left(2\sqrt{2}a^{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/30*((3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(2*sqrt(2)*a^2*s
in(3/2*d*x + 3/2*c) + 30*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 3*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*B
*sqrt(a))/d
```

Fricas [A] time = 0.571809, size = 1040, normalized size = 5.42

$$\left[\frac{4\left(3\,Aa^{2}\cos\left(dx+c\right)^{2}+(14\,A+5\,B)a^{2}\cos\left(dx+c\right)+(43\,A+40\,B)a^{2}\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+15\left(Ba\right.}{30\left(d\cos\left(dx+c\right)+d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/30*(4*(3*A*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + (43*A +
40*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(co
s(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)))/(d*cos(d*x + c) + d), 1/15*(2*(3*A*a^2*cos(d*x + c)^2 + (14*A
 + 5*B)*a^2*cos(d*x + c) + (43*A + 40*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*(B*a^2*cos(d*x + c) + B*a^2
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*co
s(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2
), x)
```

3.536 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=197

$$\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(2A-3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(14*A + 3*B)*Sin[c + d*x
])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(2*A - 3*B)*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[
Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.626155, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4017, 4018, 4015, 3801, 215}

$$\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(2A-3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(14*A + 3*B)*Sin[c + d*x
])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(2*A - 3*B)*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[
Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(A}{\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}$$

$$= -\frac{a^2(2A-3B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{c}}{}$$

$$= \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B)\sqrt{a+}}{3d\sqrt{}}$$

$$= \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B)\sqrt{a+}}{3d\sqrt{}}$$

$$= \frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{d}$$

Mathematica [A] time = 0.661503, size = 117, normalized size = 0.59

$$\frac{a^3\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(2A\cos(c+dx)+16A+3B\sec(c+dx)+6B)+3(2A+5B)\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-}\right.\right.}{3d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^3*(3*(2*A + 5*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqr
t[1 - Sec[c + d*x]]*(16*A + 6*B + 2*A*Cos[c + d*x] + 3*B*Sec[c + d*x]))*Sin
[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.332, size = 368, normalized size = 1.9

$$-\frac{a^2}{12\,d\sin(dx+c)}\left(6\,A\sin(dx+c)\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{-2\,(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -1/12/d*a^2*(6*A*sin(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*2^(1/2)-6*A*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*2^(1/2)+15*B*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)*sin(d*x+c)*2^(1/2)-15*B*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*s
in(d*x+c)*2^(1/2)+8*A*cos(d*x+c)^3+56*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-64*A
*cos(d*x+c)-12*B*cos(d*x+c)-12*B)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.38672, size = 3495, normalized size = 17.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3
```

*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*
x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))))*A*sqrt(a) + 3*(4*sqrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^3 - 4*sqrt(2)*a^2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(2*sqrt(2)*a^
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 5*(a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2
*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(a^2*cos(5/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 5*(a^2*cos(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) - 5*(a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) + 4*(2*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt
(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*
a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2)*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))))*B*sqrt(a)/(cos(5/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(5/4*arctan2(sin(2*d*x +

```
2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(5/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(5/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2))/d
```

Fricas [A] time = 0.694929, size = 1146, normalized size = 5.82

$$\left[\frac{4\left(2\,Aa^2\cos\left(dx+c\right)^2+2\,(8\,A+3\,B)a^2\cos\left(dx+c\right)+3\,Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left((2\,A+5\,B)a^2\right.}{12\left(d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/12*(4*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
 3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqrt(a)*
log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)),
 1/6*(2*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^
2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec\left(dx+c\right)+A)(a\sec\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

3.537 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=200

$$\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2(4A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{4d}$$

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(4*A - 9*B)*Sin[c
+ d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B
)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (a*B*(a
 + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.627095, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4018, 4015, 3801, 215}

$$\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2(4A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(4*A - 9*B)*Sin[c
+ d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B
)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (a*B*(a
 + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}(}{\sqrt{\sec(c+}}$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{a^2(4A+7B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{aB(a+a}{}$$

$$= \frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2(4A+7B)\sqrt{a}}{4d}$$

$$= \frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2(4A+7B)\sqrt{a}}{4d}$$

$$= \frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{}}{4d}$$

Mathematica [A] time = 0.922839, size = 173, normalized size = 0.86

$$\frac{a^3\sin(c+dx)\sqrt{\cos(c+dx)}(A+B\sec(c+dx))\left(\sqrt{1-\sec(c+dx)}\left((4A+11B)\sec(c+dx)+8A+2B\sec^2(c+dx)\right)+\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}(A\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^3*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(20*A*ArcSin[Sqrt[1 - Sec[c +
d*x]]]*Sqrt[Sec[c + d*x]] - 19*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[c + d*
x]] + Sqrt[1 - Sec[c + d*x]]*(8*A + (4*A + 11*B)*Sec[c + d*x] + 2*B*Sec[c +
 d*x]^2))*Sin[c + d*x])/(4*d*(B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.338, size = 376, normalized size = 1.9

$$-\frac{a^{2} \left(-1+\cos \left(d x+c\right)\right)}{8 d \left(\sin \left(d x+c\right)\right)^{2}} \left(16 A \left(\cos \left(d x+c\right)\right)^{2} \sin \left(d x+c\right) \sqrt{-2 \left(\cos \left(d x+c\right)+1\right)^{-1}}-20 A \left(\cos \left(d x+c\right)\right)^{2} \sqrt{2} \arctan \left(1/4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/8/d*a^2*(-1+cos(d*x+c))*(16*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2)-20*A*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))+20*A*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-19*B*cos(d*x+c)^2*2^(
1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)+19*B*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+22*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*B*(-2/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)/
sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.692874, size = 1160, normalized size = 5.8

$$\left[\frac{4 \left(8 A a^{2} \cos \left(d x+c\right)^{2}+\left(4 A+11 B\right) a^{2} \cos \left(d x+c\right)+2 B a^{2}\right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos \left(d x+c\right)} \sin \left(d x+c\right)+\left(\left(20 A+19 B\right) a^{2}\right.}{16 \left(d \cos \left(d x+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
((20*A + 19*B)*a^2*cos(d*x + c)^3 + (20*A + 19*B)*a^2*cos(d*x + c)^2)*sqrt(
a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c
)^2), 1/8*(2*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*
a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
```

```
) + ((20*A + 19*B)*a^2*cos(d*x + c)^3 + (20*A + 19*B)*a^2*cos(d*x + c)^2)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d
*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

3.538 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\cos^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}$$

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[
c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(2*A +
3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (a*B
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.651345, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4018, 4016, 3801, 215}

$$\frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\cos^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[
c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(2*A +
3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (a*B
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
```

```
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}(A$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{a^2(2A+3B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{aB(a+a\sec(c+dx))}{3d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(2A+3B)\sqrt{a+a\sec(c+}}{4d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(2A+3B)\sqrt{a+a\sec(c+}}{4d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}$$

Mathematica [A] time = 1.3001, size = 133, normalized size = 0.66

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(6A+17B)\cos(c+dx)+(66A+75B)\cos(2(c+dx))+66A\right.}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(38*A + 25*B)*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (66*A + 91*B + 4*(6*A + 1
7*B)*Cos[c + d*x] + (66*A + 75*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*
d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.289, size = 407, normalized size = 2.

$$-\frac{a^2(-1+\cos(dx+c))}{48\,d\,(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-114\,A(\cos(dx+c))^3\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-114*A*cos
(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*
x+c)))*2^(1/2)+114*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-75*B*cos(d*x+c)^3*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+75*B*cos(d*x+
c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)*2^(1/2)+132*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+150*B*cos
(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+24*A*cos(d*x+c)*sin(d*x+c)*(
-2/(cos(d*x+c)+1))^(1/2)+68*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)+16*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)^(5/2)
/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 21.4001, size = 8501, normalized size = 42.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)
*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2
- 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
```

2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4
c)^2 - 76(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*
c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co
s(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) +
4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c
) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*
d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*
x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x
+ 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt(a
)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos
(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) - (300*sqrt(2)*a^2*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(6*d*x + 6*c) - 28
*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 2
8*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos
(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*a

rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x +
9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(
sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*
sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*
c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d
*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) +
3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9
*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6
*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) +
3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*
cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6
*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) +
a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/

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2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)
^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c)
 + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a
^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 28*(sqrt(
2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(3/2*d*x + 3/2*c))*sin(6*d*x +
 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(11/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2
*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*cos(7/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*co
s(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*cos(6*d
*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(5/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos
(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 300*(sqrt(2)*a^2*cos(6*d*x +
 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*B*sqrt(a)/(cos(6*d*x + 6*c)^2 + 6*(cos(6*d*x + 6*c) + 3*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(8/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x + 6*c) + 1)*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d*x + 6*c)^2 + 6*(sin(6*d*x +
 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(6*d*x + 6*c) + 1))
/d
```

Fricas [A] time = 0.694851, size = 1204, normalized size = 6.02

$$\left[\frac{4\left(3\,(22\,A+25\,B)a^2\cos\left(dx+c\right)^2+2\,(6\,A+17\,B)a^2\cos\left(dx+c\right)+8\,Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+}{96\left(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(4*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x +
 c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + 3*((38*A + 25*B)*a^2*cos(d*x + c)^4 + (38*A + 25*B)*a^2*cos(d
*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3), 1/48*(2*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A
+ 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((38*A + 25*B)*a^2*cos(d*x + c)^4 + (3
8*A + 25*B)*a^2*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

3.539 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^3(200A+163B)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\sec(c+dx}}{24d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*
Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(20
0*A + 163*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(8*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Cos[c +
 d*x]^(5/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Cos[c + d
*x]^(5/2))
```

Rubi [A] time = 0.766348, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(200A+163B)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\sec(c+dx}}{24d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*
Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(20
0*A + 163*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(8*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Cos[c +
 d*x]^(5/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Cos[c + d
*x]^(5/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2(8A+11B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{aB(a+a\sec(c+dx))^{3/2}}{4d\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(8A+11B)\sqrt{a+a\sec(c+dx}}{24d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(200A+163B)\sin(c+dx}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(200A+163B)\sin(c+dx}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} +$$

Mathematica [A] time = 1.90154, size = 154, normalized size = 0.62

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((2056A+2203B)\cos(c+dx)+(544A+652B)\cos(2(c+dx)\right.}{768}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(200*A + 163*B)
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (544*A + 844*B + (2056*
A + 2203*B)*Cos[c + d*x] + (544*A + 652*B)*Cos[2*(c + d*x)] + 600*A*Cos[3*(
c + d*x)] + 489*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^
(7/2))
```

Maple [B] time = 0.302, size = 469, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/384/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(600*A*cos
(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))*2^(1/2)-600*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+489*B*cos(d*x+c)^4*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-489*B*cos(d*
x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))+1200*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+978*B*
sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+544*A*cos(d*x+c)^2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2)+652*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c
)+1))^(1/2)+128*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+368*B*cos
(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+96*B*(-2/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)^(7/2)/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 4.21488, size = 9897, normalized size = 40.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt
(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
```

```
- 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a
^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(
9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 +
9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^
2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(
6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c)
+ 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(
6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c)
+ a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c
)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c
) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(
a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*
```

cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(
a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*
cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c)
+ 3*sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
c))) + 12(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x
+ 3/2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x
+ 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*
d*x + 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(cos(6*d*x + 6*c)^2 + 6*
(cos(6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 9*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(
6*d*x + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + si
n(6*d*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*cos(6*d*x + 6*c) + 1) - (1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*
sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a
^2*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c
) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqr
t(2)*a^2*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) - 2060(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +

```
 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*co
s(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8
*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d
*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c)
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x
 + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*
cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 3
6*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^
2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x +
6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8
*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x +
 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6
*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c
))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos
(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a
^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)
^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 +
 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x
 + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d
*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(
2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*si
n(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4
*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a
^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2
+ 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c
) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(
2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*co
s(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*
cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2
+ 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6
*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos
(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(
8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*co
s(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*
a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*
d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8
*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*s
```

```
qrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*c
os(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*
sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2
)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2
*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4
*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*
a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(
sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a
^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) +
 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x +
4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6
*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x
 + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4
*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 +
36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x
 + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.832905, size = 1331, normalized size = 5.39

$$\left[\frac{4\left(3\,(200\,A+163\,B)a^2\cos{(dx+c)}^3+2\,(136\,A+163\,B)a^2\cos{(dx+c)}^2+8\,(8\,A+23\,B)a^2\cos{(dx+c)}+48\,Ba^2\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/768*(4*(3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*cos
(d*x + c)^2 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B)
*a^2*cos(d*x + c)^5 + (200*A + 163*B)*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*co
s(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x +
 c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*
(2*(3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*cos(d*x +
c)^2 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B)*a^2*co
s(d*x + c)^5 + (200*A + 163*B)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x +
```

```
c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

3.540 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{a^3(326A+283B)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B
)*Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*
(326*A + 283*B)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]) + (a^2*(10*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(40*d*Cos[c + d*x]^(7/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.856578, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(326A+283B)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2),x]
```


```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B
)*Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*
(326*A + 283*B)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]) + (a^2*(10*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(40*d*Cos[c + d*x]^(7/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(7/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{a^2(10A+13B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{40d\cos^{\frac{7}{2}}(c+dx)} + \frac{aB(a+a\sec(c+dx}{5d\cos^{\frac{7}{2}}(c}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(10A+13B)\sqrt{a+a\sec(c}}{40d\cos^{\frac{7}{2}}(c+}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A+283B)\sin(c}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,se}}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A+283B)\sin(c}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,se}}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A+283B)\sin(c}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,se}}$$

$$= \frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{128d}$$

Mathematica [A] time = 2.82132, size = 178, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(36(650A+781B)\cos(c+dx)+4(6730A+6509B)\cos(2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(326*A + 283*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (22030*A + 24863*B + 3
6*(650*A + 781*B)*Cos[c + d*x] + 4*(6730*A + 6509*B)*Cos[2*(c + d*x)] + 652
0*A*Cos[3*(c + d*x)] + 5660*B*Cos[3*(c + d*x)] + 4890*A*Cos[4*(c + d*x)] +
4245*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d*Cos[c + d*x]^(9/2))
```

Maple [B] time = 0.323, size = 531, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3840/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4890*A*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(
d*x+c)^5*2^(1/2)-4890*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
```

```
*x+c)+1-sin(d*x+c)))*cos(d*x+c)^5*2^(1/2)+4245*B*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^5*2^(1/2)-4245*B*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(
d*x+c)^5*2^(1/2)+9780*A*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^4+8
490*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^4+6520*A*sin(d*x+c)*c
os(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+5660*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(c
os(d*x+c)+1))^(1/2)+3680*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+4528*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+960*A*cos(d*x+c)
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2784*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2)+768*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/
cos(d*x+c)^(9/2)/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 6.72899, size = 12477, normalized size = 42.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/7680*(10*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*co
s(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(
8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6
*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2
*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*
x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt
(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*s
in(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)
*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(
sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a
^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(
6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^
2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^
2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 +
8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x
+ 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*
x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2
*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin
(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*
d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^
2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 +
 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x +
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4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c

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os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1) + (16980*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8
*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x +
4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(19/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*si
n(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x
 + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(17/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 81504*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^
2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4
*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 8320*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)
*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 86440*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5*sqr
t(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^
2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*a^2*sin(10*d*x + 10*c) + 5
*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt(2
)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^2*sin(10*d*x + 10*c) +
 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 81504*(sqrt(2)*a^2*sin(10*d*x + 10*c
) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*s
qrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 5660*(sqrt(2)*a^2*sin(10*d*x + 10
*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10
*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16980*(sqrt(2)*a^2*sin(10*d*x +
 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) +
 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4245*(a^2*cos(10*d*x + 10*c)
^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4
*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a
^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*
c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^
2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(
6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(
10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*
a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c)
+ 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*
c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6
*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 5
0*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a^2*cos(10*d*x + 10*c)^
2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4*
d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a^
2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c
)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2
 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6
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*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(1
0*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a
^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) +
 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c
) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*
c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50
*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 4245*(a^2*cos(10*d*x + 10*c)^2
 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4*d
*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a^2
*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c)
^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2
+ 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6*
d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(10
*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^
2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) +
5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c)
 + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*c
) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*
(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*
d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a^2*cos(10*d*x + 10*c)^2
+ 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c)^2 + 100*a^2*cos(4*d*
x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 25*a^2*
sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100*a^2*sin(4*d*x + 4*c)^
2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*sin(2*d*x + 2*c)^2 +
 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x + 8*c) + 10*a^2*cos(6*d
*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*
d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2
*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^2*cos(4*d*x + 4*c) + 5
*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5*a^2*cos(2*d*x + 2*c)
+ a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) + 2*a^2*sin(6*d*x + 6*c)
 + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(
2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16980*(sqrt(2)*a^2*cos(10*d*x +
10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) +
10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2)*sin(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 5660*(sqrt(2)*
a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*co
s(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(17/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) +
 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*c
os(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*a^2*cos(10*d*x + 10*c)

```
 + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 86440*(sqrt(2)*a^2*
cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*
d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*
c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 83
20*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sq
rt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^
2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d
*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*
c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqr
t(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^
2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16980*(sqrt(2)*a^2*cos(10*d*
x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c
) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt
(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2
*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*
d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 10*(10*cos(6*d*
x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) +
 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*
cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)^2 + 10*(sin(8*
d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*si
n(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x + 6*c) + 2*sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d*x + 8*c)^2 + 1
00*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 100*sin(6*d*x
 + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.84788, size = 1455, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/7680*(4*(15*(326*A + 283*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*
cos(d*x + c)^3 + 8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^
2*cos(d*x + c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + 15*((326*A + 283*B)*a^2*cos(d*x + c)^6 + (326*A
+ 283*B)*a^2*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*s
in(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))
/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), 1/3840*(2*(15*(326*A + 283*B)*a^2*c
os(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^3 + 8*(230*A + 283*B)*a
^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x + c) + 384*B*a^2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((326*
A + 283*B)*a^2*cos(d*x + c)^6 + (326*A + 283*B)*a^2*cos(d*x + c)^5)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c
)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

3.541 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=250

$$-\frac{2(A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(43*A - 91*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(31*A - 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.843238, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4022, 4013, 3808, 206}

$$-\frac{2(A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(43*A - 91*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(31*A - 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sq
rt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-7B)+3a}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a-}}}{7a}$$

$$= -\frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{(4\sqrt{}}{}$$

$$= \frac{2(31A-7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B)\sqrt{\cos(c+dx)}\sin(c+}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B)\sqrt{\cos(c+dx)}\sin(c+}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{}{105d}$$

Mathematica [A] time = 1.13812, size = 170, normalized size = 0.68

$$\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((91B-43A)\sec^3(c+dx)+(31A-7B)\sec^2(c+dx)-3(A-7B)\sec(c+\right.\right.}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(-105*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(7/2) + 2*Sqrt[1 - Sec[c + d*x]]*(1
5*A - 3*(A - 7*B)*Sec[c + d*x] + (31*A - 7*B)*Sec[c + d*x]^2 + (-43*A + 91*
B)*Sec[c + d*x]^3))*Sin[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Maple [A] time = 0.289, size = 217, normalized size = 0.9

$$-\frac{1}{105\,ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(30\,A(\cos(dx+c))^4+105\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/105/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(30*A*cos(d*x
+c)^4+105*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+
1))^(1/2)*A*sin(d*x+c)-36*A*cos(d*x+c)^3-105*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+42*B*cos(d*x+c)^3
+68*A*cos(d*x+c)^2-56*B*cos(d*x+c)^2-148*A*cos(d*x+c)+196*B*cos(d*x+c)+86*A
-182*B)/a/sin(d*x+c)
```

Maxima [B] time = 2.20107, size = 1011, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) + 28*(
30*sqrt(2)*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*(10*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 15*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 15*s
qrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 5*sqrt(2)*sin(3/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 30*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*B/sqrt(a))/d
```

Fricas [A] time = 0.538725, size = 1081, normalized size = 4.32

$$\left[\frac{4\left(15\,A\cos\left(dx+c\right)^{3}-3\left(A-7\,B\right)\cos\left(dx+c\right)^{2}+\left(31\,A-7\,B\right)\cos\left(dx+c\right)-43\,A+91\,B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)} \cdots}{210\left(ad\cos\left(dx+c\right)+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/210*(4*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B)*
cos(d*x + c) - 43*A + 91*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)
*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/105*(105*sqr
t(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)
) - 2*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B)*cos(
d*x + c) - 43*A + 91*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

3.542 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=207

$$-\frac{2(A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{t}}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*(13*A - 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) - (2*(A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rubi [A] time = 0.632401, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4022, 4013, 3808, 206}

$$-\frac{2(A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{t}}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*(13*A - 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) - (2*(A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-5B)+2aA\,\mathrm{se}}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}}{5a}$$

$$= -\frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(4\sqrt{\cos}\right.}{}$$

$$= \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2.}{}$$

$$= \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2.}{}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2}{15d\sqrt{c}}$$

Mathematica [A] time = 0.750765, size = 154, normalized size = 0.74

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((13A-5B)\sec^2(c+dx)-(A-5B)\sec(c+dx)+3A\right)+15\sqrt{2}(A-B)\sec^{\frac{5}{2}}\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(15*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])
/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3*A
 - (A - 5*B)*Sec[c + d*x] + (13*A - 5*B)*Sec[c + d*x]^2))*Sin[c + d*x])/(15
*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.415, size = 195, normalized size = 0.9

$$-\frac{1}{15\,ad\sin\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(6\,A\left(\cos\left(dx+c\right)\right)^{3}-15\,\arctan\left(1/2\,\sin\left(dx+c\right)\sqrt{-2\,(\cos\left(d\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(6*A*cos(d*x+c
)^3-15*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))
^(1/2)*A*sin(d*x+c)+15*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2
/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+28*A
*cos(d*x+c)-20*B*cos(d*x+c)-26*A+10*B)/a/sin(d*x+c)
```

Maxima [B] time = 2.14659, size = 784, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) + 10*(3*sqrt(2)*log(cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) - 3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)*sin(3/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a))/d
```

Fricas [A] time = 0.534067, size = 983, normalized size = 4.75

$$\left[\frac{4\left(3\,A\cos\left(dx+c\right)^{2}-\left(A-5\,B\right)\cos\left(dx+c\right)+13\,A-5\,B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{15\,\sqrt{2}((A-B)a\cos(dx+c)\cdots}{\cdots}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/30*(4*(3*A*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 13*A - 5*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(
2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(
a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a
*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*
a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x + c)^2 - (A - 5*B)*c
os(d*x + c) + 13*A - 5*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

3.543 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=162

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(}{3d\sqrt{}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]])
```

Rubi [A] time = 0.447105, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4022, 4013, 3808, 206}

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-3B)+aA\,\mathrm{se}}{\sqrt{\sec(c+dx)}\sqrt{a+a\,\mathrm{se}}}}{3a}$$

$$= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \left((A-B\right.$$

$$= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2(A-\right.}{}$$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2(}{3d\sqrt{\cos}}$$

Mathematica [A] time = 0.325584, size = 124, normalized size = 0.77

$$\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}(A\cos(c+dx)-A+3B)-3\sqrt{2}(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] ((2*(-A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*(A - B)*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*
x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.335, size = 173, normalized size = 1.1

$$-\frac{1}{3\,ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)
-3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/
2)*B*sin(d*x+c)+2*A*cos(d*x+c)^2-4*A*cos(d*x+c)+6*B*cos(d*x+c)+2*A-6*B)/a/s
in(d*x+c)
```

Maxima [B] time = 2.0904, size = 645, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)
*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a))/d
```

Fricas [A] time = 0.524162, size = 892, normalized size = 5.51

$$\left[\frac{4\,(A\cos(dx+c) - A + 3\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - \frac{3\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos\cdots}{\cos\cdots}}\cdots}{\cdots}}{\cdots}\right)}{\sqrt{a}}}{6\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/6*(4*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A -
```

```
B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sq
rt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c
)) - 2*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

3.544 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=119

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.28614, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4013, 3808, 206}

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \left((-A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$
$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(2(-A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Su}}{d}$$
$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{}{d\sqrt{\cos}}$$

Mathematica [A] time = 0.29818, size = 140, normalized size = 1.18

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(A+B\sec(c+dx))\left(\sqrt{2}(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2A\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}(A\cos(c+dx)+B)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*A*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*(A - B)*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*(A +
 B*Sec[c + d*x])*Sin[c + d*x])/(d*(B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x
]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.323, size = 142, normalized size = 1.2

$$\frac{(\cos(dx+c))^2-1}{ad(\sin(dx+c))^2}\left(A\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}-A\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)+B\,\text{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-A*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))+B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos
(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
/a/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [A] time = 1.96689, size = 263, normalized size = 2.21

$$-\frac{\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-4\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A}{\sqrt{a}} - \cdots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a))/d
```

Fricas [A] time = 0.516443, size = 813, normalized size = 6.83

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2
- 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B)*a*cos(d*x + c) + (A
 - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right)\right)\sqrt{\cos\left(c + dx\right)}}{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a),
 x)
```

3.545 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rubi [A] time = 0.342973, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx + \frac{\left(B\sqrt{\cos(c+d\right.}}{}$$

$$= -\frac{\left(2(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+d}{\sqrt{a+a\sec(c+dx)}}\right.}{d}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{}$$

Mathematica [A] time = 0.21388, size = 115, normalized size = 0.82

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{2}(B-A)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2B\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((-2*B*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*(-A + B)*ArcTan[(Sqrt[2]*Sqrt[S
ec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.311, size = 201, normalized size = 1.4

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2 a}\left(-B\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)+B\sqrt{2}\arctan\left(\frac{\sqrt{2}(c}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(-B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))+B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+2*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))-2*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos(d*
x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2/a/(-2/(cos(d*x+
c)+1))^(1/2)
```

Maxima [B] time = 2.08023, size = 944, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (sqrt(2)*log(cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - sqrt(2)*log(cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 2) + log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + log(2*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*B/sqrt(a))/d
```

Fricas [A] time = 0.590442, size = 945, normalized size = 6.75

$$\left[-\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos}{\cos(}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*co
s(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - B*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)))/(a*d), -(sqrt(2)*(A - B)*a*sqrt(-1/a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
c))/sin(d*x + c)) - B*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x
 + c) - 2*a)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx + c) + A}{\sqrt{a\sec(dx + c) + a}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

3.546 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\right.}{\sqrt{a}d}$$

```
[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqr
t[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Cos
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.503087, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2955, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqr
t[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Cos
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}dx$$

$$= \frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}\right.}{\sqrt{a+}}}{a}$$

$$= \frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{a}}$$

$$= \frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(2(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subs}}{d}$$

$$= \frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A-B)\tanh^{-1}}{}$$

Mathematica [A] time = 0.529088, size = 114, normalized size = 0.63

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\cos(c+dx)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(2A-B)\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] -((Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x] - Sqr
t[2]*(2*A - B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] - 2*B*Sin[(c
+ d*x)/2]))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [B] time = 0.319, size = 342, normalized size = 1.9

$$-\frac{-1+\cos(dx+c)}{2d(\sin(dx+c))^2 a}\left(2A\cos(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-2*A*cos(d*x+c)*2^(1/2)*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-B*cos(d*x+c
)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))+B*cos(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1-sin(d*x+c)))-4*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)+2*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*B*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/a
```

Maxima [B] time = 2.28476, size = 2037, normalized size = 11.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*(2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) + (4*sqrt(2)*cos(3/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/4*a
```

```
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 -
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
qrt(a)))/d
```

Fricas [A] time = 0.753037, size = 1507, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - ((2*A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*sqrt(a)*log((a
*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)) - 2*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A
- B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c)
 - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 +
 a*d*cos(d*x + c)), 1/2*(2*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*co
s(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*B*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A - B)*cos(d*x + c)
^2 + (2*A - B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

3.547 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=230

$$\frac{(4A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(4A-7B)\sqrt{\cdots}}{}$$

```
[Out] -((4*A - 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh
[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(
2*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((4*A - B)*Sin[c + d*x])
/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.700317, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2955, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(4A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(4A-7B)\sqrt{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((4*A - 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh
[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(
2*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((4*A - B)*Sin[c + d*x])
/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+}{}}{2a}$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - $$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + $$

$$= -\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} + \frac{\sqrt{2}(A-B)\,\mathrm{t}}{}$$

Mathematica [A] time = 1.02288, size = 137, normalized size = 0.6

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)((4A-B)\cos(c+dx)+2B)+8(A-B)\cos^2(c+dx)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(4A\right.}{4d\cos^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] (Cos[(c + d*x)/2]*(8*(A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - Sqr
t[2]*(4*A - 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*B
+ (4*A - B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Cos[c + d*x]^(5/2)*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.354, size = 413, normalized size = 1.8

$$-\frac{-1+\cos(dx+c)}{8\,ad(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-4\,A(\cos(dx+c))^2\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-4*A*cos(d*x+c)
^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d
*x+c)))+4*A*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))+7*B*cos(d*x+c)^2*2^(1/2)*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-7*B*cos(d*x+c)^2*2^(1/2
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+8
*A*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*A*cos(d*x+c)^2*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-2*B*cos(d*x+c)*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2)-16*B*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2))+4*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/a/(-2/(cos(d*x+c)+1))
^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.44751, size = 3650, normalized size = 15.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
```

s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - (4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)

```
+ 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*c
os(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(2)*cos(
4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.765574, size = 1615, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) - ((4*A - 7*B)*cos(d*x + c)^3 + (4*A -
 7*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqr
t(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(-(cos(d*x +
c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/8*(8*sqrt(2)
*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*sqrt(-1/a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))
/sin(d*x + c)) - 2*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A - 7*B)*cos(d*x + c)^
3 + (4*A - 7*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

3.548 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)s}{2d(a}$$

```
[Out] -((15*A - 11*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2
]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c
+ d*x])^(3/2)) + ((147*A - 95*B)*Sin[c + d*x])/(30*a*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(30*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.866346, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)s}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((15*A - 11*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2
]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c
+ d*x])^(3/2)) + ((147*A - 95*B)*Sin[c + d*x])/(30*a*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(30*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9A-5}{\sec^{\frac{5}{2}}(c}}{2a^2}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(39A-35B)\sqrt{\cos(c+dx)}\sin(c+dx)}{30ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B)\sin(c+dx)}{30ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B)\sin(c+dx)}{30ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(A}{}$$

Mathematica [A] time = 1.29232, size = 178, normalized size = 0.66

$$\frac{2\tan(c+dx)\sqrt{1-\sec(c+dx)}(3(39A-20B)\cos(c+dx)+(10B-6A)\cos(2(c+dx))+3A\cos(3(c+dx))+141A-85B}{60d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (30*Sqrt[2]*(15*A - 11*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[
c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 2*(141*A -
85*B + 3*(39*A - 20*B)*Cos[c + d*x] + (-6*A + 10*B)*Cos[2*(c + d*x)] + 3*A*
Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x])/(60*d*Sqrt[-1 + Cos[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.403, size = 329, normalized size = 1.2

$$-\frac{-1+\cos(dx+c)}{60\,d\,(\sin(dx+c))^3\,a^2}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(225\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/60/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(225*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*(-2/(cos(d*x+c)+1))^(1/2)-24*A*cos(d*x+c)^4-165*B*sin(d*x+c)*cos(d*x+c
)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2
)+225*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^
(1/2)*A*sin(d*x+c)+48*A*cos(d*x+c)^3-165*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-40*B*cos(d*x+c)^3-240
*A*cos(d*x+c)^2+160*B*cos(d*x+c)^2-78*A*cos(d*x+c)+70*B*cos(d*x+c)+294*A-19
0*B)/sin(d*x+c)^3/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.547027, size = 1283, normalized size = 4.75

$$\left[-\frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^2+2(15A-11B)\cos(dx+c)+15A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)-}{\cos(dx+c)}}}{\cos(dx+c)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/120*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x
 + c) + 15*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(12*A*cos(d*
x + c)^3 - 4*(3*A - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B)*cos(d*x + c) + 147
*A - 95*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/60*(15*sq
rt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x + c) + 15*A -
 11*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*x + c)^3 - 4*(3*A
 - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B)*cos(d*x + c) + 147*A - 95*B)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.549 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{(19A}{6ad\sqrt{\cos(c}$$

```
[Out] ((11*A - 7*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c +
d*x])^(3/2)) - ((19*A - 15*B)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) + ((7*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d
*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.686667, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{(19A}{6ad\sqrt{\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((11*A - 7*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c +
d*x])^(3/2)) - ((19*A - 15*B)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) + ((7*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d
*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(7A-3}{\sec^{\frac{3}{2}}(c}}{2a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(A-}{}$$

Mathematica [A] time = 1.23248, size = 155, normalized size = 0.7

$$\frac{\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(\sec(c+dx)(2A\cos(2(c+dx))-17A+15B)+12(B-A))-3\sqrt{2}(11A-7B)\cos^2\left(\frac{1}{2}(c\right.\right.}{6d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] ((-3*Sqrt[2]*(11*A - 7*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[
c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec[c + d*x]]*(
12*(-A + B) + (-17*A + 15*B + 2*A*Cos[2*(c + d*x)])*Sec[c + d*x]))*Sin[c +
d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.315, size = 307, normalized size = 1.4

$$\frac{-1+\cos(dx+c)}{12\,da^2(\sin(dx+c))^3}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(33\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
*(33*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*(-2/(cos(d*x+c)+1))^(1/2)-21*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+33*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+8*A
*cos(d*x+c)^3-21*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(
d*x+c)+1))^(1/2)*B*sin(d*x+c)-32*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-14*A*cos(
d*x+c)+6*B*cos(d*x+c)+38*A-30*B)/a^2/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.542424, size = 1172, normalized size = 5.26

$$\left[-\frac{3\sqrt{2}\left((11\,A-7\,B)\cos(dx+c)^2+2\,(11\,A-7\,B)\cos(dx+c)+11\,A-7\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(d}}{\cos(dx+c)^2+2\,\cos(}\right.}{24\left(a^2d\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c
) + 11*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d
*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)
^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11
*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2
*(4*A*cos(d*x + c)^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.550 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(A}{2d\sqrt{\cos(c}$$

```
[Out] -((7*A - 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c +
d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]])
```

Rubi [A] time = 0.489672, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4020, 4013, 3808, 206}

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(A}{2d\sqrt{\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```



```
[Out] -((7*A - 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c +
d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
```

```
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a}{\sqrt{}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{}{2d\sqrt{}}$$

Mathematica [A] time = 1.8762, size = 198, normalized size = 1.12

$$\frac{2\tan(c+dx)\sqrt{1-\sec(c+dx)}\left(2A^2\cos(2(c+dx))+2A^2+A(5A+3B)\cos(c+dx)+5AB-B^2\right)+4\sqrt{2}(7A-3B)\mathrm{si}}{4d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}(A\,\mathrm{co}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (4*Sqrt[2]*(7*A - 3*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Cos[(c + d*x)/2]^3*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[(c +
 d*x)/2] + 2*(2*A^2 + 5*A*B - B^2 + A*(5*A + 3*B)*Cos[c + d*x] + 2*A^2*Cos[
2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x])/(4*d*Sqrt[-1 + Cos[c + d
*x]]*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.385, size = 235, normalized size = 1.3

$$\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\left(4\,A(\cos(dx+c))^2\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+A\cos(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+7\,A\,\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(-1+cos(d*x+c))*(4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+A*cos(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2)+7*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))-B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*x+c)*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*A*(-2/(cos(d*x+c)+1))^(1/2
)+B*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [B] time = 2.42527, size = 11081, normalized size = 62.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c
)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
- 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*
x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d
*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/
2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/
2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin
(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3
/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
```

```
*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin
(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))
*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*
sin(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/
2*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*
c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 +
(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x
+ 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x +
 1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + (8*(7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 259*(log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 91*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
^2 - 104*sin(1/2*d*x + 1/2*c)^3 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2
*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 8
*(37*cos(1/2*d*x + 1/2*c)^2 + 21)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c
)^2 + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 7*(log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*c
os(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 8*cos(1/2*d*x + 1/2*c)*sin(1/2
*d*x + 1/2*c)^3 + 13*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*
d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + (2*(7*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
```

```
*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(
3/2*d*x + 3/2*c)^2 + 2*(84*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 7*(lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 - 16*(6*co
s(1/2*d*x + 1/2*c)^2 + 1)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7
*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*
d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*
x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/
2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*cos(3/2*d*x + 3/2*c) - 8*cos(1/2*d*x +
 1/2*c))*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^3 + 2*cos(1/2*d*x
 + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(147*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(1/2*d*x + 1/2*c)^3 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*si
n(1/2*d*x + 1/2*c)^2 - 40*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 - 56*
(3*cos(1/2*d*x + 1/2*c)^3 + cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos
(3/2*d*x + 3/2*c) + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin
(3/2*d*x + 3/2*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d
*x + 1/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 8*sin(1/2*d*x +
1/2*c)^4 + (7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*
c)^2 - 4)*cos(3/2*d*x + 3/2*c)^2 + (35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*
c) - 40*sin(1/2*d*x + 1/2*c)^2 - 36)*sin(3/2*d*x + 3/2*c)^2 - 4*(18*cos(1/2
*d*x + 1/2*c)^2 + 5)*sin(1/2*d*x + 1/2*c)^2 + 6*(7*(log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/
2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
1/2*c)^2 - 4*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 36*cos(1/2*d*x +
1/2*c)^2 + 2*((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x +
3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 14*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(1/2*d*x + 1/2*c)^2 - 16*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
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1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos
(3/2*d*x + 3/2*c) - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c))
*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(133*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 21*(log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
+ 1/2*c)^3 - 24*sin(1/2*d*x + 1/2*c)^4 + 2*(21*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*
x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*cos(3/2*d*x + 3/2*c)^2 - 8*(19
*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c)^2 + 16*(7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1
/2*d*x + 1/2*c)^2 - 5*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 80*cos(1
/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^4 + 11*
cos(1/2*d*x + 1/2*c)^2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a)/(4*sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)^4 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3*cos(1/2*d*x + 1/
2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^4 + 4*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c)^4 + 12*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3*sin(1/2*d*x + 1/2*c) + 10*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c)^4 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*co
s(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(5/2*d*x +
5/2*c)^2 + (61*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^
2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2
*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^
2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2
*c)^2)*sin(5/2*d*x + 5/2*c)^2 + (8*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 28*
sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 37*sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + 13*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3
/2*c)^2 + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3 + 13*sqrt(2)*a^2*cos(3/2*
d*x + 3/2*c)^2*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3
+ sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + (2*sqrt(2)*a^2*
cos(3/2*d*x + 3/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*
c)^2 + 2*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x +
1/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin
(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*
sin(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c) + 2*(21*sqrt(2)*a^2*cos(1/2*d*x
+ 1/2*c)^3 + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2)*cos
(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3 + sqrt(2)*a^2*c
os(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x
+ 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2*sin(
1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 + 2*(sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c
) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c)^2)*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(6*sqrt(2)*a^2*cos(3/2
*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 16*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*
cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*a^2*cos(1/2*d*x + 1/
2*c)^2*sin(1/2*d*x + 1/2*c) + 3*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3)*sin(3/2
*d*x + 3/2*c)) - (3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log

```
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
 1))*cos(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c
) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2
*d*x + 3/2*c)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) + 4*sin(1/2*d*x + 1/2*c))*B/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*
sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2
*d*x + 2*c)*sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x
 + c) + 2*(2*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)
*a)*sqrt(a)))/d
```

Fricas [A] time = 0.53, size = 1068, normalized size = 6.07

$$\left[-\frac{\sqrt{2}\left((7\,A-3\,B)\cos\left(dx+c\right)^2+2\,(7\,A-3\,B)\cos\left(dx+c\right)+7\,A-3\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin}{\cos(dx+c)^2+2\,\cos(dx+c)+}\right.}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7
*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c) + 5*A
 - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((7
*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A - 3*B)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c) + 5*A - B)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2
), x)
```

3.551 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((3*A + B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(
3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^(3/2))
```

Rubi [A] time = 0.31737, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2955, 4012, 3808, 206}

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(
3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^(3/2))
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left((3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{4a}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left((3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= \frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{}{2d\,\mathrm{cc}}$$

Mathematica [A] time = 0.50512, size = 86, normalized size = 0.68

$$\frac{\frac{1}{2}(B-A)\sin(c+dx)+(3A+B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] ((3*A + B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((-A + B)*Sin[c +
 d*x])/2)/(a*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [A] time = 0.287, size = 209, normalized size = 1.7

$$-\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\left(A\cos(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+3\,A\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*A*sin(d*x+
c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-B*cos(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2)+B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))-A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2
)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a^2/sin(d*x+c)^3/(-2/(cos(d*x+c)+1))^
(1/2)
```

Maxima [B] time = 2.18236, size = 2924, normalized size = 23.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
 + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin
(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2
*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c
)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
+ 4*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*co
s(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*
sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2
*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a))
 + (4*(sin(3/2*d*x + 3/2*c) - sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 8*(sin(3/2*d*x + 3/2*c) - sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + (2*(2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 4*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*l
og(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (2*(2*cos(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
```

```
n(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(2/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*log(cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - 4*(cos(3/2*d*x + 3/2*c) - cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 8*(cos(3/2*d*x + 3/2*c) - cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*sin(3/2*d*x + 3/2*c) - 4*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B/((sqrt(2)*a*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*cos(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sqrt(2)*a*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*cos(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(2*sqrt(2)*a*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*cos(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sqrt(a)))/d
```

Fricas [A] time = 0.521389, size = 995, normalized size = 7.83

$$\left[\frac{\sqrt{2}\left((3\,A+B)\cos\left(dx+c\right)^{2}+2\,(3\,A+B)\cos\left(dx+c\right)+3\,A+B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(d}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*
(3*A + B)*cos(d*x + c) + 3*A + B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A
 - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

3.552 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2
*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.53118, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2955, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2
*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+}}{}}{2a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left((A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{4a}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left((A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{}}\right.}{}$$

Mathematica [A] time = 0.923604, size = 113, normalized size = 0.61

$$\frac{(A-B)\tan\left(\frac{1}{2}(c+dx)\right)+(A-5B)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\sqrt{2}B\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}\right.\right.}{2ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] ((A - 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + 4*Sqrt[2]*B*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B)*Tan[(c + d*x)/2])/(2*
a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.298, size = 303, normalized size = 1.6

$$-\frac{-1+\cos(dx+c)}{2d(\sin(dx+c))^3a^2}\sqrt{\cos(dx+c)}\left(-2B\sin(dx+c)\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(-2*B*sin(d*x+c)*2^(1/2)*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*B*sin(d*x+c
)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))+A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*cos(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))+B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+A*(-2/(cos(d*x
+c)+1))^(1/2)-B*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3/a^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.655233, size = 1577, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5
*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 +
```

```
2*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d
*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((A - 5*B)*c
os(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(-a)*arctan(sqrt(2)
*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin
(d*x + c))) - 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c) + A}{(a \sec(dx+c) + a)^{\frac{3}{2}} \cos(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

3.553 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\cdots\right.}{a^{3/2}d}$$

```
[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[
a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Si
n[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.744347, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2955, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\cdots\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[
a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Si
n[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec}{}}{2a}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{(5A-9B)\,\mathrm{t}}{}$$

Mathematica [A] time = 2.16972, size = 288, normalized size = 1.22

$$-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(4(A-3B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-2\sqrt{2}(5A-9B)\cos^2\left(\frac{1}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(4*(A - 3*B)*ArcSin[Sqrt[1 - Sec[c
+ d*x]]]*Cos[(c + d*x)/2]^2 + 20*A*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)
/2]^2 - 36*B*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 2*Sqrt[2]*(5*A
 - 9*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c
+ d*x)/2]^2 - 4*B*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 2*A*Cos[c + d
*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] - 6*B*Cos[c + d*x]*Sqrt[(-1 +
Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a
*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.321, size = 467, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*
sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
```

```
+1+sin(d*x+c)))-2*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-3*B*cos(d*x+c)*sin(d*x+c)*2^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))+3*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-
5*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-
3*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+9*B*cos(d*x+c)*sin(d*x+c)*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*cos(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2)+B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2*B*(-2/(cos(d*x+c)+1))^(1/2)
)/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(1/2)
```

Maxima [B] time = 3.47852, size = 9527, normalized size = 40.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*
cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqr
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
 + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
```

```
, cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*
a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)) - (12*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4*d*x + 4*c)
 + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x +
4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 +
4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4
*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sq
rt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(2)*cos(4*d*
x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
```

*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d
*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*
x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqr
t(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*s
qrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) + 4(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)
*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqr
t(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*s
qrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sq
rt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x +
4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 9*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4*d*x + 4*c)
 + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4*c) + 2*cos
(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4

```
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*x + 4*c) +
2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 9*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4*d*x + 4*c)
 + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4*c) + 2*cos
(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*x + 4*c) +
2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((sqrt
(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*x + 4*c)^
2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x +
 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*c
os(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2
*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sq
rt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*
a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) +
2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)))/d
```

Fricas [A] time = 0.841485, size = 1862, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^3 + 2*(5*A - 9*B)*cos(d*x + c)^2 +
 (5*A - 9*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
- 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A -
3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) + 2*((2*A - 3*B)*cos(d*x + c)^3 + 2*(2*A - 3*B)*cos(d
*x + c)^2 + (2*A - 3*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x +
c)), 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^3 + 2*(5*A - 9*B)*cos(d*x + c)^
2 + (5*A - 9*B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((A -
3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) + 2*((2*A - 3*B)*cos(d*x + c)^3 + 2*(2*A - 3*B)*cos(d
*x + c)^2 + (2*A - 3*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)
^2 + a^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

3.554 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=287

$$\frac{(9A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(12A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}}{4a^{3/2}d}$$

```
[Out] -((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) + ((9*A - 13*B)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*
Sin[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A - 2
*B)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((6
*A - 7*B)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.94735, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2955, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(9A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(12A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}}{4a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```



```
[Out] -((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) + ((9*A - 13*B)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*
Sin[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A - 2
*B)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((6
*A - 7*B)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec}{}}{2a}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{3/2}d} + \frac{(9A-1}{}$$

Mathematica [A] time = 3.30096, size = 328, normalized size = 1.14

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(2(6A-7B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-2\sqrt{2}(9A-13B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(2*(6*A - 7*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c +
d*x)/2]^2 + 36*A*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 52*B*ArcSi
n[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 2*Sqrt[2]*(9*A - 13*B)*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 + 2*B
*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*A*Sqrt[-((-1 + Sec[c + d*x])
*Sec[c + d*x])] - 3*B*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 6*A*Cos[c
 + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] - 7*B*Cos[c + d*x]*Sqrt[(-
1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[-1 + Cos[c + d*x
]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.291, size = 531, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(12*A*sin(d*x+c)*
cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin
(d*x+c)))*2^(1/2)-12*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-19*B*sin(d*x+c)*cos(d*x
+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*2^(1/2)+19*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-36*A*sin(d*x+c)*cos(d*x+c)^2*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+12*A*cos(d*x+c)^3*(-2/(cos(
d*x+c)+1))^(1/2)+52*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))-14*B*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x
+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+8*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-
8*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+10*B*cos(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2)-4*B*(-2/(cos(d*x+c)+1))^(1/2))/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*
x+c)^3/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.862492, size = 1989, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(2)*((9*A - 13*B)*cos(d*x + c)^4 + 2*(9*A - 13*B)*cos(d*x + c
)^3 + (9*A - 13*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(
2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
4*((6*A - 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A - 19*
B)*cos(d*x + c)^4 + 2*(12*A - 19*B)*cos(d*x + c)^3 + (12*A - 19*B)*cos(d*x
+ c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos
(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^
4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), -1/8*(2*sqrt(2)*((9*A -
 13*B)*cos(d*x + c)^4 + 2*(9*A - 13*B)*cos(d*x + c)^3 + (9*A - 13*B)*cos(d*
x + c)^2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((6*A - 7*B)*cos(d*x + c)^
2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A - 19*B)*cos(d*x + c)^4 + 2*(12*A
- 19*B)*cos(d*x + c)^3 + (12*A - 19*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sq
```

```
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^4 + 2*a
^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/
2)), x)
```

3.555 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(157A-85B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)}{240a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)}}$$

```
[Out] -((283*A - 163*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqr
t[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Sec
[c + d*x])^(5/2)) - ((21*A - 13*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d
*(a + a*Sec[c + d*x])^(3/2)) + ((2671*A - 1495*B)*Sin[c + d*x])/(240*a^2*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((787*A - 475*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((157*A - 85*B
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 1.12291, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(157A-85B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)}{240a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((283*A - 163*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqr
t[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Sec
[c + d*x])^(5/2)) - ((21*A - 13*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d
*(a + a*Sec[c + d*x])^(3/2)) + ((2671*A - 1495*B)*Sin[c + d*x])/(240*a^2*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((787*A - 475*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((157*A - 85*B
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(13A-5B)\cdots}{\sec^{\frac{5}{2}}(c+dx)\cdots}}{4a^2}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \cdots$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \cdots$$

Mathematica [A] time = 2.09538, size = 207, normalized size = 0.65

$$\frac{2\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)(5(887A-479B)\cos(c+dx)+16(52A-25B)\cos(2(c+dx))-40A\cos(3(c+\cdots}{480d\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (60*Sqrt[2]*(283*A - 163*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*(3491*A
 - 1895*B + 5*(887*A - 479*B)*Cos[c + d*x] + 16*(52*A - 25*B)*Cos[2*(c + d*
x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)] + 12*A*Cos[4*(c + d*x)]
)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(480*d*Sqrt[-1 + Cos[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.325, size = 461, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/480/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(4245*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-192*A*cos(d*x+c)^5-2445*B*sin(d*x+c)*co
s(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)
+1))^(1/2)+8490*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+
c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+512*A*cos(d*x+c)^4-4890*B*sin(d*x+c
)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+
c)+1))^(1/2)-320*B*cos(d*x+c)^4+4245*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*
B*sin(d*x+c)+1920*B*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*x+c)^2+37
68*A*cos(d*x+c)-2040*B*cos(d*x+c)+5342*A-2990*B)/sin(d*x+c)^5/a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.566558, size = 1565, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/960*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos
(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(a)*log(-
(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(96*A*cos(d*x + c)^4 - 160*(A - B)*cos(d*x + c
)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 + 5*(911*A - 503*B)*cos(d*x + c) + 26
71*A - 1495*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(28
3*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*
B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^4 - 160*(A - B
)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 + 5*(911*A - 503*B)*cos(
d*x + c) + 2671*A - 1495*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.556 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(95A-39B)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\ldots}}{16\sqrt{2}\ldots}$$

```
[Out] ((163*A - 75*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c
 + d*x])^(5/2)) - ((17*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a
 + a*Sec[c + d*x])^(3/2)) - ((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((95*A - 39*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.909944, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4020, 4022, 4013, 3808, 206}

$$\frac{(95A-39B)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\ldots}}{16\sqrt{2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```



```
[Out] ((163*A - 75*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c
 + d*x])^(5/2)) - ((17*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a
 + a*Sec[c + d*x])^(3/2)) - ((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((95*A - 39*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(11A-3B)\cdots}{\sec^{\frac{3}{2}}(c+dx)\cdots}}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(9\cdots}{\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\cdots}{48\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\cdots}{48\cdots}$$

$$= \frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{(A-\cdots}{4\cdots}$$

Mathematica [A] time = 1.66488, size = 183, normalized size = 0.68

$$\frac{2\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)((255B-479A)\cos(c+dx)+(48B-80A)\cos(2(c+dx))+8A\cos(3(c+d}{96d\sqrt{\cos(c+dx)-1}(a(\sec(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (-12*Sqrt[2]*(163*A - 75*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*(-379*A
 + 195*B + (-479*A + 255*B)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)]
+ 8*A*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(
96*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.325, size = 439, normalized size = 1.6

$$-\frac{(-1+\cos(dx+c))^2}{96\,da^3(\sin(dx+c))^5}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(489\,A\sin(dx+c)(\cos(dx+c))^2\arctan\left(1/2\,\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(489*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-225*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+978*A*sin(
d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos
(d*x+c)+1))^(1/2)+64*A*cos(d*x+c)^4-450*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+489*arctan(
1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d
*x+c)-384*A*cos(d*x+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+192*B*cos(d*x+c)^3-686*A*cos(d*x+
c)^2+318*B*cos(d*x+c)^2+408*A*cos(d*x+c)-216*B*cos(d*x+c)+598*A-294*B)/a^3/
sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.558409, size = 1457, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*
x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^
2 - (503*A - 255*B)*cos(d*x + c) - 299*A + 147*B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163
*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + 3*(163*A - 75
*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(32
*A*cos(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^2 - (503*A - 255*B)*cos(d*x
 + c) - 299*A + 147*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.557 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16aa}$$

```
[Out] -((75*A - 19*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c
 + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a
 + a*Sec[c + d*x])^(3/2)) + ((49*A - 9*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.699583, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4020, 4013, 3808, 206}

$$\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16aa}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((75*A - 19*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c
 + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a
 + a*Sec[c + d*x])^(3/2)) + ((49*A - 9*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(9A\cdots}{\sqrt{\sec(c\cdots}}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{\cdots}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{\cdots}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{\cdots}}$$

$$= -\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\cdots}{4d\sqrt{\cdots}}$$

Mathematica [A] time = 2.46083, size = 228, normalized size = 1.02

$$\frac{\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)\left(2\left(73A^2+76AB-13B^2\right)\cos(c+dx)+16A^2\cos(3(c+dx))+85A^2+A(85A\cdots\right.}{32d\sqrt{\cos(c+dx)-\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (8*Sqrt[2]*(75*A - 19*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Cos[(c + d*x)/2]^5*(B + A*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[(c
 + d*x)/2] + (85*A^2 + 117*A*B - 18*B^2 + 2*(73*A^2 + 76*A*B - 13*B^2)*Cos[
c + d*x] + A*(85*A + 19*B)*Cos[2*(c + d*x)] + 16*A^2*Cos[3*(c + d*x)])*Sqrt
[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(32*d*Sqrt[-1 + Cos[c + d*x]]
*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.444, size = 365, normalized size = 1.6

$$-\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(32\,A(\cos(dx+c))^3\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+75\,A\cos(dx+c)\sin(dx+c)\arctan\left(1/2\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(32*A*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+75*A
*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+53*
A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-19*B*cos(d*x+c)*sin(d*x+c)*arctan(
1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-13*B*cos(d*x+c)^2*(-2/(cos(d*x+c)
+1))^(1/2)+75*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))
-36*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-19*B*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+4*B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-4
9*A*(-2/(cos(d*x+c)+1))^(1/2)+9*B*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/
2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a^3/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*
x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.546412, size = 1338, normalized size = 6.

$$\left[-\frac{\sqrt{2}\left((75\,A-19\,B)\cos(dx+c)^3+3\,(75\,A-19\,B)\cos(dx+c)^2+3\,(75\,A-19\,B)\cos(dx+c)+75\,A-19\,B\right)\sqrt{a}\log}{64\left(a^3d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c
)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*log(-(a*cos(d*x +
 c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(32*A*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 49*A -
 9*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
```

```
c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d), 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(
d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(-a)*arctan(sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(
a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 49
*A - 9*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2
), x)
```

3.558 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(9A-B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(19A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{}{16a}$$

```
[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c +
 d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^(3/2)) - ((9*A - B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.711566, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2955, 4019, 4020, 4013, 3808, 206}

$$-\frac{(9A-B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(19A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c +
 d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^(3/2)) - ((9*A - B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A}{\sqrt{\sec(}}}{4a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

$$= \frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{}{4d\sqrt{c}}$$

Mathematica [A] time = 0.988327, size = 108, normalized size = 0.48

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((5B-13A)\cos(c+dx)-9A+B)+8(19A+5B)\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{64ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*(8*(19*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2
]^4 + 4*(-9*A + B + (-13*A + 5*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(64*a*d*
Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.324, size = 339, normalized size = 1.5

$$\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(19\,A\cos(dx+c)\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)+13\,A\,(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(19*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))+13*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+5*B*
cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*B*
cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+19*A*sin(d*x+c)*arctan(1/2*sin(d*x+c
)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+5*B*s
in(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+4*B*cos(d*x+c)*(
-2/(cos(d*x+c)+1))^(1/2)-9*A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a^3/sin(d*x+c
)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 4.8777, size = 7997, normalized size = 35.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
```

```
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2
+ 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x
+ 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
 + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x +
4*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7
/2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10
*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*s
in(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*
sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(
1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
 57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)
+ 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
 - 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*c
os(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2
*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
 + 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
 + 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(
1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x +
3/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*si
n(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
```

```
(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a
^2*cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*
cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x
+ 4*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2
*c)^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d
*x + c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos
(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c)
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sq
rt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*
cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x +
 3*c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*
d*x + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c
))*sin(3*d*x + 3*c))*sqrt(a)) + (4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 40*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 24*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 24*(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 16*(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3
*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*si
n(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x
+ 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4
*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 +
 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
```

```
x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*
c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*lo
g(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)
*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3
*cos(3/2*d*x + 3/2*c) + 5*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 3*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3
*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*co
s(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c)
 + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c)
 - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3
/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x
 + 3*c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 12*sin(3/2*d*x + 3/2*c))*B/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a
^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt
(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16
*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*
c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2
)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sq
rt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c
) + 6*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + sqrt(2)*a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 12*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x
 + 3*c) + sqrt(2)*a^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sq
rt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)))/d
```

Fricas [A] time = 0.53028, size = 1257, normalized size = 5.64

$$\left[\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\log\left(-\frac{a\cos}{\ }\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((13*A - 5*B)*cos(d*x + c) + 9*A - B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*A
+ 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*(19*A + 5*B)*cos(
d*x + c) + 19*A + 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 5*B)
*cos(d*x + c) + 9*A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

3.559 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A+3B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\quad}{4d\cos^{\frac{5}{2}}(}$$

```
[Out] ((5*A + 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.403405, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2955, 4012, 3810, 3808, 206}

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A+3B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\quad}{4d\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((5*A + 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^(3/2))
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
```

```
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left((5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{8a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{}{4d\,c}$$

Mathematica [A] time = 0.808088, size = 108, normalized size = 0.61

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)((5A+3B)\cos(c+dx)+A+7B)+(5A+3B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c\right.\right.}{4d\cos^{\frac{5}{2}}(c+dx)(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((5*A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]
^3 + ((A + 7*B + (5*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/2))/(4*d*Cos[c
 + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.338, size = 340, normalized size = 1.9

$$-\frac{(-1+\cos(dx+c))^2}{16\,d\,(\sin(dx+c))^5\,a^3}\sqrt{\cos(dx+c)}\left(5\,A\,(\cos(dx+c))^2\,\sqrt{-2\,(\cos(dx+c)+1)^{-1}}-5\,A\cos(dx+c)\sin(dx+c)\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))^2*(5*A*cos(d*x+c)^2*(-2/(cos(d*x+c
)+1))^(1/2)-5*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2))+3*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2)-5*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))+4*B*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*x+c)*arctan(1/
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*(-2/(cos(d*x+c)+1))^(1/2)-7*B*(-2
/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/(-
2/(cos(d*x+c)+1))^(1/2)/a^3
```

Maxima [B] time = 4.13796, size = 7231, normalized size = 41.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*si
n(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x
+ 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*
c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x
+ 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*
cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*
d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8
*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
```

s(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))*A/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2

```
*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^
2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)) - (12*(sin(4*d*x + 4*c) + 6*sin(2
*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 16*(11*sin(5/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 11*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x +
2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d
*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(4*d*x + 4*c) + 6*si
n(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2
*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*
d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) + 3*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 3
6*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c
) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 1) - 12*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 16*(11*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*cos(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos
```

```
(2*d*x + 2*c))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 48*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(4*d*
x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 48*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((sqrt(2)*a^2*cos(4*d*x
 + 4*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2*sin(4*d*x + 4*c)^2 +
12*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*a^2*sin(2*d*x
 + 2*c)^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 12*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(6*sqrt(2)*a^2*cos(
2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(sqrt(2)*a^2*cos(4*d*x + 4
*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*c
os(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c)
 + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x
 + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*sqrt(a)))/d
```

Fricas [A] time = 0.527843, size = 1243, normalized size = 7.06

$$\left[\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+\cdots}{\cdots}\right.}{64\left(a^3d\cos(dx+c)^3+3a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) + 4*((5*A + 3*B)*cos(d*x + c) + A + 7*B)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A + 3*B)
*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A + 3*B)*cos(d*x + c)
 + 5*A + 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((5*A + 3*B)*cos(d*x +
 c) + A + 7*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

3.560 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=234

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c\cdot}{\sqrt{a\sec(c+d}}\right.}{a^{5/2}d}$$

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x]
)/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.713895, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2955, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c\cdot}{\sqrt{a\sec(c+d}}\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x]
)/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+}{}}{4a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A-43B)\tanh^{-1}\left(\right.}{}$$

Mathematica [B] time = 6.15611, size = 965, normalized size = 4.12

$$\frac{3A\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)(\sec(c+dx)+1)^2}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}-\frac{11B\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{16d\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] -(B*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (
A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*
B*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c +
 d*x]))^(5/2)) - (A*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/
2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/
(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (A*(1 + Sec[c + d*
x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2))
 + (7*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(5/2)*(a*(1 +
Sec[c + d*x]))^(5/2)) + (3*A*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c
 + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*B*(1 + Sec[c + d*x])^2*Si
n[c + d*x])/(16*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A*A
rcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Se
c[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*
x]))^(5/2)) - (11*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x
]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt
[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) - (43*B*ArcSin[Sqrt[Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c +
 d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*A*Ar
cTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]
]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[
1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (43*B*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3
/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]
*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.321, size = 540, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(16*B*cos(d*x
+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))-16*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+3*A*cos(d*x+c)^2*(-2/(c
os(d*x+c)+1))^(1/2)-3*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))-11*B*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+43*B*cos(d*
x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+16*B*sin(d
*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))-16*B*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1+sin(d*x+c)))+4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-3*
```

```
A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*B*cos(d*x+c
)*(-2/(cos(d*x+c)+1))^(1/2)+43*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))-7*A*(-2/(cos(d*x+c)+1))^(1/2)+15*B*(-2/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)^(1/2)/a^3/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.688488, size = 1906, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^
2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(d*x + c)^
2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 4*((3*A - 11*B)*cos(d*x + c) + 7*A - 15*B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(B*cos(d*x + c)^3 +
 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 -
 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A -
 43*B)*cos(d*x + c)^2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))/(a*sin(d*x + c))) - 2*((3*A - 11*B)*cos(d*x + c) + 7*A - 15*B)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(B
*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(-a)*arcta
n(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

3.561 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=286

$$-\frac{(11A-35B)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(43A-115B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2A\cdot}{}$$

```
[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sq
rt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Si
n[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 1
5*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) -
 ((11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
 + d*x]])
```

Rubi [A] time = 0.957811, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2955, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(11A-35B)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(43A-115B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2A\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sq
rt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Si
n[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 1
5*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) -
 ((11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
 + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+}{}}{4a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{(43A-115B)\,\mathrm{t}}{}$$

Mathematica [B] time = 6.17207, size = 1061, normalized size = 3.71

$$-\frac{11A\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)(\sec(c+dx)+1)^2}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}} + \frac{35B\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{16d\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] -(B*Sin[c + d*x])/(4*d*Cos[c + d*x]^(11/2)*(a*(1 + Sec[c + d*x]))^(5/2)) -
(A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (7
*B*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(11/2)*(a*(1 + Sec[c
 + d*x]))^(5/2)) + (3*A*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]
^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (7*B*(1 + Sec[c + d*x])^2*Sin[c + d*
x])/(16*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*A*(1 + Sec[
c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(
5/2)) + (11*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)*(
a*(1 + Sec[c + d*x]))^(5/2)) + (7*A*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*
d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (15*B*(1 + Sec[c + d*x
])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)) -
 (11*A*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(3/2)*(a*(1 +
Sec[c + d*x]))^(5/2)) + (35*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[
c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*ArcSin[Sqrt[1 - Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c +
 d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (35*B*A
rcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Se
c[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*
```

```
x]))^(5/2)) - (43*A*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2)) + (115*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c
 + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1
 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (43*A*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/
2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*
(a*(1 + Sec[c + d*x]))^(5/2)) - (115*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/
Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c +
d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c +
d*x]))^(5/2))
```

Maple [B] time = 0.333, size = 821, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-16*A*sin(d*x+c)
*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))*2^(1/2)-40*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+40*B*sin(d*x+c)*cos(d*
x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))*2^(1/2)-43*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))+16*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-16*A*cos(d*x+c)*sin(d*x+c)*2
^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))+11*A*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+115*B*sin(d*x+c)*cos(d*x+c)
^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-40*B*cos(d*x+c)*sin(d*x
+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))+40*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-35*B*cos(d*x+c)^3*(-2/(cos(d*x+c)+
1))^(1/2)-43*A*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2))+4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+115*B*cos(d*x+c)*sin(
d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-20*B*cos(d*x+c)^2*(
-2/(cos(d*x+c)+1))^(1/2)-15*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+39*B*cos
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*B*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d
*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-2/(cos(d*x+c)+1)
)^(1/2)/a^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.924742, size = 2234, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^4 + 3*(43*A - 115*B)*cos(d*x +
 c)^3 + 3*(43*A - 115*B)*cos(d*x + c)^2 + (43*A - 115*B)*cos(d*x + c))*sqrt
(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*
A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + 16*((2*A - 5*B)*cos(d*x + c)^4 + 3*(2*A - 5*B
)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))
*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^4 + 3*a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/32*(s
qrt(2)*((43*A - 115*B)*cos(d*x + c)^4 + 3*(43*A - 115*B)*cos(d*x + c)^3 + 3
*(43*A - 115*B)*cos(d*x + c)^2 + (43*A - 115*B)*cos(d*x + c))*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))/(a*sin(d*x + c))) - 2*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*A - 11*B)*co
s(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 16*((2*A - 5*B)*cos(d*x + c)^4 + 3*(2*A - 5*B)*cos(d*x +
c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c
)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/
2)), x)
```

3.562 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2(5aA+7bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(5aA+$$

```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A + 7*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.23104, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(5aA+7bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(5aA+7bB)\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```


```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A + 7*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))(B+A\cos(c+dx))\,dx$$

$$= \int \cos^{\frac{3}{2}}(c+dx)\left(bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c\right.$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(5a\right.$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (Ab+aB)\int \cos^{\frac{5}{2}}(c+dx$$

$$= \frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(Ab+aB)\cos}{}$$

$$= \frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5aA+7bB)F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

Mathematica [A] time = 0.892786, size = 103, normalized size = 0.74

$$\frac{10(5aA+7bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+126(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(42(aB+Ab)\cos(}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(5*a*A + 7*b*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(65*a*A + 70*b*B + 42*(A*b + a*B)*Cos[c
 + d*x] + 15*a*A*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 1.888, size = 413, normalized size = 3.

$$-\frac{2}{105 d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,Aa\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-360\,Aa-168\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a-168*A*b-168*B*a)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+25*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+35*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^{3}\sec(dx+c)^{2}+Aa\cos(dx+c)^{3}+(Ba+Ab)\cos(dx+c)^{3}\sec(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3*sec(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A
*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.563 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2(aB+Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(\ldots}{\ldots}$$

```
[Out] (2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.213619, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(c+dx)\,\mathrm{c}\ldots}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}(b+a\cos(c+dx))(B+A\cos(c+dx))\,dx$$
$$= \int \sqrt{\cos(c+dx)}\left(bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c\right.$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}(3a\right.$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+(Ab+aB)\int \cos^{\frac{3}{2}}(c+dx$$
$$= \frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}}{3d}$$
$$= \frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A] time = 0.440875, size = 86, normalized size = 0.8

$$\frac{2\left(5(aB+Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3aA\cos(c+dx)+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Si
n[c + d*x]))/(15*d)
```

Maple [B] time = 1.894, size = 371, normalized size = 3.4

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,Aa\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(24\,Aa+20\,Ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a+20*A*b+20*B*a)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5
*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos(dx+c)^2\sec(dx+c)^2+Aa\cos(dx+c)^2+(Ba+Ab)\cos(dx+c)^2\sec(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A
*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.564 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{2(aA+3bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.193081, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2954, 2968, 3023, 2748, 2641, 2639}

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(aA+3bB)+\frac{3}{2}(A\ldots}{\sqrt{\cos(c\ldots}}$$
$$= \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + (Ab+aB)\int \sqrt{\cos(c+d\ldots}$$
$$= \frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\ldots\right.}{3d}$$

Mathematica [A] time = 0.25861, size = 67, normalized size = 0.89

$$\frac{2\left((aA+3bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+aA\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (a*A + 3*b*B)*EllipticF[(c +
d*x)/2, 2] + a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 1.946, size = 326, normalized size = 4.4

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,Aa\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+Aa\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b-2*A*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
```

```
d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)\sec(dx+c)^2+Aa\cos(dx+c)+(Ba+Ab)\cos(dx+c)\sec(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)*sec(d*x + c)^2 + A*a*cos(d*x + c) + (B*a + A*b)*
cos(d*x + c)*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.565 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=71

$$\frac{2(aB+Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c +
 d*x)/2, 2])/d + (2*b*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.193214, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2954, 2968, 3021, 2748, 2641, 2639}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c +
 d*x)/2, 2])/d + (2*b*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{1}{2}(Ab+aB)+\frac{1}{2}(aA-bB)\cos(c+dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+(Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+(aA-bE\\ &= \frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\end{aligned}$$

Mathematica [A] time = 0.368513, size = 64, normalized size = 0.9

$$\frac{2\left((aB+Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{bB\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*((a*A - b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c + d*x)
/2, 2] + (b*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [B] time = 2.018, size = 244, normalized size = 3.4

$$-2\,\frac{Ab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] -2*(A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+B*a*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-2*B*b*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec(dx+c)^2 + Aa + (Ba+Ab)\sec(dx+c)\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(cos(d*x
 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))(a+b\sec(c+dx))\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.566 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=103

$$\frac{2(3aA+bB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.212193, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3}{2}(Ab+aB)+\frac{1}{2}(3aA+bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+(Ab+aB)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\frac{1}{3}(3aA+bB)\int \frac{}{\sqrt{}} \\
&= \frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+}{d\sqrt{\cos(c+dx)}} \\
&= -\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bB\,\mathrm{s}}{3d\cos}
\end{aligned}$$

Mathematica [A] time = 0.49008, size = 107, normalized size = 1.04

$$\frac{2\left((3aA+bB)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-3(aB+Ab)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+3aB\sin(c+dx)+3\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (3*a*A +
b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x] + b*B*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 4.615, size = 428, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*b
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/sqrt(cos(d*x
 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))(a+b\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.567 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{2(aB+Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+3bB)\sin(c+d}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a*A + 3*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.231375, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2954, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+3bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2b}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a*A + 3*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\
&= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\
&= \frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\frac{5}{2}(Ab+aB)+\frac{1}{2}(5aA+3bB)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+(Ab+aB)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{5}(5aA+3bB)\int \text{-}\\
&= \frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+3bB)\sin(c+dx}{5d\sqrt{\cos(c+dx)}}\\
&= -\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bB}{5d\,\text{c}}\end{aligned}$$

Mathematica [A] time = 0.842185, size = 134, normalized size = 0.96

$$\frac{10(aB+Ab)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-6(5aA+3bB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+15aA\sin(2(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*(5*a*A + 3*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b
+ a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] +
 10*a*B*Sin[c + d*x] + 15*a*A*Sin[2*(c + d*x)] + 9*b*B*Sin[2*(c + d*x)] + 6
*b*B*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 5.971, size = 663, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*B*b/(8*sin(1/2*d*x+1/2*c)^6-
12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))/cos(d*x + c)
^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.568 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=182

$$\frac{2\left(5a^2A+7b(2aB+Ab)\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2\left(3a^2B+6aAb+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2A+7b(2aB+Ab\right.}{}$$

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a
^2*A + 7*b*(A*b + 2*a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*a^2*A +
 7*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(9*A*b +
 7*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(3/2)
*(b + a*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.368102, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2990, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(3a^2B+6aAb+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2A+7b(2aB+Ab)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(5a^2A+7b(2aB+Ab)\right)\sin(c}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a
^2*A + 7*b*(A*b + 2*a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*a^2*A +
 7*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(9*A*b +
 7*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(3/2)
*(b + a*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \int \sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2(B+A\cos(c+dx))\,d\\ &= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))\sin(c+dx)}{7d}+\frac{2}{7}\int\\ &= \frac{2a(9Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2aA\cos^{\frac{3}{2}}(c+\\ &= \frac{2a(9Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2aA\cos^{\frac{3}{2}}(c+\\ &= \frac{2\left(6aAb+3a^2B+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2A+7\right.}{}\\ &= \frac{2\left(6aAb+3a^2B+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2A+7\right.}{}\end{aligned}$$

Mathematica [A] time = 1.21557, size = 139, normalized size = 0.76

$$\frac{10\left(5a^2A+14abB+7Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+42\left(3a^2B+6aAb+5b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (42*(6*a*A*b + 3*a^2*B + 5*b^2*B)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2*A +
 7*A*b^2 + 14*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*a*(
2*A*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 14*A*b^2 + 28*a*b*B + 3*a^2*A*Cos
[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] time = 1.966, size = 548, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a
^2+280*B*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-84*A*a*b-7
0*A*b^2-42*B*a^2-140*B*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-126*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b+25*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*A*b^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-105*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3\sec(dx+c)^3 + Aa^2\cos(dx+c)^3 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^3\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (2*B
*a*b + A*b^2)*cos(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
```

```
)^3*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x
)
```

3.569 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(3a^2A+5b(2aB+Ab)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5aB+7Ab)\sin(c+d}{15d}$$

```
[Out] (2*(3*a^2*A + 5*b*(A*b + 2*a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2*a
*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(7*A*b + 5*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Cos[c + d*x]]*(b
 + a*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.332282, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2990, 3023, 2748, 2641, 2639}

$$\frac{2\left(a^2B+2aAb+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^2A+5b(2aB+Ab)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5aB+7Ab)\sin(c+dx)\sqrt{\cos}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a^2*A + 5*b*(A*b + 2*a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2*a
*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(7*A*b + 5*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Cos[c + d*x]]*(b
 + a*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\cos(c+dx)}(b+a\cos(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int$$

$$= \frac{2a(7Ab+5aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2aA\sqrt{\cos(c+}}{}$$

$$= \frac{2a(7Ab+5aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2aA\sqrt{\cos(c+}}{}$$

$$= \frac{2\left(3a^2A+5b(Ab+2aB)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(2aAb+\right.}{}$$

Mathematica [A] time = 0.621987, size = 106, normalized size = 0.76

$$\frac{2\left(5\left(a^2B+2aAb+3b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\left(3a^2A+10abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)\sqrt{\cos(}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(2*a*A*b
 + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(10*A*
b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 2.096, size = 487, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^2+40*A*a*b+20*B*a^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^2-20*A*a*b-10*B*a^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+10*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+5*B*a^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+15*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^2\sec(dx+c)^3 + Aa^2\cos(dx+c)^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (2*B
*a*b + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^2*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

3.570 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{2(a^2A+6abB+3Ab^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(a^2B+2aAb-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)\sqrt{\cos(c+d}}{3d}$$

```
[Out] (2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*A
*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]]) + (2*a^2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.317962, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2988, 3023, 2748, 2641, 2639}

$$\frac{2(a^2A+6abB+3Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(a^2B+2aAb-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*A
*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]]) + (2*a^2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{-\frac{1}{2}b(Ab+2aB) - \frac{1}{2}\left(2aAb+a^2B\right.$$

$$= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{4}{3}\int$$

$$= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}($$

$$= \frac{2\left(2aAb+a^2B-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2A+3Ab^2\right.}{}$$

Mathematica [A] time = 0.662784, size = 102, normalized size = 0.84

$$\frac{2\left(\left(a^2A+6abB+3Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(a^2A\cos(c+dx)+3b^2B\right.}{\sqrt{\cos(c+dx)}}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^
2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + ((3*b^2*B + a^2*A*Cos[c + d*x])*Si
n[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)
```

Maple [B] time = 2.04, size = 404, normalized size = 3.3

$$-\frac{2}{3d}\left(4Aa^2\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^4+a^2A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\text{EllipticF}\left(\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/3*(4*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
-2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-6*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)\sec(dx+c)^3+Aa^2\cos(dx+c)+\left(2\,Bab+Ab^2\right)\cos(dx+c)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\text{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b
 + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(
d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

3.571 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=126

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b(2aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+$$

```
[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b + 3
*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rubi [A] time = 0.335161, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 2988, 3021, 2748, 2641, 2639}

$$\frac{2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b(2aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\,\mathrm{si}}{3d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b + 3
*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB) - \frac{1}{2}\left(6aAb+3a\right.}{}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{4}{3}\int \frac{1}{4}($$

$$= \frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \left(-a^2A+\right.$$

$$= \frac{2\left(a^2A-Ab^2-2abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(6aAb+3a^2\right.}{}$$

Mathematica [A] time = 1.19733, size = 105, normalized size = 0.83

$$\frac{2\left(\left(3a^2B+6aAb+b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)(3(2aB+Ab)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3*a^
2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + (b*(b*B + 3*(A*b + 2*a*B)*Cos[c +
d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 4.912, size = 677, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*A*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
B*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b+2*B*a)*(-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2\sec(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\sec(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

3.572 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2\left(5a^2B+10aAb+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2B+10aAb+3b^2B\right.}{5d\sqrt{\cos(c+a}}$$

```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3
*a^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rubi [A] time = 0.374806, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2988, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^2B+10aAb+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sin(c}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3
*a^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB) - \frac{1}{2}\left(10aAb+5a^2B+3b^2B\right)}{\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}\left(10aAb+5\right.}{}$$

$$= \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{3}\left(-3a^2A-Ab^2-2a\right.$$

$$= \frac{2\left(3a^2A+Ab^2+2abB\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab}{3}$$

$$= -\frac{2\left(10aAb+5a^2B+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(3a^2A+Ab^2+2abB\right.}{3d}$$

Mathematica [A] time = 1.12825, size = 175, normalized size = 1.02

$$\frac{10\left(3a^2A+2abB+Ab^2\right)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-6\left(5a^2B+10aAb+3b^2B\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (-6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2
, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x
)/2, 2] + 10*A*b^2*Sin[c + d*x] + 20*a*b*B*Sin[c + d*x] + 30*a*A*b*Sin[2*(c
 + d*x)] + 15*a^2*B*Sin[2*(c + d*x)] + 9*b^2*B*Sin[2*(c + d*x)] + 6*b^2*B*T
an[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 6.644, size = 750, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5
*B*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(A*b+2*B*a)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+2*a*(2*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*
c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\sec(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c) + A)(b\sec(dx+c) + a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

3.573 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{2\left(7a^2B+14aAb+5b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{2\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a^2B+14aAb+5b^2\right.}{21d\cos^{\frac{3}{2}}(c+}$$

```
[Out] (-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(14
*a*A*b + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b^2*B*Si
n[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21
*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c + d*x])/(5*
d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.396129, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2988, 3021, 2748, 2636, 2641, 2639}

$$\frac{2\left(7a^2B+14aAb+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a^2B+14aAb+5b^2B\right)\sin(c}{21d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(14
*a*A*b + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b^2*B*Si
n[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21
*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c + d*x])/(5*
d*Sqrt[Cos[c + d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB) - \frac{1}{2}\left(14aAb+7a^2B+5b^2B\right)}{\cos^{\frac{7}{2}}(c+dx}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}\left(14aAb+\right.}{}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{5}\left(-5a^2A-3Ab^2-\right.$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(14aAb+7a^2B+\right.}{21d\cos^{\frac{3}{2}}(}$$

$$= -\frac{2\left(5a^2A+3Ab^2+6abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(14aAb+7a^2B+5b^2\right.}{21d}$$

Mathematica [A] time = 4.6096, size = 191, normalized size = 0.89

$$\frac{2\left(5\left(7a^2B+14aAb+5b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-21\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{5\left(7a^2B+14aAb+5b^2B\right)}{\cos^{\frac{3}{2}}(c+dx)}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (2*(-21*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(14*a*A
*b + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + (15*b^2*B*Sin[c + d*x])
/Cos[c + d*x]^(7/2) + (21*b*(A*b + 2*a*B)*Sin[c + d*x])/Cos[c + d*x]^(5/2)
+ (5*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21*
(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)
```

Maple [B] time = 8.288, size = 859, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b^2*(-1/56*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(2*A
*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b*(A*b+2*B*a)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a^2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bb^2\sec(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c) + A)(b\sec(dx+c) + a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

3.574 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=182

$$-\frac{2\left(a^2+3b^2\right)(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d}+\frac{2\left(3a^2A-5abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2b^3(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\right.}{a^4d(a+b)}$$

```
[Out] (2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*
(a^2 + 3*b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^3*(A*
b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*(A
*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (2*A*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.855298, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2990, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(a^2+3b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2\left(3a^2A-5abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2b^3(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a^4d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] (2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*
(a^2 + 3*b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^3*(A*
b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*(A
*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (2*A*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*a*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{b+a\cos(c+dx)}\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3Ab}{2}+\frac{3}{2}aA\cos(c+dx)-\frac{5}{2}(Ab-aB)\cos^2(c+dx)\right.}{b+a\cos(c+dx)}}{5a}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{4\int \frac{-\frac{5}{4}}{}}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{4\int \frac{\frac{5}{4}ab}{}}{}$$

$$= \frac{2\left(3a^2A+5Ab^2-5abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+d}{3a^2d}$$

$$= \frac{2\left(3a^2A+5Ab^2-5abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2\left(a^2+3b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+dx\right.}{3a^4d}$$

Mathematica [A] time = 2.54348, size = 263, normalized size = 1.45

$$\frac{\frac{6\left(3a^2A-5abB+5Ab^2\right)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b\sqrt{\sin^2(c+dx)}}+2a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2*(9*a^2*A + 5*A*b^2 - 5*a*b*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b) + 2*a^2*(4*A*b + 5*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*a^2*Sqrt[Cos[c + d*x
]]*(-5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] + (6*(3*a^2*A + 5*A*b
^2 - 5*a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a +
b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a
/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2
]))/(30*a^4*d)
```

Maple [B] time = 2.38, size = 1074, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*A*a^4+2
4*A*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^4-44*A*a^3*b+20*
A*a^2*b^2+20*B*a^4-20*B*a^3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*
A*a^4+16*A*a^3*b-10*A*a^2*b^2-10*B*a^4+10*B*a^3*b)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
```

```
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b
-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-
15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^4+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2
-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)*a*b^3)/a^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec{(dx+c)}+A)\cos{(dx+c)}^{\frac{5}{2}}}{b\sec{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.575 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{2\left(a^2A-3abB+3Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^3d}-\frac{2b^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-$$

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2*A + 3*A*b^2 -
3*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.586128, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2990, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(a^2A-3abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```


```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2*A + 3*A*b^2 -
3*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d)
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx &= \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{b+a\cos(c+dx)}\,dx \\
&= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{Ab}{2}+\frac{1}{2}aA\cos(c+dx)-\frac{3}{2}(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{3a} \\
&= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{2\int \frac{-\frac{1}{2}aAb-\frac{1}{2}\left(a^2A+3Ab^2-3abB\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{3a^2} - \frac{(Ab-
\\
&= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(b^2(Ab-aB)\right)\int
\\
&= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\left(a^2A+3Ab^2-3abB\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d} - \frac{2b^2
\end{aligned}$$

Mathematica [A] time = 1.20836, size = 210, normalized size = 1.54

$$\frac{-\frac{6(Ab-aB)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}+4A\left(\text{Ellipti}\right.}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(-(A*b) + 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 4
*A*(EllipticF[(c + d*x)/2, 2] - (b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a + b)) + 4*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(A*b - a*B)*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/(6*a*d)
```

Maple [B] time = 2.597, size = 786, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*A*a^3-4*A*
a^2*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^3+2*A*a^2*b)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2*b+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^3-3*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))*a*b^2)/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.576 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{2(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(a^2*d) + (2*b*(A*b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a + b)*d)
```

Rubi [A] time = 0.275883, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 3002, 2639, 2803, 2641, 2805}

$$-\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(a^2*d) + (2*b*(A*b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a + b)*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+A\cos(c+dx))}{b+a\cos(c+dx)}\,dx$$
$$= \frac{A\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{(Ab-aB)\int\frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx}{a}$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^2} + \frac{(b(Ab-aB))\int\frac{1}{\sqrt{\cos(c+dx)}(b+\ldots}}{a^2}$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2b(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\ldots\right.}{a^2(a+b)d}$$

Mathematica [A] time = 0.964181, size = 131, normalized size = 1.47

$$\frac{aB\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)-\frac{2A\sin(c+dx)\left(-(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-b\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\ldots}{\sqrt{\sin^2(c+dx)}}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*B*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*A*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - b*EllipticPi[-(a/b), -ArcS
in[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(a^2*d)
```

Maple [A] time = 2.124, size = 295, normalized size = 3.3

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\,a^2\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(A\text{EllipticF}\left(\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
```

```
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b)/a^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\sec(c+dx))\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.577 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=61

$$\frac{2A\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] (2*A*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2])/(a*(a + b)*d)
```

Rubi [A] time = 0.218543, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2954, 3002, 2641, 2805}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*A*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2])/(a*(a + b)*d)
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx$$
$$= \frac{A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a}$$
$$= \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}$$

Mathematica [A] time = 0.221744, size = 58, normalized size = 0.95

$$\frac{2\left(A(a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(aB-Ab)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*(A*(a + b)*EllipticF[(c + d*x)/2, 2] + (-(A*b) + a*B)*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]))/(a*(a + b)*d)
```

Maple [A] time = 1.973, size = 217, normalized size = 3.6

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{a\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\Big(A\text{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec{(c + dx)}}{(a + b\sec{(c + dx)})\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec{(dx + c)} + A}{(b\sec{(dx + c)} + a)\sqrt{\cos{(dx + c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

3.578 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=86

$$\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(b*(a + b)*d) + (2*B*Sin[c + d*x])/(b*d*Sqrt[Cos[c
+ d*x]])
```

Rubi [A] time = 0.393183, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2954, 3000, 3059, 2639, 12, 2805}

$$\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(b*(a + b)*d) + (2*B*Sin[c + d*x])/(b*d*Sqrt[Cos[c
+ d*x]])
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
```

```
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}\,dx \\
&= \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(Ab-aB)-\frac{1}{2}bB\cos(c+dx)-\frac{1}{2}aB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{b} \\
&= \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{2\int -\frac{a(Ab-aB)}{2\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{ab} - \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b} \\
&= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{b} \\
&= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 2.49994, size = 208, normalized size = 2.42

$$\frac{\frac{2B\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}} - \frac{2bB\left(2\text{EllipticF}\left(\frac{1}{2}\right.\right.}{}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*(2*A*b - 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (2
*b*B*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b)))/a + (4*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*B*(2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[
```

```
c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*b*d)
```

Maple [B] time = 4.391, size = 325, normalized size = 3.8

$$-\frac{1}{d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-2\,\frac{a\,(Ab-Ba)\,\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+}}{b\left(a^2-ab\right)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a)/b/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+2*B/b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*
c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

3.579 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*B*EllipticF[(c + d*
x)/2, 2])/(3*b*d) - (2*a*(A*b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(b^2*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) + (2*(
A*b - a*B)*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.835672, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2B}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*B*EllipticF[(c + d*
x)/2, 2])/(3*b*d) - (2*a*(A*b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(b^2*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) + (2*(
A*b - a*B)*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{3}{2}(Ab-aB)+\frac{1}{2}bB\cos(c+dx)+\frac{1}{2}aB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}dx}{3b}$$

$$= \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}(-3aAb+3a^2B+b^2B)-\frac{1}{4}b(3Ab-4aB)\cos \ldots}{\sqrt{\cos(c+dx)}(b+a \ldots}}{3b^2}$$

$$= \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{\frac{1}{4}a(3aAb-3a^2B-b^2B)-\frac{1}{4}a^2bB\cos(c+dx \ldots}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{3ab^2}$$

$$= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \ldots B$$

$$= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd} - \frac{2a(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+ \ldots\right.}{b^2(a+b)d}$$

Mathematica [A] time = 2.31937, size = 263, normalized size = 1.75

$$-\frac{\frac{6(Ab-aB)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-(a^2-2b^2)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a\sqrt{\sin^2(c+dx)}}+\frac{b(8abB-6Ab^2) \ldots}{\ldots}}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*b*(-9*a*A*b + 9*a^2*B + 2*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b) + (b*(-6*A*b^2 + 8*a*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (4*b^2*B*Sin[c + d
*x])/Cos[c + d*x]^(3/2) + (12*b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x]
] - (6*(A*b - a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*
(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticP
i[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d
*x]^2]))/(6*b^3*d)
```

Maple [B] time = 5.868, size = 466, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
```

```
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(A*b-B*a)/b^2/(a^2-a*b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+2*(A*b-B*a)/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2
*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

3.580 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=217

$$\frac{2(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d}+\frac{2\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{b^3d(a+b)}$$

```
[Out] (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2*
(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^2*d) + (2*a^2*(A*b - a*B)*Ellip
ticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sin[c + d*x])/
(5*b*d*Cos[c + d*x]^(5/2)) + (2*(A*b - a*B)*Sin[c + d*x])/(3*b^2*d*Cos[c +
d*x]^(3/2)) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sin[c + d*x])/(5*b^3*d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 1.18846, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2\left(-5a^2B+5aAb-3b^2B\right)\sin(c+}{5b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```


```
[Out] (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2*
(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^2*d) + (2*a^2*(A*b - a*B)*Ellip
ticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sin[c + d*x])/
(5*b*d*Cos[c + d*x]^(5/2)) + (2*(A*b - a*B)*Sin[c + d*x])/(3*b^2*d*Cos[c +
d*x]^(3/2)) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sin[c + d*x])/(5*b^3*d*Sqrt[
Cos[c + d*x]])
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx=\int \frac{B+A\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b+a\cos(c+dx))}\,dx$$

$$=\frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)}+\frac{2\int \frac{\frac{5}{2}(Ab-aB)+\frac{3}{2}bB\cos(c+dx)+\frac{3}{2}aB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))}dx}{5b}$$

$$=\frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{4\int \frac{-\frac{3}{4}\left(5aAb-5a^2B-3b^2B\right)+\frac{1}{4}b(5Ab+4}{\cos^{\frac{3}{2}}(c+dx)}}{1}$$

$$=\frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\left(5aAb-5a^2B-3b^2B\right)\sin(c}{5b^3d\sqrt{\cos(c+dx)}}$$

$$=\frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{2\left(5aAb-5a^2B-3b^2B\right)\sin(c}{5b^3d\sqrt{\cos(c+dx)}}$$

$$=\frac{2\left(5aAb-5a^2B-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab-aB)\,\mathrm{s}}{3b^2d\cos^{\frac{3}{2}}}$$

$$=\frac{2\left(5aAb-5a^2B-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d}+\frac{2a}{}$$

Mathematica [A] time = 4.58338, size = 328, normalized size = 1.51

$$\frac{-\frac{b^2\left(20a^2B-20aAb+9b^2B\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{3\left(5a^2B-5aAb+3b^2B\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+(\right.}{a\sqrt{\sin^2(c}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((b*(45*a^2*A*b + 10*A*b^3 - 45*a^3*B - 19*a*b^2*B)*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2])/(a + b) - (b^2*(-20*a*A*b + 20*a^2*B + 9*b^2*B)*(2*Elli
pticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
+ b)))/a + (6*b^3*B*Sin[c + d*x])/Cos[c + d*x]^(5/2) + (10*b^2*(A*b - a*B)*
Sin[c + d*x])/Cos[c + d*x]^(3/2) + (6*b*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[
c + d*x])/Sqrt[Cos[c + d*x]] + (3*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*(2*a*b*Ell
ipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(15*b^4*d)
```

Maple [B] time = 7.738, size = 785, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b^2
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5/b*B/(8*sin(1/2*d*x+1/2*c)^6-1
2*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)-2*(A*b-B*a)*a^3/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^3*(-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x
)
```

3.581 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=305

$$\frac{\left(16a^2Ab^2+2a^4A-12a^3bB+9ab^3B-15Ab^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d\left(a^2-b^2\right)}-\frac{\left(4a^2Ab-2a^3B+3ab^2B-5Ab^3\right)E\left(\frac{1}{2}(c+dx)\right|$$

```
[Out] -(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(
a^3*(a^2 - b^2)*d)) + ((2*a^4*A + 16*a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*
a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*b
 - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.018, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2989, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(16a^2Ab^2+2a^4A-12a^3bB+9ab^3B-15Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d\left(a^2-b^2\right)}-\frac{\left(4a^2Ab-2a^3B+3ab^2B-5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}-\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(
a^3*(a^2 - b^2)*d)) + ((2*a^4*A + 16*a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*
a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*b
 - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}b(Ab-aB)-a(Ab-aB)\cos(c+dx)+\frac{1}{2}\right.}{b+a\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin}{a\left(a^2-b^2\right)d(b+a\cos(c}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin}{a\left(a^2-b^2\right)d(b+a\cos(c}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2+3abB\right)\sqrt{}}{3a^2\left(a^2\right.}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4A+16a^2Ab^2-15A\right.}{3}$$

Mathematica [A] time = 3.39989, size = 320, normalized size = 1.05

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3b^2(Ab-aB)}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}+2A\right)-\frac{\frac{8\left(a^2A-3abB+2Ab^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-6\left(-4a^2Ab+2a^3B-3ab^2B+5\right.}{}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*A + (3*b^2*(A*b - a*B))/((-a^2 + b^2)*(b + a*Cos[c
 + d*x])))*Sin[c + d*x] - ((2*(-8*a^2*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^2*A + 2*A*b^2 -
3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2]))/(a + b) - (6*(-4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*
(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a +
b)*(a + b)))/(12*a^2*d)
```

Maple [B] time = 7.568, size = 1059, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*
```

```
A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*b^3*(A*b-B*a)/a^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2
/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a
^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^
3*b^2*(4*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x
)
```

3.582 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=223

$$-\frac{\left(4a^2Ab-2a^3B+ab^2B-3Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b\left(5a^2Ab-3a^3\right.}{}$$

```
[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)
 - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B + a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(a
^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.702195, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2989, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(4a^2Ab-2a^3B+ab^2B-3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b\left(5a^2Ab-3a^3B+ab^2\right.}{a^3d(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)
 - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B + a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(a
^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}b(Ab-aB)-a(Ab-aB)\cos(c+dx)+\frac{1}{2}\left(2a^2A-3Ab\right.}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}ab(Ab-aB)+\frac{1}{2}\left(4a^2Ab-3Ab^3-2a^3B+ab^2B\right)\cos}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{a^2\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{\left(4a^2Ab-3Ab^3-2a^3B+ab^2B\right)F\left(\frac{1}{2}\right.}{a^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 2.81339, size = 283, normalized size = 1.27

$$\frac{\frac{-\frac{2\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}+\frac{8(aB-Ab)\left((a+b)\text{EllipticF}\left(\right.\right.}{}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[
c + d*x])) + ((2*(2*a^2*A - A*b^2 - a*b*B)*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/2, 2] -
 b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*(2*a^2*A - 3*A*
b^2 + a*b*B)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)
*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b
), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]
^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [B] time = 6.337, size = 843, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2*b^2*(A*b-B*a)/a^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^
2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
-2/a^2*b*(3*A*b-2*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sqrt{\cos(c + dx)}}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\sqrt{\cos(dx + c)}}{(b\sec(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

3.583 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{\left(2a^2A-abB-Ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(3a^2Ab+a^3(-B)-ab^2B-Ab^3\right)\Pi\left(\frac{2a}{a+b};\right.}{a^2d(a-b)(a+b)^2}$$

```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b
^2 - a*b*B)*EllipticF[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b -
A*b^3 - a^3*B - a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a
 - b)*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.613571, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 2999, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(2a^2A-abB-Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(3a^2Ab+a^3(-B)-ab^2B-Ab^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+d\right.}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b
^2 - a*b*B)*EllipticF[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b -
A*b^3 - a^3*B - a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a
 - b)*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx=\int \frac{\sqrt{\cos(c+dx)}(B+A\cos(c+dx))}{(b+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}(-Ab+aB)+(aA-bB)\cos(c+dx)+\frac{1}{2}(Ab-aB)\cos}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{a^2-b^2}$$

$$=-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))}-\frac{\int \frac{\frac{1}{2}a(Ab-aB)-\frac{1}{2}\left(2a^2A-Ab^2-abB\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a\left(a^2-b^2\right)}$$

$$=\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))}+\frac{\left(2a^2A-A\right.}{}$$

$$=\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}+\frac{\left(2a^2A-Ab^2-abB\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d}-\frac{\left(3a^2Ab-\right.}{}$$

Mathematica [A] time = 2.506, size = 263, normalized size = 1.3

$$\frac{\frac{4(aB-Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\frac{2(Ab-aB)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}+\frac{(4a}{}}{(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos
[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])
/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(A*b - a*B)*(-2*a*b*Ellipt
icE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/
(4*d)
```

Maple [B] time = 5.03, size = 802, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b-B*a)/a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-2*A*b+B*a)/a
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

3.584 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$-\frac{(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2Ab+a^3B-3ab^2B+Ab^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{abd(a-b)(a+b)^2}$$

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B -
 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b
 + a*Cos[c + d*x]))
```

Rubi [A] time = 0.666893, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2954, 3000, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2Ab+a^3B-3ab^2B+Ab^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}+\frac{a(A}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B -
 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b
 + a*Cos[c + d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-aAb-a^2B+2b^2B\right)+b(Ab-aB)\cos(c+dx)+\frac{1}{2}a(A}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a\left(aAb+a^2B-2b^2B\right)-\frac{1}{2}ab(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{ab\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{(Ab-aB}{2a}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{\left(a^2Ab+Ab^3+a^3B\right.}{a(a}$$

Mathematica [A] time = 2.7206, size = 276, normalized size = 1.4

$$\frac{-\frac{\frac{2(Ab-aB)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{4b(aB-Ab)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b}{}\right.}{a}}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*
Cos[c + d*x])) + ((2*(a*A*b + 3*a^2*B - 4*b^2*B)*EllipticPi[(2*a)/(a + b),
(c + d*x)/2, 2])/(a + b) + (4*b*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2]
 - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a - (2*(A*b -
a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ellipt
icF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -Arc
Sin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a
 - b)*(a + b)))/(4*b*d)
```

Maple [B] time = 4.72, size = 715, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*
(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+
3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*A/(a^2-a*b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

3.585 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=255

$$\frac{(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(-3a^2B+aAb+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{\left(a^2Ab-3a^3B+5ab^2B-3Ab^3\right)\Pi\left(\frac{2}{a+}\right.}{b^2d(a-b)(a+b)^2}$$

```
[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)
 + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a^2*A*b -
3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^2*(a + b)^2*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.944142, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(-3a^2B+aAb+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{\left(a^2Ab-3a^3B+5ab^2B-3Ab^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)
 + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a^2*A*b -
3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^2*(a + b)^2*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(aAb-3a^2B+2b^2B\right)+b(Ab-aB)\cos}{\cos^{\frac{3}{2}}(c+dx)(b+}}{b\left(a^2\right.}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c}$$

$$= \frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} -$$

$$= \frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(a^2A\right.}{}$$

Mathematica [A] time = 4.37372, size = 319, normalized size = 1.25

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{a^2(aB-Ab)\sin(c+dx)}{\left(a^2-b^2\right)(a\cos(c+dx)+b)}+2B\tan(c+dx)\right) - \frac{-\frac{8b\left(-2a^2B+aAb+b^2B\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)} - \frac{2\left(3a^2B-aAb-2b^2B\right.}{}}{}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] (-(((2*(-3*a^2*A*b + 4*A*b^3 + 9*a^3*B - 10*a*b^2*B)*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2])/(a + b) - (8*b*(a*A*b - 2*a^2*B + b^2*B)*((a + b)*Elli
pticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a
 + b)) - (2*(-(a*A*b) + 3*a^2*B - 2*b^2*B)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos
[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
(a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c +
d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]
*((a^2*(-(A*b) + a*B)*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) + 2*
B*Tan[c + d*x]))/(4*b^2*d)
```

Maple [B] time = 6.725, size = 877, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b*(
a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3
/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*B*a^2/b^2/(a^2-a*b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1
/2))+2*B/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx + c) + A}{(b \sec(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

3.586 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(-5a^2B+3aAb+2b^2B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2Ab-5a^3B+4ab^2B-2Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{a\left(3a^2Ab-5a^3\right.}{}$$

```
[Out] -(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(
b^3*(a^2 - b^2)*d)) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*EllipticF[(c + d*x)/2,
 2])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) - ((3
*a*A*b - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Cos[c + d*x]
^(3/2)) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^3*(
a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b
^2)*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.31062, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(-5a^2B+3aAb+2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2Ab-5a^3B+4ab^2B-2Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{a\left(3a^2Ab-5a^3B+7ab\right.}{b^3d(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(
b^3*(a^2 - b^2)*d)) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*EllipticF[(c + d*x)/2,
 2])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) - ((3
*a*A*b - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Cos[c + d*x]
^(3/2)) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^3*(
a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b
^2)*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x]))
```

Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
```

```
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx=\int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))^2}\,dx$$

$$=\frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}-\frac{\int \frac{\frac{1}{2}\left(3aAb-5a^2B+2b^2B\right)+b(Ab-aB)\cos(c}{\cos^{\frac{5}{2}}(c+dx)(b+a\,c}}{b\left(a^2-b\right.}$$

$$=-\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}+\frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+}$$

$$=-\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sin(c+}{b^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$=-\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sin(c+}{b^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$=-\frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d}-\frac{\left(3aAb-5a^2B+2b^2B\right)\mathrm{si}}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c}$$

$$=-\frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d}-\frac{\left(3aAb-5a^2B+2b^2B\right)F}{3b^2\left(a^2-b^2\right)}$$

Mathematica [A] time = 6.90735, size = 429, normalized size = 1.24

$$\frac{\frac{\left(-24a^2Ab^2+40a^3bB-28ab^3B+12Ab^4\right)\left(2\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{\left(-9a^3Ab-12a^2b^2B+15a^4B+6aAb^3\right)\sin(c+dx)\cos(2(c+dx))\left(4b(a+b)\mathrm{Elli}\right.}{a}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((2*(-27*a^3*A*b + 30*a*A*b^3 + 45*a^4*B - 44*a^2*b^2*B - 4*b^4*B)*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + ((-24*a^2*A*b^2 + 12*A*b^4 + 4
0*a^3*b*B - 28*a*b^3*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + ((-9*a^3*A*b + 6*a*A*b^3 + 15*a^4
*B - 12*a^2*b^2*B)*Cos[2*(c + d*x)]*(-4*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + 4*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*(a^2
 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x]
)/(a^2*b*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(12*(a - b)*b^3
*(a + b)*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(A*b*Sin[c + d*x] - 2*a*
B*Sin[c + d*x]))/b^3 + (-(a^3*A*b*Sin[c + d*x]) + a^4*B*Sin[c + d*x])/(b^3*
(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(3*b^2
)))/d
```

Maple [B] time = 9.885, size = 1024, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a)*a/
b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*(A*b-2*B*a)/b^3/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))+2/b^2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b
-2*B*a)/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)),
 x)
```

3.587 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=461

$$\frac{\left(128a^4Ab^2-223a^2Ab^4+8a^6A+99a^3b^3B-72a^5bB-45ab^5B+105Ab^6\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^5d\left(a^2-b^2\right)^2}-\frac{\left(-65a^2Ab^3+24\right.}{}$$

```
[Out] -((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4
*B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^
4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*
B)*EllipticF[(c + d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b -
 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A
- 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b*(A*b - a*B)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b*(13*a^2*A*b - 7*
A*b^3 - 9*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 -
 b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.58464, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2954, 2989, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(128a^4Ab^2-223a^2Ab^4+8a^6A+99a^3b^3B-72a^5bB-45ab^5B+105Ab^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d\left(a^2-b^2\right)^2}-\frac{\left(-65a^2Ab^3+24a^4Ab+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4
*B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^
4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*
B)*EllipticF[(c + d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b -
 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A
- 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b*(A*b - a*B)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b*(13*a^2*A*b - 7*
A*b^3 - 9*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 -
 b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}b(Ab-aB)-2a(Ab-aB)\cos(c+\cdots\right.}{(b+a\cos(c+\cdots}}{2a\left(a^2-\cdots\right.}$$

$$= \frac{b(Ab-aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b\left(13a^2Ab-7Ab^3-9a^3B+3ab^2B\right)\cdots}{4a^2\left(a^2-b^2\right)^2 d(b+a\,\mathrm{c}\cdots}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \cdots$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \cdots$$

$$= -\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 6.01117, size = 463, normalized size = 1.

$$\frac{\frac{16\left(14a^2Ab^2+2a^4A-12a^3bB+3ab^3B-7Ab^4\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{6\left(65a^2Ab^3-24a^4Ab-29a^3b^2B+8a^5B+15ab^4B-35Ab^5\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\mathrm{si}\cdots\right.\right.}{a^2\cdots}}{(a-b)^2(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(4*a^6*A - 57*a^2*A*b^4 + 35*A*b^6 + 33*a^3*b^3*B -
15*a*b^5*B + a*b*(16*a^4*A - 83*a^2*A*b^2 + 49*A*b^4 + 39*a^3*b*B - 21*a*b^
3*B)*Cos[c + d*x] + 4*A*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/((a
```

```
^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35*
A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c
+ d*x)/2, 2])/(a + b) + (16*(2*a^4*A + 14*a^2*A*b^2 - 7*A*b^4 - 12*a^3*b*B
+ 3*a*b^3*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2]))/(a + b) - (6*(-24*a^4*A*b + 65*a^2*A*b^3 - 35*A*b^5 +
8*a^5*B - 29*a^3*b^2*B + 15*a*b^4*B)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d
*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 -
 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/
(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(48*a^3*d)
```

Maple [B] time = 12.205, size = 2216, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^5*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18
*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+2/a^5*b^3*(5*A*b-4*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^
2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
-2*b^4*(A*b-B*a)/a^5*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4
*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2
-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a
^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
```

```
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/
8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)
/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4/a^4*b^2*(5*A*b-3*B*a)/(a^2-a*b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x
)
```

3.588 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=367

$$-\frac{\left(-33a^2Ab^3+24a^4Ab+5a^3b^2B-8a^5B-3ab^4B+15Ab^5\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2Ab^2+8a^4A+9a^3bB-\right.}{4a^3d\left(\right.}$$

```
[Out] ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c +
 d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b
^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*a^4*(
a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a
^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a
- b)^2*(a + b)^3*d) + (b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B
+ a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*C
os[c + d*x]))
```

Rubi [A] time = 1.10805, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2989, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(-33a^2Ab^3+24a^4Ab+5a^3b^2B-8a^5B-3ab^4B+15Ab^5\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2Ab^2+8a^4A+9a^3bB-3ab^3B\right.}{4a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c +
 d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b
^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*a^4*(
a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a
^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a
- b)^2*(a + b)^3*d) + (b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B
+ a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*C
os[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}b(Ab-aB)-2a(Ab-aB)\cos(c+\ldots\right.}{(b+a\cos(c+\ldots}}{2a\left(a^2-\ldots\right.}$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)\sqrt{\ldots}}{4a^2\left(a^2-b^2\right)^2 d(b+a\,\mathrm{c}\ldots}$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+ab^2B\right)\sqrt{\ldots}}{4a^2\left(a^2-b^2\right)^2 d(b+a\,\mathrm{c}\ldots}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b(Ab-a\ldots}{2a\left(a^2-\ldots\right.}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} - \frac{\left(24a^4Ab-\ldots\right.}{\ldots}$$

Mathematica [A] time = 4.70144, size = 394, normalized size = 1.07

$$\frac{\frac{16\left(-4a^2Ab+2a^3B+ab^2B+Ab^3\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{\frac{2\left(-29a^2Ab^2+8a^4A+9a^3bB-3ab^3B+15Ab^4\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\ldots\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((-4*b*Sqrt[Cos[c + d*x]]*(b*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) +
a*(-13*a^2*A*b + 7*A*b^3 + 9*a^3*B - 3*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])
/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(8*a^4*A - 7*a^2*A*b^2 + 5*A*
b^4 - 5*a^3*b*B - a*b^3*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a +
b) + (16*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*((a + b)*EllipticF[(c + d
*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*(8*a
^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]],
-1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16
*a^2*d)
```

Maple [B] time = 10.316, size = 2000, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2/a^4*b^2*(4*A*b-3*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^
2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2)))+2*b^3*(A*b-B*a)/a^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2
+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(
a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-6/a^3*b*(2*A*b-B*a)/(a^2-a*b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
```

```
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x
)
```

3.589 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$\frac{\left(-5a^2Ab^2+8a^4A-7a^3bB+ab^3B+3Ab^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-$$

```
[Out] ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a^
2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*
B)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a^
2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) - ((9
*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.10186, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2989, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-5a^2Ab^2+8a^4A-7a^3bB+ab^3B+3Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-6a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a^
2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*
B)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a^
2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) - ((9
*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
```

```
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}b(Ab-aB)-2a(Ab-aB)\cos(c+dx)+\frac{1}{2}\left(4a^2A-3A\right.}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\cos(c}}{4a\left(a^2-b^2\right)^2 d(b+a\cos(c}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\cos(c}}{4a\left(a^2-b^2\right)^2 d(b+a\cos(c}$$

$$= \frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\sin}{2a\left(a^2-b^2\right)d(b+a\cos(c}$$

$$= \frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4A-5a^2Ab^2+3Ab^4-\right.}{4a^3\left(a\right.}$$

Mathematica [A] time = 4.49601, size = 361, normalized size = 1.04

$$\frac{\frac{8\left(2a^2A-3abB+Ab^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(-9a^2Ab+5a^3B+ab^2B+3Ab^3\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{c}\right.\right.\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(b*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + a*(-
9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2
- b^2)^2*(b + a*Cos[c + d*x])^2) + (((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*
B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(2*a^2*A + A*b^2
 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + ((-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*(
2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqr
t[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)
^2*(a + b)^2))/(8*a*d)
```

Maple [B] time = 8.999, size = 1959, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
```

```
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a
^3*b*(3*A*b-2*B*a)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b^2*(A*b-B
*a)/a^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b
^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)
/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^
2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-3*A*b+B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

3.590 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=338

$$-\frac{(7a^2Ab-3a^3B-3ab^2B-Ab^3)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2d\,(a^2-b^2)^2}-\frac{(5a^2Ab+a^3(-B)-5ab^2B+Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\,(a^2-b^2)^2}+\frac{(10\ldots}{}$$

```
[Out] -((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a*b
*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*EllipticF[(c
 + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((5*a^2*A*b + A*b^3 -
a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b
 + a*Cos[c + d*x]))
```

Rubi [A] time = 1.0062, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 2999, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(7a^2Ab-3a^3B-3ab^2B-Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\,(a^2-b^2)^2}-\frac{(5a^2Ab+a^3(-B)-5ab^2B+Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\,(a^2-b^2)^2}+\frac{(10a^2Ab^3+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] -((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a*b
*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*EllipticF[(c
 + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((5*a^2*A*b + A*b^3 -
a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b
 + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+dx)}(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(-Ab+aB)+2(aA-bB)\cos(c+dx)-\frac{1}{2}(Ab-aB}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2}}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos}}{4b\left(a^2-b^2\right)^2 d(b+a\cos}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos}}{4b\left(a^2-b^2\right)^2 d(b+a\cos}$$

$$= -\frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\,s}{2\left(a^2-b^2\right)d(b+a\cos(}$$

$$= -\frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} - \frac{\left(7a^2Ab-Ab^3-3a^3B-\right.}{4a^2\left(a^2-\right.}$$

Mathematica [A] time = 4.71943, size = 368, normalized size = 1.09

$$\frac{\frac{16b\left(a^2B-3aAb+2b^2B\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)} - \frac{\frac{2\left(-5a^2Ab+a^3B+5ab^2B-Ab^3\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin\right.\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(b*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) - a*(-5
*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b
^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*
B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*b*(-3*a*A*b + a
^2*B + 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]))/(a*(a + b)) - (2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*
b^2*B)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -Ar
cSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/
((a - b)^2*(a + b)^2))/(16*b*d)
```

Maple [B] time = 8.612, size = 1872, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-2*A*b+B*a)/
a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*b*(A*b-B*a)/a^2*(1/2*a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)
^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a
^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/
(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/
(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2)))-2*A/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

3.591 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=342

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2Ab\right.}{}$$

```
[Out] ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*b^
2*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(
c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((a^4*A*b - 10*a^2*A*b^3 - 3*A*b^
5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) + (a*(A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) - (a*(a^2*A*b + 5
*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2
- b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.09625, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2Ab^3+a^4A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*b^
2*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(
c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((a^4*A*b - 10*a^2*A*b^3 - 3*A*b^
5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) + (a*(A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) - (a*(a^2*A*b + 5
*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2
- b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
```

```
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(-aAb-3a^2B+4b^2B\right)+2b(Ab-aB)\cos(c+dx)-\frac{1}{2}a}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(}}{4b^2\left(a^2-b^2\right)^2 d(b+a\cos(c}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(}}{4b^2\left(a^2-b^2\right)^2 d(b+a\cos(c}$$

$$= \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\sin}{2b\left(a^2-b^2\right)d(b+a\cos(c}$$

$$= \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7a\right.}{4ab\left(a^2-b\right.}$$

Mathematica [A] time = 4.89262, size = 387, normalized size = 1.13

$$\frac{\frac{16b\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)} - \frac{2\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\right.\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((-4*a*Sqrt[Cos[c + d*x]]*(b*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B)
+ a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(
(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(3*a^3*A*b - 9*a*A*b^3 + 9*a^4*
B - 19*a^2*b^2*B + 16*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
+ b) + (16*b*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*((a + b)*EllipticF[(c
+ d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) -
(2*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*(2*a*b*EllipticE[ArcSin[Sqrt[C
os[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^2*d)
```

Maple [B] time = 8.816, size = 1768, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(a^2/b/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*(-A*b+B*a)/a*(1/2*a^2/b/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/
b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

3.592 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=420

$$\frac{\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(3a^3Ab+29a^2b^2B-15a^4B-9aAb^3-8b^4B\right)E\left(\frac{1}{2}(c+dx\right.}{4b^3d\left(a^2-b^2\right)^2}$$

```
[Out] ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c +
 d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a
*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b -
6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b
- 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sin[c + d*x])/(4*b^3*(a^2
- b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 -
5*a^3*B + 11*a*b^2*B)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x
]]*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.47785, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(3a^3Ab+29a^2b^2B-15a^4B-9aAb^3-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c +
 d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a
*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b -
6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b
- 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sin[c + d*x])/(4*b^3*(a^2
- b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 -
5*a^3*B + 11*a*b^2*B)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x
]]*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
```

```
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(aAb-5a^2B+4b^2B\right)+2b(Ab-a\ldots}{\cos^{\frac{3}{2}}(c+dx)\ldots}}{2b\left(\ldots\right.}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+\ldots\right.}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+\ldots}}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{a(\ldots}{2b\left(a^2-b^2\right)d\ldots}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{a(\ldots}{2b\left(a^2-b^2\right)d\ldots}$$

$$= \frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{\left(3a^3Ab-9\ldots\right.}{\ldots}$$

$$= \frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} + \frac{\left(a^2Ab-7A\ldots\right.}{\ldots}$$

Mathematica [A] time = 5.69809, size = 463, normalized size = 1.1

$$\frac{2\sqrt{\cos(c+dx)}\left(a^2\left(-3a^3Ab-29a^2b^2B+15a^4B+9aAb^3+8b^4B\right)\sin(2(c+dx))+2ab\left(-5a^3Ab-47a^2b^2B+25a^4B+11aAb^3+16b^4B\right)\sin(c+dx)+16B\left(b^3-a^2b\right)^2\tan(c+d\ldots\right.}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] (-((((2*(-9*a^4*A*b + 19*a^2*A*b^3 - 16*A*b^5 + 45*a^5*B - 95*a^3*b^2*B + 56
*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*b*(-(a^3
*A*b) + 4*a*A*b^3 + 5*a^4*B - 10*a^2*b^2*B + 2*b^4*B)*((a + b)*EllipticF[(c
 + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) -
 (2*(-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(2*a*b*Ell
ipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^
2)) + (2*Sqrt[Cos[c + d*x]]*(2*a*b*(-5*a^3*A*b + 11*a*A*b^3 + 25*a^4*B - 47
*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x] + a^2*(-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*
B - 29*a^2*b^2*B + 8*b^4*B)*Sin[2*(c + d*x)] + 16*(-(a^2*b) + b^3)^2*B*Tan[
c + d*x]))/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(16*b^3*d)
```

Maple [B] time = 10.895, size = 2024, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*a/b^2*(a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*(A*b-B*a)/b*(1/2*a^2/b/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b
^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
+2*B*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*B/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin
(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)),
 x)
```

3.593 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=523

$$-\frac{\left(15a^3Ab+61a^2b^2B-35a^4B-33aAb^3-8b^4B\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12b^3d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2Ab^3+15a^4Ab+65a^3b^2B-35a^5B-\right.}{4b^4d\left(a^2-b^2\right.}$$

```
[Out] -((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4
*B)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*
a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*EllipticF[(c + d*x)/2, 2])/(12
*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B
 + 86*a^3*b^2*B - 63*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4
*(a - b)^2*b^4*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2
*b^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
 + ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b
^4*B)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b -
a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x
])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/(4*b^
2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.9797, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2954, 3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(15a^3Ab+61a^2b^2B-35a^4B-33aAb^3-8b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^3d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2Ab^3+15a^4Ab+65a^3b^2B-35a^5B-24ab^4B\right.}{4b^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] -((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4
*B)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*
a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*EllipticF[(c + d*x)/2, 2])/(12
*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B
 + 86*a^3*b^2*B - 63*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4
*(a - b)^2*b^4*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2
*b^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
 + ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b
^4*B)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b -
a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x
])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/(4*b^
2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x]))
```


Rule 2954

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(3aAb-7a^2B+4b^2B\right)+2b(Ab-aB)\,\mathrm{c}}{\cos^{\frac{5}{2}}(c+dx)(b+}}{2b\left(a^2\right.}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2} + \frac{a\left(3a^2Ab-9Ab^3-7a^3B+13\right.}{4b^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)(}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{a(A}{2b\left(a^2-b^2\right)d\,\mathrm{c}}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(15a^4Ab-29a\right.}{}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(15a^4Ab-29a\right.}{}$$

$$= -\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2d} -$$

$$= -\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2d} -$$

Mathematica [A] time = 7.30124, size = 572, normalized size = 1.09

$$\frac{\frac{\left(240a^2Ab^4-120a^4Ab^2-512a^3b^3B+280a^5bB+160ab^5B-48Ab^6\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{\left(87a^3Ab^3-45a^5Ab-195a^4b^2B+72a^2b^4B+105a^6\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((2*(-135*a^5*A*b + 285*a^3*A*b^3 - 168*a*A*b^5 + 315*a^6*B - 641*a^4*b^2*B
 + 328*a^2*b^4*B + 16*b^6*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
+ b) + ((-120*a^4*A*b^2 + 240*a^2*A*b^4 - 48*A*b^6 + 280*a^5*b*B - 512*a^3*
b^3*B + 160*a*b^5*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2])/(a + b)))/a + ((-45*a^5*A*b + 87*a^3*A*b^3 - 24*a*
A*b^5 + 105*a^6*B - 195*a^4*b^2*B + 72*a^2*b^4*B)*Cos[2*(c + d*x)]*(-4*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 4*b*(a + b)*EllipticF[ArcSin[Sq
```

```
rt[Cos[c + d*x]]], -1] - 2*(a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Co
s[c + d*x]^2)))/(48*(a - b)^2*b^4*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((2*Se
c[c + d*x]*(A*b*Sin[c + d*x] - 3*a*B*Sin[c + d*x]))/b^4 + (-(a^3*A*b*Sin[c
+ d*x]) + a^4*B*Sin[c + d*x])/(2*b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) +
 (7*a^5*A*b*Sin[c + d*x] - 13*a^3*A*b^3*Sin[c + d*x] - 11*a^6*B*Sin[c + d*x
] + 17*a^4*b^2*B*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))
+ (2*B*Sec[c + d*x]*Tan[c + d*x])/(3*b^3)))/d
```

Maple [B] time = 16.947, size = 2178, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(A*b-2*B*a
)/b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(A*b-B*a)*a/b^2*(1/2*a^
2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/
(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b
)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
```

```
*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2)))+2/b^3*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(A*b-
3*B*a)/b^4/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*(A*b-3*B*a)/b^4*(-(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)),
 x)
```

3.594 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=343

$$\frac{2(a^2-b^2)(25a^2A-14abB+8Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{c}}{105a^2d}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^3*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A -
 4*A*b^2 + 7*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(105*a^2*d) + (2*(A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(35*a*d) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.21888, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105a^2d}+\frac{2(a^2-b^2)(25a^2A-14abB+8Ab^2)\sqrt{\frac{a\cos(c+dx}{a+b}}}{105a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^3*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A -
 4*A*b^2 + 7*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(105*a^2*d) + (2*(A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(35*a*d) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(7*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
```

```
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(\right.$$

$$= \frac{2(Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad} + 2$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{105a^2d}$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{105a^2d}$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{105a^2d}$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{105a^2d}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A+8Ab^2-14abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+d\right.}{105a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 17.2825, size = 455, normalized size = 1.33

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(\frac{\left(115a^2A+28abB-16Ab^2\right)\sin(c+dx)}{210a^2}+\frac{(7aB+Ab)\sin(2(c+dx))}{35a}+\frac{1}{14}A\sin(3(c+dx))\right)}{d} - \frac{2\cos^{\frac{3}{2}}(c+dx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(((115*a^2*A - 16*A*b^2 + 28*a
*b*B)*Sin[c + d*x])/(210*a^2) + ((A*b + 7*a*B)*Sin[2*(c + d*x)])/(35*a) + (
A*Sin[3*(c + d*x)])/14))/d - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((-I)*(a + b)*(19*a^2*A*b + 8*A*b^
3 + 63*a^3*B - 14*a*b^2*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] + I*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*El
lipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*S
qrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (19*a^2*A*b + 8*A*
b^3 + 63*a^3*B - 14*a*b^2*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/(105*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.603, size = 2364, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c)
)*(cos(d*x+c)+1)*(19*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^3*b-7*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c
)+1))^(1/2)-19*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1
/2)+20*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-8*
A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+35*B*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+15*A*cos(d*x+c)^5*(
(a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+10*A*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^4*(1/(cos(d*x+c)+1))^(1/2)+21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/
(cos(d*x+c)+1))^(1/2)+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x
+c)+1))^(1/2)+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/
2)-8*A*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)+14*B*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-14*B*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+49*B*sin(d*x+c)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+14*B*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-14*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+14*B*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+28*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3
*b*(1/(cos(d*x+c)+1))^(1/2)+26*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/
(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*
x+c)+1))^(1/2)-19*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^2*b^2+8*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a*b^3-19*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3*b+2*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b^2-8*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^3+18*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(
1/2)-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-25*A*
((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-19*A*((a-b)/(a+b))^(1/2)
*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+4*A*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x
+c)+1))^(1/2)-63*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-7*B*(
(a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+14*B*((a-b)/(a+b))^(1/2
)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-63*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^4-8*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/
```

```
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*b^4+25*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^4)/a^3/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c
)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^{3}\sec(dx+c)+A\cos(dx+c)^{3}\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2),
 x)
```

3.595 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=267

$$-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+5abB-2Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]) + (2*(A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.913874, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4032, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+5abB-2Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]) + (2*(A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(5*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(\right.$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2}{}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2}{}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2}{}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2}{}$$

$$= -\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2}{}$$

Mathematica [C] time = 14.7522, size = 353, normalized size = 1.32

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(a\sin(c+dx)(3aA\cos(c+dx)+5aB+Ab)-\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ia(a+b)(9aA+5aB-2A\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(A*b + 5*a*B + 3*a*A*Cos[
c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a
+ b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2)/(a + b)] + I*a*(a + b)*(9*a*A - 2*A*b + 5*a*B)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2*A - 2*A*b^2 + 5*a*b*B)*(b + a
*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c
+ d*x])*Sec[c + d*x]^(3/2))))/(15*a^2*d)
```

Maple [B] time = 0.487, size = 1701, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
*(cos(d*x+c)+1)*(2*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a*b^2-9*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1
/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+9*A*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+2*A*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-2*A*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+5*B*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-5*B*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-5*B*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+7*A*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+5*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(
cos(d*x+c)+1))^(1/2)+5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c
)+1))^(1/2)+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(
1/2)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+10*B
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+5*B*cos(d*
x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+5*B*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-5*B*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^3*(1/(cos(d*x+c)+1))^(1/2)-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(co
s(d*x+c)+1))^(1/2)-9*A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-A
*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*B*((a-b)/(a+b))^(1/2)
*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c
)+1))^(1/2)+2*A*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2))/a^2/((a-b
)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec (dx+c)+A)\sqrt{b\sec (dx+c)+a}\cos (dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

3.596 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=201

$$\frac{2A\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A
*b + 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.62034, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2955, 4032, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A
*b + 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}(A+B}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{c\right.$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\left(A\left(a^2\right.\right.}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\left(A\left(a^2\right.\right.}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\left(A\left(a^2\right.\right.}{3}$$

$$= \frac{2A\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(Ab+3a}{}$$

Mathematica [C] time = 9.0131, size = 305, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(A\sin(c+dx) + \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia(a+b)(A+3B)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+d}{a+b}}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A*Sin[c + d*x] + ((Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(A*b + 3*a*B)*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(A + 3*B)*EllipticF[I*A
rcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (A*b + 3*a*B)*(b + a*Cos[c +
d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*(b + a*Cos[c + d*x])
*Sec[c + d*x]^(3/2))))/(3*d)
```

Maple [B] time = 0.38, size = 1162, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(cos(d*x+c)+1)*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(
1/2)+2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+3*B*
```

```
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+A*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a^2-A*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+A*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-A*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*
(1/(cos(d*x+c)+1))^(1/2)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c
)+1))^(1/2)+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)-3
*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2+3*B*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b+3*B*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-3*B*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-3*B*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c
)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)-3*B*((a-b)/(
a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2))/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*
x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

3.597 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=208

$$\frac{2aB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2bB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rubi [A] time = 0.683206, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {2955, 4037, 3854, 3858, 2663, 2661, 3859, 2807, 2805, 3856, 2655, 2653}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4037

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\operatorname{se}}{\sqrt{\sec(c+dx)}}$$

$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \Big(I$$

$$= \left(aB\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \Big($$

$$= \frac{\left(aB\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(bB\sqrt{b+a\cos\right.}{\sqrt{\cos(c+}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{\left(aB\right.}{}$$

$$= \frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2bB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\Big(2}{d\sqrt{\cos(c+dx)}\sqrt{a+}}$$

Mathematica [C] time = 29.4073, size = 25347, normalized size = 121.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.389, size = 822, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-A*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
```

```
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+A*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b+A*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*b+2*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*b-A*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)
/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec{(dx+c)}+A)\sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec{(dx+c)}+A)\sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec{(c+dx)})\sqrt{a+b\sec{(c+dx)}}\sqrt{\cos{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

3.598 $\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{(2aA+bB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\ldots}}{d\sqrt{\cos\ldots}}$$

```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.935146, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2955, 4031, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2aA+bB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{a+b\sec\ldots}}{d\sqrt{\cos(c+dx)}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
```

```
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}(A+B$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left((2aA+bB)\sqrt{b+a\cos(c+dx)}\right)\int}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

$$= \frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{B\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+d}}$$

$$= \frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx}{a+b}}}{d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [C] time = 32.2891, size = 52603, normalized size = 207.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

[Out] Result too large to show

Maple [C] time = 0.421, size = 789, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b+2*A*cos(d*x+c)*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+B*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2)+2*B*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-B*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+B
*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2)+B*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*b*(1/(cos(d*x+c)+1))^(1/2)-B*((a-b)/(a+b))^(1/2)*b*(1/(
cos(d*x+c)+1))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/
2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\sqrt{b\sec(dx + c) + a}}{\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

3.599 $\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=336

$$\frac{(3aB+4Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(a\ldots}{}$$

```
[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((4*a*A
*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((4*A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + ((4*A*b +
 a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.26203, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4031, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4bd\sqrt{\cos(c+dx)}}+\frac{(3aB+4Ab)\ldots}{4d\sqrt{\cos(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((4*a*A
*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((4*A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + ((4*A*b +
 a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
```

```
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}(A+B$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\sec(c+dx)}\sin(}{4bd\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\sec(c+dx)}\sin(}{4bd\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\sec(c+dx)}\sin(}{4bd\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\sec(c+dx)}\sin(}{4bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(4aAb-a^2B+4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{B\sqrt{a+b\sec}}{2d}$$

$$= \frac{(4Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(4aAb-a^2B+4b^2B\right)}{4bd\sqrt{\cos(c+}}$$

Mathematica [C] time = 32.9809, size = 77879, normalized size = 231.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.417, size = 1475, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*sin(d*x+c)*cos(d*x+c)^2
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-8*A*sin(d*x+c)*cos
```

```
(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))
*a*b-4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+B*si
n(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2-B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b-2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2-2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b+4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*B*sin(d*x+c)*
cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*b^2-B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)-2*B
*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1
/(cos(d*x+c)+1))^(1/2)-2*B*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+
b))^(1/2)*b^2+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/
2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+2*B*((a-
b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(
d*x+c)^3/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

3.600 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=427

$$\frac{2\left(a^2-b^2\right)\left(39a^2Ab+75a^3B-18ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(49a^2A+72abB+3Ab^2\right)\text{si}}{}$$

```
[Out] (2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^3*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 +
8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*
A + 3*A*b^2 + 72*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(315*a*d) + (2*(10*A*b + 9*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(63*d) + (2*a*A*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.70772, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad}+\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+}{315a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^3*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 +
8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*
A + 3*A*b^2 + 72*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(315*a*d) + (2*(10*A*b + 9*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(63*d) + (2*a*A*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(9*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}(A+}{\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d} - \frac{1}{9}\left(2\sqrt{c}\right.$$

$$= \frac{2(10Ab+9aB)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d}$$

$$= \frac{2\left(49a^2A+3Ab^2+72abB\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx}}{315ad}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b}}{315a^2d}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b}}{315a^2d}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b}}{315a^2d}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b}}{315a^2d}$$

$$= \frac{2\left(a^2-b^2\right)\left(39a^2Ab+8Ab^3+75a^3B-18ab^2B\right)\sqrt{\frac{b+a\cos(c+}{a+b}}}{315a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}}$$

Mathematica [C] time = 18.3851, size = 540, normalized size = 1.26

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{\left(402a^2Ab+345a^3B+36ab^2B-16Ab^3\right)\sin(c+dx)}{630a^2}+\frac{\left(133a^2A+144abB+6Ab^2\right)\sin(2(c+dx))}{630a}+\frac{1}{126}(9aB+10A\right.}{d(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(((402*a^2*A*b - 16*A*b^3 +
345*a^3*B + 36*a*b^2*B)*Sin[c + d*x])/(630*a^2) + ((133*a^2*A + 6*A*b^2 + 1
44*a*b*B)*Sin[2*(c + d*x)])/(630*a) + ((10*A*b + 9*a*B)*Sin[3*(c + d*x)])/1
```

```
26 + (a*A*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])) - (2*Cos[c + d*x]
^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(
(-I)*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B
)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]
^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8
*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4*A + 33*a
^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(b + a*Cos[c + d*x])*(Sec[(c
 + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^3*d*(b + a*Cos[c + d*x])^2*Se
c[c + d*x]^(3/2))
```

Maple [B] time = 0.702, size = 3068, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] -2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c
))*(cos(d*x+c)+1)*(147*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^5-147*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^5+8*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^5-98*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/
2)+147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2)-8*A*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*b^5*(1/(cos(d*x+c)+1))^(1/2)+147*A*((a-b)/(a+b
))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+88*A*((a-b)/(a+b))^(1/2)*a^3*b^2*(1
/(cos(d*x+c)+1))^(1/2)+33*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^
(1/2)-4*A*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)+75*B*((a-b)/(a
+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+246*B*((a-b)/(a+b))^(1/2)*a^3*b^2
*(1/(cos(d*x+c)+1))^(1/2)+33*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^3*b^2-2*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^2*b^3+8*A*((a-b)/(a+b))^(1/2)*b^5*(1/(cos(d*x+c)+1))^(
1/2)+8*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b
^4+147*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4
*b-33*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*
b^2+33*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
*b^3-8*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b
^4-53*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)-1
17*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-52*A*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)
^3*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-81*B*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)-85*A*cos(d*x+c)^5*((a-b
)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-68*A*cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)-204*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
```

```
^4*b*(1/(cos(d*x+c)+1))^(1/2)+9*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3*
(1/(cos(d*x+c)+1))^(1/2)-10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(
d*x+c)+1))^(1/2)+33*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)
+1))^(1/2)-34*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(
1/2)+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)+246*
B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-165*B*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)-18*B*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)+18*B*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)+246*B*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-153*B*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-18*B*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-246*B*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+246*B*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+18*B*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-18*B*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-186*A*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+9*B*((a-b)/(a+b))^(1/2)*a
^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-18*B*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+
c)+1))^(1/2)-45*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(
1/2)-30*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2)-75*
B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5+75*B*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2)-35*A*cos(d*x+c)^
6*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2)-14*A*cos(d*x+c)^4*((a-b)
/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2))/a^3/((a-b)/(a+b))^(1/2)/(b+a*co
s(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^4\sec(dx+c)^2+Aa\cos(dx+c)^4+(Ba+Ab)\cos(dx+c)^4\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\sqrt{c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4*sec(d*x + c)^2 + A*a*cos(d*x + c)^4 + (B*a + A
*b)*cos(d*x + c)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.601 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=342

$$\frac{2\left(a^2-b^2\right)\left(25a^2A+21abB-6Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{}}{105ad}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A +
 3*A*b^2 + 42*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.30429, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{2\left(a^2-b^2\right)\left(25a^2A+21abB-6Ab^2\right)\sqrt{\frac{a\cos(c+}{a+b}}}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```



```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A +
 3*A*b^2 + 42*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(7*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
```

```
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}(A+}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d} - \frac{1}{7}\left(2\sqrt{c}\right.$$

$$= \frac{2(8Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{105ad}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{105ad}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{105ad}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{105ad}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A-6Ab^2+21abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 16.9422, size = 466, normalized size = 1.36

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{\left(115a^2A+168abB+12Ab^2\right)\sin(c+dx)}{210a}+\frac{1}{35}(7aB+8Ab)\sin(2(c+dx))+\frac{1}{14}aA\sin(3(c+dx))\right)}{d(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(((115*a^2*A + 12*A*b^2 + 16
8*a*b*B)*Sin[c + d*x])/(210*a) + ((8*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a
*A*Sin[3*(c + d*x)])/14))/(d*(b + a*Cos[c + d*x])) - (2*Cos[c + d*x]^(3/2)*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*((-I)*(a
 + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*EllipticE[I*ArcSinh[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A +
7*B) + a^2*(25*A + 63*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)] - (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*(b + a*Cos[c + d*x
])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a^2*d*(b + a*Cos[c +
```

```
d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.493, size = 2326, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c)
)*(cos(d*x+c)+1)*(82*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^3*b+63*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+
c)+1))^(1/2)-82*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(
1/2)+55*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+6
*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+15*A*cos(d
*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+10*A*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2
)*a^4*(1/(cos(d*x+c)+1))^(1/2)+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1
/(cos(d*x+c)+1))^(1/2)-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c
)+1))^(1/2)+6*A*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)-21*B*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+21*B*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+84*B*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-21*B*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+21*B*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-21*B*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+27*A*cos(d*x+c)^3*((a-b)/
(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+63*B*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+68*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/
2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^
3*(1/(cos(d*x+c)+1))^(1/2)-82*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a^2*b^2-6*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a*b^3-82*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3*b+51*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^3+39*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos
(d*x+c)+1))^(1/2)-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1)
)^(1/2)-25*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-82*A*((a-b)
/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*a*b^
3*(1/(cos(d*x+c)+1))^(1/2)-63*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1)
)^(1/2)-42*B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-21*B*((a-
b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-63*B*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*sin(d*x+c)*EllipticE((-1+c
```

```
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+6*A*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+25*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4)/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))
/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^3\sec(dx+c)^2+Aa\cos(dx+c)^3+(Ba+Ab)\cos(dx+c)^3\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3*sec(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A
*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2
), x)
```

3.602 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=266

$$\frac{2(a^2-b^2)(5aB+3Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+20abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c-}{15ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(15*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]) + (2*(6*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.970073, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4025, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2(a^2-b^2)(5aB+3Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+20abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{15ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(15*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]) + (2*(6*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(5*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
```

LeQ[n, -1]

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}(A+}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} - \frac{1}{5}\left(2\sqrt{c}\right.$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d}$$

$$= \frac{2\left(a^2-b^2\right)(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + $$

Mathematica [C] time = 14.2602, size = 369, normalized size = 1.39

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2\sin(c+dx)(a\cos(c+dx)+b)(3aA\cos(c+dx)+5aB+6Ab) - \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+d\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(6*A
*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*((-I)*(a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*EllipticE[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(3*b*(A + 5*B) + a
*(9*A + 5*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[
(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9
*a^2*A + 3*A*b^2 + 20*a*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(15*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.481, size = 1749, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
*(cos(d*x+c)+1)*(-3*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a*b^2-9*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(
1/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+9*A*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-3*A*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+3*A*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+20*B*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-20*B*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-20*B*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+12*A*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-3*A*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-20*B*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b
^2*(1/(cos(d*x+c)+1))^(1/2)+9*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(
cos(d*x+c)+1))^(1/2)+9*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x
+c)+1))^(1/2)+25*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1)
)^(1/2)+5*B*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+5
*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-5*B*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+3*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(
d*x+c)+1))^(1/2)-6*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*B
*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-20*B*((a-b)/(a+b))^(1/2
)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)
+1))^(1/2)+15*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a*b^2)/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^3/(1/(cos(d*x+
c)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2
```

```
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2\sec(dx+c)^2 + Aa\cos(dx+c)^2 + (Ba+Ab)\cos(dx+c)^2\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A
*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2
), x)
```

3.603 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=276

$$\frac{2\left(a^2A+3abB-Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+4Ab)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]) + (2*b^2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b + 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*A*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.09329, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2955, 4025, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A+3abB-Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+4Ab)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a\cdot}\right.}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]) + (2*b^2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b + 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*A*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}(A}{\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\Big(2$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\Big(2$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\Big(($$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\Big((-a}{}$$

$$= \frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2aA\sqrt{\cos(c+}}{}$$

$$= \frac{2\left(a^2A-Ab^2+3abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} -$$

Mathematica [C] time = 33.8954, size = 45958, normalized size = 166.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.364, size = 1429, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(cos(d*x+c)+1)*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(
1/2)+5*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+3*B*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+A*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a^2-4*A*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+3*A*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2+4*A*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-4*A*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
2*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d
*x+c)+1))^(1/2)+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(
1/2)-3*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2
+6*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b-3*B
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2+3*B*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-3*B*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+6*B*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-3*B*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b*(
1/(cos(d*x+c)+1))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2
)-3*B*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2))/((a-b)/(a+b))^(1/2)
/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

3.604 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=272

$$\frac{(2a^2B+2aAb+b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2aA-bB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] ((2*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (b*(2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
((2*a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.02288, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2955, 4026, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2a^2B+2aAb+b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2aA-bB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (b*(2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
((2*a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
```

```
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}(A+}{\sqrt{\sec(c+dx}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left((2aA-bB)\sqrt{\cos(c\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(\left(-2aAb-2a^2B-b^2\right.\right.}{2\sqrt{\cos(}}$$

$$= \frac{b(2Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{bB\sqrt{a-}}{}$$

$$= \frac{\left(2aAb+2a^2B+b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b(}{}$$

Mathematica [C] time = 32.9488, size = 66581, normalized size = 244.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.439, size = 1410, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(-2*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a^2*(1/(cos(d*x+c)+1))^(1/2)+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2-4*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b+2*A*sin(d*x+c)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2-2*A*sin(d*x+c)*cos(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*sin(d*x+c)*cos
(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*sin(d*x
+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*b^2+2*A*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^2
-2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-2*B*sin(
d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2+2
*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*b+B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b-B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^2-6*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b
*(1/(cos(d*x+c)+1))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*
x+c)+1))^(1/2)+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2
)-B*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*b^2+B*((a-b)/(a
+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^
(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

3.605 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=339

$$\frac{(8a^2A+7abB+4Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2B+12aAb+4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+a\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) - ((4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(
3/2)) + ((4*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[C
os[c + d*x]])
```

Rubi [A] time = 1.41758, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(8a^2A+7abB+4Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2B+12aAb+4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) - ((4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(
3/2)) + ((4*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[C
os[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
```

```
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}(A+$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(12aAb+3a^2B+4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{bB\sqrt{a+b}}{2}$$

$$= \frac{\left(8a^2A+4Ab^2+7abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(12aAb+3a}{4}$$

Mathematica [C] time = 32.9551, size = 79375, normalized size = 234.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.414, size = 1659, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(8
*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^2-8*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a*b-4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos
```

```
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+24*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+4*A*cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+2*B*sin(d*x+c)*cos(d*x+c)
^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*B*sin(d*x+c)*c
os(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*B*sin
(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^
2-5*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2+5*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b+6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)^2*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+5*B*cos(d
*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+2*B*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(
cos(d*x+c)+1))^(1/2)+5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c
)+1))^(1/2)+2*B*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*b
^2-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)-7*B*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-2*B*((a-b)/(a+b))^(
1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2))/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/
(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

3.606 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=421

$$\frac{\left(17a^2B+42aAb+16b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\sec}}{24bd\sqrt{\cos(c+dx)}}$$

```
[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(8*b*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]
])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + ((6*A*b + 7*a*B)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((30*a*A*b + 3*a^2*B
+ 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x
]])
```

Rubi [A] time = 1.80199, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4026, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\left(17a^2B+42aAb+16b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(8*b*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]
])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + ((6*A*b + 7*a*B)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((30*a*A*b + 3*a^2*B
+ 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x
]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin}{12d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin}{12d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin}{12d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin}{12d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin}{12d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(6a^2Ab+8Ab^3-a^3B+12ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{\left(42aAb+17a^2B+16b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(6a^2Ab+}{}$$

Mathematica [C] time = 33.3855, size = 104716, normalized size = 248.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.508, size = 2351, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)+1)*
(-3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-6*B*c
```

```
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+3*B*sin(d*x+
c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-16*
B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*b^3-72*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
-b),I/((a-b)/(a+b))^(1/2))*a*b^2+30*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-30*A*sin(d*x+c)*cos(d*x+c)^3*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-12*A*sin(d*x+c)*co
s(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*A*s
in(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b^2-36*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*a^2*b-3*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+16*B*sin(d*x+c)*cos(d*x+c)^3*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-14*B*sin(d*x+c)*cos(
d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+20*B*sin
(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^2+22*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+30*A
*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+42*A*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+17*B*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)^3*(1/(c
os(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+12*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-6*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*B*sin(d*x+c)*cos(d*x+c)^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+24*A*sin(
d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3
-48*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/
((a-b)/(a+b))^(1/2))*b^3-30*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b*(1/(co
s(d*x+c)+1))^(1/2)-12*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+
c)+1))^(1/2)-14*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))
^(1/2)-16*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)
-30*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+8*B*(
(a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)+8*B*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-12*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*b^3-3*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/
(cos(d*x+c)+1))^(1/2)-16*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x
+c)+1))^(1/2))/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1
/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

3.607 $\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=519

$$\frac{2\left(a^2-b^2\right)\left(285a^2Ab^2+675a^4A+1254a^3bB-110ab^3B+40Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3465a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(81a^2\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a
*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(3465*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*
a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^
4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(3465*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(675*a^4*
A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b +
 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Cos[
c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(693*d) + (2*a*(14*A*
b + 11*a*B)*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(99*d
) + (2*a*A*Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*
d)
```

Rubi [A] time = 2.17391, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2955, 4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{693d}+\frac{2\left(1145a^2Ab+539a^3B+825ab^2B+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a
*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(3465*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*
a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^
4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(3465*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(675*a^4*
A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b +
 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Cos[
c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(693*d) + (2*a*(14*A*
b + 11*a*B)*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(99*d
) + (2*a*A*Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*
d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}(A+}{\sec^{\frac{11}{2}}(c+dx}$$

$$= \frac{2aA\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{11d} - \frac{1}{11}\Big(2$$

$$= \frac{2a(14Ab+11aB)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{99d}$$

$$= \frac{2\left(81a^2A+113Ab^2+209abB\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}{693d}$$

$$= \frac{2\left(1145a^2Ab+15Ab^3+539a^3B+825ab^2B\right)\cos^{\frac{3}{2}}(c+dx)}{3465ad}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B\right)}{3465a^2d}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B\right)}{3465a^2d}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B\right)}{3465a^2d}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+55ab^3B\right)}{3465a^2d}$$

$$= \frac{2\left(a^2-b^2\right)\left(675a^4A+285a^2Ab^2+40Ab^4+1254a^3bB-1\right.}{3465a^3d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

Mathematica [C] time = 19.966, size = 626, normalized size = 1.21

$$\frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{\left(513a^2A+836abB+452Ab^2\right)\sin(3(c+dx))}{5544}+\frac{\left(3095a^2Ab+1463a^3B+1650ab^2B+30Ab^3\right)\sin(2(c+dx))}{6930a}+\frac{(9330a^2}{}\right.}{d(a\cos(c+dx)+b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
]),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((6525*a^4*A + 9330*a^2*A*b
^2 - 160*A*b^4 + 16434*a^3*b*B + 440*a*b^3*B)*Sin[c + d*x])/(13860*a^2) + (
(3095*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a*b^2*B)*Sin[2*(c + d*x)])/(69
30*a) + ((513*a^2*A + 452*A*b^2 + 836*a*b*B)*Sin[3*(c + d*x)])/5544 + (a*(2
3*A*b + 11*a*B)*Sin[4*(c + d*x)])/396 + (a^2*A*Sin[5*(c + d*x)])/88))/(d*(b
 + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d
*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I)*(a + b)*(3705*a^4*A*b + 255*a^2
*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*EllipticE[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(40*A*b^4 - 10*a
*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3
```

```
*a^4*(225*A + 539*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*
B - 110*a*b^4*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d
*x)/2]))/(3465*a^3*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 1.01, size = 3816, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/3465/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c
))*(cos(d*x+c)+1)*(-55*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d
*x+c)+1))^(1/2)-40*A*((a-b)/(a+b))^(1/2)*b^6*(1/(cos(d*x+c)+1))^(1/2)-3705*
A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)+385*B*cos(d
*x+c)^6*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)+154*B*cos(d*x+c)^4
*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)+1078*B*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)+40*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b^6*(1/(cos(d*x+c)+1))^(1/2)-1617*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^6
*(1/(cos(d*x+c)+1))^(1/2)-675*A*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1)
)^(1/2)-3705*A*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)-1025*A*
((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)-255*A*((a-b)/(a+b))^(1
/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)+20*A*((a-b)/(a+b))^(1/2)*a*b^5*(1/(cos
(d*x+c)+1))^(1/2)-1617*B*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)
-1793*B*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)-3069*B*((a-b)/
(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)-55*B*((a-b)/(a+b))^(1/2)*a^2*
b^4*(1/(cos(d*x+c)+1))^(1/2)+110*B*((a-b)/(a+b))^(1/2)*a*b^5*(1/(cos(d*x+c)
+1))^(1/2)+315*A*cos(d*x+c)^7*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1
/2)+90*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)+270*
A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)-675*A*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)-1617*B*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6+1617*B*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+675*A*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6-40*A*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6-3705*A*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b+3315*A*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+430*A*cos(d*x+c)^4*((a-
b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)+580*A*cos(d*x+c)^4*((a-b)/(a
+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)+1870*B*cos(d*x+c)^4*((a-b)/(a+b
))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)+800*A*cos(d*x+c)^3*((a-b)/(a+b))^
(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)-255*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+10*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-40*A*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5+3705*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
```

```
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-3705*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+255*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-255*A*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4+40*A*sin(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+2871*B*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-3069*B*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+1705*B*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+110*B*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-1617*B*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+3069*B*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2-3069*B*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-110*B*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4+110*B*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+1370*A*cos(d*x
+c)^5*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)+1430*B*cos(d*x+c
)^5*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)+1535*A*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)-255*A*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)+260*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)-40*A*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a*b^5*(1/(cos(d*x+c)+1))^(1/2)-715*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b
*(1/(cos(d*x+c)+1))^(1/2)-3069*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/
(cos(d*x+c)+1))^(1/2)+2189*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos
(d*x+c)+1))^(1/2)+110*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d*x+
c)+1))^(1/2)-110*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^5*(1/(cos(d*x+c)+1))^
(1/2)+1120*A*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2
)+902*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)+880
*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)+2830*A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)+700*A*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)+20*A*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*b^5*(1/(cos(d*x+c)+1))^(1/2)+2992*B*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2))/a^3/((a-b)/(a+b))^(1
/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Bb^2\cos(dx+c)^5\sec(dx+c)^3+Aa^2\cos(dx+c)^5+\left(2Bab+Ab^2\right)\cos(dx+c)^5\sec(dx+c)^2+\left(Ba^2+2A\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^5*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^5 + (2*B
*a*b + A*b^2)*cos(d*x + c)^5*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^5*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/
2), x)
```

3.608 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=425

$$\frac{2\left(a^2-b^2\right)\left(114a^2Ab+75a^3B+45ab^2B-10Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(49a^2A+135abB+75Ab\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49
*a^2*A + 75*A*b^2 + 135*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(315*d) + (2*a*(4*A*b + 3*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.71555, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2955, 4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)\sin}{315a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49
*a^2*A + 75*A*b^2 + 135*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(315*d) + (2*a*(4*A*b + 3*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}(A+}{\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{9d} - \frac{1}{9}\left(2\right.$$

$$= \frac{2a(4Ab+3aB)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d}$$

$$= \frac{2\left(49a^2A+75Ab^2+135abB\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{315d}$$

$$= \frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a}}{315ad}$$

$$= \frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a}}{315ad}$$

$$= \frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a}}{315ad}$$

$$= \frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{\cos(c+dx)}\sqrt{a}}{315ad}$$

$$= \frac{2\left(a^2-b^2\right)\left(114a^2Ab-10Ab^3+75a^3B+45ab^2B\right)\sqrt{\frac{b+a\cos}{a+}}}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}$$

Mathematica [C] time = 18.3075, size = 542, normalized size = 1.28

$$\frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{630}\left(133a^2A+270abB+150Ab^2\right)\sin(2(c+dx))+\frac{\left(747a^2Ab+345a^3B+540ab^2B+20Ab^3\right)\sin(c+}{630a}\right.}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((747*a^2*A*b + 20*A*b^3 +
345*a^3*B + 540*a*b^2*B)*Sin[c + d*x])/(630*a) + ((133*a^2*A + 150*A*b^2 +
270*a*b*B)*Sin[2*(c + d*x)])/630 + (a*(19*A*b + 9*a*B)*Sin[3*(c + d*x)])/12
6 + (a^2*A*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d
*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2
)*((-I)*(a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*
b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*
x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a +
b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A
 + 60*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
 d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a
^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*(b + a*Cos[c +
d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^2*d*(b + a*Cos[c
 + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.689, size = 3069, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c)
)*(cos(d*x+c)+1)*(-147*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^5+147*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^5+10*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*b^5+98*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1
/2)-147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2)-10*A*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5*(1/(cos(d*x+c)+1))^(1/2)-147*A*((a-b)/(a
+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-163*A*((a-b)/(a+b))^(1/2)*a^3*b^2
*(1/(cos(d*x+c)+1))^(1/2)-279*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+
1))^(1/2)-5*A*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)-75*B*((a-b
)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-435*B*((a-b)/(a+b))^(1/2)*a^3
*b^2*(1/(cos(d*x+c)+1))^(1/2)-279*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a^3*b^2+155*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+10*A*((a-b)/(a+b))^(1/2)*b^5*(1/(cos(d*x+
c)+1))^(1/2)+10*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a*b^4-147*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^4*b+279*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*b^2-279*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^3-10*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
```

```
)^(1/2))*a*b^4+170*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c
)+1))^(1/2)+180*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))
^(1/2)+82*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)
+80*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)+270
*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)+130*A*
cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+272*A*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*A*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)+330*B*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+180*B*cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-65*A*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-279*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)+199*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b
^3*(1/(cos(d*x+c)+1))^(1/2)+10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(c
os(d*x+c)+1))^(1/2)-435*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+
c)+1))^(1/2)+165*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1)
)^(1/2)-45*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2
)+45*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)-435*B*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+405*B*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-45*B
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+435
*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-435
*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+4
5*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-
45*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+2
61*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-1
35*B*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-45*B*((a-b)/(a+b)
)^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)+45*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2
)*a^5*(1/(cos(d*x+c)+1))^(1/2)+30*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*(1
/(cos(d*x+c)+1))^(1/2)+75*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^5-75*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+
1))^(1/2)+35*A*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2
)+14*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5*(1/(cos(d*x+c)+1))^(1/2))/a^2/(
(a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+Aa^2\cos\left(dx+c\right)^4+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+\left(Ba^2+2\,A\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (2*B
*a*b + A*b^2)*cos(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.609 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=340

$$\frac{2\left(a^2-b^2\right)\left(25a^2A+56abB+15Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx}{105d}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Sec[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A +
 45*A*b^2 + 77*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(105*d) + (2*a*(10*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.32245, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2955, 4025, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105d}+\frac{2\left(a^2-b^2\right)\left(25a^2A+56abB+15Ab^2\right)\sqrt{\frac{a\cos(}{}}}{105ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Sec[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A +
 45*A*b^2 + 77*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(105*d) + (2*a*(10*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(7*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
```

```
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}(A+}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{7d} - \frac{1}{7}\left(2\sqrt{}\right.$$

$$= \frac{2a(10Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{35d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{105d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{105d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{105d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{105d}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A+15Ab^2+56abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{105ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 17.262, size = 470, normalized size = 1.38

$$\frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{210}\left(115a^2A+308abB+180Ab^2\right)\sin(c+dx)+\frac{1}{14}a^2A\sin(3(c+dx))+\frac{1}{35}a(7aB+15\right.}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((115*a^2*A + 180*A*b^2 + 3
08*a*b*B)*Sin[c + d*x])/210 + (a*(15*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a
^2*A*Sin[3*(c + d*x)])/14))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I
)*(a + b)*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*EllipticE[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(15*b^2*(A + 7*B) +
 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*
(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a*d
*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.597, size = 2450, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c)
)*(cos(d*x+c)+1)*(145*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^3*b+238*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*
x+c)+1))^(1/2)-145*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1)
)^(1/2)+55*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2
)-15*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-35*B*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+15*A*cos(d*x+c
)^5*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+10*A*cos(d*x+c)^3*((a-
b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^
4*(1/(cos(d*x+c)+1))^(1/2)+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(co
s(d*x+c)+1))^(1/2)+15*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1
))^(1/2)-15*A*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)-161*B*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+161*B*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+119*B*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-161*B*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-63*B*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+161*B*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-161*B*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+90*A*cos(d*x+c)^3*((a
-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+98*B*cos(d*x+c)^3*((a-b)/
(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+110*A*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+60*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-145*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+15*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-145*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+135*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
```

```
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+60*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^
3*b*(1/(cos(d*x+c)+1))^(1/2)+105*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a*b^3-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(co
s(d*x+c)+1))^(1/2)-25*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-
145*A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-45*A*((a-b)/(a+b
))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-63*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(
cos(d*x+c)+1))^(1/2)-77*B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1
/2)-161*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-63*B*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-15*A*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+25*A*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4)/a/((a-b)/(a+b))^(1/2)/(b+
a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3\sec(dx+c)^3 + Aa^2\cos(dx+c)^3 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^3\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (2*B
*a*b + A*b^2)*cos(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2
), x)
```

3.610 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=342

$$\frac{2\left(8a^2Ab+5a^3B+10ab^2B-8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+35abB+23Ab^2\right)\sqrt{\cos(c+dx)}}{15d\sqrt{\frac{a\cos(c}{a}}}$$

```
[Out] (2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Sec[c + d*x]]) + (2*b^3*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*a*(8*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c + d*x]^(3/2)*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.39308, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4025, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(8a^2Ab+5a^3B+10ab^2B-8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A+35abB+23Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\,\text{se}}}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] (2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Sec[c + d*x]]) + (2*b^3*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*a*(8*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c + d*x]^(3/2)*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}(A}{\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{5d} - \frac{1}{5}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{15d}$$

$$= \frac{2b^3B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a(8Ab+5a}{}$$

$$= \frac{2\left(8a^2Ab-8Ab^3+5a^3B+10ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 34.447, size = 49609, normalized size = 145.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.43, size = 2052, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
*(cos(d*x+c)+1)*(-23*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a*b^2-9*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^
(1/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
```

```
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+9*A*sin
(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-23*A*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+23*A*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+35*B*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-35*B*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-35*B*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+17*A*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-5*A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-23*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^2*b*(1/(cos(d*x+c)+1))^(1/2)+35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1
/(cos(d*x+c)+1))^(1/2)+14*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(
d*x+c)+1))^(1/2)+34*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)
+1))^(1/2)+40*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(
1/2)+5*B*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+5*B*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-5*B*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+23*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*
x+c)+1))^(1/2)-11*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*B*
((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-35*B*((a-b)/(a+b))^(1/2)
*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+15*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*b^3-15*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*b^3+30*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3-23*A*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d
*x+c)+1))^(1/2)+45*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a*b^2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/
2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2
), x)
```

3.611 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=349

$$\frac{\left(2a^3A+12a^2bB+4aAb^2+3b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(6a^2B+14aAb-3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+\ldots}}{3d\sqrt{\frac{a\cos(c+dx)\ldots}{a+b}}}$$

```
[Out] ((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) + (b^2*(2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]) - (b*(2*a*A - 3*b*B)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*(a + b*Sec
[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.42568, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4025, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^3A+12a^2bB+4aAb^2+3b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(6a^2B+14aAb-3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+\ldots}}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) + (b^2*(2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]) - (b*(2*a*A - 3*b*B)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*(a + b*Sec
[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}(A}{\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} - \frac{1}{3}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + 2aA\sqrt{}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + 2aA\sqrt{}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + 2aA\sqrt{}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + 2aA\sqrt{}$$

$$= \frac{b^2(2Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - b(2$$

$$= \frac{\left(2a^3A+4aAb^2+12a^2bB+3b^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 33.5737, size = 73332, normalized size = 210.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.444, size = 2073, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] 1/3/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(2*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*
a^3*(1/(cos(d*x+c)+1))^(1/2)-2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(c
os(d*x+c)+1))^(1/2)-14*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b^2-14*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+18*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
```

```
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-6*B*sin(d*x+c)*cos(d*x+c)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-3*B*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+30*B*si
n(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a*b^2+18*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b-12*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+14*A*sin(d*x+c)*cos(d*x+c)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-2*A*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-14*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
*b^2*(1/(cos(d*x+c)+1))^(1/2)-6*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(
cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c
)+1))^(1/2)+16*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^
(1/2)+14*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+
6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+6*B*cos
(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+3*B*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+6*B*sin(d*x+c)*cos(d*x+
c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+3*B*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3-6*B*sin(d
*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+12
*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^3+2*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^3-6*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-14*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
*b*(1/(cos(d*x+c)+1))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^
(1/2)-6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+3*B
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2))*((b+a*cos(d*x
+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+
1))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\Big(\big(Bb^2\cos(dx+c)\sec(dx+c)^3 + Aa^2\cos(dx+c) + \big(2\,Bab + Ab^2\big)\cos(dx+c)\sec(dx+c)^2 + \big(Ba^2 + 2\,Aab\big)\,\text{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b
 + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(
d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

3.612 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=359

$$\frac{\left(16a^2Ab+8a^3B+11ab^2B+4Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(8a^2A-9abB-4Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+}}{4d\sqrt{\frac{a\cos(c+dx)}{a+b}}}$$

```
[Out] ((16*a^2*A*b + 4*A*b^3 + 8*a^3*B + 11*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (b*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(4*A*b + 7*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c +
d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.42883, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4026, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(16a^2Ab+8a^3B+11ab^2B+4Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(8a^2A-9abB-4Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{4d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```


```
[Out] ((16*a^2*A*b + 4*A*b^3 + 8*a^3*B + 11*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (b*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(4*A*b + 7*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c +
d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}(A}{\sqrt{\sec(c+}}$$

$$= \frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB(a+}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB(a+}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB(a+}{}$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB(a+}{}$$

$$= \frac{b\left(20aAb+15a^2B+4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(16a^2Ab+4Ab^3+8a^3B+11ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 33.9063, size = 97208, normalized size = 270.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.605, size = 2216, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(-9*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1
/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-8*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3
*(1/(cos(d*x+c)+1))^(1/2)-8*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
```

```
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-8*B*sin(d*x+c)*cos(d*x+c)^2*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+4*B*sin(d*x+c)*cos(d*x
+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3-8*B*sin(d*x+c
)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*b^3+11*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2
)-8*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+9*B*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-9*B*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+6*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-2*B*sin(d*x+c)*cos(d*x+c)^
2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-30*B*sin(d*x+c)
*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*a^2*b+9*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b-9*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+16*A*sin(d*x+c)*cos(d*x+c)^2*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-40*A*sin(d*x+c)*co
s(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*a*b^2+8*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^2+8*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1
/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d
*x+c)+1))^(1/2)+2*B*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)+8*A*co
s(d*x+c)^3*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-4*A*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3-2*B*cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c
)^(3/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\sec(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

3.613 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=422

$$\frac{\left(48a^3A+59a^2bB+66aAb^2+16b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{}}{24d\sqrt{\cos(c+dx)}}$$

```
[Out] ((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((54*a*A*b + 33*a^2*B
+ 16*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(2*A*b +
 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (
(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24
*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.79516, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2955, 4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\left(48a^3A+59a^2bB+66aAb^2+16b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] ((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((54*a*A*b + 33*a^2*B
+ 16*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(2*A*b +
 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (
(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24
*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}(A$$

$$= \frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{bB(a+b\sec(c+dx)}{3d\cos^{\frac{3}{2}}(c}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb+33a^2B+1}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb+33a^2B+1}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb+33a^2B+1}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb+33a^2B+1}{}$$

$$= \frac{(30a^2Ab+8Ab^3+5a^3B+20ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(48a^3A+66aAb^2+59a^2bB+16b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 33.6204, size = 106199, normalized size = 251.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.528, size = 2441, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)+1)*
(-33*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-18*B
*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+33*B*sin(d
```

```
*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-
16*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*b^3-120*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+54*A*sin(d*x+c)*cos(d*x+c)^3*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-54*A*sin(d*x+c)*cos(d*x+c)
^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+36*A*sin(d*x+c
)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12
*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b^2-180*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-33*B*sin(d*x+c)*cos(d*x+c)^3*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+16*B*sin(d*x+c)*cos(d*x+c)
^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-26*B*sin(d*x+c
)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+44
*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b^2-48*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3+34*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c
)+1))^(1/2)+54*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^
(1/2)+66*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+
59*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+33*B*c
os(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+12*A*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-18*B*sin(d*x+c)*cos(d*x+
c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*B*sin(d*x+c
)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*a^3+24*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^3-48*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3-54*A*cos(d*x+c)^4*((a-b)/(a+b))
^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-12*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)
*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-26*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b
*(1/(cos(d*x+c)+1))^(1/2)-16*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2*(1/(c
os(d*x+c)+1))^(1/2)-54*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x
+c)+1))^(1/2)+8*B*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)+8*B*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-12*A*cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3-33*B*cos(d*x+c)^4*((a-b)/
(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-16*B*cos(d*x+c)^3*((a-b)/(a+b))^(
1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2))/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/
(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

3.614 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=513

$$\frac{\left(472a^2Ab+133a^3B+356ab^2B+128Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{192d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(59a^2B+104aAb+36b^2B\right)\sin(c}{96d\cos^{\frac{3}{2}}(c+}$$

```
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(192*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a
^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)])/(64*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(8*A*b + 11*a*B)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(24*d*Cos[c + d*x]^(5/2)) + ((104*a*A*b + 59*a^2*B + 36
*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*d*Cos[c + d*x]^(3/2)) +
((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 2.24829, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2955, 4026, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(59a^2B+104aAb+36b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{96d\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(264a^2Ab+15a^3B+284ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{a+}}{192bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```


```
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(192*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a
^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)])/(64*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(8*A*b + 11*a*B)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(24*d*Cos[c + d*x]^(5/2)) + ((104*a*A*b + 59*a^2*B + 36
*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*d*Cos[c + d*x]^(3/2)) +
((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2))
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
```

```
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A$$

$$= \frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{bB(a+b\sec(c+dx)}{4d\cos^{\frac{5}{2}}(c}$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(104aAb+59a^2B\right.}{}$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(104aAb+59a^2B\right.}{}$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(104aAb+59a^2B\right.}{}$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(104aAb+59a^2B\right.}{}$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(104aAb+59a^2B\right.}{}$$

$$= \frac{\left(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(\right.}{64bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(472a^2Ab+128Ab^3+133a^3B+356ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+d\right.}{192d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 34.2534, size = 131553, normalized size = 256.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.724, size = 3175, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

[Out] -1/192/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)+1)
*(254*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+1
5*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+64*A*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)-264*A*cos(d*x+c)^4*
((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-144*A*cos(d*x+c)^4*((a
-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-15*B*cos(d*x+c)^4*((a-b)/(a
+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+30*B*cos(d*x+c)^4*((a-b)/(a+b))^(
1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-284*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2
)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+172*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b
^3*(1/(cos(d*x+c)+1))^(1/2)-208*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^2*b^2*
(1/(cos(d*x+c)+1))^(1/2)-128*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a*b^3*(1/(c
os(d*x+c)+1))^(1/2)-118*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*
x+c)+1))^(1/2)-284*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c
)+1))^(1/2)-72*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^
(1/2)-128*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^4*b^4-264*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c
)+1))^(1/2)+15*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^4*a^4-30*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^4*a^4+144*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^4+30*B*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4-288*B*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^4*b^4+76*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^4*a^2*b^2-72*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3-720*B*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^2-240
A(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^4*a^3*b-960*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/
((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3-144*A*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b-208*A*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*
b^2+352*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^4*a*b^3+264*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^4*a^3*b-264*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^2+128*A*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^3-15*B*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b+284*B
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
4*a^2*b^2-284*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x

```
+c)*cos(d*x+c)^4*a*b^3-118*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b+48*B*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*
x+c)+1))^(1/2)-72*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))
^(1/2)+24*B*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*b^4-1
5*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-128*A*cos
(d*x+c)^4*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)+64*A*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)+184*B*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+472*A*cos(d*x+c)^3*((a-b)/(a+b)
)^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+133*B*cos(d*x+c)^3*((a-b)/(a+b))^(
1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+272*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a*b^3*(1/(cos(d*x+c)+1))^(1/2)+264*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b
*(1/(cos(d*x+c)+1))^(1/2))/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d
*x+c)+1))^(1/2)/cos(d*x+c)^(7/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

3.615 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=280

$$-\frac{2\left(7a^2Ab-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A-10abB+8Ab^2\right)\sqrt{\cos(c+d}}{15a^3d\sqrt{}}$$

```
[Out] (-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15
*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b - 5*a*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (2*A*Cos[c + d*
x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.916042, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4034, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(7a^2Ab-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2A-10abB+8Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+}}{15a^3d\sqrt{\frac{a\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15
*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b - 5*a*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (2*A*Cos[c + d*
x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+ \right.}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+}{}$$

$$= -\frac{2\left(7a^2Ab+8Ab^3-5a^3B-10ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(9}{}$$

Mathematica [C] time = 14.7894, size = 363, normalized size = 1.3

$$\frac{2a\sin(c+dx)(a\cos(c+dx)+b)(3aA\cos(c+dx)+5aB-4Ab) + \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia\left(a^2(9A+5B)+2ab(A-5B)+8Ab^2\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x
] + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(9*a^2*A + 8*A*b^
2 - 10*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[
(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*
a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*EllipticF[I*ArcSinh[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^2*A + 8*A*b^2 - 10*a*b*B)*(b + a*Cos[c
 + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2))/
(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.445, size = 1701, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
*(cos(d*x+c)+1)*(-8*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a*b^2-9*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(
1/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+9*A*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-8*A*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+8*A*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-10*B*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+10*B*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+10*B*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+2*A*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+10*A*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-8*A*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+10*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*
b*(1/(cos(d*x+c)+1))^(1/2)-10*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(co
s(d*x+c)+1))^(1/2)-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+
1))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/
2)-5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+5*B*
cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3+5*B*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-5*B*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-9*A*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*
(1/(cos(d*x+c)+1))^(1/2)-9*A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(
1/2)+4*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-5*B*((a-b)/(a+b
))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+10*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(
cos(d*x+c)+1))^(1/2)-8*A*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2))/
a^3/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c
)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(cos(d*x +
c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

3.616 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=212

$$\frac{2\left(a^2A-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*a*d)
```

Rubi [A] time = 0.63214, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2955, 4034, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*a*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{3}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left((2Ab-3aB)\sqrt{\cos(c+dx)}\right.}{3}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\left(\frac{a^2A}{2}+\frac{1}{2}b(2Ab-3aB)\right)\right.}{3a^2\sqrt{\cos(c+}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\left(\frac{a^2A}{2}+\frac{1}{2}b(2Ab-3aB)\right)\right.}{3a^2\sqrt{\cos(c+dx}}$$

$$= \frac{2\left(a^2A+2Ab^2-3abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2(2Ab-3aB)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 8.96496, size = 311, normalized size = 1.47

$$\frac{2\left(aA\sin(c+dx)(a\cos(c+dx)+b) - \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ia(a(A+3B)-2Ab)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}}\text{EllipticF}\left(i\sin\right.\right.}{}\right.}{3a^2d\sqrt{\text{co}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*(a*A*(b + a*Cos[c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(3/2)*((-I)*(a + b)*(-2*A*b + 3*a*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] + I*a*(-2*A*b + a*(A + 3*B))*EllipticF[I*ArcSinh[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*A*b - 3*a*B)*(b + a*Cos[c + d*x])*(S
ec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*a^2*d*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.384, size = 1080, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(cos(d*x+c)+1)*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(
```

```
1/2)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+3*B*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+A*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a^2+2*A*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-2*A*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+2*A*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^2*(1/(cos(d*x+c)+1))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos
(d*x+c)+1))^(1/2)-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))
^(1/2)-3*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^2+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-3
*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-3*B*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b*(
1/(cos(d*x+c)+1))^(1/2)+2*A*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2
)-3*B*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2))/a^2/((a-b)/(a+b))^(
1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

3.617 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=150

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Rubi [A] time = 0.43489, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2955, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{\left((Ab-aB)\sqrt{\cos(c+dx)}\sqrt{s\ldots}\right.}{\ldots}$$

$$= -\frac{\left((Ab-aB)\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{a+b\,s\ldots}\right.}{a\sqrt{b\ldots}}$$

$$= -\frac{\left((Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(\ldots}\right.}{a\sqrt{\frac{b+\ldots}{\ldots}}}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+\ldots}\right.}{ad\sqrt{\frac{b+a\cos(c+\ldots}{a+b}}}$$

Mathematica [C] time = 6.7387, size = 260, normalized size = 1.73

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))}\left(-ia(A+B)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}}\text{EllipticF}\left(i\sinh\ldots\right.\right.}{ad\sqrt{\sec\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c +
d*x])*(I*A*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(A + B)*Elli
pticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + A*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*
x)/2]^2]*Tan[(c + d*x)/2]))/(a*d*(B + A*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.382, size = 564, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+A*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-A*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+B*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-A*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/a/((a-b)/(a
+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx))\sqrt{\cos(c + dx)}}{\sqrt{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx + c) + A)\sqrt{\cos(dx + c)}}{\sqrt{b\sec(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

3.618 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.52561, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2955, 4036, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```


```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4036

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (
A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec \right. \\
&= \frac{\left(A\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{b+a\cos(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+}}}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}} \\
&= \frac{\left(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 28.3339, size = 16611, normalized size = 120.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.361, size = 257, normalized size = 1.9

$$-2\frac{\sqrt{\cos(dx+c)}}{d(b+a\cos(dx+c))\sqrt{(\cos(dx+c)+1)^{-1}}}\left(A\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)-B\mathrm{EllipticF}\left(\frac{-1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/((a-b)/(
a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

3.619 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=256

$$\frac{B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{a+b\sec(}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sq
rt[Cos[c + d*x]])
```

Rubi [A] time = 0.888688, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2955, 4033, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```


```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sq
rt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
```

```
0] && !IGtQ[m, 1]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aB}{2}+\frac{1}{2}(2Ab-}{\sqrt{\sec(c+dx)}}}{b}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\left(aB\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\sec(c+d}}}{2b}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{s}}{\sqrt{a+}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+}{bd\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\right.}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 32.2606, size = 51168, normalized size = 199.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.376, size = 776, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-4*A*cos(d*x+c)*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-B*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2)-2*B*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+2*B*cos(d*x+c)*sin
(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a
+B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a-B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2)-B*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*b
*(1/(cos(d*x+c)+1))^(1/2))/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d
*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

3.620 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=344

$$\frac{(4Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(4Ab}{}$$

```
[Out] ((4*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((4*a*A
*b - 3*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]) - ((4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)) +
 ((4*A*b - 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[
c + d*x]])
```

Rubi [A] time = 1.28119, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4033, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}-\frac{(4Ab-3aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```


```
[Out] ((4*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((4*a*A
*b - 3*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]) - ((4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)) +
 ((4*A*b - 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[
c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
```

```
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(}{}}{2b}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(4aAb-3a^2B-4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{B\sqrt{a+b\sec(c}{2bd\cos}$$

$$= \frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-3a^2B-4b^2B\right)\sqrt{\frac{b+a\cos}{a+}}}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [C] time = 32.6537, size = 77909, normalized size = 226.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.409, size = 1568, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)*(4
*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+8*A*sin(d*
x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b-8
*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
```

```
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*a*b-4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*a^2*(1/(cos(d*x+c)+1))^(1/2)+2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/
(cos(d*x+c)+1))^(1/2)-6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B
*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(c
os(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)
+1))^(1/2)-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2
)+2*B*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*b^2-4*A*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-2*B*((a-b)/(a+b))^(1/2)*b^2*(1/
(cos(d*x+c)+1))^(1/2))/b^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x
+c)+1))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

3.621 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{2\left(12a^2Ab-5a^3B-40ab^2B+48Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+5abB-6Ab^2\right)\sin(c+dx)\,\text{cc}}{5a^2d\left(a^2-\right.}$$

```
[Out] (-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^4*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*
a^3*b*B + 40*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (2*b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*
a*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*
(a^2 - b^2)*d) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.40441, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A+5abB-6Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(9a^2Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^4*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*
a^3*b*B + 40*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (2*b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*
a*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*
(a^2 - b^2)*d) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(-a^2A}{}}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-6Ab^2+5abB\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{}}{5a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(9a^2Ab-24Ab^3-5a^3B+20ab^2B\right)\sqrt{}}{15a^3(}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(9a^2Ab-24Ab^3-5a^3B+20ab^2B\right)\sqrt{}}{15a^3(}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(9a^2Ab-24Ab^3-5a^3B+20ab^2B\right)\sqrt{}}{15a^3(}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(9a^2Ab-24Ab^3-5a^3B+20ab^2B\right)\sqrt{}}{15a^3(}$$

$$= -\frac{2\left(12a^2Ab+48Ab^3-5a^3B-40ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(9a}{}$$

Mathematica [C] time = 18.9577, size = 533, normalized size = 1.26

$$\frac{(a\cos(c+dx)+b)^2\left(\frac{2\left(Ab^4\sin(c+dx)-ab^3B\sin(c+dx)\right)}{a^3\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2(5aB-9Ab)\sin(c+dx)}{15a^3}+\frac{A\sin(2(c+dx))}{5a^2}\right)}{d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(\cos^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*(-9*A*b + 5*a*B)*Sin[c + d*x])/(15*a^3) + (2*(A
*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(b + a*Cos[c +
d*x])) + (A*Sin[2*(c + d*x)])/(5*a^2)))/(d*Cos[c + d*x]^(3/2)*(a + b*Sec[c
+ d*x])^(3/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3
```

```
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(9*a^4*A + 24*a^2
*A*b^2 - 48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 - 6*a^2*b*(2*A + 5*B) +
 a^3*(9*A + 5*B) + 4*a*b^2*(9*A + 10*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - (9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^3*b*B +
40*a*b^3*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2
]))/(15*a^4*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.435, size = 2084, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
*(cos(d*x+c)+1)^2*(-20*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d
*x+c)+1))^(1/2)+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))
^(1/2)-18*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)
+20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-9*A*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+48*A*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)+24*A*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-20*B*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a
^3*b*(1/(cos(d*x+c)+1))^(1/2)-48*A*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1
))^(1/2)+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4+
9*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*A*
cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)+6*A*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-40*B*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+30*B*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+40*B*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-25*B*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+40*B*sin(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-6*A*cos(d*x+c)^3*((a-b)/(a+b))^(
1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a^3*b*(1/(cos(d*x+c)+1))^(1/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(
1/(cos(d*x+c)+1))^(1/2)+24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos
(d*x+c)+1))^(1/2)+24*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a^2*b^2-12*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3*b-36*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2-48*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^3+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)
+1))^(1/2)-5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-
9*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-24*A*((a-b)/(a+b))^(
1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d
```

```
*x+c)+1))^(1/2)+20*B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+4
0*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+5*B*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-48*A*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-9*A*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4)*((a-b)/(a+b))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)/a^4/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.622 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$\frac{2\left(a^2A-6abB+8Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+}}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a
^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a
^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.03192, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4030, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A-\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a
^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a
^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
```

```
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(-a^2A}{}}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{\cos(c+dx)}}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{\cos(c+dx)}}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{\cos(c+dx)}}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{\cos(c+dx)}}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(a^2A+8Ab^2-6abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(5a^2Ab-8Ab^3-3\right.}{}$$

Mathematica [C] time = 17.014, size = 417, normalized size = 1.28

$$\frac{2(a\cos(c+dx)+b)\left(a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(b\left(a^2A+3abB-4Ab^2\right)+aA\left(a^2-b^2\right)\cos(c+dx)\right)+\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(a*(b*(a^2*A - 4*A*b^2 + 3*a*b*B) + a*A*(a^2 - b^2)
*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x] + (Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(3/2)*(I*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Ellip
ticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt
[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^
2)*(-4*A*b + a*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.342, size = 1460, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(cos(d*x+c)+1)^2*(8*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a*b^2-5*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b+A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^3+8*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b^3-6*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2-6*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+
6*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+4*
A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+6*B*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a^3-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(
d*x+c)+1))^(1/2)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1
/2)-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b
)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*((a-b)/(a+b))^(1/2)*a*b^2
*(1/(cos(d*x+c)+1))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^
(1/2)-6*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+8*A*((a-b)/(a+b))^(1/
2)*b^3*(1/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*a^3+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c
)+1))^(1/2)+3*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^3)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/a^3/(b+a*cos(d*x+c))/
(a-b)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.623 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{2(2Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+abB\right.}{}$$

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(
a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.719584, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2955, 4030, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(
a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{b+a\cos(c+\right.}{a^2\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)}{a^2\sqrt{\cos(c+dx)}\sqrt{a+}}$$

$$= -\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2A-2Ab^2+abB\right)\sqrt{\mathrm{co}}}{a^2\left(\right.}$$

Mathematica [C] time = 15.1463, size = 365, normalized size = 1.55

$$\frac{2(a\cos(c+dx)+b)(A+B\sec(c+dx))\left(-ab(aB-Ab)\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia(a+b)(a(A+B)-2Ab)\sec^2\left(\frac{1}{2}(\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*(-(a*b*(-(A*b) + a*B)*Sin[c +
d*x]) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(a^2*A - 2*A*b^
2 + a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(
a + b)*(-2*A*b + a*(A + B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] + (a^2*A - 2*A*b^2 + a*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2
]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a^2*(a^2 - b^2)*d*Sqrt[
Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.407, size = 889, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)^2*(A*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/
2)*((a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d
*x+c)+1))^(1/2)+A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^2-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*b^2-A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a^2-
2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-A*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)+2*A*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+B*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/
(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-2*
A*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*a*
b*(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/a^2/(b+a*cos(d*x+c))/(a-
b)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec{(dx+c)}+A)\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.624 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{a}}{ad\left(a^2-b^2\right.}$$

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(a*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b - a*B)*S
in[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.665158, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2955, 4027, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sqrt{a\cos}}{ad\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(a*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b - a*B)*S
in[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{a^2}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{a}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{b+a\cos(c+dx)}\right)\int \frac{}{\sqrt{b+a}}}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos}{}}}}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(}{a+b}}}$$

Mathematica [C] time = 10.3634, size = 328, normalized size = 1.53

$$\frac{2(a\cos(c+dx)+b)\left(\frac{(aB-Ab)\sin(c+dx)}{a^2-b^2} + \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia(a+b)(A-B)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}}\text{EllipticF}\left(i\sinh\right.\right.}{}\right.}{d\cos^{\frac{3}{2}}(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(((-(A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) + ((Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(-(A*b) + a*B)*EllipticE[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(A - B)*Ellipti
cF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[(
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (A*b - a*B)*(b + a*Cos[
c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)*Sec[
c + d*x]^(3/2))))/(d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.386, size = 564, normalized size = 2.6

$$2\,\frac{(-1+\cos(dx+c))(\cos(dx+c)+1)^2\sqrt{\cos(dx+c)}\sqrt{(\cos(dx+c)+1)^{-1}}}{ad(b+a\cos(dx+c))(a-b)(\sin(dx+c))^3}\left(A\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)^2*(A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b+A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*b-B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+B
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+B*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2)+A*((a-b)/(a+b))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*b-B*((a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)/a/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)\sec(dx+c)^2+2\,ab\cos(dx+c)\sec(dx+c)+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c) + a^2*cos
(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

3.625 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\ldots}}{bd\sqrt{\ldots}}$$

```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
 + b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
 d*x]])/(b*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b -
a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]])
```

Rubi [A] time = 0.787558, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4029, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\ldots}}{bd\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
 + b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
 d*x]])/(b*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b -
a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{b}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left((-Ab+aB)\sqrt{\cos(c+dx)}\right.}{b(}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{\sec(}{\sqrt{\frac{b}{a+b}+}}}{b\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+bs}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+a\right.}{b\left(a^2-b^2\right)d\sqrt{\frac{b+}{}}}$$

Mathematica [C] time = 32.167, size = 50122, normalized size = 227.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.336, size = 840, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(A
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b-A*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-2*B*sin(d*x+c)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
```

```
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*b+B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a+2*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
-b),I/((a-b)/(a+b))^(1/2))*a+2*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*(1/
(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+B*((
a-b)/(a+b))^(1/2)*a*(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*((a-b)/(a+b)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/b/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

3.626 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=371

$$\frac{B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c}{b^2d\left(a^2-b^2\right)\sqrt{c}}$$

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - 3*a*B)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a*A*b - 3*a^2*B +
b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*
a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.44624, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4029, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{\left(-3a^2B+2aAb-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - 3*a*B)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a*A*b - 3*a^2*B +
b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*
a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
```

```
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+}}{b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+}}{b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+}}{b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+}}{b^2\left(a^2-b^2\right)d}$$

$$= \frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sin}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c\right.}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

Mathematica [C] time = 33.4809, size = 95694, normalized size = 257.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.367, size = 1441, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(
2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d
*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b+2*
```

```
A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*b^2-4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*a*b-4*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-2*A*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)-6*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2-4*B*cos(d*x+c)*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b+6*B*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+6*B*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2-B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b^2-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(cos(d
*x+c)+1))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(cos(d*x+c)+1))^(
1/2)-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)
/(a+b))^(1/2)*a*b*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*b^2*(1/(co
s(d*x+c)+1))^(1/2))*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/b^2/(b+a*c
os(d*x+c))/cos(d*x+c)^(1/2)/(a-b)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

3.627 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=487

$$\frac{(4Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\left(-5a^2B+4aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2b^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{}{bd\left(a^2-\right.}$$

```
[Out] ((4*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((1
2*a*A*b - 15*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) - ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^3
*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]) - ((
4*a*A*b - 5*a^2*B + b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b
^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[
c + d*x]])
```

Rubi [A] time = 1.85873, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4029, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{\left(-5a^2B+4aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2b^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\left(12a^2Ab-15a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] ((4*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((1
2*a*A*b - 15*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) - ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^3
*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]) - ((
4*a*A*b - 5*a^2*B + b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b
^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[
c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-5a^2B+b^2B\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-5a^2B+b^2B\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-5a^2B+b^2B\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-5a^2B+b^2B\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(4aAb-5a^2B+b^2B\right)\sqrt{a+}}{2b^2\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12aAb-15a^2B-4b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{}{b\left(a^2-b^2\right)}$$

$$= \frac{(4Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(12aAb-15a^2B-4b^2B\right)\sqrt{}}{4b^3d\sqrt{\cos(c+dx}}$$

Mathematica [C] time = 34.8707, size = 140027, normalized size = 287.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.449, size = 2295, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*
(-5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+2*B*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+8*B*cos(d*x+
c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)*a*b^2-24*A*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a^2*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-30*B*sin(d*x+c)*cos(d*x+c)^
2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-4*B*sin(d*x+c)*co
s(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+8*B*sin(
d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*b^3-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^
(1/2)+5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+5*B
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+12*A*cos(d*x
+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-15*B*cos(d*x+c)^3*
(1/(cos(d*x+c)+1))^(1/2)*((a-b)/(a+b))^(1/2)*a^3-4*A*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)-2*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos
(d*x+c)+1))^(1/2)-5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)
+1))^(1/2)-20*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b-2*B*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+30*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-7*B*sin(d*x+c)*
cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+24*A
*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a^2*b+16*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2-12*A*sin(d*x+c)*cos(d*x+c)^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*A*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)-2*B*((a-b)/(a+b))^(1/2)*b^3*(1
/(cos(d*x+c)+1))^(1/2)+15*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*a^3+30*B*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a^3+4*A*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+15*B*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*b^3*(1/(cos(d*x+c)+1))^(1/2))*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)/b^3/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/(a-b)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/
2)), x)
```

3.628 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2\left(116a^2Ab^3+17a^4Ab-80a^3b^2B-5a^5B+80ab^4B-128Ab^5\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(-71a^2Ab^2\right.}{}$$

```
[Out] (-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(15*a^5*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
 (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*
a^3*b^3*B - 80*a*b^5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a
^3*B + 5*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
qrt[a + b*Sec[c + d*x]]) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5
*B + 65*a^3*b^2*B - 40*a*b^4*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A
*b^4 + 50*a^3*b*B - 30*a*b^3*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 2.06686, antiderivative size = 588, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2955, 4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(-71a^2Ab^2+3a^4A+50a^3bB-30ab^3B+48Ab^4\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^3d\left(a^2-b^2\right)^2}+\frac{2b\left(12a^2Ab-9a^3B+5\right.}{3a^2d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(15*a^5*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
 (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*
a^3*b^3*B - 80*a*b^5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a
^3*B + 5*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
qrt[a + b*Sec[c + d*x]]) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5
*B + 65*a^3*b^2*B - 40*a*b^4*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A
*b^4 + 50*a^3*b*B - 30*a*b^3*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(\cdots}{\cdots}}{\cdots}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9a^3B+5ab^2B\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9a^3B+5ab^2B\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9a^3B+5ab^2B\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9a^3B+5ab^2B\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9a^3B+5ab^2B\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(12a^2Ab-8Ab^3-9a^3B+5ab^2B\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= -\frac{2\left(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B\right)\sqrt{\frac{b+a\cos(c+\cdots}{a+b}}}{15a^5\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

<u>Mathematica</u> [C] time = 24.6379, size = 4179, normalized size = 7.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*(-14*A*b + 5*a*B)*Sin[c + d*x])/(15*a^4) - (2*(
A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*Cos[c
 + d*x])^2) - (2*(-15*a^2*A*b^4*Sin[c + d*x] + 11*A*b^6*Sin[c + d*x] + 12*a
^3*b^3*B*Sin[c + d*x] - 8*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b +
a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(5*a^3)))/(d*Cos[c + d*x]^(5/2)*(a
+ b*Sec[c + d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*((3
*a^2*A*Sqrt[Cos[c + d*x]])/(5*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]) + (11*A*b^2*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (212*A*b^4*Sqrt[Cos[c + d*x]])/(15*a^2*(
a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (128*A*b^6*Sqrt
[Cos[c + d*x]])/(15*a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) - (8*a*b*B*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]) + (28*b^3*B*Sqrt[Cos[c + d*x]])/(3*a*(a^2 - b^2)^
```

2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (16*b^5*B*Sqrt[Cos[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a*A*b*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (44*A*b^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*a*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (32*A*b^5*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (7*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*b^4*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^5*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*(-(Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)) + (Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(5*a^5*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + a*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(Sec[(c + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a +

```
b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)*
(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B
) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]
^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] - (a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3
*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*Sec[(c
+ d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqr
t[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])
+ ((a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B
 + 140*a^3*b^3*B - 80*a*b^5*B)*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a +
b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(15*a^5*(a^2 - b^2)^2*Sqrt[b + a*Co
s[c + d*x]]) - (Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((
-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*
B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A +
5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45
*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^6*A +
55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*
a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*
(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec
[c + d*x]*Tan[c + d*x]))/(5*a^5*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 0.734, size = 5675, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.629 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=472

$$\frac{2\left(16a^2Ab^2+a^4A-9a^3bB+8ab^3B-16Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB\right.}{}$$

```
[Out] (2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2
*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (
2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^4*A - 13*a^
2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.54015, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2955, 4030, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2b\left(10a^2Ab-7a^3B+3a\right.}{3a^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2
*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (
2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^4*A - 13*a^
2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3}{2}}{\ldots}}{\ldots}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2\ldots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\ldots}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2\ldots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\ldots}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2\ldots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\ldots}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2\ldots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\ldots}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+3ab^2\ldots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+\ldots}}$$

$$= \frac{2\left(a^4A+16a^2Ab^2-16Ab^4-9a^3bB+8ab^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\ldots\right.}{3a^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 20.0776, size = 626, normalized size = 1.33

$$\frac{(a\cos(c+dx)+b)^3\left(\frac{2\left(Ab^4\sin(c+dx)-ab^3B\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{2\left(-12a^2Ab^3\sin(c+dx)+9a^3b^2B\sin(c+dx)-5ab^4B\sin(c+dx)+8Ab^5\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2A\sin(c+d}{3a^3}\right.}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*A*Sin[c + d*x])/(3*a^3) + (2*(A*b^4*Sin[c + d*x
] - a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (2*
(-12*a^2*A*b^3*Sin[c + d*x] + 8*A*b^5*Sin[c + d*x] + 9*a^3*b^2*B*Sin[c + d*
x] - 5*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/
(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(
b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
(3/2)*((-I)*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^
3*b^2*B + 8*a*b^4*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + I*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a
*b^3*(3*A + 2*B) + a^4*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*
b^2*B + 8*a*b^4*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c +
 d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.548, size = 4480, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*
(cos(d*x+c)+1)^2*(-A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c
)+1))^(1/2)-12*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d*x+c)+1)
)^(1/2)-16*A*((a-b)/(a+b))^(1/2)*b^6*(1/(cos(d*x+c)+1))^(1/2)-6*A*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)-2*A*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)+A*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^6+3*B*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^6-3*B*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^6-3*B*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)+16*A*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*b^6*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a^4*b^2
*(1/(cos(d*x+c)+1))^(1/2)+7*A*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1)
)^(1/2)+20*A*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)-8*A*((a-b
)/(a+b))^(1/2)*a*b^5*(1/(cos(d*x+c)+1))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*a^4*b
^2*(1/(cos(d*x+c)+1))^(1/2)-11*B*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)
+1))^(1/2)+4*B*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)+8*B*((a
-b)/(a+b))^(1/2)*a*b^5*(1/(cos(d*x+c)+1))^(1/2)-16*A*sin(d*x+c)*(1/(a+b)*(b
```

+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6+A*sin(d*x+c)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b+6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/
2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^
4*b^2*(1/(cos(d*x+c)+1))^(1/2)-6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b^2
*(1/(cos(d*x+c)+1))^(1/2)+8*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(
cos(d*x+c)+1))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x
+c)+1))^(1/2)-4*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1
))^(1/2)+9*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
x+c),(-(a+b)/(a-b))^(1/2))(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a^4*b^2+A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)-
A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)-6*A*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)+6*A*cos(d*x+
c)^3*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)+16*A*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3-12*A*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-16*A*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-8*A*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+28*A*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-3*B*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-9*B*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+6*B*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+8*B*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+3*B*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-15*B*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3+8*B*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+14*A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)+22*A*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)-34*A*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)-16*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b^5*(1/(cos(d*x+c)+1))^(1/2)-6*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b*(1/(c
os(d*x+c)+1))^(1/2)-12*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(cos(d*x
+c)+1))^(1/2)+18*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1)
)^(1/2)+8*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(cos(d*x+c)+1))^(1/2)
-8*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^5*(1/(cos(d*x+c)+1))^(1/2)+3*B*cos(
d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)+7*A*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a^5*b*(1/(cos(d*x+c)+1))^(1/2)-34*A*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a^3*b^3*(1/(cos(d*x+c)+1))^(1/2)+24*A*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a*b^5*(1/(cos(d*x+c)+1))^(1/2)+18*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a^4*b^2*(1/(cos(d*x+c)+1))^(1/2)-12*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^3-16*A*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^4-15*B*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^4*b^2+8*B*E

```
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^
2*b^4-9*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^5*b+6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^4*b^2+8*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^3-8*A*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^5*b+28*A*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^3*b^3-16*A*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a*b^5+9*A*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*
b+16*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d
*x+c)*a^4*b^2+3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(
1/2)+A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2)-A*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^6*(1/(cos(d*x+c)+1))^(1/2))*((a-b)/(a+b))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)/a^4/(a+b)/(a-b)^2/(b+a*cos(d*x+c))^2/sin(d*x+c
)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)\sec(dx+c)+A\cos(dx+c))\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a
^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.630 $\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{2\left(9a^2Ab-3a^3B+2ab^2B-8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}$$

```
[Out] (-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^
4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*
a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.10523, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4030, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2\left(9a^2Ab-3a^3B+\right.}{3a^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^
4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*
a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
```

```
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= -\frac{2\left(9a^2Ab-8Ab^3-3a^3B+2ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4A\right.}{}$$

Mathematica [C] time = 18.5396, size = 621, normalized size = 1.69

$$\frac{(a\cos(c+dx)+b)^3(A+B\sec(c+dx))\left(-\frac{2\left(Ab^3\sin(c+dx)-ab^2B\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}-\frac{2\left(-9a^2Ab^2\sin(c+dx)+6a^3bB\sin(c+dx)-2ab^3B\sin(c+dx)+5Ab^4\mathrm{s}\right.}{3a^2\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}\right.}{d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A\cos(c+dx)+B)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x])*((-2*(A*b^3*Sin[c + d*x] - a*b
^2*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) - (2*(-9*a^2
*A*b^2*Sin[c + d*x] + 5*A*b^4*Sin[c + d*x] + 6*a^3*b*B*Sin[c + d*x] - 2*a*b
^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c +
 d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) - (2*Cos[c + d
*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*((-I)*(a + b)*(3*a^4*A - 15*a^2*A*b^2
 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
```

```
x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A +
B) - 2*a*b^2*(3*A + B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*(b + a*C
os[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a*(a^3 - a*b^
2)^2*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.677, size = 3337, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)^2*(-8*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a^5*(1/(cos(d*x+c)+1))^(1/2)+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5*(1/(cos
(d*x+c)+1))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)+
11*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-4*A*((a-b)/(a+b))
^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(c
os(d*x+c)+1))^(1/2)+9*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^3*b^2-6*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^2*b^3-3*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a^5-8*A*((a-b)/(a+b))^(1/2)*b^5*(1/(cos(d*x+c)+1))^(1/2)+9*A*sin(d*x
+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-6*
A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^3*b^2-8*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^3+15*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^3*b^2-8*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+3*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+2*B*sin(d*x+c)*cos(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-6*B*sin(d*x+c)
*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+2*B*s
in(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^3-8*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*
b^4-3*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*
b+15*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^3-3*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+3*A*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-18*A*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)+12*A*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)+6*B*cos(d*x+c)^2*((
```

```
a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)+6*A*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a^4*b*(1/(cos(d*x+c)+1))^(1/2)+12*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*
b^2*(1/(cos(d*x+c)+1))^(1/2)-18*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1
/(cos(d*x+c)+1))^(1/2)-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x
+c)+1))^(1/2)-6*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(
1/2)+6*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(1/2)+2*
B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(1/2)-2*B*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*x+c)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+3*B*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+2*B*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-6*B*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+2*B*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+3*A*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+3*A*cos(d*x+c)^3*((a-b)/
(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+3*A*sin(d*x+c)*cos(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*A*sin(d*x+c)*cos(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*A*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b+4*A*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3-B*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+B*((a-b)/(a+b))^(1/2)*a^2*b
^3*(1/(cos(d*x+c)+1))^(1/2)+2*B*((a-b)/(a+b))^(1/2)*a*b^4*(1/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/a^3/(a+b)/(a-b)^2/(b+a*cos(d*x+c))^2/sin(d*
x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sec(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.631 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$\frac{2\left(3a^2A-abB-2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\left(5a^2Ab-2a^3B-2ab^2B-Ab^3\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\ldots}{3d(\ldots}$$

```
[Out] (2*(3*a^2*A - 2*A*b^2 - a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b - a*B
)*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3
/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*Sin[c + d*x])/(3*a*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.00769, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4027, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(5a^2Ab-2a^3B-2ab^2B-Ab^3\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2\left(3a^2A-abB-2A\ldots\right.}{3a^2d\left(a^2-b^2\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (2*(3*a^2*A - 2*A*b^2 - a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b - a*B
)*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3
/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*Sin[c + d*x])/(3*a*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
```

```
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + B\sec(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(5a^2Ab-Ab^3-2a^3B-\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(5a^2Ab-Ab^3-2a^3B-\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(5a^2Ab-Ab^3-2a^3B-\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(5a^2Ab-Ab^3-2a^3B-\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= \frac{2\left(3a^2A-2Ab^2-abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(6a^2Ab-2Ab^3-\right.}{}$$

Mathematica [C] time = 16.3687, size = 463, normalized size = 1.34

$$\frac{(a\cos(c+dx)+b)^2\left(\frac{2\sin(c+dx)\left(a\left(-6a^2Ab+3a^3B+ab^2B+2Ab^3\right)\cos(c+dx)+b\left(-5a^2Ab+2a^3B+2ab^2B+Ab^3\right)\right)}{a\left(a^2-b^2\right)^2(a\cos(c+dx)+b)} + \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia(a+b\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*(b*(-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B) +
 a*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(
a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(3/2)*((-I)*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticE[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(-2*A*b^2 + 3*
a^2*(A - B) + a*b*(3*A - B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)] - (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*(b + a*Cos[c + d*x]
)*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*
```

```
x]^(3/2))))/(3*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.495, size = 2416, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)^2*(3*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+2*A*cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+B*cos(d*x+c)*s
in(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+B*cos(d
*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^
2-6*A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^3*b+2*A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a*b^3-3*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^4-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos
(d*x+c)+1))^(1/2)+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)
+1))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2
)-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-3*B*cos
(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-2*A*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)-A*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+2*A*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+
1))^(1/2)+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)
-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+B*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*B*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+B*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+6*A*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-3*A*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-6*A*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-3*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+3*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+2*A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*B*cos(d*x+c)*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+3*B*cos(d*x+c)*((a-b)/(a
```

```
+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-5*A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/
(cos(d*x+c)+1))^(1/2)+A*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+
2*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-B*((a-b)/(a+b))^(1/2
)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+
c)+1))^(1/2)+2*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*b^4)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/a^2/(a+b)/(a-b)^2/(b+a*cos(d*x+c))^2/sin(d
*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c) + A}{(b \sec(dx+c) + a)^{\frac{5}{2}} \sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B \sec(dx+c) + A)\sqrt{b \sec(dx+c) + a}\sqrt{\cos(dx+c)}}{b^3 \cos(dx+c) \sec(dx+c)^3 + 3\,ab^2 \cos(dx+c) \sec(dx+c)^2 + 3\,a^2 b \cos(dx+c) \sec(dx+c) + a^3 \cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sec(d*x + c)^2 + 3*a
^2*b*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c) + A}{(b \sec(dx+c) + a)^{\frac{5}{2}} \sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

3.632 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2Ab+a^3B-5ab^2B+2Ab^3\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a(}{3bd\left(a^2-b^2\right)\sqrt{}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(2*a^2*A*b + 2*A*b^
3 + a^3*B - 5*a*b^2*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.04262, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2955, 4029, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(2a^2Ab+a^3B-5ab^2B+2Ab^3\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a\cos(}{a}}}{3ad\left(a^2-b^2\right)\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(2*a^2*A*b + 2*A*b^
3 + a^3*B - 5*a*b^2*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
```

```
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-\right.}{3b\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-\right.}{3b\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-\right.}{3b\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(2a^2Ab+2Ab^3+a^3B-\right.}{3b\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx}}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(3a^2A+Ab^2-4abB\right)\sqrt{c}}{3a\left(a\right.}$$

Mathematica [C] time = 16.0318, size = 487, normalized size = 1.48

$$\frac{(a\cos(c+dx)+b)^3\left(\frac{2(Ab\sin(c+dx)-aB\sin(c+dx))}{3\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{2\left(3a^2A\sin(c+dx)-4abB\sin(c+dx)+Ab^2\sin(c+dx)\right)}{3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*(-a^2
 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(3*a^2*A*Sin[c + d*x] + A*b^2*Sin[c +
d*x] - 4*a*b*B*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*
Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/
2)*((-I)*(a + b)*(3*a^2*A + A*b^2 - 4*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Ellipt
icF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2*A + A*b^2 - 4*a
*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3
```

```
*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.454, size = 1921, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2
*(A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+3*
A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-3*A*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+A*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-4*B*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-B*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-3*A*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-3*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b
^2*(1/(cos(d*x+c)+1))^(1/2)-4*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(co
s(d*x+c)+1))^(1/2)+4*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+
1))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/
2)+3*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)-A*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(1/2)+2*A*((a-b)/(a+b))^(1/
2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c
)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)-4*B*((a-b)
/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^2*b*(1/(cos(d*x+c)+1))^(1/2)+A*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c
)+1))^(1/2)+3*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^3-B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3+A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+A*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*B*sin(d*x+c)*cos(d*x+c)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+3*B*sin(d*x+c)*cos(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+3*B*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-B*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^3*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*a^3
*(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)/a/(a+b)/(a-b)^2/(b+a*cos(d*x+c))^2/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^2\sec(dx+c)^3+3\,ab^2\cos(dx+c)^2\sec(dx+c)^2+3\,a^2b\cos(dx+c)^2\sec(dx+c)+a^3\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2*sec(d*x + c)^2 +
 3*a^2*b*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

3.633 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=399

$$\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a\left(3a^3B\right.}{3b^2d\left(a^2-b^2\right.}$$

```
[Out] (2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*S
ec[c + d*x]])
```

Rubi [A] time = 1.50043, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2955, 4029, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cdots}}{3bd\left(a^2-b^2\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*S
ec[c + d*x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7a\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7a\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7a\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4Ab^3+3a^3B-7a\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+d}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(}$$

$$= \frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

Mathematica [C] time = 33.5951, size = 97528, normalized size = 244.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.535, size = 3159, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*
(A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b^2-4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
```

))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-7*B*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+4*A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)-3*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+3*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-6*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+4*A*((a-b)/(a+b))^(1/2)*b^4*(1/(cos(d*x+c)+1))^(1/2)-7*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)+7*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)-6*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-4*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-7*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+A*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*B*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+9*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b-6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2-6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3-3*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+9*B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+4*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(cos(d*x+c)+1))^(1/2)+B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(cos(d*x+c)+1))^(1/2)-7*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(cos(d*x+c)+1))^(1/2)+4*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/

```
(a+b))^(1/2))*a^4+6*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*a^3*b+6*B*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2-6*B*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3)*cos(d*x+c)^(1/2)*(
(a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/b^2/(a+b)/(a-b)^2/(b+a*cos(d*x+
c))^2/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

3.634 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=526

$$-\frac{\left(-5a^2B+2aAb+3b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\cdots$$

```
[Out] -((2*a*A*b - 5*a^2*B + 3*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]) + ((2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B -
 3*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
 b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a
 + b*Sec[c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
+ d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
x]])
```

Rubi [A] time = 1.99028, antiderivative size = 526, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2955, 4029, 4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{\left(6a^3Ab+26a^2b^2B\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] -((2*a*A*b - 5*a^2*B + 3*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]) + ((2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B -
 3*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
 b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a
 + b*Sec[c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
+ d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
x]])
```


Rule 2955

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n)/(g*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a\right.}{3b^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a\right.}{3b^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a\right.}{3b^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a\right.}{3b^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a\right.}{3b^2\left(a^2-b^2\right)^2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab-aB)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+d}$$

$$= -\frac{\left(2aAb-5a^2B+3b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-5aB)\sqrt{}}{b^3d\sqrt{\cos(}}$$

Mathematica [C] time = 35.9732, size = 184379, normalized size = 350.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.714, size = 5358, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/
2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```